Towards a new interpretation of upper-ocean dynamics using Surface Quasi-Geostrophy

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Upper oceanic layers at mesoscale

Classical paradigm

- QG turbulence driven by interior potential vorticity
- Kinetic Energy in $k^{-3}$ at mesoscales (Charney, 1971)
- The altimeter sees 1st baroclinic mode (Stammer, 1997)
- Transfer of surface (baroclinic) KE towards small scales
Upper oceanic layers at mesoscale

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In contradiction with recent results for ocean surface
- Kinetic energy spectra in $k^{-5/3}$ (Le Traon et al. 2008)
- Transfer of surface Kinetic Energy towards large scales (Scott et Wang 2005)

⇒ need to better understand surface dynamics
Stratified turbulence with baroclinic unstable front
Earth Simulator (Japan) (Klein et al. 2008)
Surface ocean dynamics

Towards a new interpretation

- Dynamics driven by **surface density** and not by **interior potential vorticity**

- The altimeter sees a surface-intensified mode (Lapeyre 2007, submitted)

- **Surface Quasi-Geostrophic model**
  - KE spectra in $k^{-5/3}$ (Held et al. 1995)
  - Same spectra for surface KE and density
  - Inverse transfer of surface KE (Capet et al. 2008)

$\Rightarrow$ **consistent with surface observations**
Potential vorticity inversion

QG PV inversion $\equiv$ invert an **elliptic equation**: 

$$
\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} + \frac{\partial}{\partial z} \left( \frac{f_0^2}{N^2} \frac{\partial \psi}{\partial z} \right) = PV
$$

relative vorticity  

vortex stretching

with **surface boundary condition**

$$
f_0 \frac{\partial \psi}{\partial z} \bigg|_{z=0} = b \big|_{z=0}
$$

$b = -\frac{g \rho}{\rho_0}$

**Important remark:**

$b \big|_{z=0}$ plays the same role as interior PV!
**Surface vs interior decomposition**

\[
\text{total inversion} = \text{inversion (PV)} + \text{inversion } (b|_{z=0}^{-1})
\]

vertical distribution of \(\hat{\psi}\) for an horizontal mode \(k = \frac{2\pi}{80} \text{ km}^{-1}\) using data from realistic simulation (POP model)

⇒ **Effective SQG solution** with constant \(N^2\) may represent upper layer dynamics
**Surface QG model**

Solution with constant $N^2$ using surface density

$$\hat{\psi}(k, z) = \frac{1}{N} \frac{\hat{b}_s(k)}{|k|} \exp \left( N \frac{|k|}{f_0} z \right)$$

- link between SSH and SST in Fourier space:
  $$\text{SSH} \propto k^{-1} \text{SST}$$

- same spectra for surface KE and SST

- Reconstruction of upper-layer dynamics using surface density only

$\Rightarrow$ Test of the SQG solution in different models
surface density

Reconstruction of vorticity field at the surface (Earth Simulator simulations)

relative vorticity ($s^{-1}$)

SQG prediction
Reconstruction relatively accurate down to 500 m
Realistic simulation of the North Atlantic
(Isern-Fontanet et al. 2007, submitted)

**POP, 1/10°**

surface relative vorticity
SQG reconstruction of relative vorticity using SST as a proxy for surface density of observed vorticity with SQG reconstruction.
Comparison altimeter/SST (Isern-Fontanet et al. 2006)
Conclusions

- Importance of the surface-intensified mode driven by surface density
- Surface Quasi-Geostrophic dynamics

Reconstruction of 3D dynamics from SST
- accurate for the upper 500 meters

Coupling surface/interior dynamics?

