Scale interactions and scaling laws in rotating flows at moderate Rossby numbers and large Reynolds numbers

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Computer time provided by NCAR

Moderate Rossby numbers

- The Rossby number $$Ro = \frac{U}{2\Omega L_F}$$

for mid-latitude synoptic scales in the atmosphere is $Ro \approx 0.1$.

- In the sun, the typical Rossby number in the convective zone is $Ro \approx 0.1$-1.
- The Reynolds number $$Re = \frac{L_F U}{\nu}$$

in these systems is very large.

- In DNS, the study of rotating flows is constrained by the cost of resolving inertial waves and resonant interactions (Embid & Majda 1996; Cambon & Jacquin 1989; Smith & Lee 2005), together with small scale fluctuations when *Re* is large.
- We study the effect of rotation in turbulent flows (scaling laws, energy transfer, and intermittency) in 256^3 and 512^3 DNS (Bartello, Métais, and Lesieur 1994; Smith & Waleffe 1999; Yeung & Zhou 1998; Müller & Thiele 2007).

The Navier-Stokes equation

- Momentum equation

$$\frac{\partial \mathbf{u}}{\partial t} + \omega \times \mathbf{u} + 2\Omega \times \mathbf{u} = -\nabla \mathcal{P} + \nu \nabla^2 \mathbf{u} + \mathbf{F} \qquad \nabla \cdot \mathbf{u} = 0$$

P is the pressure, **F** an external force, ν the kinematic viscosity, $\mathbf{\Omega}$ the angular velocity, and **u** the velocity; incompressibility is assumed.

- Quadratic invariants ($\mathbf{F} = 0$, $\nu = 0$):

$$E = \int \mathbf{u}^2 \, d^3x$$

$$H = \int \mathbf{u} \cdot \boldsymbol{\omega} \, d^3x \qquad \boldsymbol{\omega} = \nabla \times \mathbf{u}$$

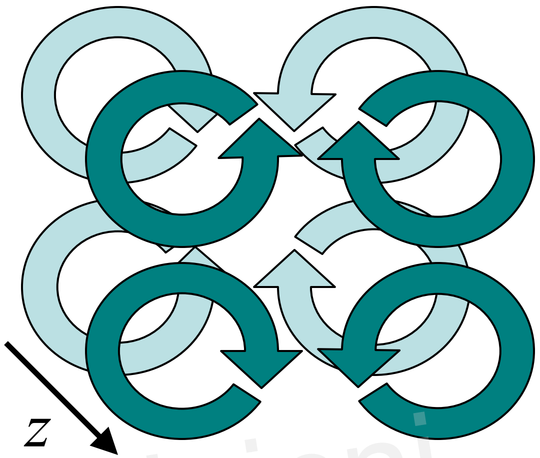
- Reynolds, Rossby, and Ekman numbers

$$Re = \frac{L_F U}{\nu} \qquad Ro = \frac{U}{2\Omega L_F} \qquad Ek = \frac{Ro}{Re} = \frac{\nu}{2\Omega L_F^2}$$

where L is the forcing scale.

Taylor-Green forcing

$$\mathbf{F} = \begin{bmatrix} \sin(k_0 x)\cos(k_0 y)\cos(k_0 z) \\ -\cos(k_0 x)\sin(k_0 y)\cos(k_0 z) \\ 0 \end{bmatrix}$$

