Towards a new interpretation of upper-ocean dynamics using Surface Quasi-Geostrophy

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Upper oceanic layers at mesoscale

Classical paradigm

- QG turbulence driven by **interior potential vorticity**
  - Kinetic Energy in $\mathbf{k^{-3}}$ at mesoscales (Charney, 1971)
- The altimeter sees **1st baroclinic mode** (Stammer, 1997)
  - Transfer of surface (baroclinic) KE towards **small scales**

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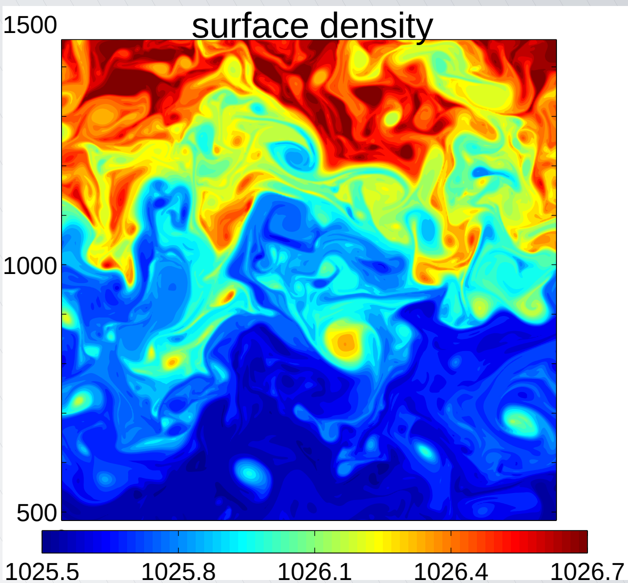
In contradiction with recent results for ocean surface

- Kinetic energy spectra in $\mathbf{k^{-5/3}}$ (Le Traon et al. 2008)
- Transfer of surface Kinetic Energy towards **large scales** (Scott et Wang 2005)

$\Rightarrow$ **need to better understand surface dynamics**

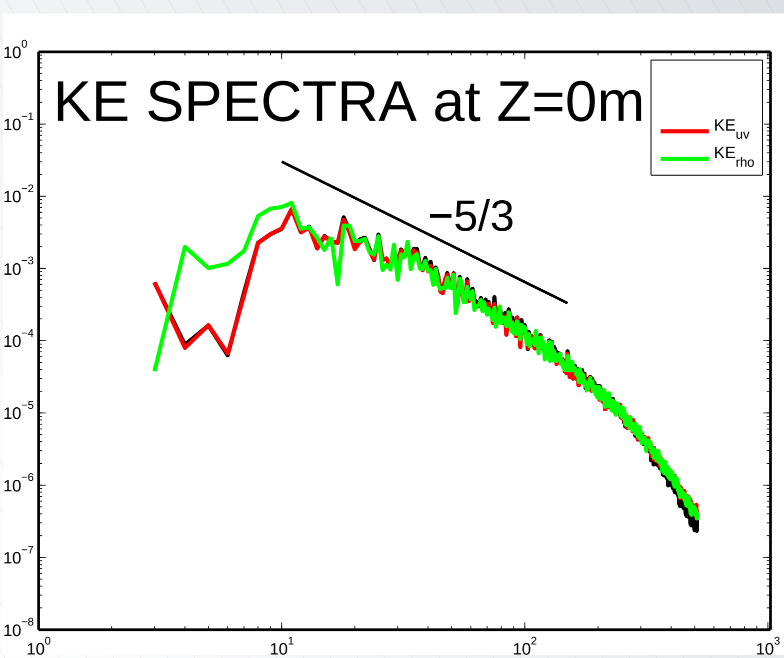
PE simulations at very high resolution

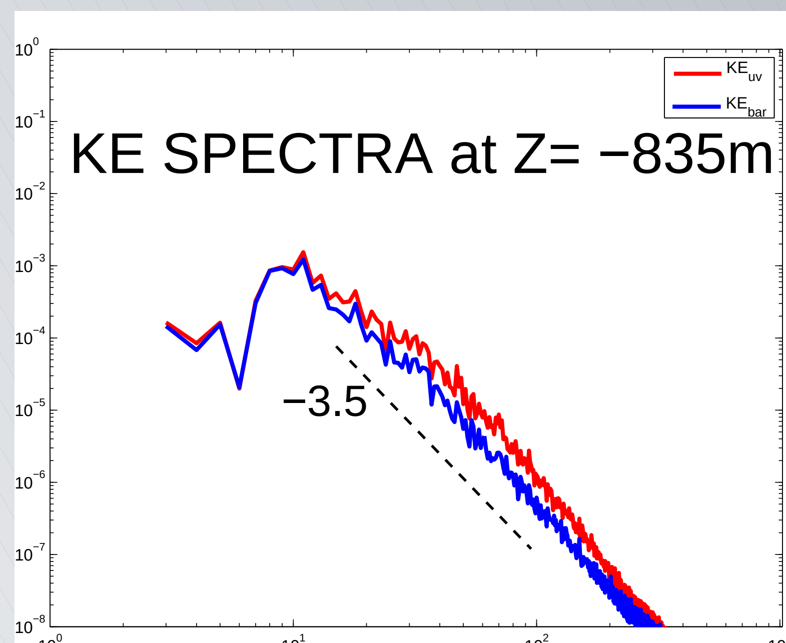


Stratified turbulence with baroclinic unstable front

Earth Simulator (Japan)

(Klein et al. 2008)





Surface ocean dynamics

Towards a new interpretation

- Dynamics driven by **surface density**
  and not by **interior potential vorticity**
- The altimeter sees a surface-intensified mode
  (Lapeyre 2007, submitted)
- **Surface Quasi-Geostrophic model**
  - KE spectra in $\mathbf{k^{-5/3}}$ (Held et al. 1995)
  - Same spectra for surface KE and density
  - Inverse transfer of surface KE (Capet et al. 2008)

  $\Rightarrow$ **consistent with surface observations**

Potential vorticity inversion

QG PV inversion $\equiv$ invert an **elliptic equation** :

$$\underbrace{\frac{\partial^2\psi}{\partial x^2}+\frac{\partial^2\psi}{\partial y^2}}_{\text{relative vorticity}}+\underbrace{\frac{\partial}{\partial z}\left(\frac{f_0^2}{N^2}\frac{\partial\psi}{\partial z}\right)}_{\text{vortex stretching}}=PV$$

with **surface boundary condition**

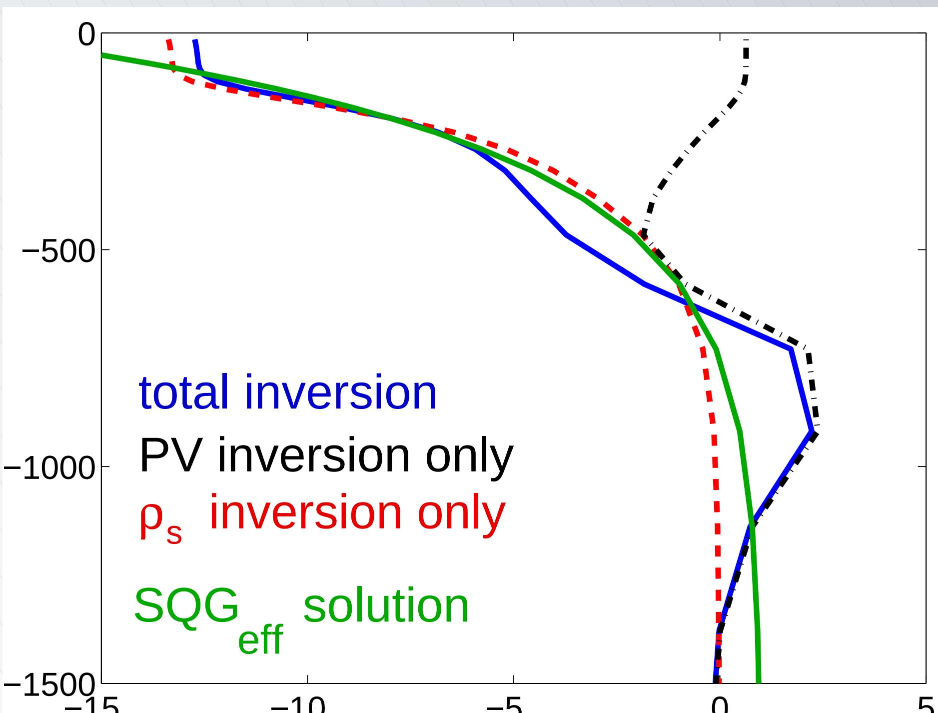
$$f_0\left.\frac{\partial\psi}{\partial z}\right|_{z=0}=b|_{z=0} \qquad\qquad b=-\frac{g\rho}{\rho_0}$$

