Theory and Computation of Wavenumber-2 Vortex Rossby Wave Instabilities in Hurricane-like Vortices

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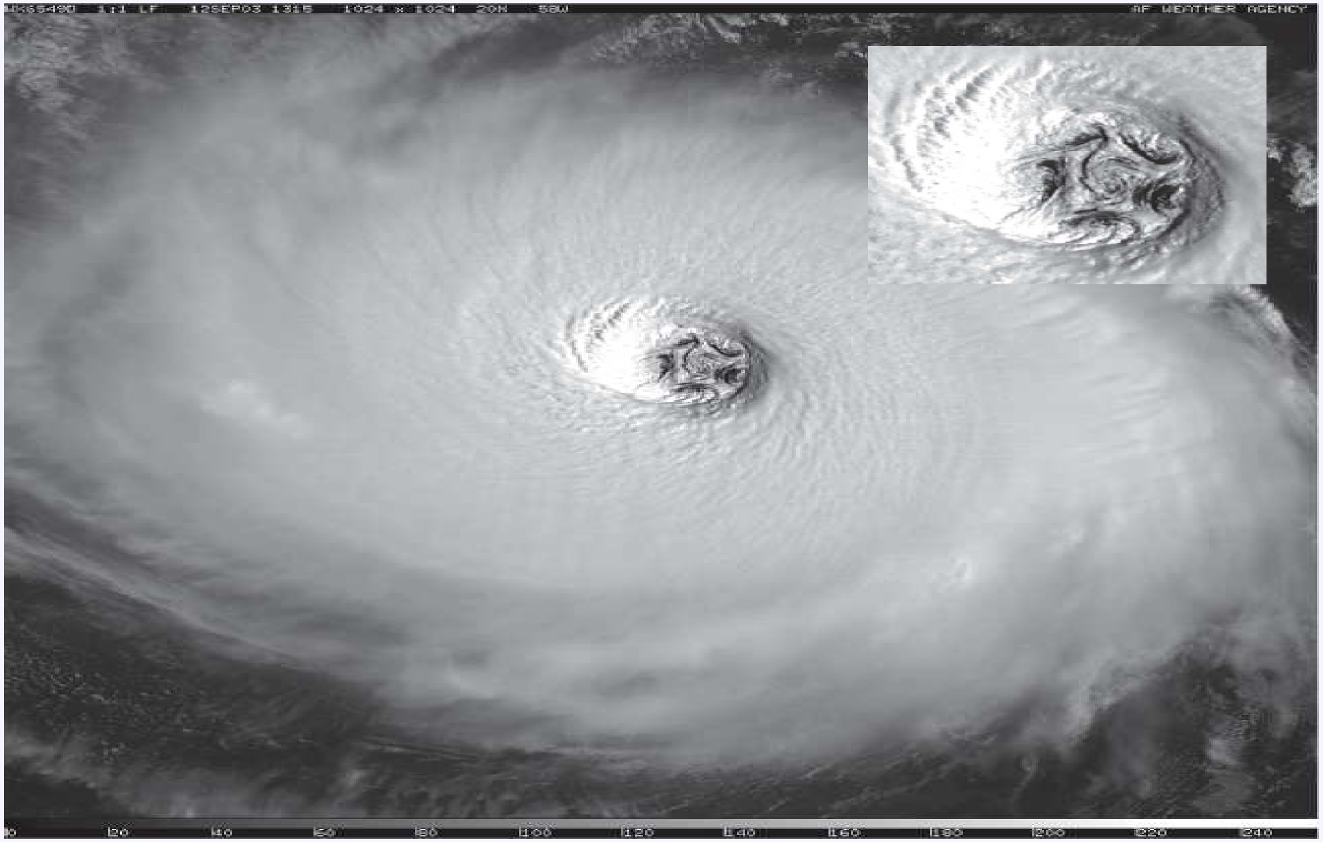
Polygonal Eyewalls



DMSP image of Hurricane Isabel, 12 Sept. 2003.

- First satellite imagery of hurricanes revealed a **surprise**!
- Fascinating dynamical feature: hurricane eyewalls are often **polygonal** in appearance (square, pentagonal and hexagonal).

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Why do polygonal eyewalls form?

- Schubert et al. (1999) suggested that polygonal eyewalls form from perturbations that can interpreted as two, discrete, phase-locked, **Vortex-Rossby Waves** (VRWs) that live on the inner and outer vorticity gradients of the eyewall.
- They formulated a discontinuous, **2D**, three-region vortex model (piece-wise constant vorticity) with dispersion relation

$$\nu = \frac{1}{2}(\nu_1 + \nu_2) \pm \frac{1}{2}\left[(\nu_1 - \nu_2)^2 + \xi_1 \xi_2 (r_1/r_2)^{2l}\right]^{1/2}$$

  for the unstable mode with non-interacting VRW frequencies $\{\nu_1, \nu_2\}$ and vorticity jumps $\{\xi_1, \xi_2\}$ at $\{r_1, r_2\}$.
- Instability occurs for azimuthal wave-number $\boldsymbol{l \geq 3}$; this barotropic instability may cause spinup and maintenance of eye vorticity, thereby increasing hurricane maximum intensity.

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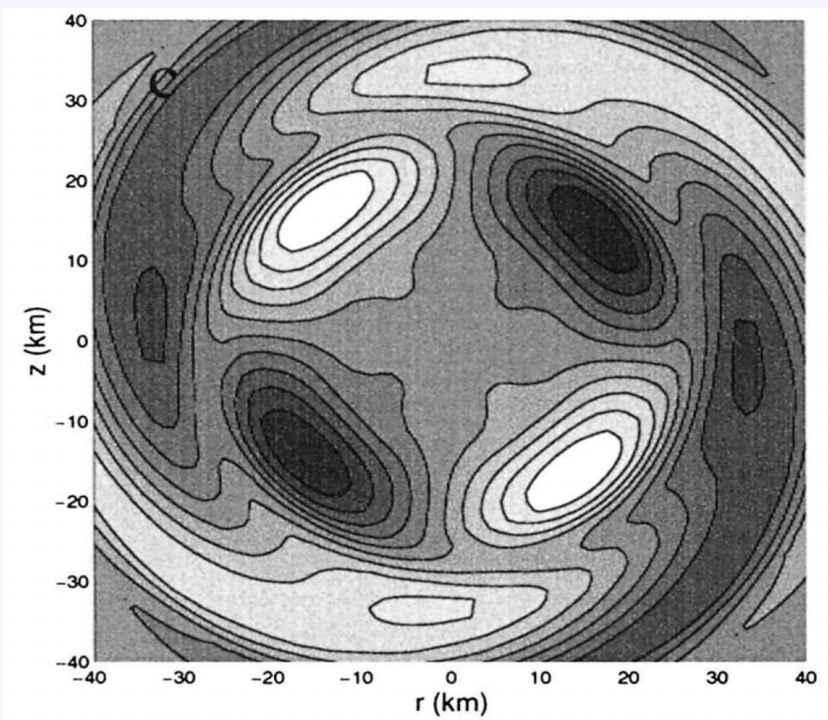
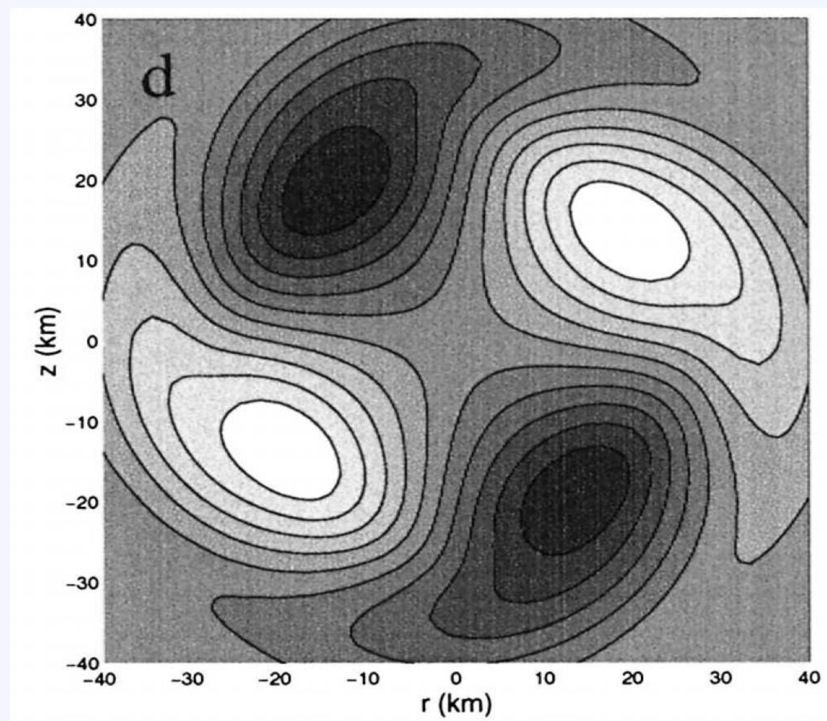
The elusive $l = 2$ instability

- Nolan & Montgomery (2002) computed fully **3D**, nonhydrostatic modes for hurricane-like vortices.
- They found a **quasi-2D**, $l = 2$ instability that **disappears** as eye vorticity vanishes (hurricane category $\uparrow$).
- Two concentric sets of perturbations in eyewall $\rightarrow$ VRWs!