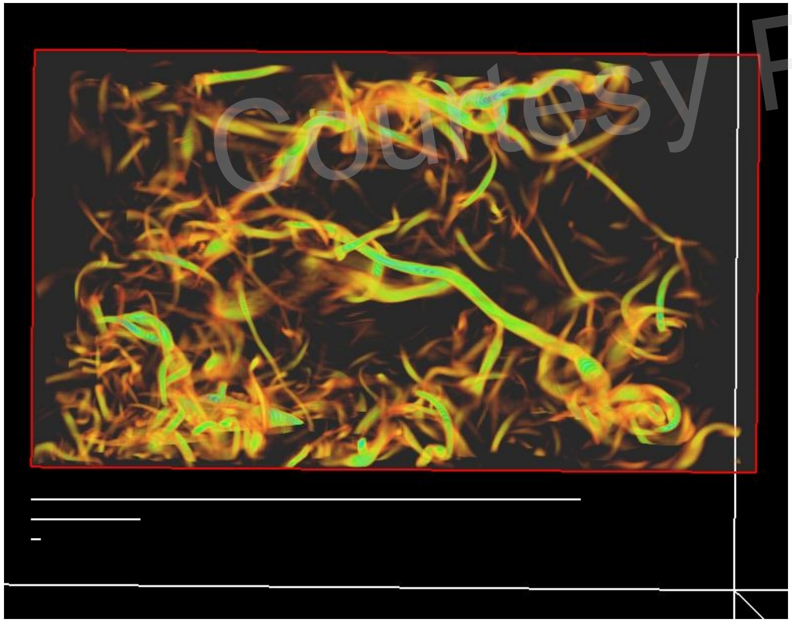


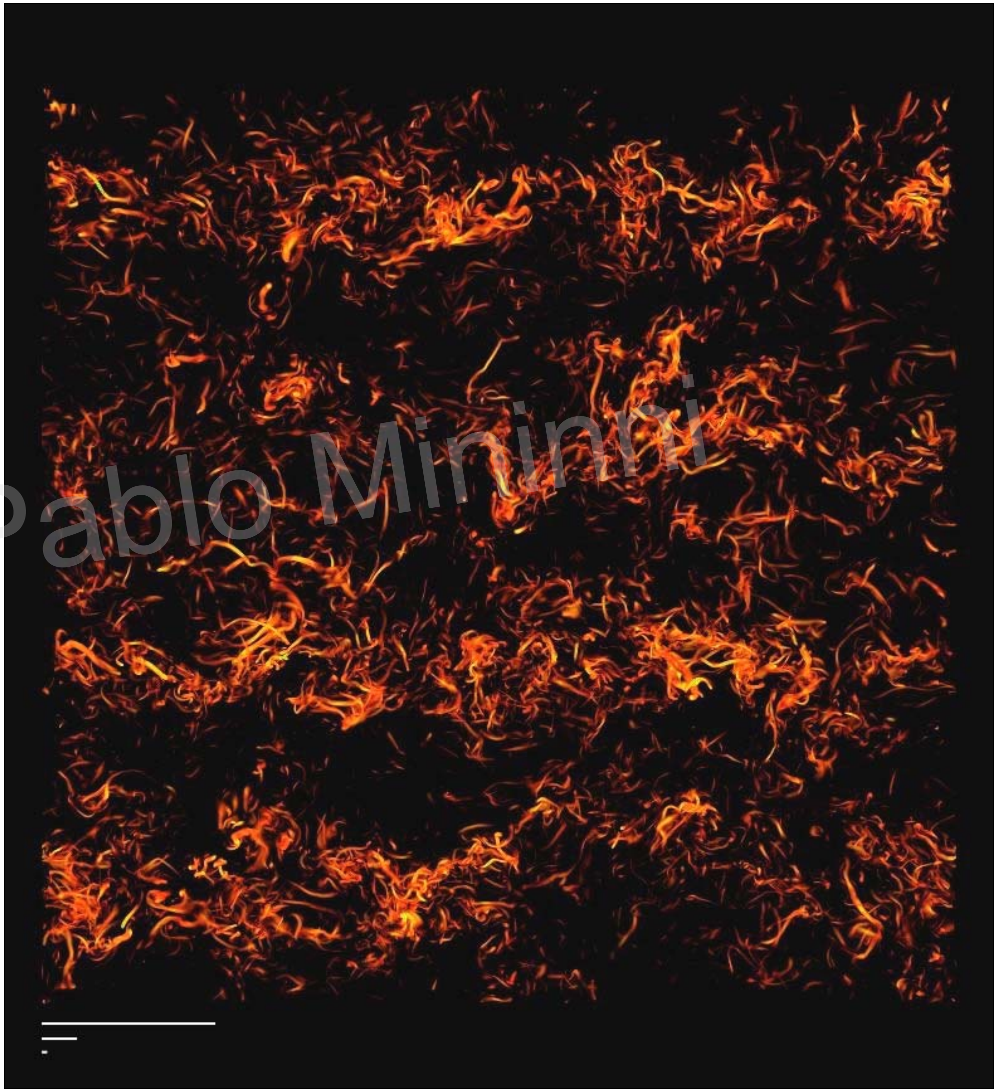
- Non-helical force; generates a fully 3D flow without rotation; injects no energy in modes with $k_z = 0$.
- Proposed as a paradigm of turbulence: eddies at the scale $1/k_0$ cascade down to smaller eddies until reaching the viscous scale.
  Taylor & Green, *Proc. Roy. Soc.* A **151**, 421 (1937)
- Pseudospectral code with periodic boundary conditions, Runge-Kutta in time, 2/3-rule for dealiasing.
- $Ro = 0.07 - 5$, $Re = 900 - 1100$, $Ek = 5\times10^{-3} - 6\times 10^{-5}$

Mininni, Alexakis, & Pouquet, arXiv:physics/0802.3714 (2008).