Important remark:

$b|_{z=0}$ **plays the same role as interior PV!**

Surface vs interior decomposition

total inversion $=$ **inversion (PV)** $+$ **inversion (**$b|_{z=0}$**)**



vertical distribution of $\widehat{\psi}$
for an horizontal mode
$\mathbf{k} = \mathbf{2}\pi/\mathbf{80}$ **km**$^{-1}$
using data from
realistic simulation
(POP model)

$\Rightarrow$ **Effective SQG solution** with constant N^2
may represent upper layer dynamics

Surface QG model

Solution with constant N^2 using surface density

$$\widehat{\psi}(\boldsymbol{k}, z) \;=\; \frac{1}{N}\,\frac{\widehat{\mathbf{b}}_{\mathbf{s}}(\boldsymbol{k})}{|\mathbf{k}|}\;\exp\left(\frac{\mathbf{N}}{\mathbf{f_0}}|\mathbf{k}|\,\mathbf{z}\right)$$

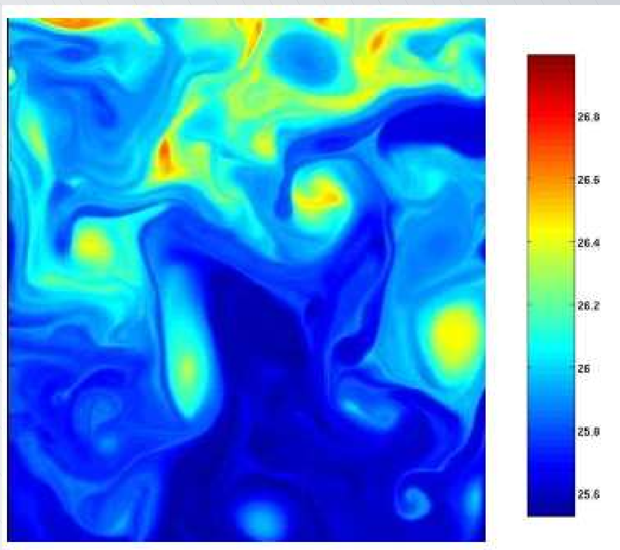
- link between SSH and SST in Fourier space:

$$\mathbf{SSH} \;\propto\; \mathbf{k^{-1}}\,\mathbf{SST}$$

- same spectra for surface KE and SST
- Reconstruction of upper-layer dynamics
  using surface density only

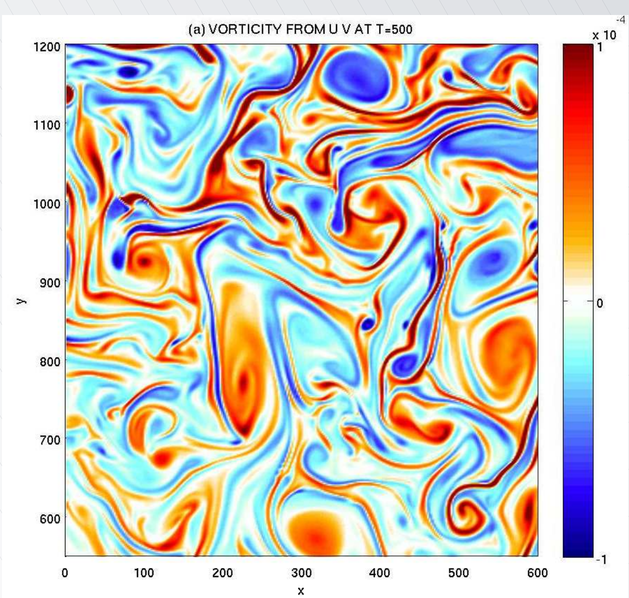
$\Rightarrow$ Test of the SQG solution in different models

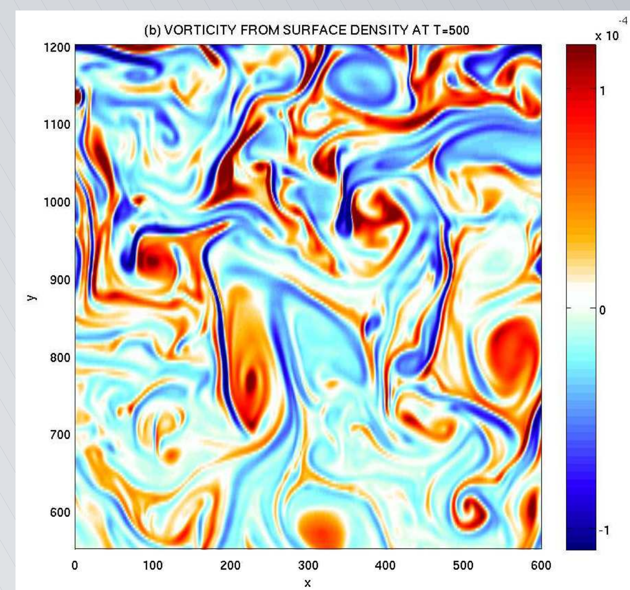
surface density



Reconstruction of vorticity field at the surface

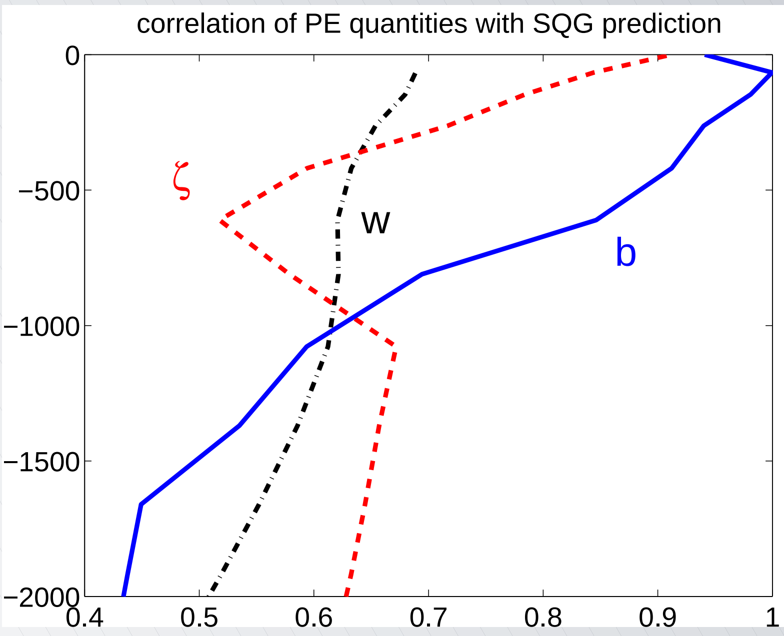
(Earth Simulator simulations)



