Themes in Turbulence Research (Unsolved Problems?)

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1. Isotropic turbulence: 2 & 3D

Why, in early days, such a focus on isotropic turbulence. Split flow into mean $<\mathbf{u}>$ + fluctuation $\mathbf{u}' = \mathbf{u} - <\mathbf{u}>$ Belief that interactions $\mathbf{u} <\mathbf{u}>$ is *easy*, $<\mathbf{u}><\mathbf{u}>$ *trivial*, $\mathbf{u}'\mathbf{u}'$ *difficult*.

• Isotropic simplest context of $\mathbf{u}'\mathbf{u}'$. Solve first, then $\mathbf{u}' <\mathbf{u}>$ included.

Unanticipated problems:

•(a) Intermittency (its multi-fractal nature) makes "closures" difficult, and

•(b) Boundaries may be important in creating new structures, unanticipated from homogeneity.

2. Two dimensional turbulence

Meteorological (and plasma) interest. Again Intermittency but *in extrema*. Dissipation of enstrophy $\to 0$ with Reynolds $\to \infty$. The decay of enstrophy very different from that predicted by "closure", whereas for 3-D, closure prediction not bad. Mention Loitsyansky etc.

- A problem for closures: Given the observation that Enstrophy decays like t^{-p}, $p \leq 1$, give a closure that is consistent with this. Recall most closures (EDQNM, TFM, LHDIA, LRA, etc.) give Enstrophy$\sim t^{-2}$ (Batchelor). Extreme Intermittency (a la McWilliams).

- Dritschel: Enstrophy decay (its dissipation) $\to 0$ as $\nu \to 0$. Recall for 3D $\epsilon \to Const.$ as $\nu \to 0$

Constitine's Analysis, His notation too!

The math: the correct bound that the time average of the square of the L^2 norm of the Laplacian of vorticity diverges at most like ν^{-2} (given bounds on vorticity in $L^2 \cap L^\infty$) was known to me (and many others I presume). It requires three integrations by parts and a Schwartz inequality. The fact that the time average of the square of the L^2 norm of the gradients of vorticity is bounded by the a constant times the square root of the previous, (again using bounds on enstrophy and L^∞ norm of vorticity) is just one integration by parts.

Now "physics". Write $G^2 =< |\nabla\omega|^2 > and M^2 =< |\nabla^2\omega|^2 >$ and $\eta = \nu G^2$.

The more ore less correct statement is that (νM) is bounded apriori by a constant. (In fact there is a term involving the initial data, that can be dealt with by either taking long averaging time T, or allow the constants to depend on the norm of the initial vorticity in H^1, hmm, not so physical...).

Write $\eta = (\nu M)(\frac{G^2}{M})$ The content of their paper is that η will converge to zero if $\lim \frac{G^2}{M} = 0$

They show $\lim_{Re\to\infty} \frac{G^2}{M} = 0$ by computing G, M using an energy spectrum. For any power spectrum that would predict infinite enstrophy, they put a constant C so that the enstrophy is finite, so the spectra look like $Ck^{-\beta}$, but C depends on Reynolds number, and is adjusted so that the enstrophy is finite.

By necessity this C needs to go to zero when Reynolds goes to infty. Using the spectrum you get, with k_d the dissipation wavenumber, $G^2 \sim Ck_d^{(3-\beta)}$ and $M \sim (\sqrt{C})k_d^{\frac{5-\beta}{2}}$4 The ratio G^2/M is $(\sqrt{C})k_d^{\frac{1-\beta}{2}}$

4.Stably Stratified turbulence

Equations to be studied:

$$\{\partial_t - \nu\nabla^2\}\mathbf{u} = -\mathbf{u}\cdot\nabla\mathbf{u} - \nabla p + \hat{\mathbf{g}}\theta - 2\boldsymbol{\Omega}\times\mathbf{u} \quad (1)$$

$$\{\partial_t - \kappa\nabla^2\}\theta = -N^2 w - \mathbf{u}\cdot\nabla\theta \quad (2)$$

$$\nabla\cdot\mathbf{u} = 0 \quad (3)$$

Frequency of Linear part:

$$\omega = \sqrt{N^2 sin^2(\vartheta) + 4\Omega^2\cos^2(\vartheta)} \quad (4)$$

•Question: For $N = 0$, $E(k) \sim k^{5/3}$, for $N \to \infty$, $E(k) \sim \sqrt{N\epsilon}k^{-2}$?