Large and small scale structures in TG

- 3D visualization of vorticity intensity in the tail of the PDF in a 1024^3 TG simulation.
- Turbulent fluctuations and a large scale pattern.
- Visualizations using VAPOR.





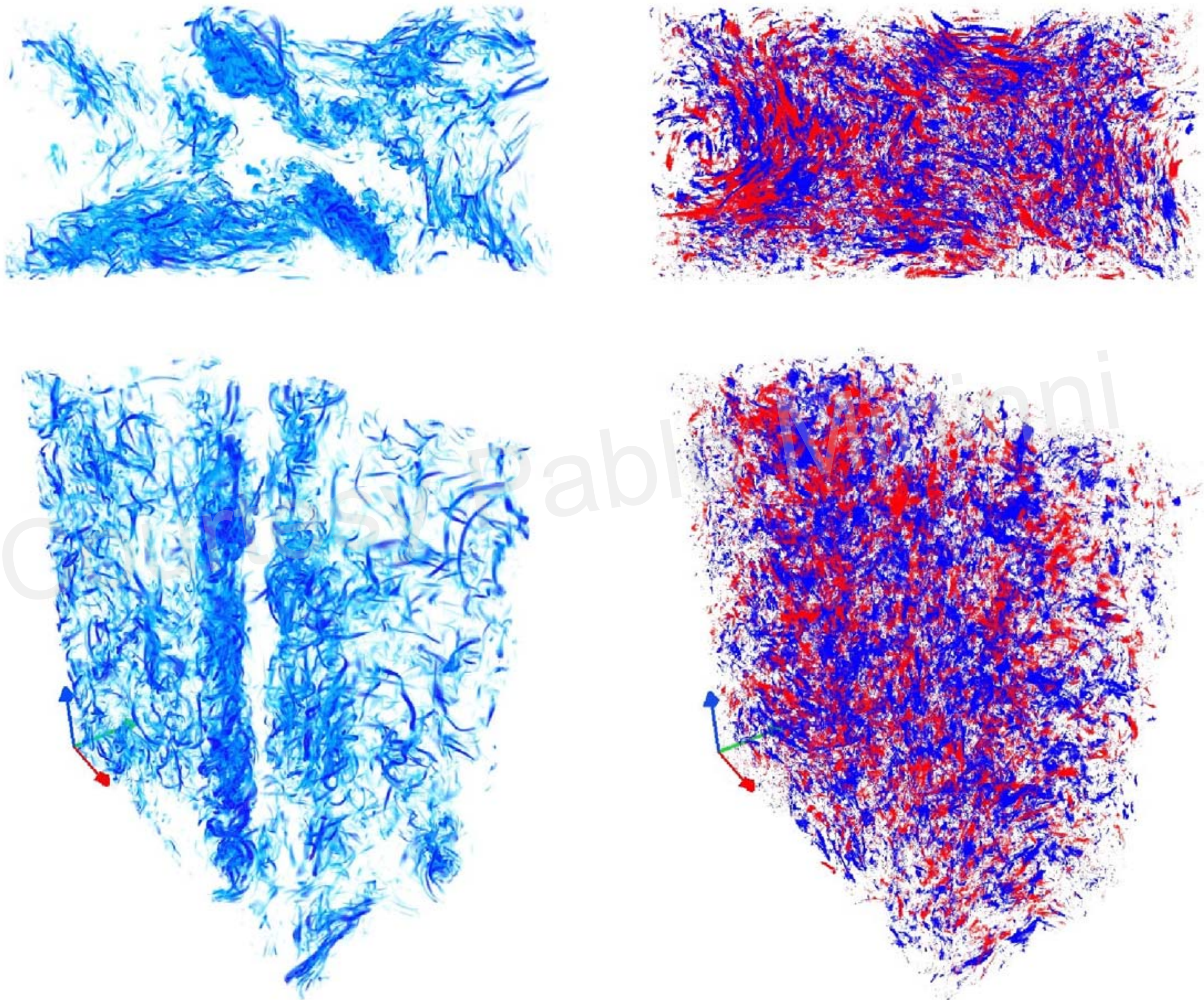
Large and small scale structures (TG)