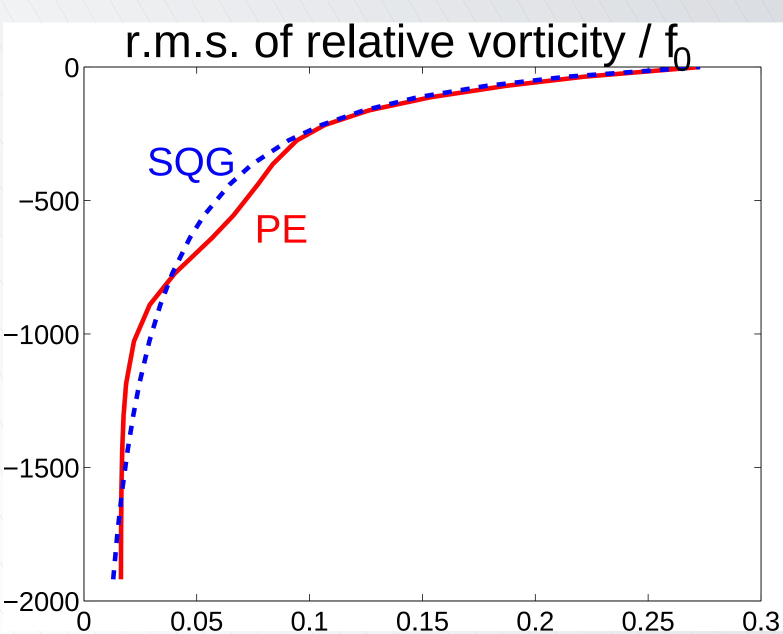


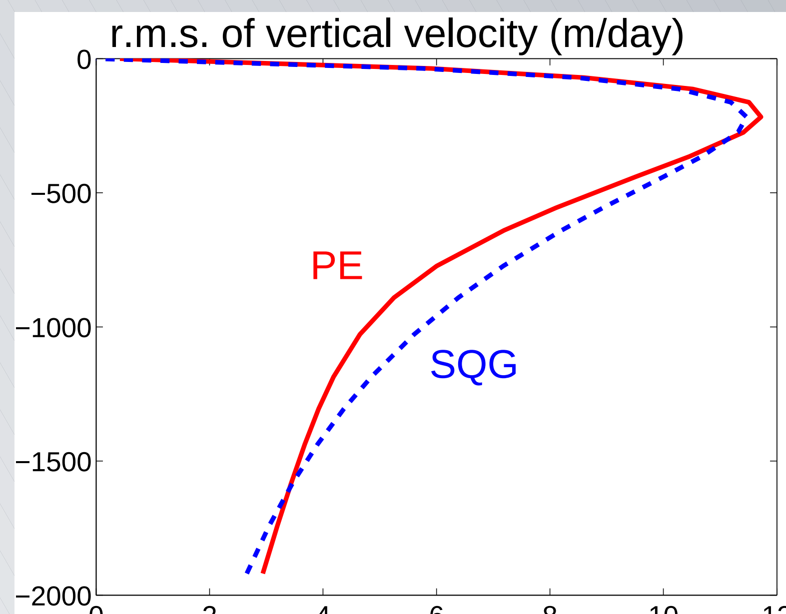
relative vorticity (s^{-1})

SQG prediction



Reconstruction relatively accurate down to 500 m



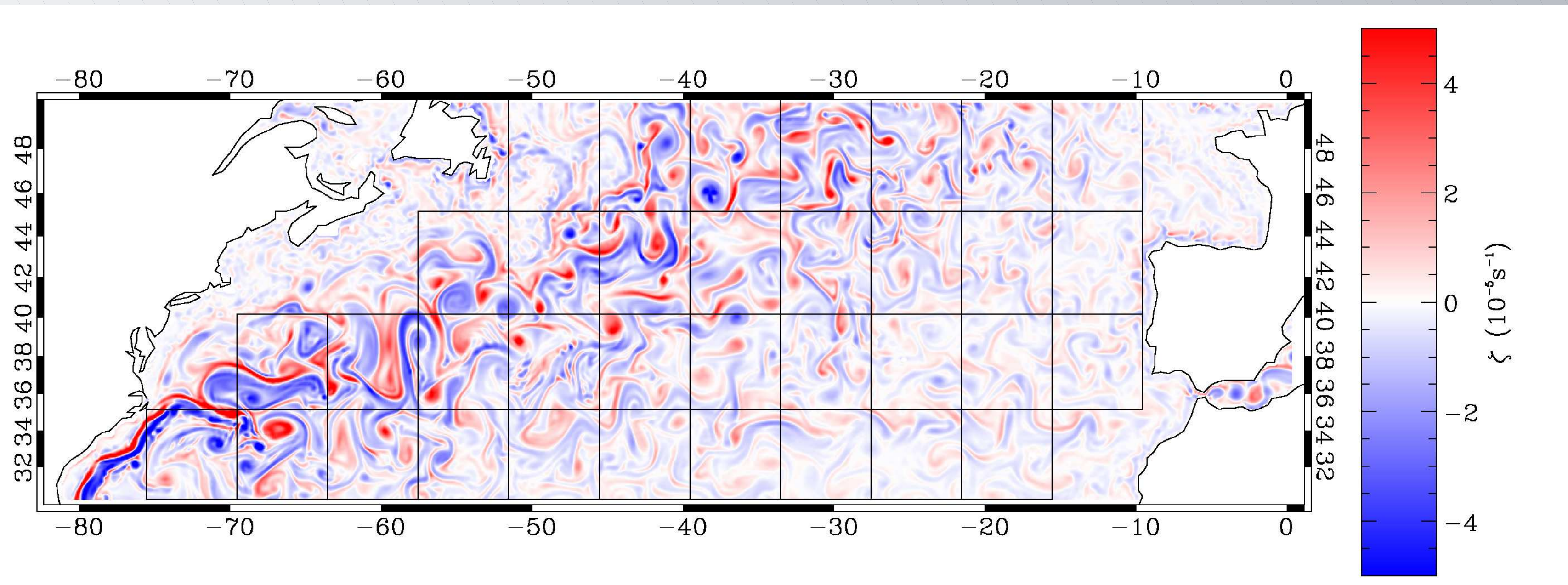


Realistic simulation of the North Atlantic

(Isern-Fontanet et al. 2007, submitted)