PV



Pressure



Despite this. . .

Terwey and Montgomery (2002) note the **unsatisfactory absence** of **wavenumber-2** instabilities in normal-mode models with small hurricane eye vorticity:

> It is strange to think that an instability as common in continuous models and observed phenomenon . . . would require that the core's vorticity must be negative.

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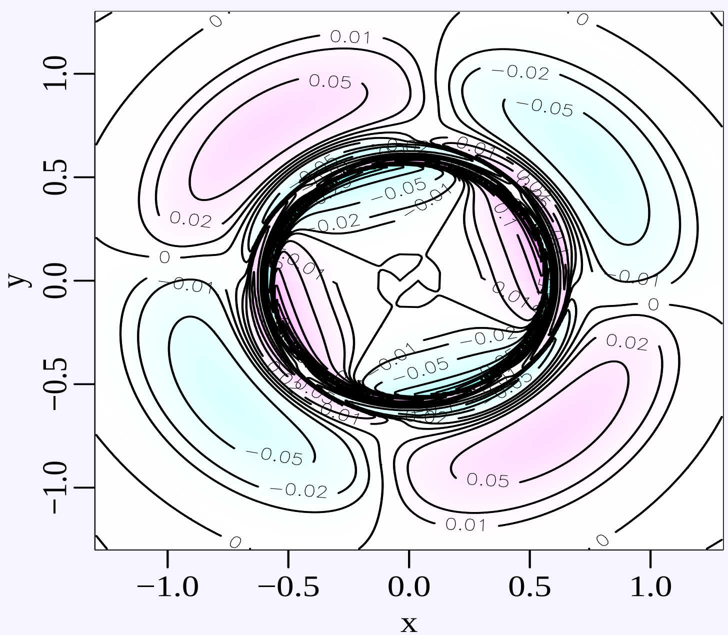
An new $l = 2$ instability?

- Our new numerical routine produces unstable, $l = 2$, modes that appear to be distinct from Nolan & Montgomery (2002)'s **quasi-2D** instability:

N= 16 , Ro= 38 , Fr= 3



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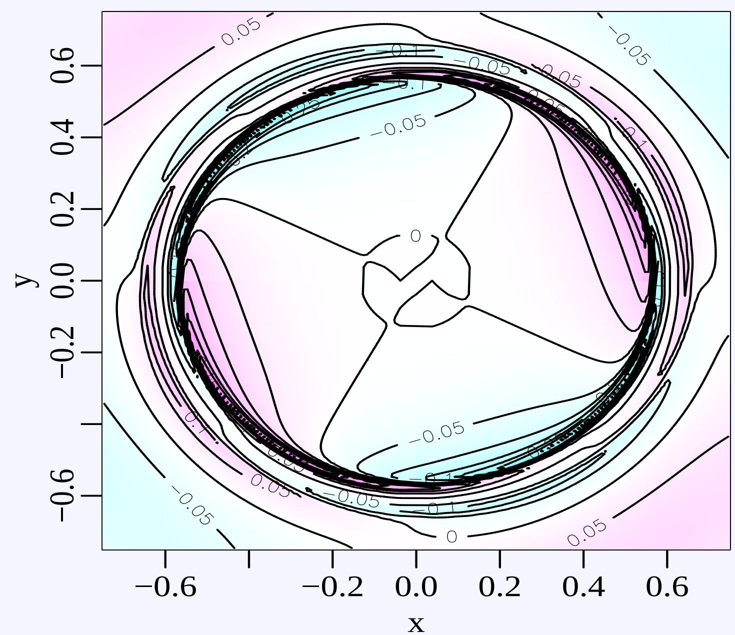
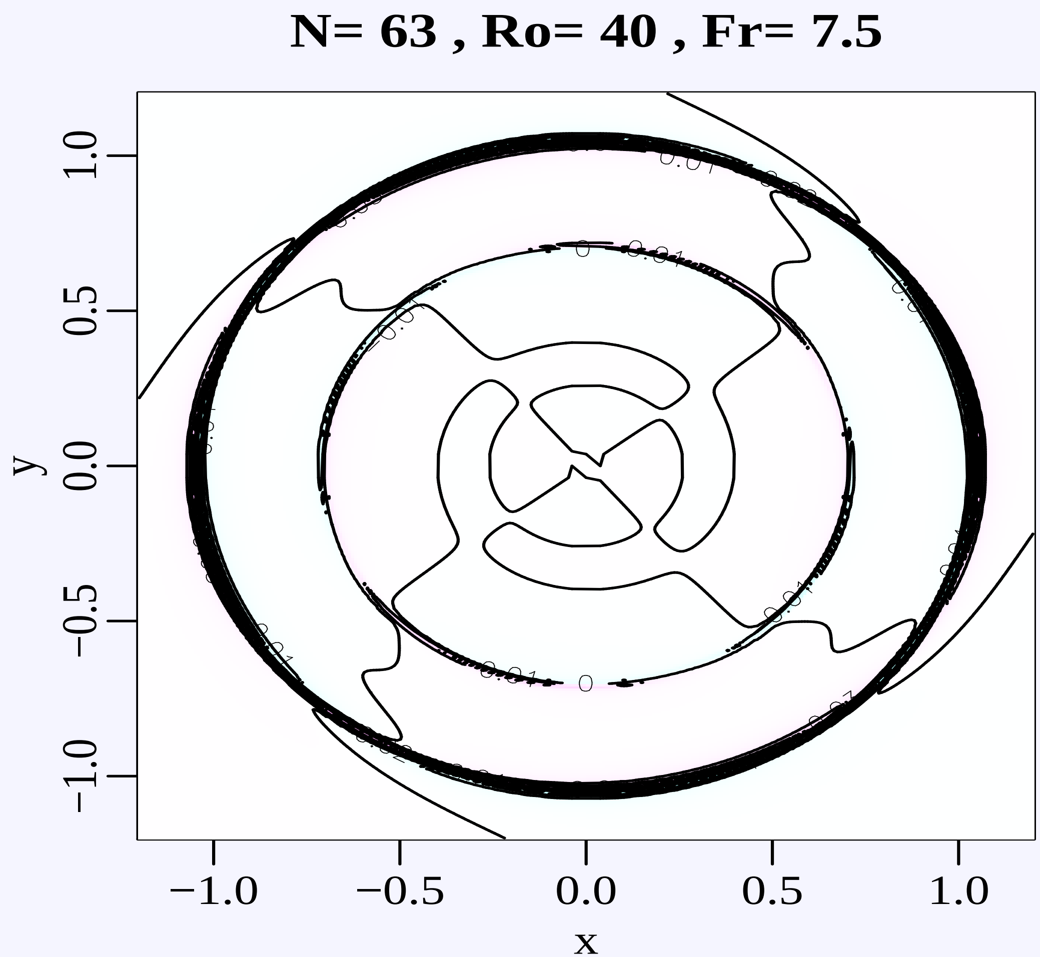


- Note the two inner consecutive rings of vertical vorticity.

An new $l = 2$ instability?

- Example with larger stratification.



- Note the two rings which dominate the vertical vorticity.

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Questions

- Can we interpret this instability as two, discrete, phase-locked, Vortex-Rossby Waves (VRWs)?
- How is this instability different from the "Category 1" instability discovered by Nolan and Montgomery (2002)?
- A **2D** wavenumber-2 instability is **not** predicted by simple, discontinuous, three-region vortex models [e.g. Schubert et al., 1999]. What is the salient **3D** feature that allows instability?
- Can we capture this class of instability in a **3D** extension of Schubert et al. (1999)'s discontinuous, three-region vortex model?
- What the implications of this class of instability for hurricane intensification.