- Relative helicity $h = \mathbf{v}\cdot\boldsymbol{\omega} / \langle|\mathbf{v}||\boldsymbol{\omega}|\rangle$ and vorticity intensity.
- Local beltramization. Tsinober & Levich, Phys. Lett. **99A**, 321 (1983); Moffat, J. Fluid Mech. **150**, 359 (1985); Farge, Pellegrino, & Schneider, PRL 87, 054501 (2001).

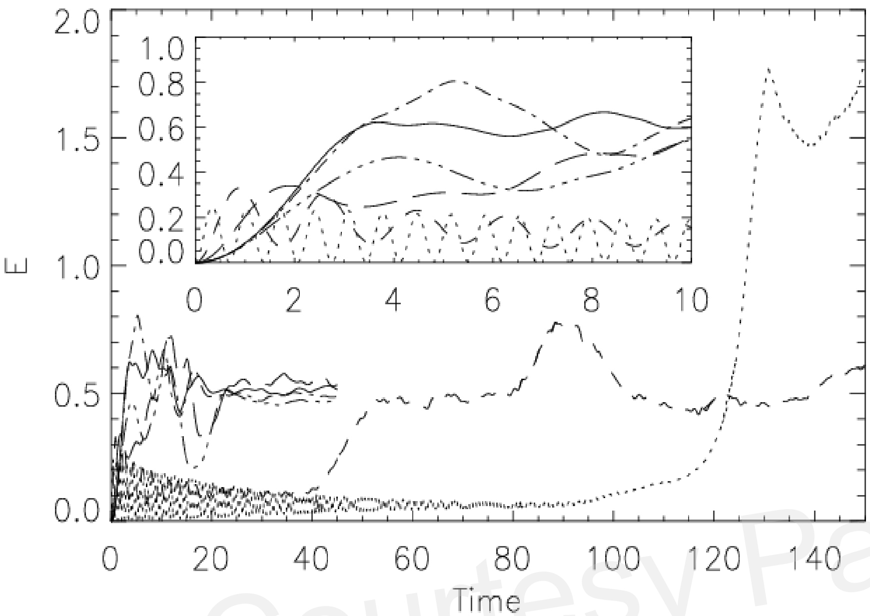
-1 red

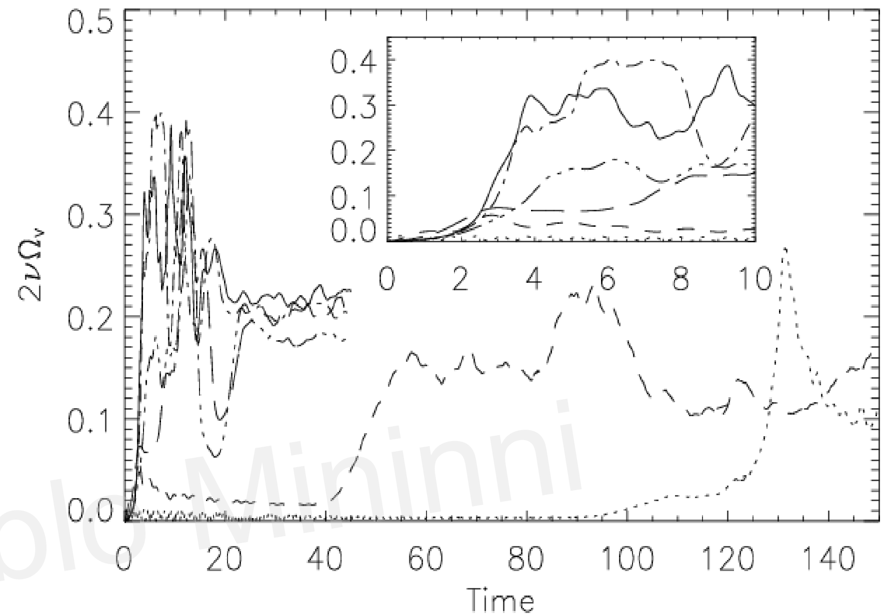
+1 blue

Results with rotation



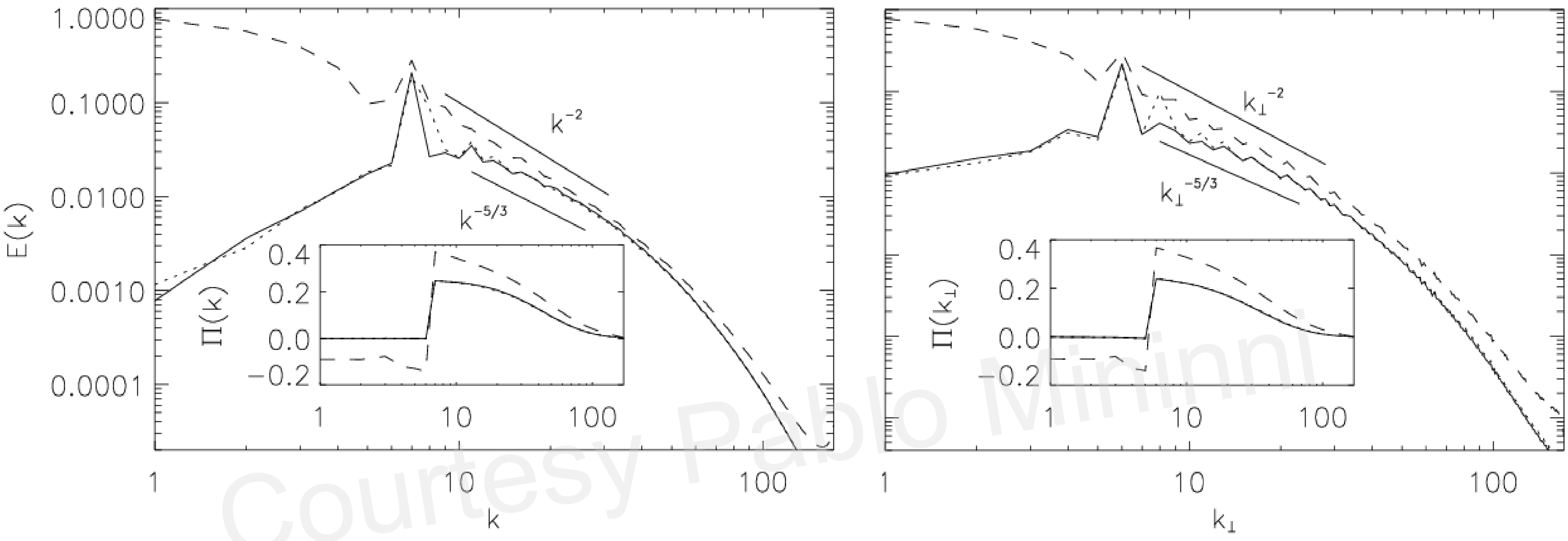
Time evolution





- Long transient as the Rossby number is decreased.
- At early times, the energy spectrum is steep and the energy flux is negligible.
- Before the inverse cascade starts, the energy spectrum scales as $\sim k^{-3}$, but the spectrum evolves towards $\sim k^{-2}$ at late times.
- The energy dissipation rate decreases as the Rossby number is decreased.

Energy spectrum



- Inverse cascade of energy at large scales with constant negative energy flux.
- Spectrum dominated by contributions with wavectors perpendicular to the axis of rotation.
- The small scale energy spectrum seems to scale as $\sim k^{-2}$.
- No clear scaling in the direction parallel to the axis of rotation.