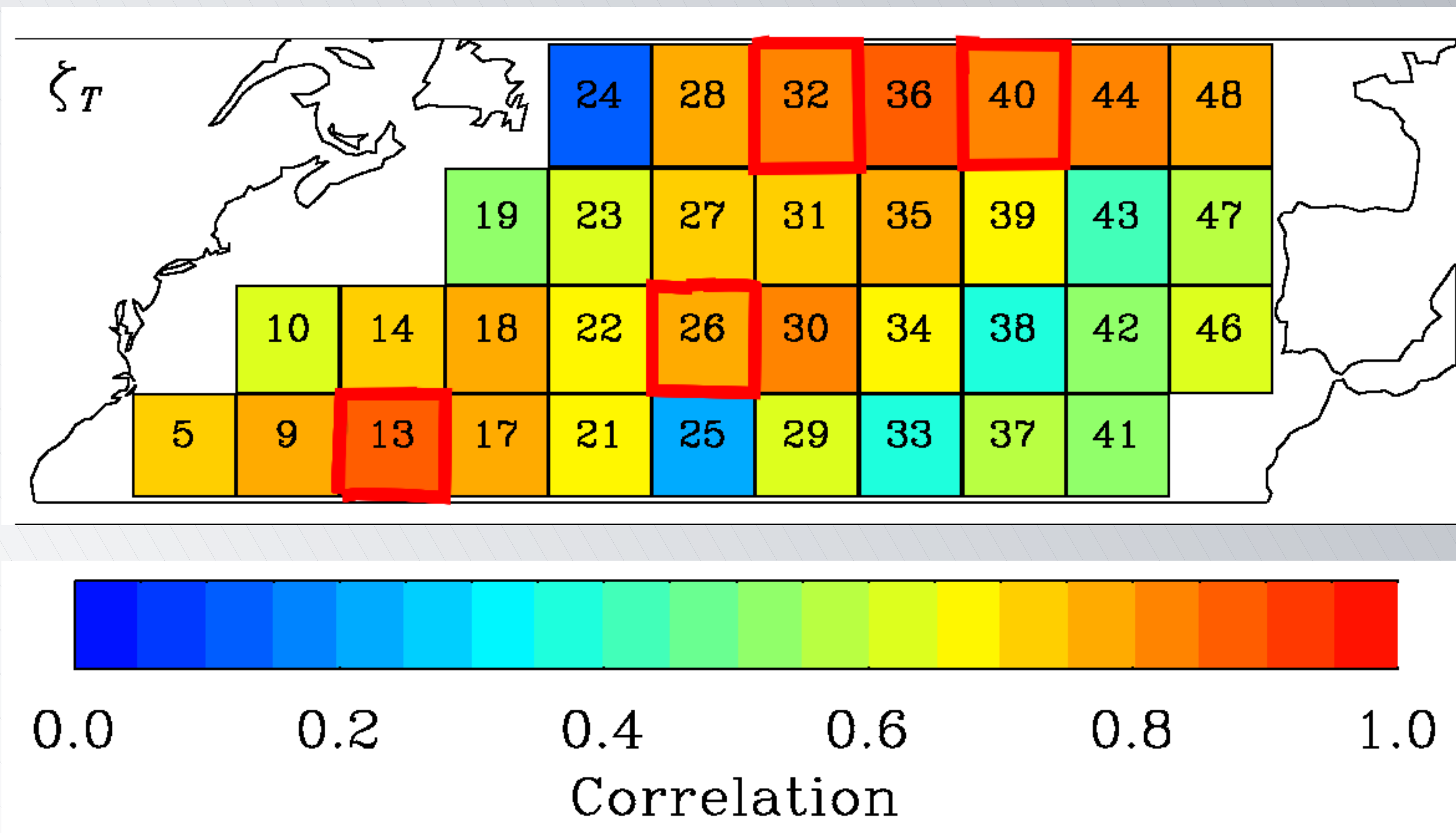
POP, $1/10^{\circ}$

surface relative vorticity



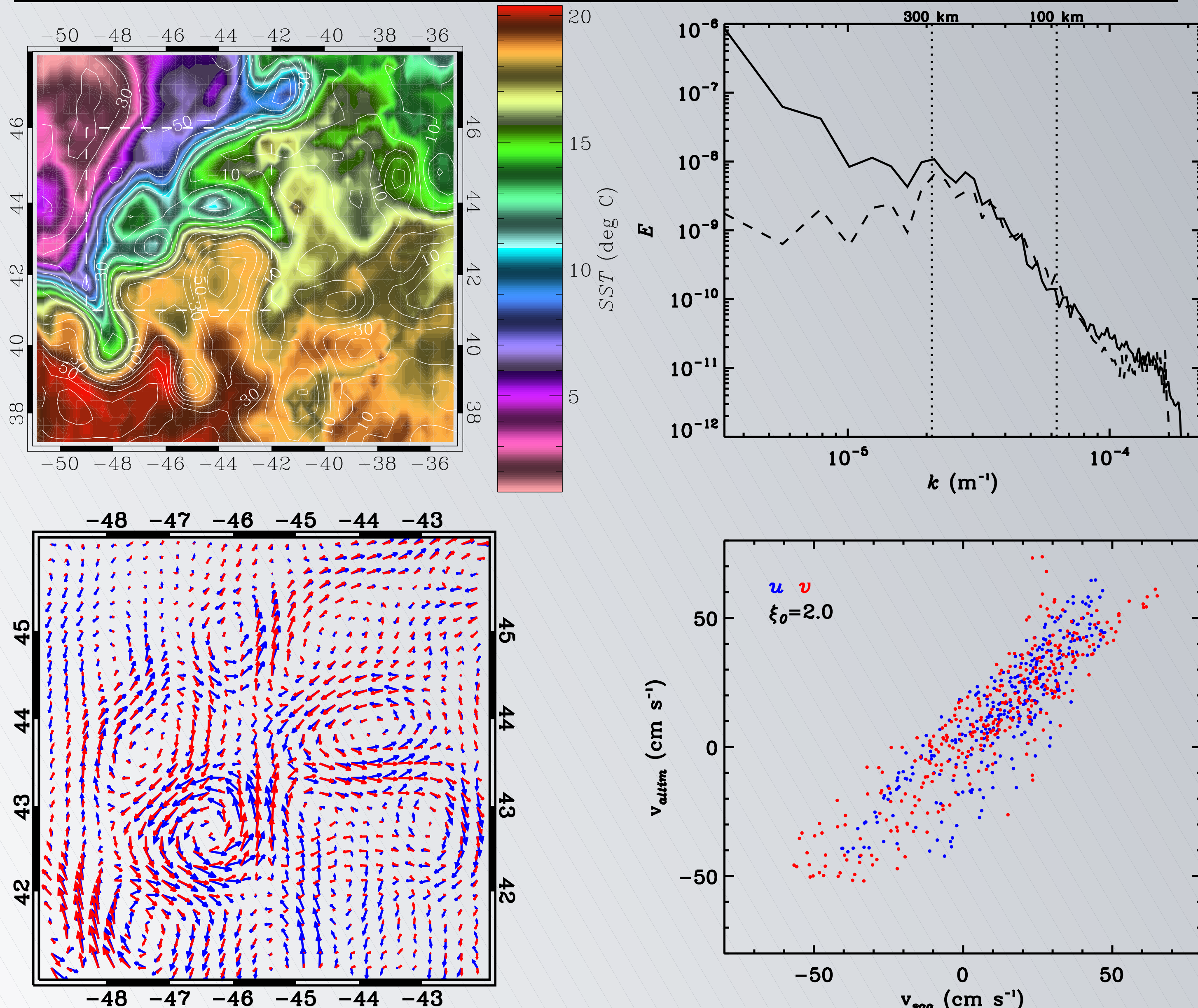
SQG reconstruction of relative vorticity

using SST as a proxy for surface density



of observed vorticity with SQG reconstruction

Comparison altimeter/SST (Isern-Fontanet et al. 2006)



Conclusions

- **Importance of the surface-intensified mode**
  - driven by surface density
  - Surface Quasi-Geostrophic dynamics
- **Reconstruction of 3D dynamics from SST**
  - accurate for the upper 500 meters
- **Coupling surface/interior dynamics?**

References :

Lapeyre et Klein, J.P.O. 2006; Isern-Fontanet et al., G.R.L 2006

Lapeyre et al. J.P.O. 2006; Klein et al. 2008, J.P.O. in press

Capet et al. J. Fluid Mech. 2008, in press.