• Recent DNS (Linborg, Riley, 2006) & theory (Sukoriansky, S. & B. Galperin, 2005), give $k^{-5/3}$, without N.

• DNS Suggests more Gaussian distribution of vorticity. Less Intermittency?

• See & hear Smith & Sukoriansky, this conference.

• Kimura's calculations for $\Omega = 0$, $N^2 = 1, 10, 100$. Uses Chollet & Lesieur eddy viscosity (1981) (Kraichnan, 1976),really)

$$\nu(k|k_c) = \nu_0(E(k_c)\{1 + C\exp(k_c/k)\} \quad (5)$$

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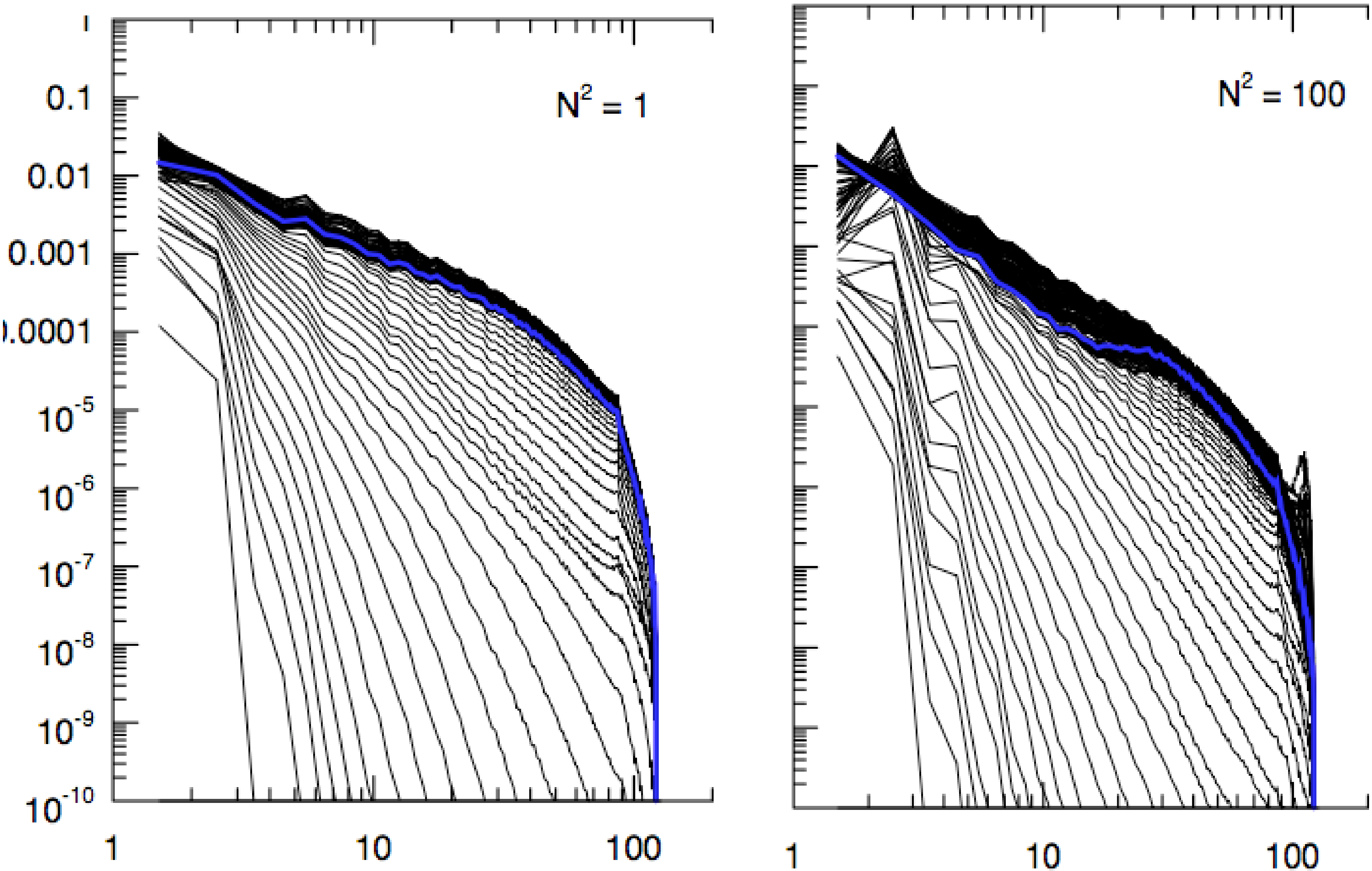