References:
Lapeyre et Klein, J.P.O. 2006; Isern-Fontanet et al., G.R.L 2006
Mathematical equivalence

\[
\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} + \frac{\partial}{\partial z} \left( \frac{f_0^2}{N^2} \frac{\partial \psi}{\partial z} \right) = PV
\]

\[
f_0 \left. \frac{\partial \psi}{\partial z} \right|_{z=0} = b\mid_{z=0}
\]

and

\[
\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} + \frac{\partial}{\partial z} \left( \frac{f_0^2}{N^2} \frac{\partial \psi}{\partial z} \right) = PV + f_0 b\mid_{z=0} \text{ dirac}(z)
\]

\[
f_0 \left. \frac{\partial \psi}{\partial z} \right|_{z=0} = 0
\]

(Bretherton 1966)
Coupling between interior and surface inversions

\[ \text{total inversion} = \text{inversion (PV)} + \text{inversion } \left( b \bigg|_{z=0} \right) \]
**Coupling between interior and surface inversions**

Total inversion = inversion (PV) + inversion ($b|_{z=0}$)

However, coupling between PV and surface density for baroclinically unstable flows

\[
\frac{DPV'}{Dt} = -v \frac{\partial PV}{\partial y} \quad \text{and} \quad \frac{Db'|_{z=0}}{Dt} = -v_s \frac{\partial y b}{\partial y}|_{z=0}
\]

\[
\frac{D}{Dt} \left( PV' - \frac{\partial y PV}{\partial y b_s} b'|_{z=0} \right) = 0
\]

\[\Rightarrow PV'(x, y, z) = \frac{\partial y PV}{\partial y b_s} b'(x, y, z = 0) = G(z)b_s(x, y)\]
Coupling between interior and surface inversions

total inversion = inversion (PV) + inversion \( (b|_{z=0}) \)

For baroclinically unstable flows

\[ PV'(x, y, z) = \frac{\partial_y PV}{\partial y b_s} b'(x, y, z = 0) = G(z)b_s(x, y) \]

⇒ inversion (PV) \( \approx \) \( \gamma(z) \) inversion \( (b|_{z=0}) \)

total solution \( \approx \) “effective SQG” solution \( (N = cst) \)
Correlation between PV and surface density

Potential vorticity at 200 m

Surface density

Regression PV' b' prediction d PV/dy / d b/dy

Simulation POP
Decomposing into surface and baroclinic modes

Surface vorticity and surface mode show the distribution of vorticity and stream function, respectively, across the ocean. The barotropic mode and the 1st baroclinic mode illustrate the different components of the oceanic flow, with the barotropic mode representing the slow, large-scale circulation and the baroclinic modes capturing the more rapid, complex flow patterns.
Decomposing into surface and interior dynamics

ratio rms vorticity interior modes vs surface mode

correlation SQG reconstruction and observed vorticity
**Surface QG model**

\[
\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} + \frac{\partial}{\partial z} \left( \frac{f_0^2}{N^2} \frac{\partial \psi}{\partial z} \right) = 0 \quad \text{with} \quad f_0 \frac{\partial \psi}{\partial z} \bigg|_{z=0} = b \bigg|_{z=0}
\]

\[
\left( \frac{\partial}{\partial t} + \mathbf{u} \bigg|_{z=0} \cdot \nabla \right) b \bigg|_{z=0} = 0
\]

**Solution with constant** \(N^2\)

\[
\hat{\psi}(\mathbf{x}, z) = \frac{1}{N} \frac{\hat{b}_s(\mathbf{k})}{|k|} \exp \left( \frac{N}{f_0} |k| z \right)
\]

\[
\hat{b}_s(\mathbf{x}, z) = \hat{b}_s(\mathbf{k}) \exp \left( \frac{N}{f_0} |k| z \right)
\]
Vertical velocities

\[ w = -\frac{1}{N^2} \frac{Db}{Dt} = -\frac{1}{N^2} \left( \frac{\partial b}{\partial t} + \mathbf{u} \cdot \nabla_H b \right) \]

\[ \hat{w} = \frac{1}{N^2} \left( -J(\psi_s, b_s) \exp \left( \frac{N}{f_0} |k| z \right) + J(\psi, b) \right) \]

vertical velocities (m/day)  SQG prediction
at \( z = -220 \text{ m} \)
**Idealized simulation** *(Klein et al. 2008)*

baroclinically unstable front

- Primitive equations model (ROMS) on the Earth Simulator (Japan)
- $1000 \text{ km} \times 2000 \text{ km} \times 4000 \text{ m}$
- $\Delta x = 2 \text{ km}$
- 100 vertical levels
- forcing by restoring on large-scale density gradient
Surface reconstruction

Vorticity and horizontal velocity

SQG reconstruction
500 m reconstruction

Vorticity and horizontal velocity

SQG reconstruction