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Equations and Numerics

For a "hurricane-like" base-state with

$$\begin{aligned}\text{Azimuthal Angular Velocity} &= \Omega(r)\\ \text{Vertical Vorticity} &= Z(r)\end{aligned}$$

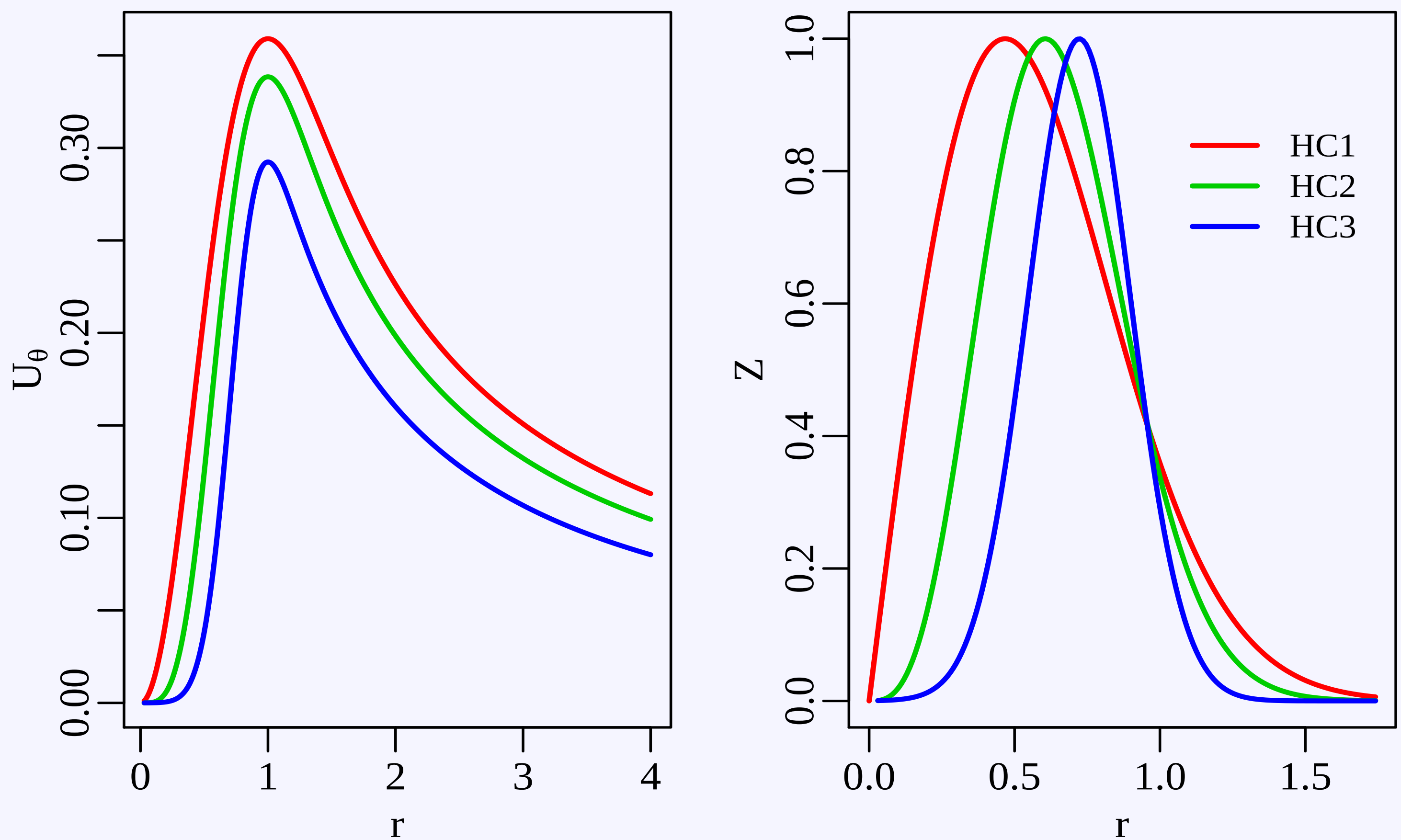
and constant stratification, N, we solve the 3D, linearized, Boussinesq equations in cylindrical coordinates:

$$\begin{aligned}\partial_t u_r + \Omega\partial_\theta u_r - (2\Omega + f)u_\theta &= -\partial_r \pi\\ \partial_t u_\theta + \Omega\partial_\theta u_\theta + (Z + f)u_r &= -\frac{\partial_\theta}{r}\pi\\ \partial_t u_z + \Omega\partial_\theta u_z &= b - \partial_z \pi\\ \partial_t b + \Omega\partial_\theta b + u_z N^2 &= 0\\ \frac{1}{r}\partial_r\left(r u_r\right) + \frac{1}{r}\partial_\theta u_\theta + \partial_z u_z &= 0\end{aligned}$$

for perturbations, $\phi(r,\theta,z,t) = A(r)\exp\left[i(l\theta + mz - \nu t)\right]$.

"Hurricane-Like" Base States

- Increasing hurricane strength (category) is roughly analogous with decreasing eye vorticity.



Numerics

We use a new numerical approach.

- Solve for eigenvalues using standard packages (LAPACK) but…
- We solve the equation for pressure Laplacian analytically.
- Pressure is written as a convolution over Bessel functions,

$$K_l(mr), I_l(mr)$$

Advantages:

- Pressure is computed using Nth-order stencil; **NO** numerical derivatives.
- Divergent boundary conditions (Bessel functions) are eliminated analytically.

Disadvantages:

- Numerics is “finicky”.
- Remove spurious eigenvalues using (imperfect) convergence test.

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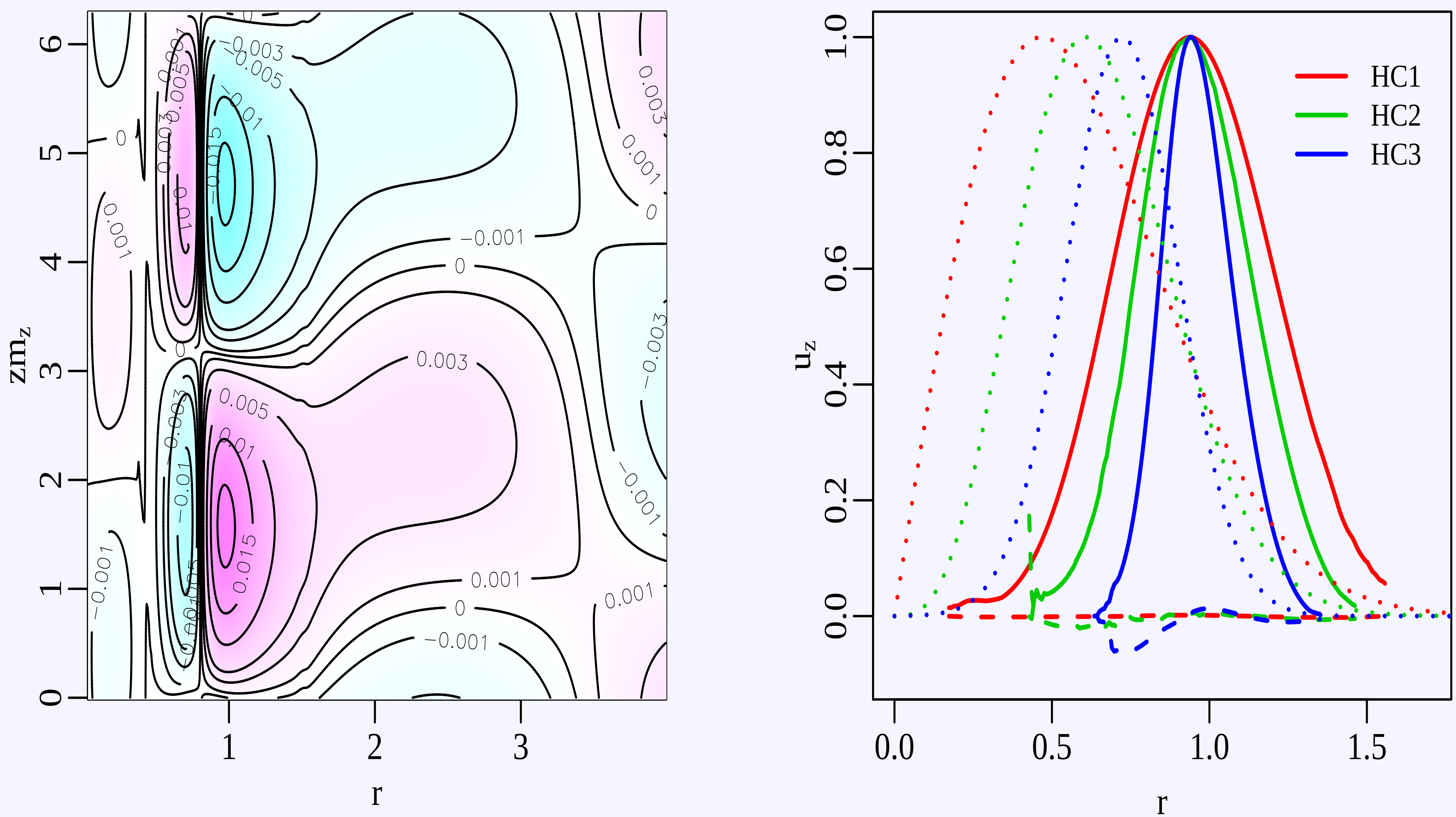
Hot Vortical Towers!