Mathematical equivalence

$$\frac{\partial^2\psi}{\partial x^2}+\frac{\partial^2\psi}{\partial y^2}+\frac{\partial}{\partial z}\left(\frac{f_0^2}{N^2}\frac{\partial\psi}{\partial z}\right)=PV$$

$$f_0\left.\frac{\partial\psi}{\partial z}\right|_{z=0}=b|_{z=0}$$

and

$$\frac{\partial^2\psi}{\partial x^2}+\frac{\partial^2\psi}{\partial y^2}+\frac{\partial}{\partial z}\left(\frac{f_0^2}{N^2}\frac{\partial\psi}{\partial z}\right)=PV+f_0\,b|_{z=0}\;dirac(z)$$

$$f_0\left.\frac{\partial\psi}{\partial z}\right|_{z=0}=0$$

(Bretherton 1966)

Coupling between interior and surface inversions

total inversion $=$ **inversion (PV)** $+$ **inversion ($b|_{z=0}$)**

Coupling between interior and surface inversions

total inversion $=$ **inversion (PV)** $+$ **inversion ($b|_{z=0}$)**

However coupling between PV and surface density for baroclinically unstable flows

$$\frac{DPV'}{Dt} = -v\,\partial_y \overline{PV} \qquad \frac{D\,b'|_{z=0}}{Dt} = -v_s\,\partial_y \bar{b}\Big|_{z=0}$$

$$\frac{D}{Dt}\left(PV' - \frac{\partial_y \overline{PV}}{\partial_y \overline{b_s}}\,b'|_{z=0}\right) = 0$$

$$\Rightarrow \boxed{\mathbf{PV'(x,y,z)} = \frac{\partial_{\mathbf{y}} \overline{\mathbf{PV}}}{\partial_{\mathbf{y}} \overline{\mathbf{b_s}}}\mathbf{b'(x,y,z=0)} = \mathbf{G(z)b_s(x,y)}}$$

Coupling between interior and surface inversions

total inversion $=$ **inversion (PV)** $+$ **inversion ($b|_{z=0}$)**

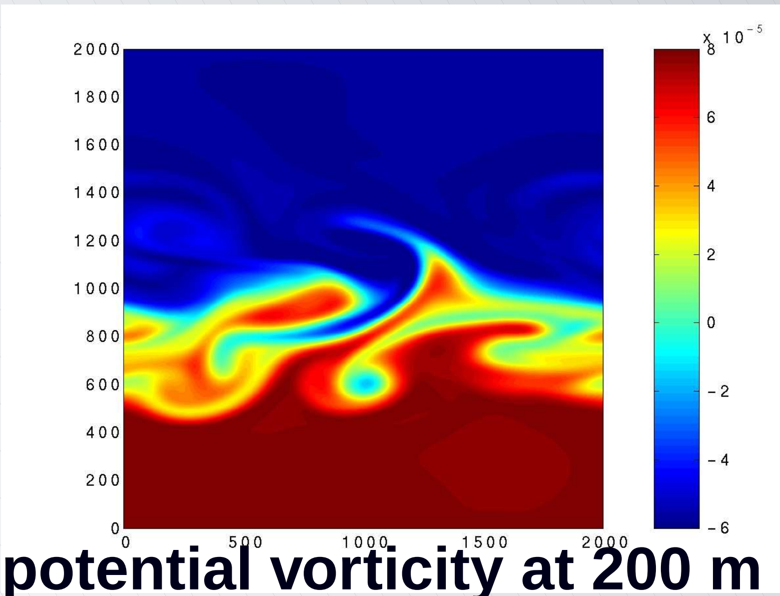
For baroclinically unstable flows

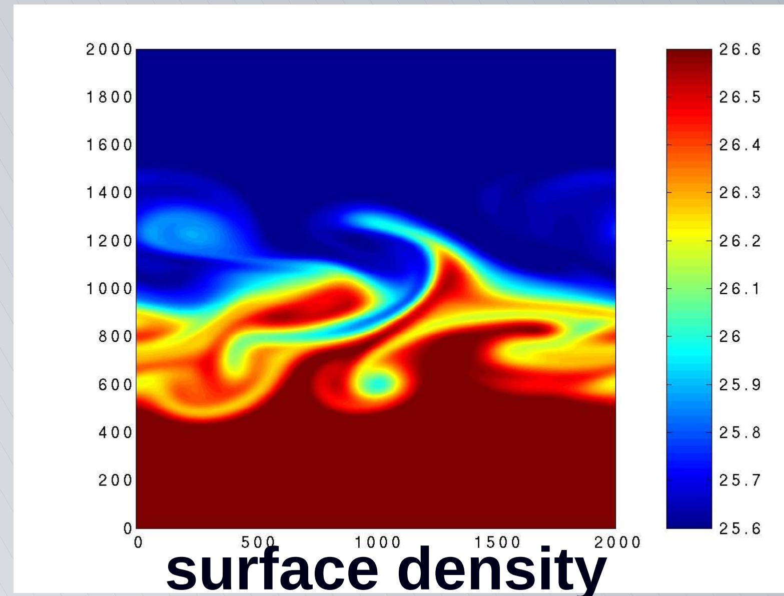
$$PV'(x,y,z) = \frac{\partial_y \overline{PV}}{\partial_y \overline{b_s}} b'(x,y,z=0) = G(z) b_s(x,y)$$

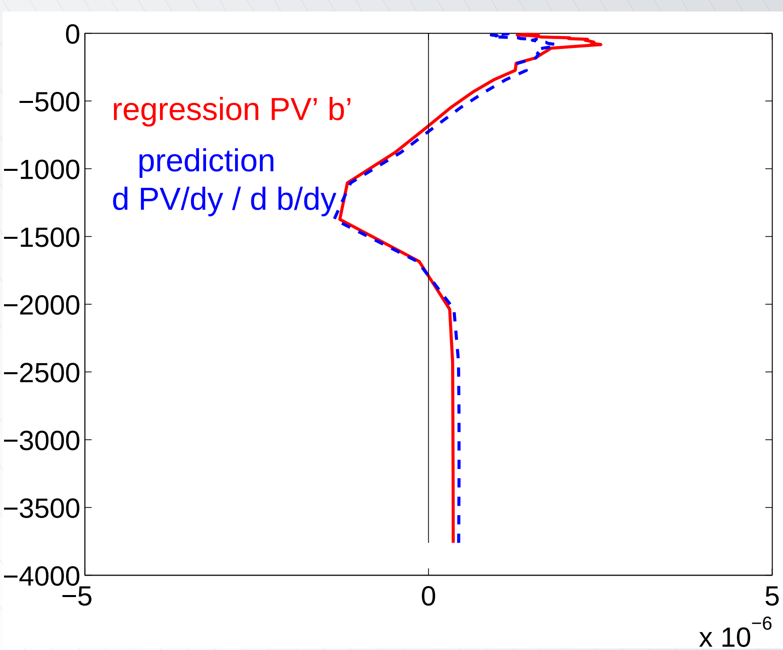
$\Rightarrow$ **inversion (PV)**$\approx \gamma(\mathbf{z})$ **inversion ($b|_{z=0}$)**

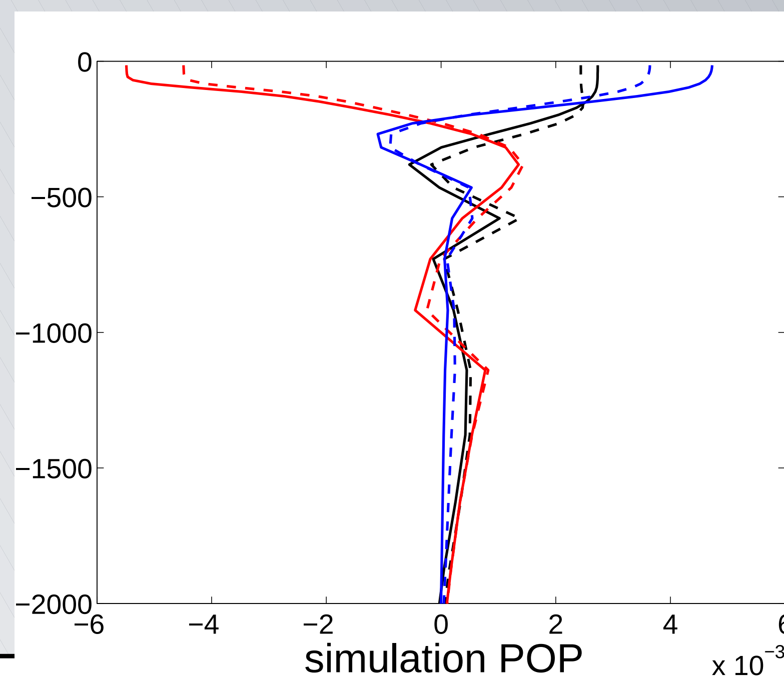
total solution $\approx$ “effective SQG” solution ($N = cst$)