Energy transfer

- Given the filtered velocity fields

$$\mathbf{u}_K(\mathbf{x}) = \sum_{K \le |\mathbf{k}| \le K+1} \tilde{\mathbf{u}}_{\mathbf{k}} e^{i\mathbf{k}\cdot\mathbf{x}} \qquad \mathbf{u}_{K_\perp}(\mathbf{x}) = \sum_{K \le |\mathbf{k}_\perp| \le K+1} \tilde{\mathbf{u}}_{\mathbf{k}} e^{i\mathbf{k}\cdot\mathbf{x}}$$

$$\mathbf{u}_{K_\parallel}(\mathbf{x}) = \sum_{K \le |\mathbf{k}_\parallel| \le K+1} \tilde{\mathbf{u}}_{\mathbf{k}} e^{i\mathbf{k}\cdot\mathbf{x}}$$

we can define the shell-to-shell transfer

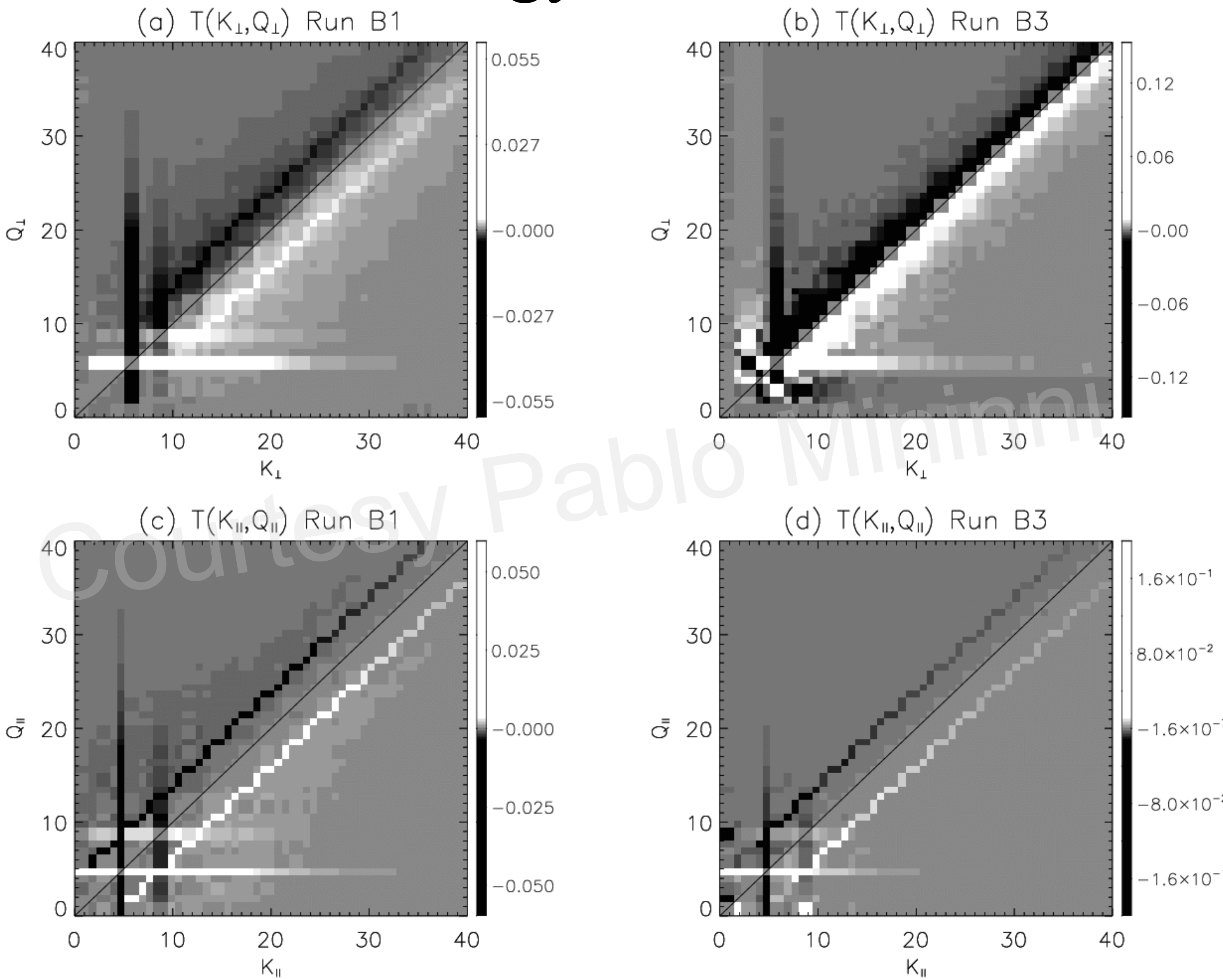
$$T(Q,K) = -\int \mathbf{u}_{\mathbf{K}}(\mathbf{u}\cdot\nabla)\mathbf{u}_{\mathbf{Q}}\, d\mathbf{x}^3$$