- Instability produces strong azimuthal vorticity in the eyewall associated with vertical convection that peaks at $r = 1$.

N= 16 , Ro= 29 , Fr= 3.3

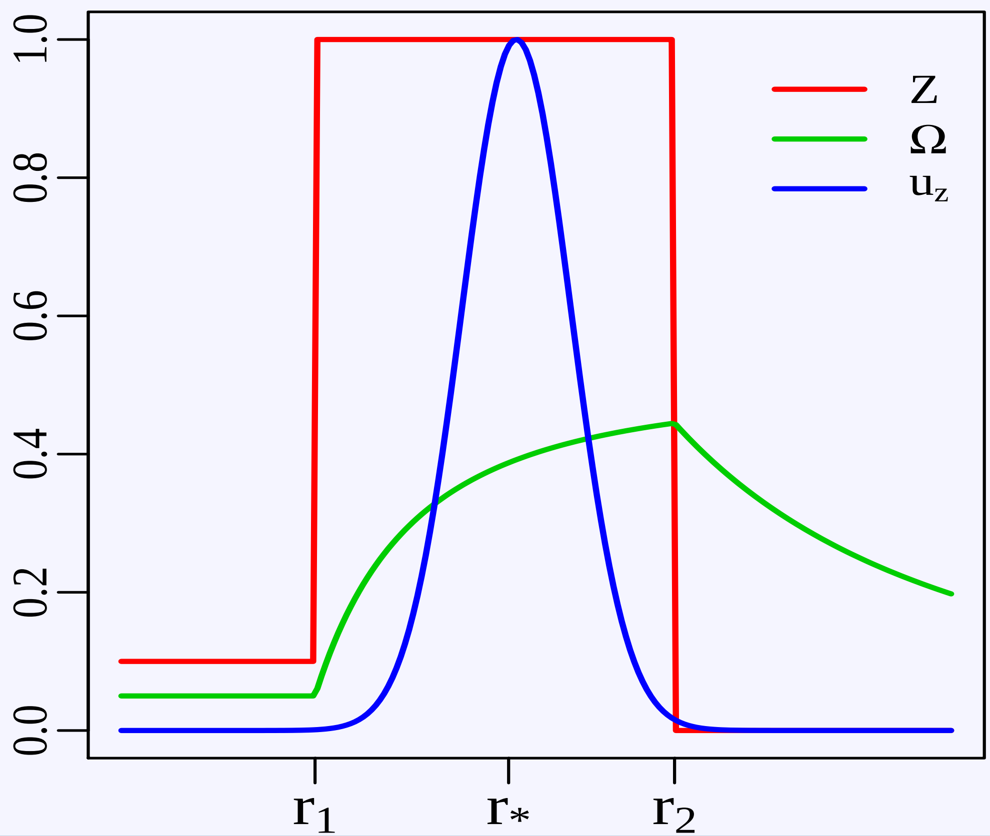


- Figure shows $R_0 \in \{0, 50\}$, $N \in \{16, 128\}$, $m \in \{0, 40\}$.

Is this a VRW?

Question: Is this 3D structure consistent with VRW theory and Schubert et al. (1999)?

Approach: Extend three region vortex model by adding skewed-Gaussian eyewall vertical velocity.



New Model

- Following Schubert et al. (1999) we use vertical vorticity (ζ_z) Eqn:

$$(\nu - l\Omega)\zeta_z + i\frac{\partial Z}{\partial r}u_r + \boldsymbol{(Z+1)mu_z} = 0$$

  Note the **Vortex Stretching!**

- We solve the u_r-equation in each region:

$$\frac{\partial}{\partial r}r\frac{\partial}{\partial r}(ru_r) - l^2u_r = -i\frac{\partial}{\partial r}(r^2mu_z) + ilr\frac{(Z+1)}{\nu - l\Omega(r)}mu_z$$

- Inner ($r < r_1$) and outer ($r > r_2$) we have $u_z = 0$:

$$u_r \sim r^{l-1} \quad r < r_1 \qquad \text{OK.}$$
$$u_r \sim r^{-l-1} \quad r > r_2 \qquad \text{Punt.}$$

- This is a **TOY MODEL**!

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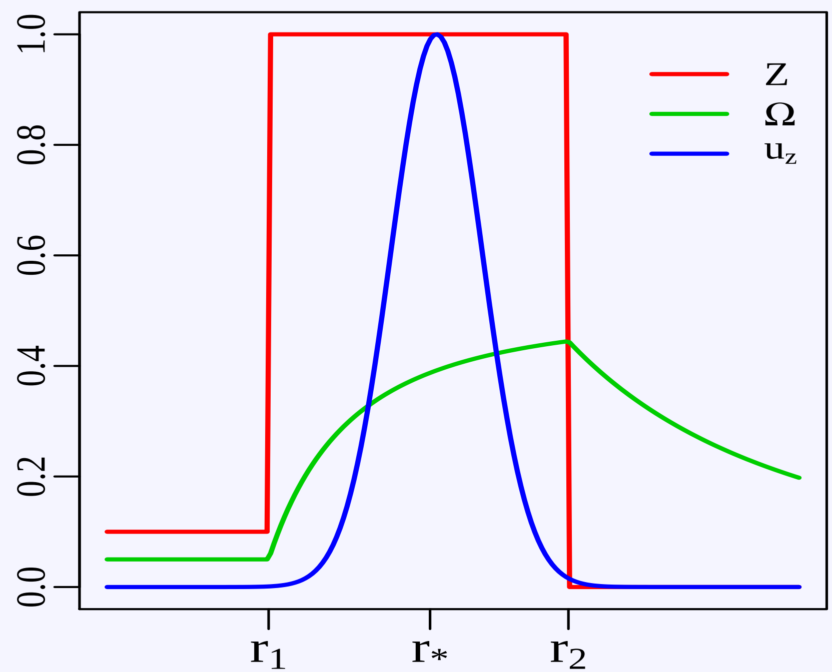
Boundary Conditions

- Usual jump conditions at $\{r_1, r_2\}$ for vorticity jumps $\{\xi_1, \xi_2\}$:

$$
\begin{aligned}
(\nu - l\Omega_1)\Delta u_\theta\big|_{r_1} &= i\xi_1 u_r(r_1) \\
(\nu - l\Omega_2)\Delta u_\theta\big|_{r_2} &= i\xi_2 u_r(r_2)
\end{aligned}
$$

- New BC: satisfy nonhydrostatic vertical momentum balance at r_*.



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Results: 1st Model

First Model:

- Given

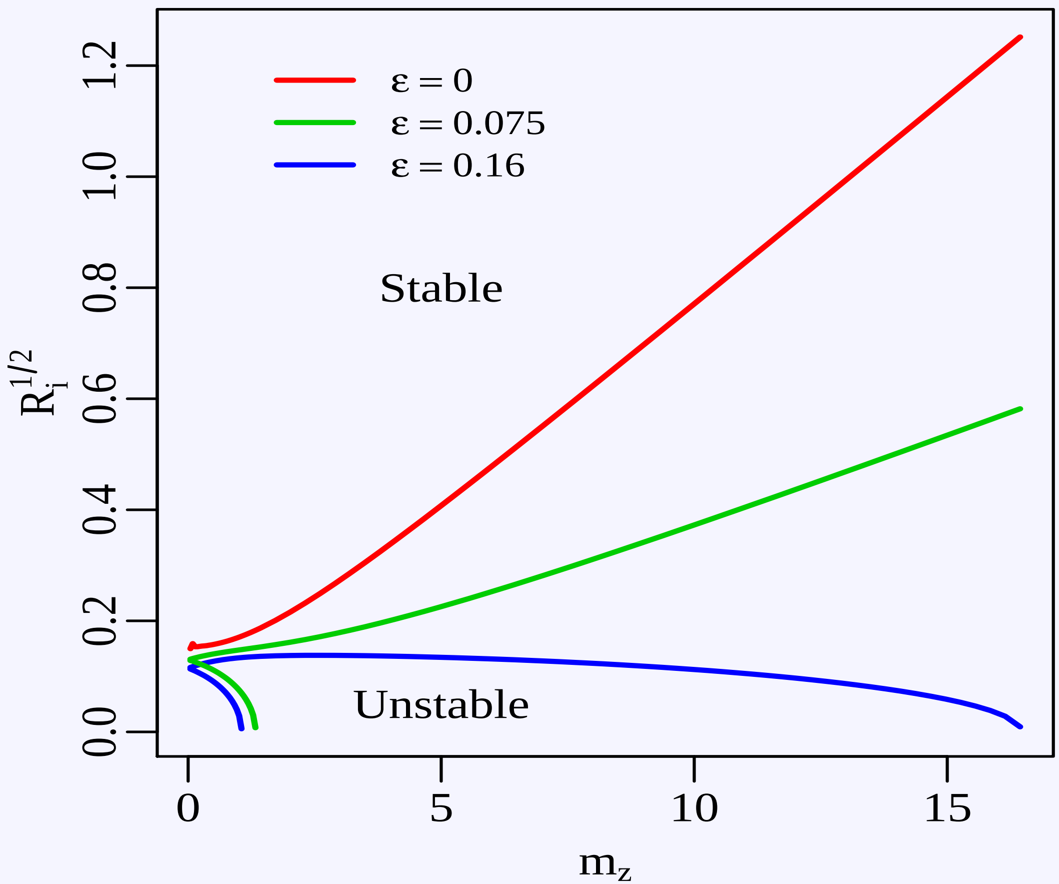
$$\frac{\partial}{\partial r} r \frac{\partial}{\partial r}(ru_r) - l^2 u_r = -i\frac{\partial}{\partial r}(r^2 m u_z) + ilr\frac{(Z+1)}{\nu - l\Omega(r)} m u_z$$

replace

$$ilr\frac{(Z+1)}{\nu - l\boldsymbol{\Omega(r)}} m u_z \rightarrow ilr\frac{(Z+1)}{\nu - l\boldsymbol{\Omega_*(r_*)}} m u_z$$

- Resulting Model: 4th order (quartic) expression for ν.
- Advantage: Provides analytic insight.

Instability Boundaries



- Unstable modes found for small $\epsilon = 1 + \xi_1/\xi_2$.
- Critical points:

$$\epsilon = 0.075 \quad : \quad R_i = 0.017$$

$$\epsilon = 0.016 \quad : \quad R_i = 0.013$$

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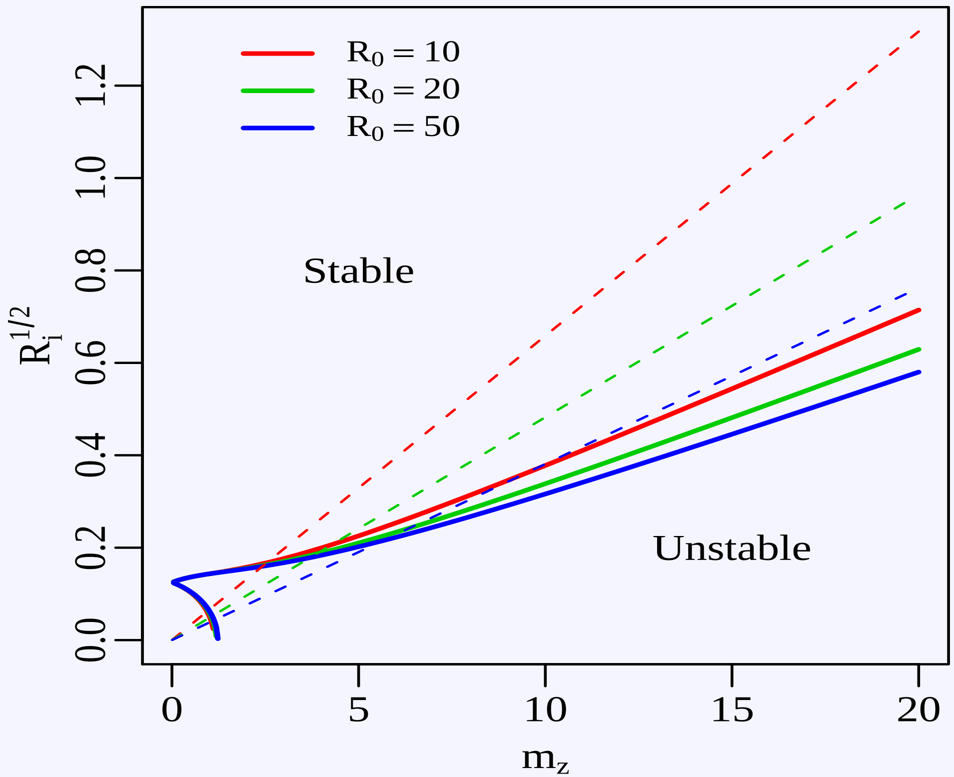
Analytic Behavior

Stability boundary of our 4th-order expression for ν can be analyzed using a new expression for the discriminant [Yang, 1999].



Critical Froude number is [Gaussian u_z of width σ]:

$$\mathrm{F_r}^2 \sim (-l\Omega_* + \xi_2 + f)(-l\Omega_* - \xi_2 - f)\frac{\sigma r_* m^2}{N^2}$$

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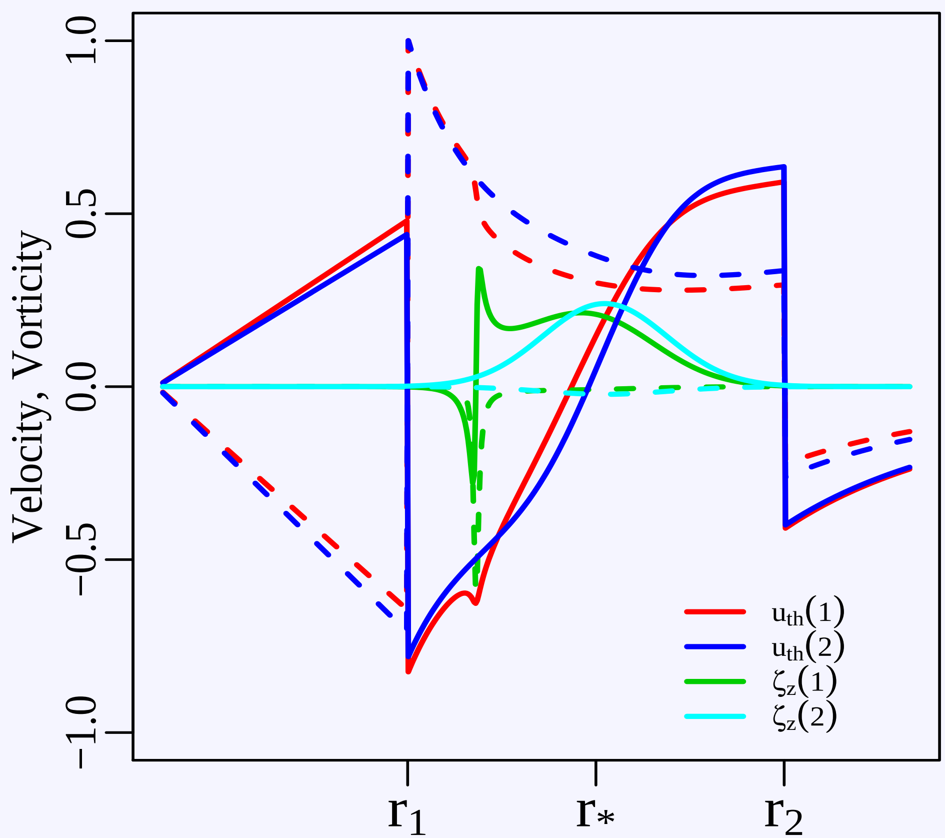
Origin of Vorticity Rings

Effect of vertical vorticity stretching:

$$ilr\frac{(Z+1)}{\nu - l\boldsymbol{\Omega(r)}}mu_z \rightarrow ilr\frac{(Z+1)}{\nu - l\boldsymbol{\Omega_*(r_*)}}mu_z$$



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Vertical vorticity stretching

- Given linearized vertical vorticity equation

$$(\nu - l\Omega)\zeta_z + i\frac{\partial Z}{\partial r}u_r + \boldsymbol{(Z+1)mu_z} = 0$$

Narrow rings of vorticity occur where

Material frequency $= Re(\nu) - l\Omega(r) = 0$

- This resonance is a fixed point in reference frame of the base state.

Conclusion: Narrow rings of enhanced vorticity seen in full numerical model **are not** VRWs, but instability can be interpreted as two, discrete, phase-locked **VRWs**.