Correlation between PV and surface density

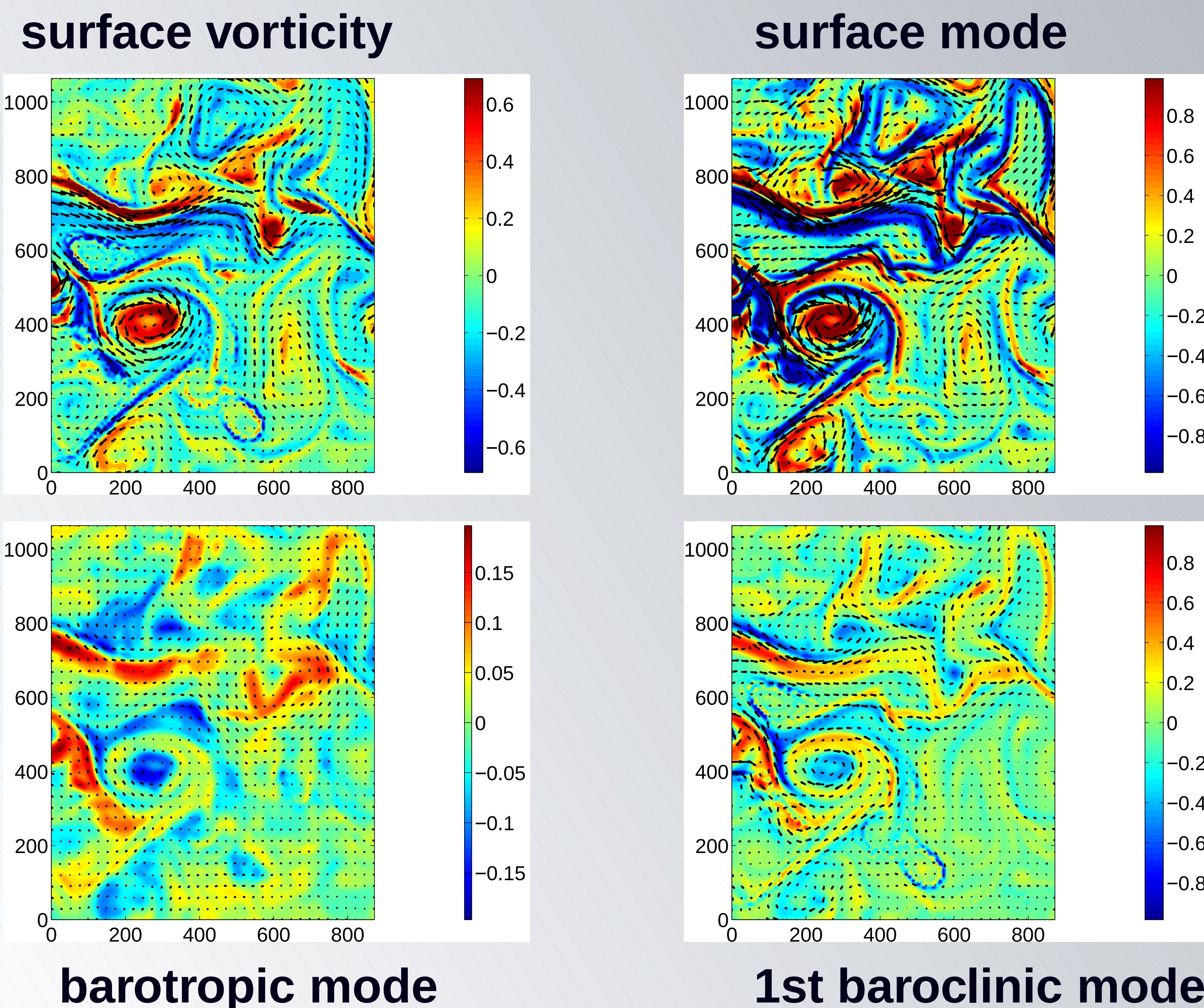






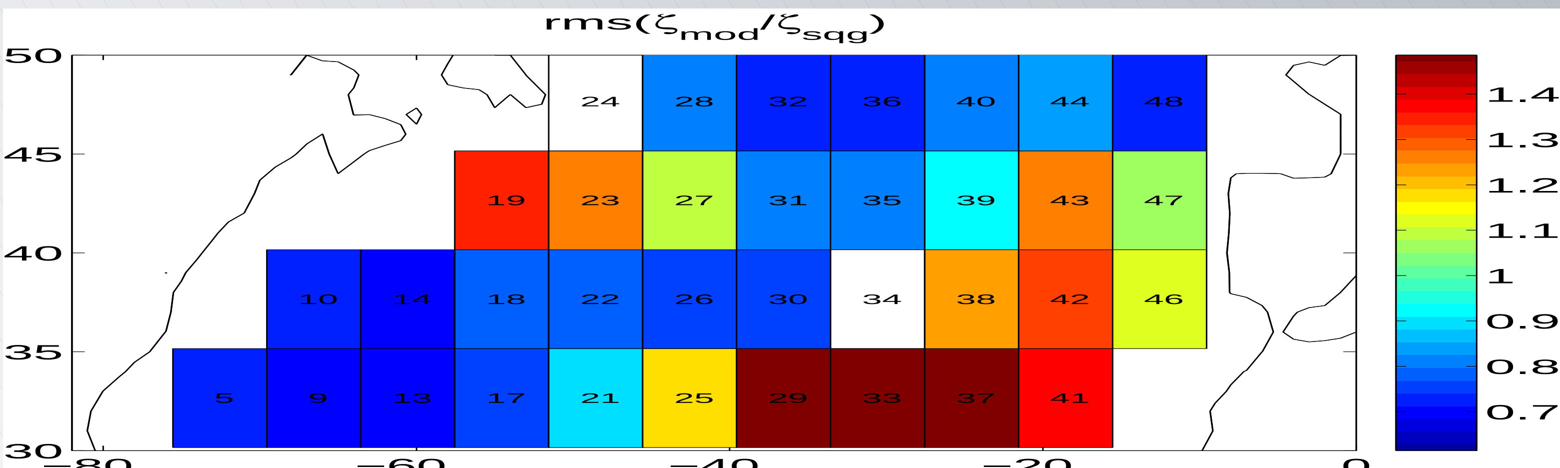


Decomposing into surface and baroclinic modes

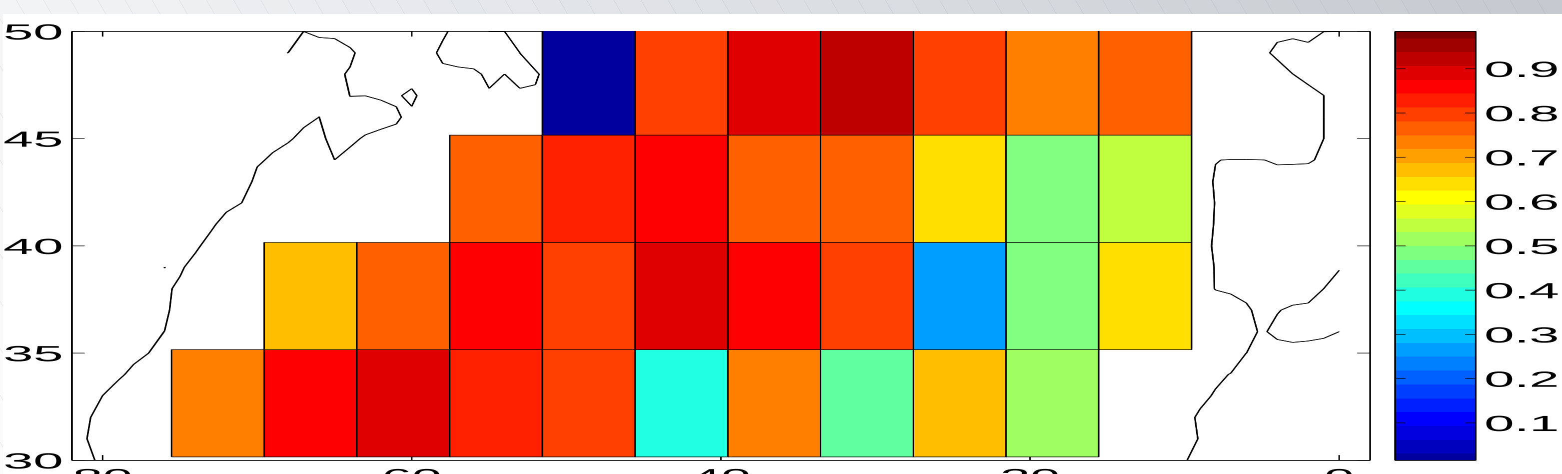


Decomposing into surface and interior dynamics

ratio rms vorticity interior modes vs surface mode



correlation SQG reconstruction and observed vorticity



Surface QG model

$$\frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} + \frac{\partial}{\partial z}\left(\frac{f_0^2}{N^2}\frac{\partial\psi}{\partial z}\right) = 0 \quad \text{with} \quad f_0\,\frac{\partial\psi}{\partial z}\bigg|_{z=0} = b|_{z=0}$$

$$\left(\frac{\partial}{\partial t} + \boldsymbol{u}|_{z=0}\cdot\nabla\right) b|_{z=0} = 0$$

Solution with constant N^2