1
0.1
0.01
0.001
).0001
10^{-5}
10^{-6}
10^{-7}
10^{-8}
10^{-9}
10^{-10}
$N^2 = 1$
1
10
100
$N^2 = 100$
1
10
100

3. Convection (aspect ratio $(A) \to \infty$)

Here, we should recall that rapid distortion theory (include only $q_i < q >_j$, $q_i = (\mathbf{u}, T)$ gives reasonable results.

- The ($N_u \sim R_a^{2/7}$) small aspect ratio!) results of Libchaber, Siggia, *et alter* How does this hold up for large A.
- Also important *rough boundaries* (Finnegan, Patton, ,,,).

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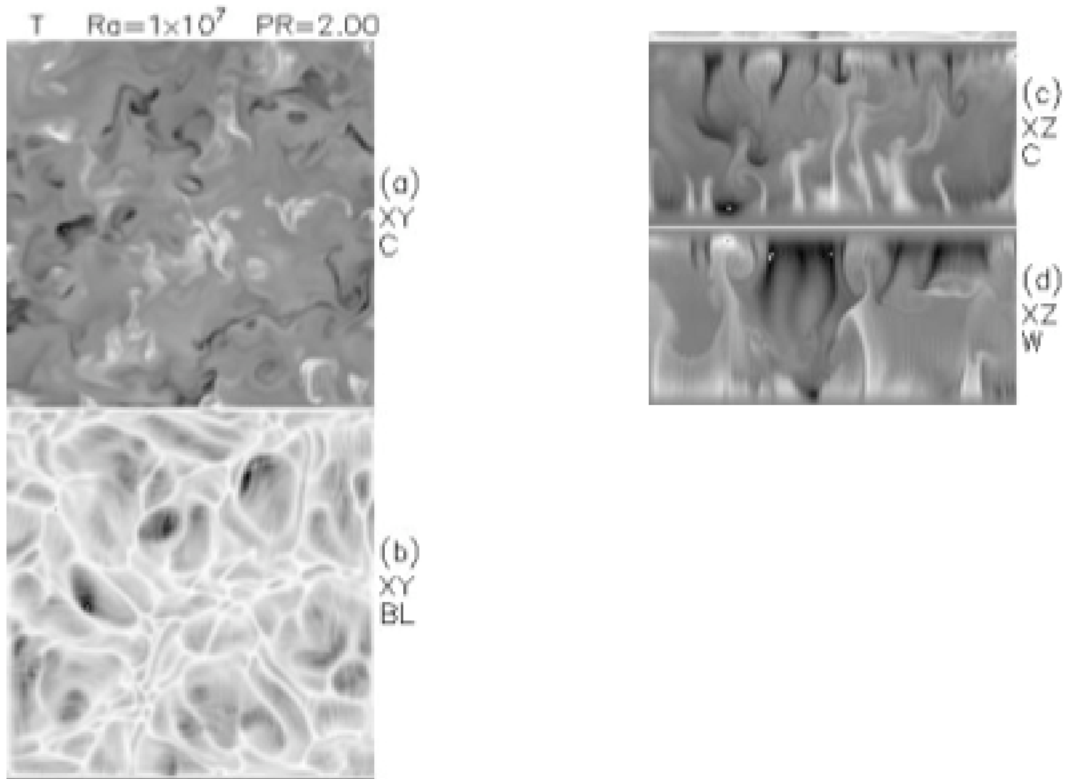

5. The role of computers in turbulence (theory?)

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- What if theory (explanation=computer code)

becomes nearly as complicated as the phenomenon?
Leibniz, Turing, Jimenez. Theory *vs* DNS LES.

In 1686 in his Discours de métaphysique, Leibniz points out that if an arbitrarily complex theory is permitted then the notion of "theory" becomes vacuous because there is always a theory. from Chaitin’s lecture