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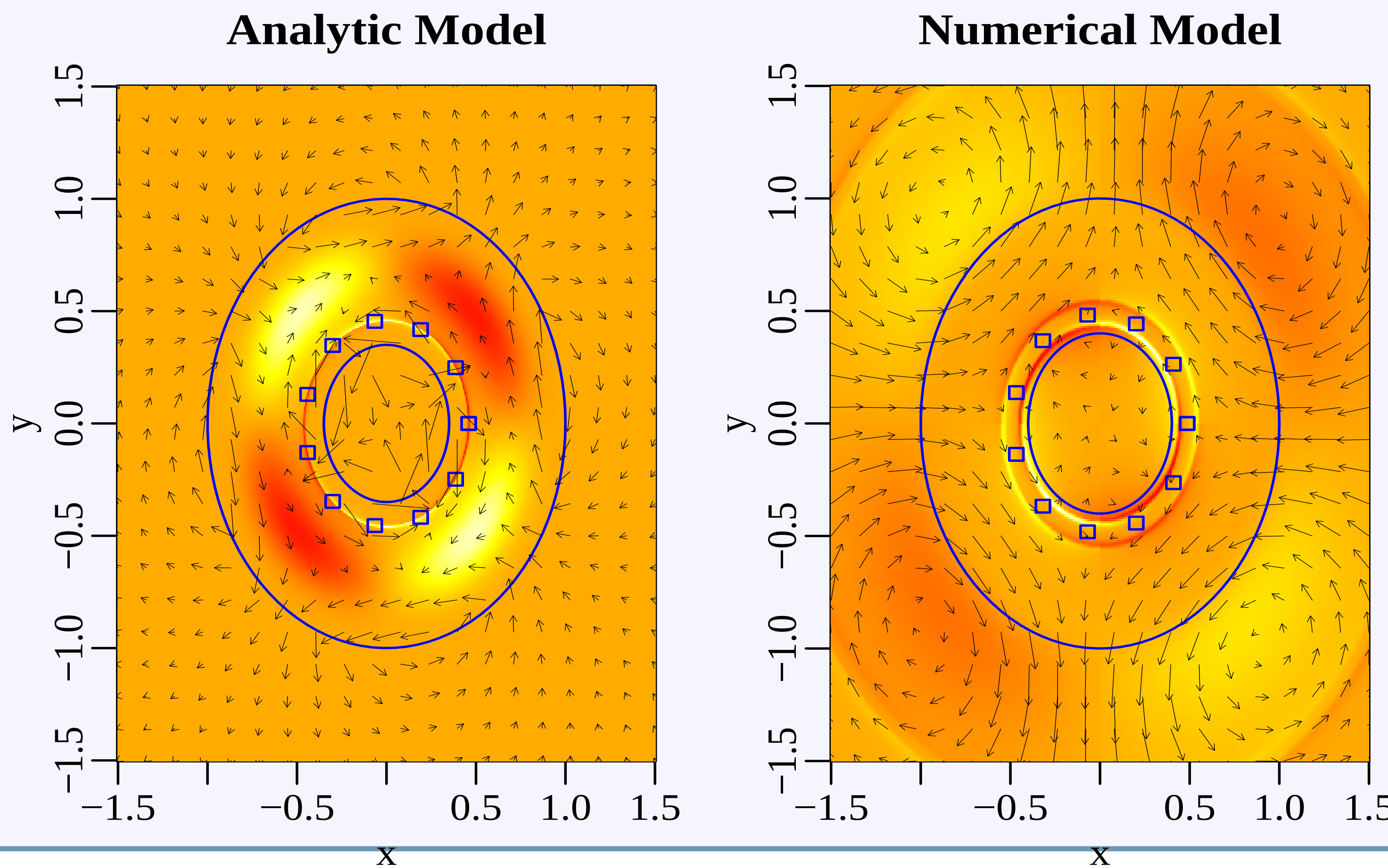
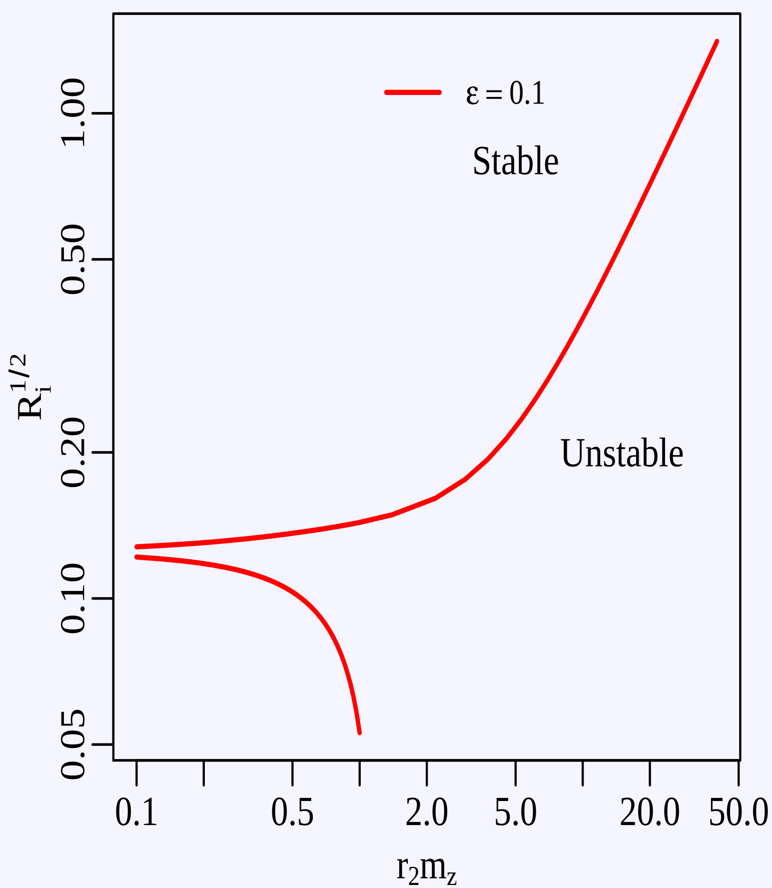
Comparison of Analytic and Numerical Models

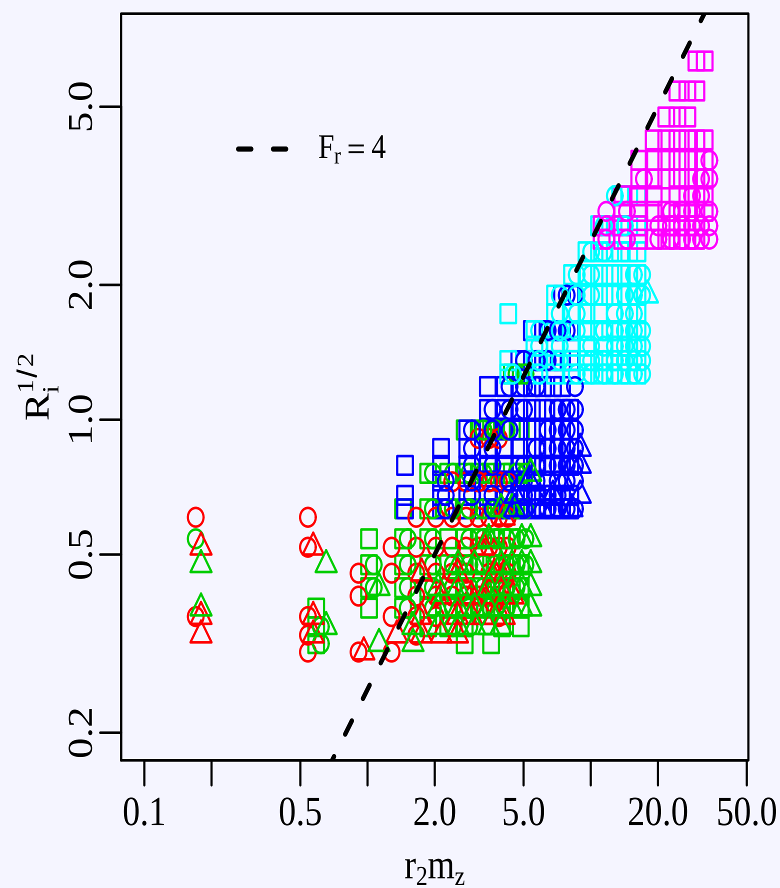
Prediction from analytic model:

- Moving radial outward: first VRW, then disturbance due to material frequency resonance, then peak in $\{u_z, \zeta_z\}$ associated with stretching, then second VRW.