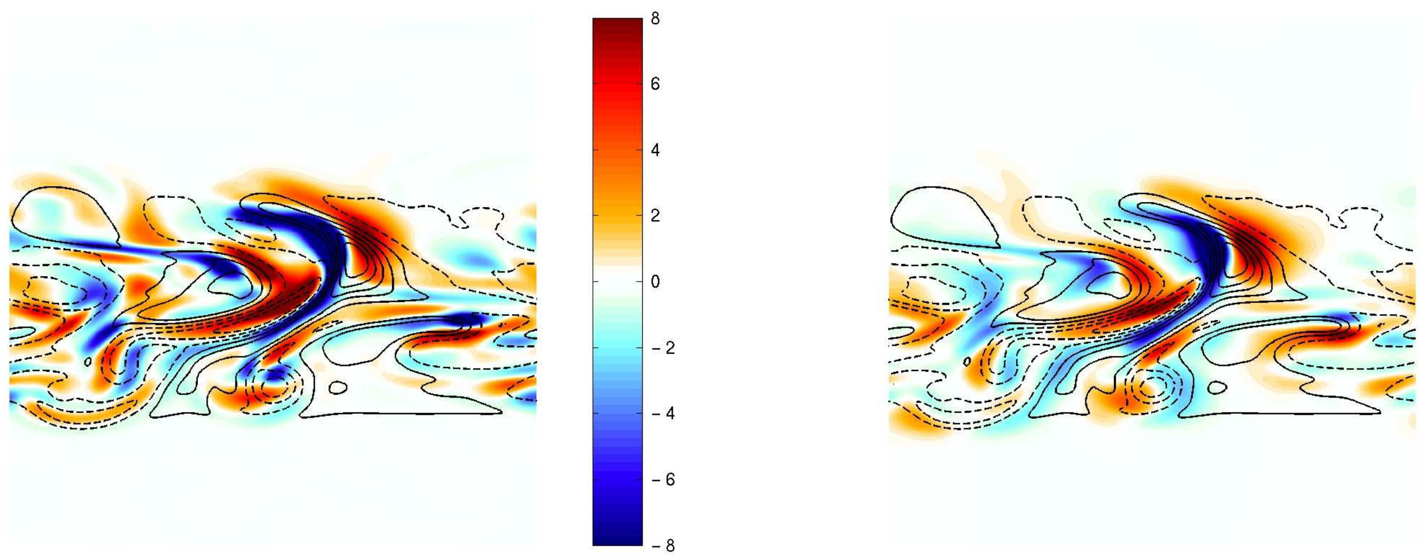
$$\widehat{\psi}(\boldsymbol{x}, z) \;=\; \frac{1}{N}\;\frac{\widehat{\mathbf{b}}_{\mathbf{s}}(\boldsymbol{k})}{|\mathbf{k}|}\;\exp\left(\frac{N}{f_0}|k|\,z\right)$$

$$\widehat{b_s}(\boldsymbol{x}, z) \;=\; \widehat{b}_s(\boldsymbol{k})\exp\left(\frac{N}{f_0}|k|\,z\right)$$

Vertical velocities

$$w = -\frac{1}{N^2}\,\frac{Db}{Dt} = -\frac{1}{N^2}\,(\frac{\partial b}{\partial t} + \boldsymbol{u}\cdot\nabla_H b)$$

$$\widehat{w} = \frac{1}{N^2}\,\left(-\widehat{J(\psi_s,\ b_s)}\exp\left(\frac{N}{f_0}|k|\,z\right) \ + \widehat{J(\psi,\ b)}\right)$$



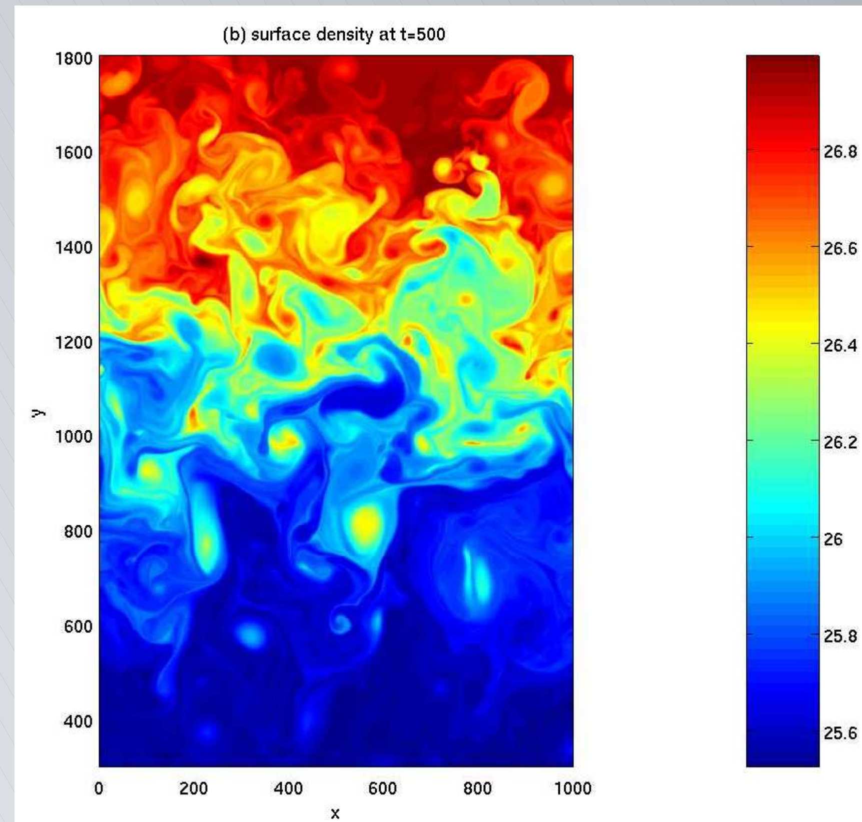
vertical velocities (m/day) **SQG prediction**

at $z =$-220 m

Idealized simulation (Klein et al. 2008)