- The transfer satisfies the relation *T(Q,K) = -T(Q,K)*, and the energy flux can be obtained from

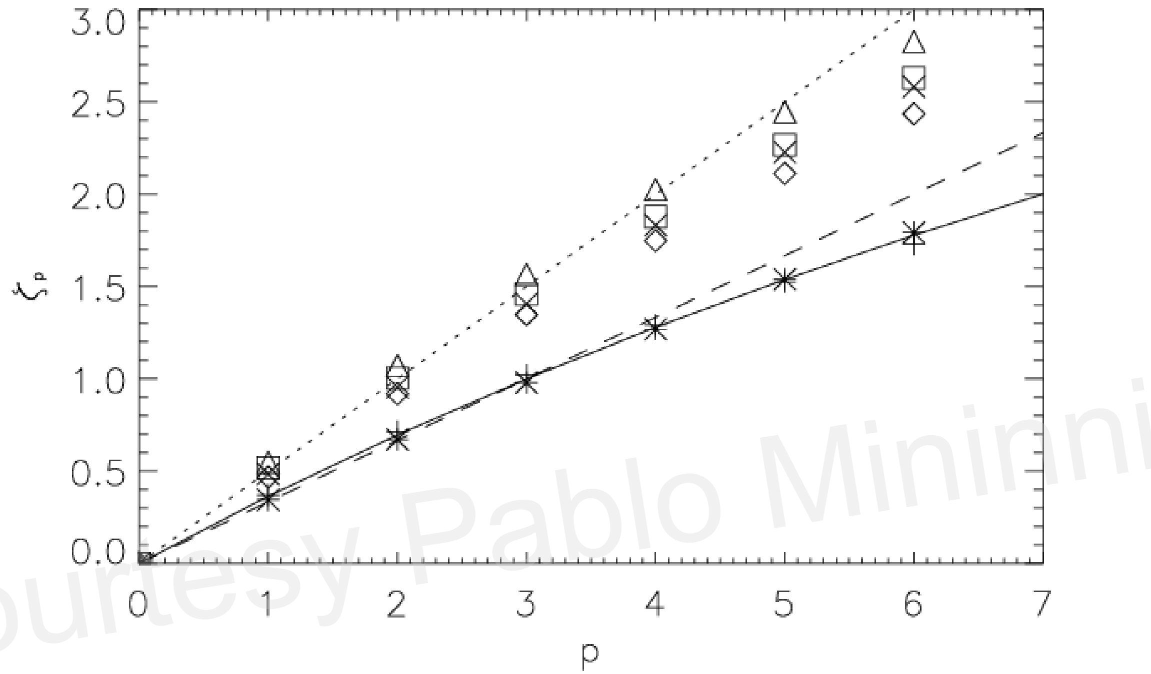
$$\Pi(k) = -\sum_{K=0}^{k} \sum_{Q} T(Q,K)$$

Alexakis, Mininni, & Pouquet, PRE **95**, 264503

Energy transfer



Intermittency



- Given the velocity increments: $\delta u(\mathbf{x}, \ell_\perp) = \hat{\mathbf{r}} \cdot [\mathbf{u}(\mathbf{x} + \ell\hat{\mathbf{r}}) - \mathbf{u}(\mathbf{x})]$ and the structure functions: $S_p(\ell_\perp) = \langle \delta u(\mathbf{x}, \ell_\perp) \rangle$ we measure the scaling exponents: $S_p(l) \sim l^{\zeta p}$.
- The level of intermittency can be measured in terms of $\mu = 2\zeta_3 - \zeta_6$ ≈ 0.25 for all runs.

Conclusions

- Several runs (256^3 and 512^3 forced simulations with rotation) were used to compute detailed transfer of energy, scaling laws, and the development of structures in real space.
- A direct cascade of energy is observed at small scales, and an inverse cascade of energy at large scales.
- The direct transfer of energy is local, although the transfer is significantly slowed down by the rotation. This transfer is mediated by interactions with the largest available scale.
- The inverse cascade of energy is non-local: the transfer takes place between disparate shells in Fourier space.
- At late times, the non-local inverse transfer superposes with a (smaller in amplitude) local and direct transfer of energy.
- The level of intermittency observed in the simulations seems to be independent of the Rossby number.