Comparison of Analytic and Numerical Models





red: $N = 11$, green: $N = 16$, blue: $N = 32$,

light blue: $N = 64$, pink: $N = 128$,

Conclusions

- We have addressed the **“missing”** wavenumber-2, VRW instability, absent from 3-region Schubert et al. model, and in Nolan and Montgomery (2002)’s analysis at large category.
- Using new numerical approach, we find a class of fundamentally **3D**, $l = 2$ instability associated with:
  - Intense vertical convection (vortical hot towers) in eyewall.
  - Rings of enhance vertical vorticity at resonance radius where material frequency is zero.
  - Froude numbers greater 4.
  - Richard numbers greater than 0.1.
- Developed a new **3D**, non-hydrostatic 3-region model:
  - 4th-order eigenvalue equation that is analytically tractable.
  - Contend is simplest analytic model which shows archetypal features of $l = 2$ instability, in agreement with numerics.

Current/Future Work

- Study fully nonlinear evolution of this $l = 2$ VRW instability.
- Assess role of instability in either
  - Transporting vorticity into the eye and the axisymmetrization process $\rightarrow$ intensification.
  - Disruption of eyewall processes $\rightarrow$ eyewall renewal cycles and de-intensification.

New Hurricane-Lightning Project:

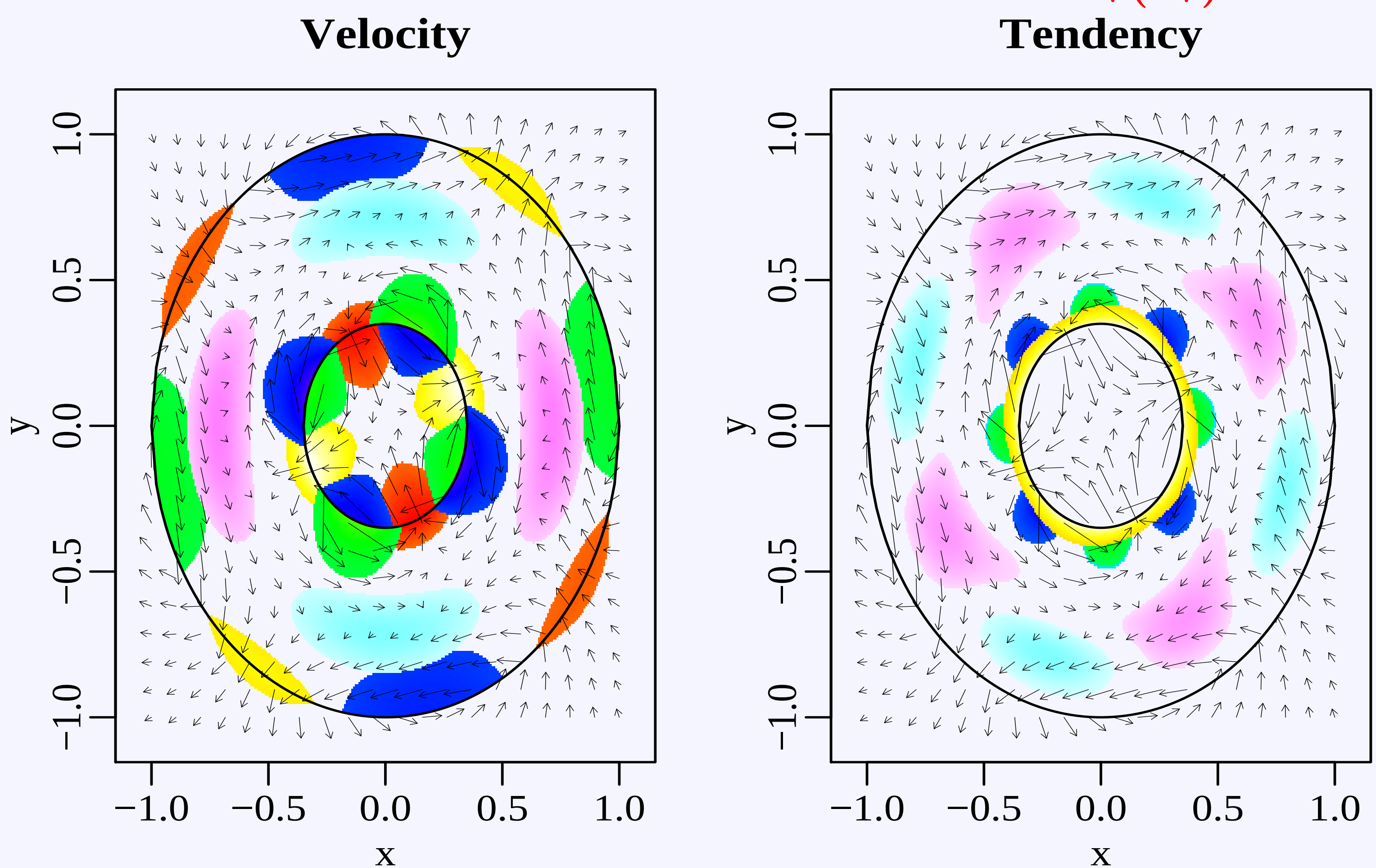
- Understand roll of VRW instabilities, and vortical hot towers in the generation of hurricane eyewall lightning.
- Experiments with cloud-resolving model and microphysics.

Other stability work in Cylindrical Geometry:

- Normal mode analysis and instability of land ice-sheets.
- Contacts: Nicole Jeffery, njeffery@lanl.gov, and Beth Wingate.

Model Comparison

Velocity and basestate tendency for VS $= ilr\frac{(Z+1)}{\nu - l\boldsymbol{\Omega}_*(\boldsymbol{r}_*)}mu_z$.



u_r (yellow,red), u_θ (green,blue), u_z (pink,light blue)

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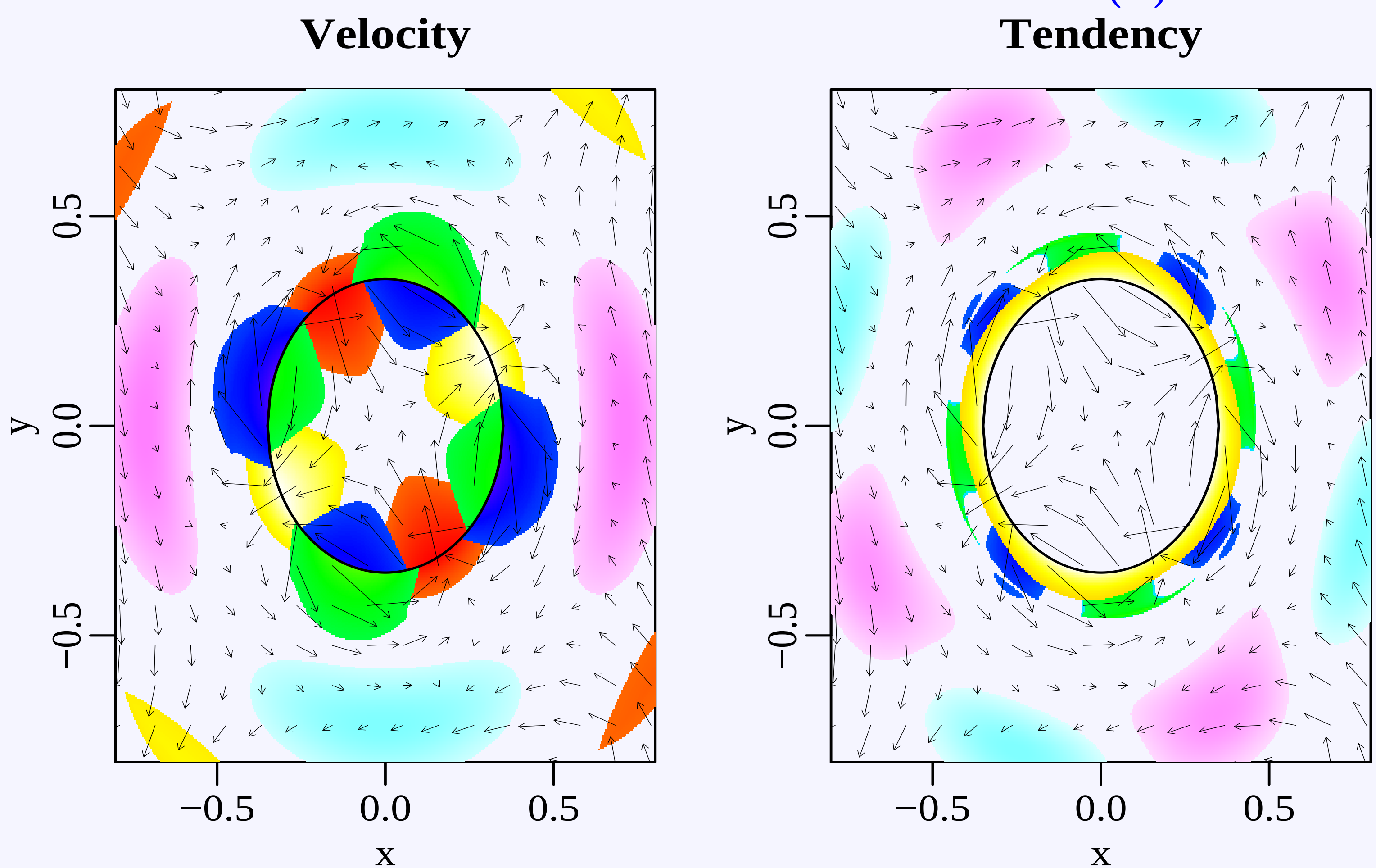