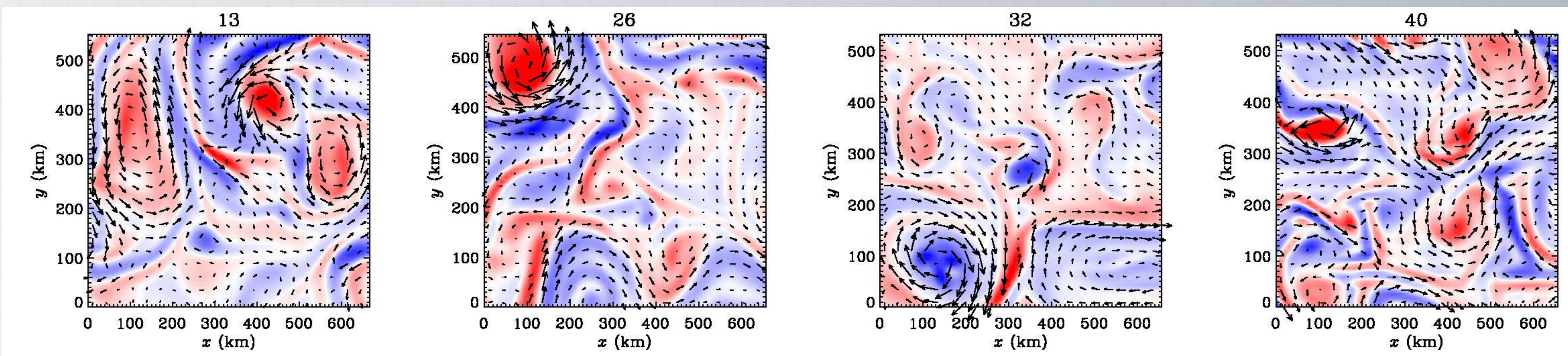
baroclinically
unstable front

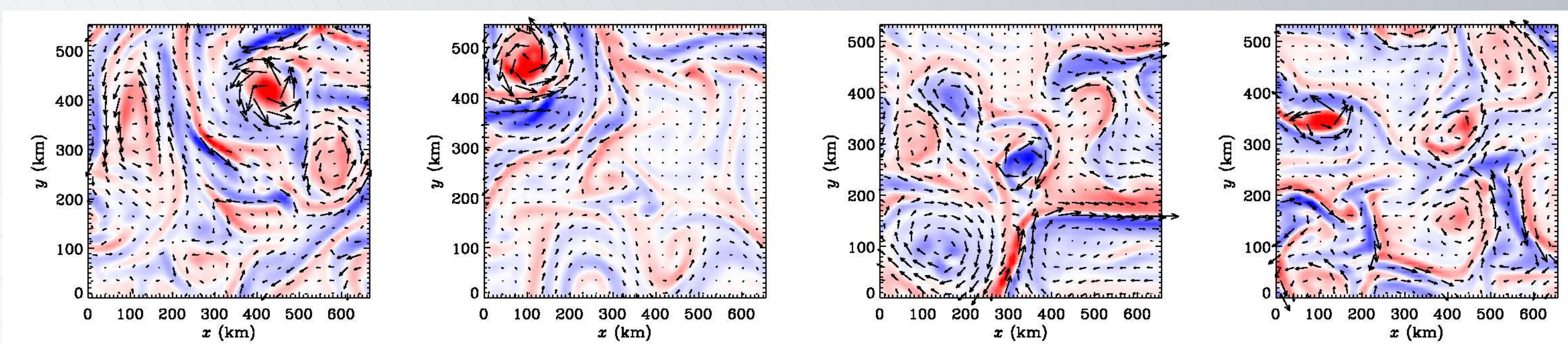
- Primitive equations model (ROMS) on the Earth Simulator (Japan)
- 1000 km $\times$ 2000 km $\times$ 4000 m
  $\Delta x = 2$ km
  100 vertical levels
- forcing by restoring
  on large-scale density gradient



Surface reconstruction

Vorticity and horizontal velocity

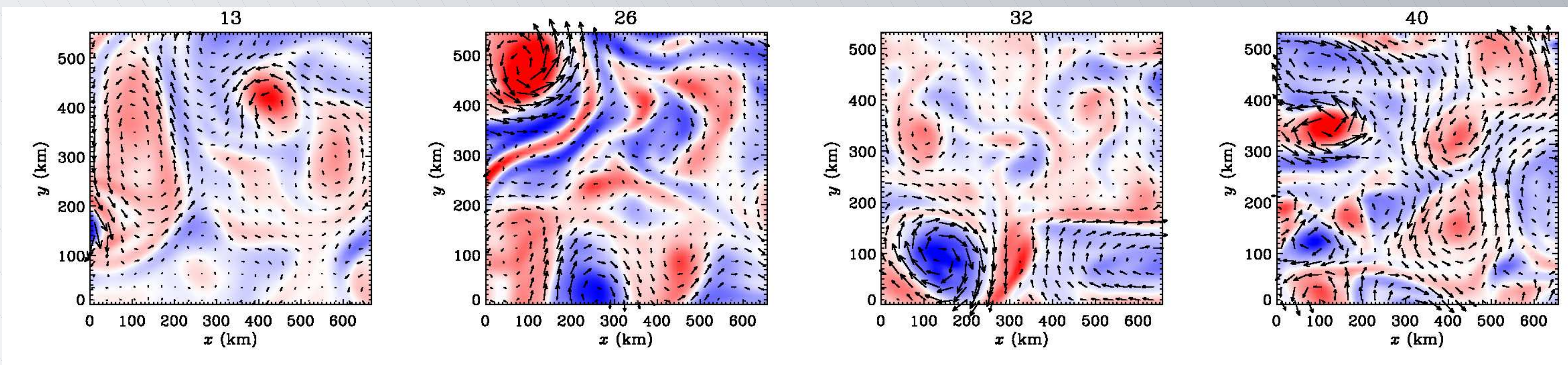


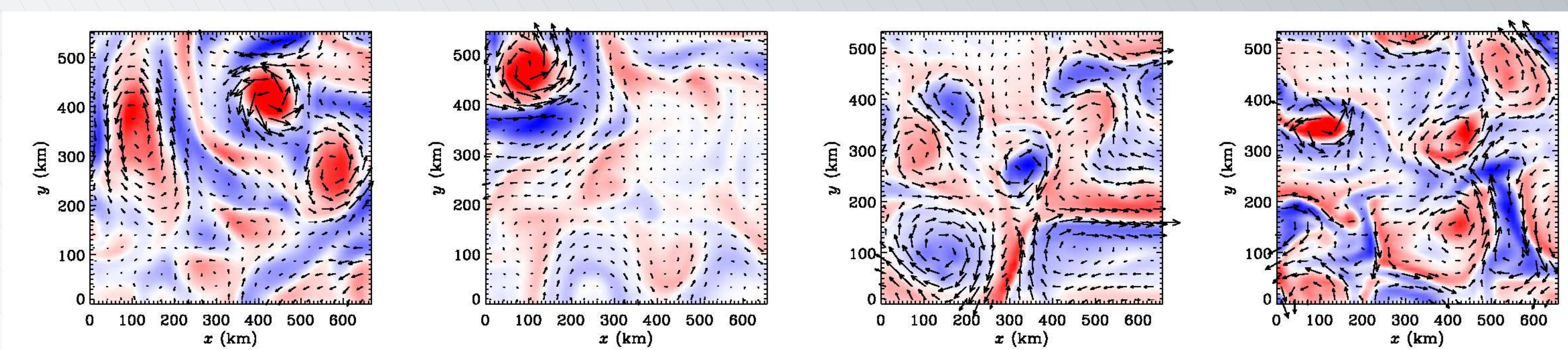


SQG reconstruction

500 m reconstruction

Vorticity and horizontal velocity





SQG reconstruction