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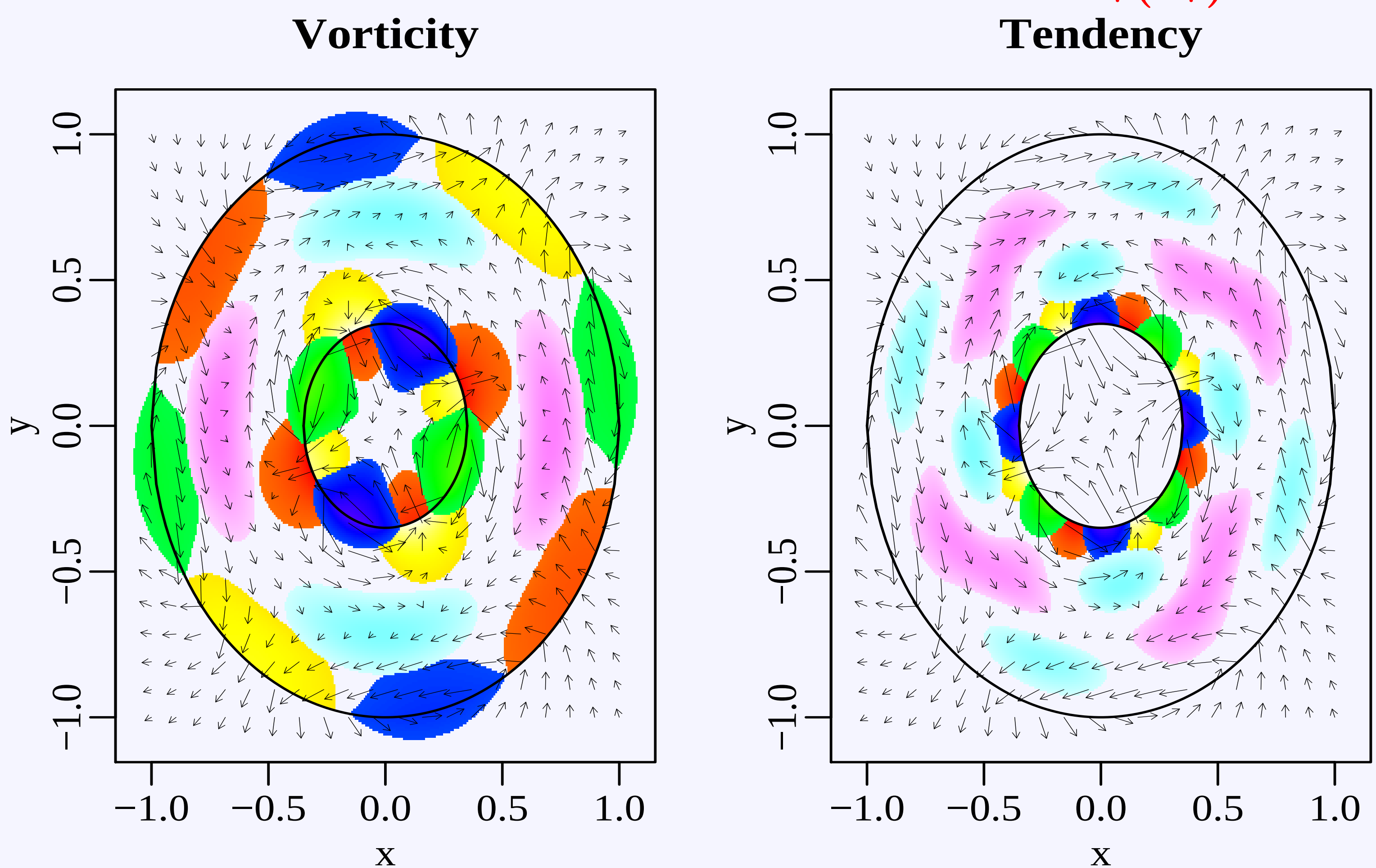

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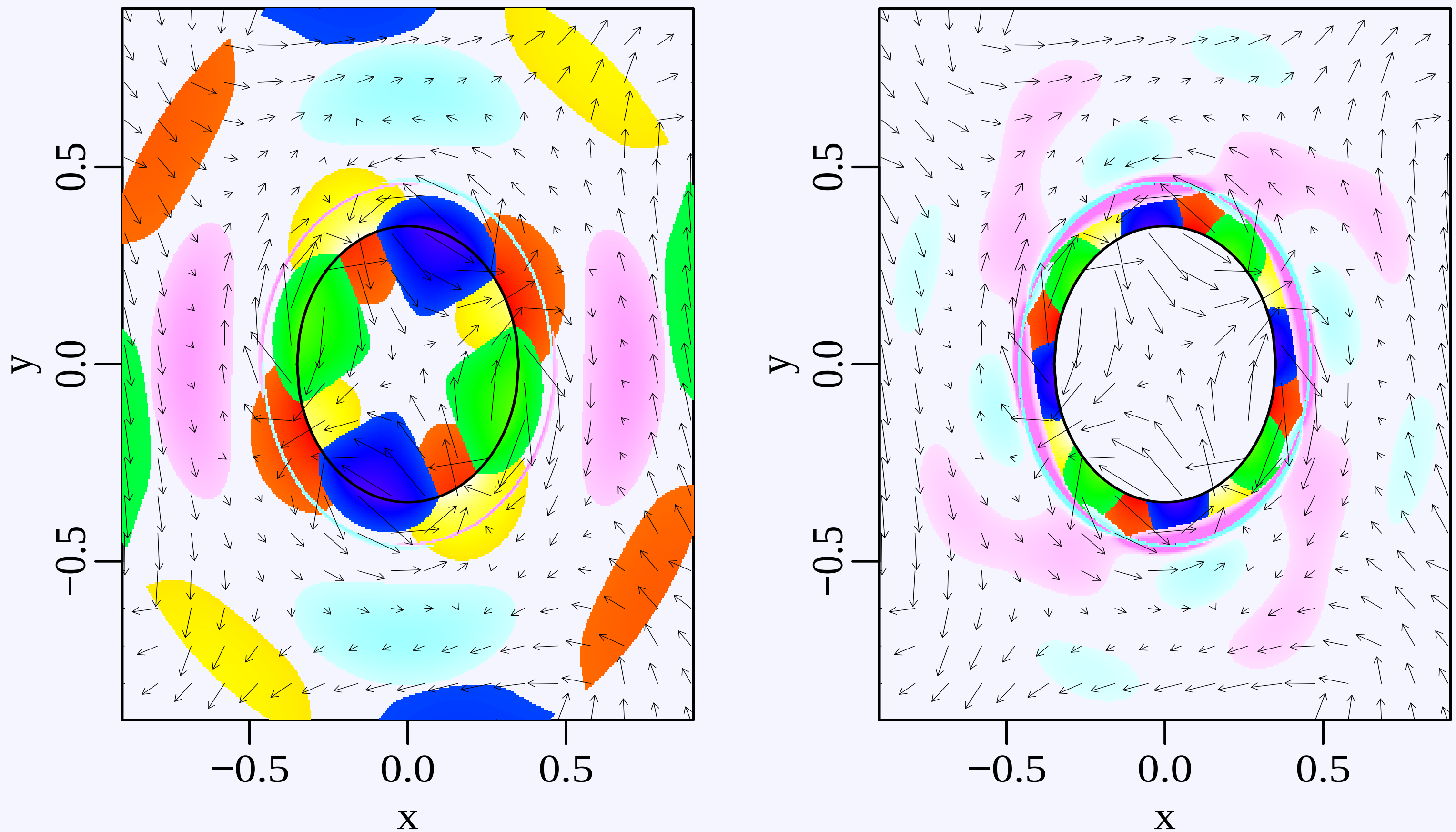

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