Dissipation of synoptic-scale flow by small-scale turbulence

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Motivation
Many atmospheric and oceanic models use large eddy simulation to represent unresolved processes. Common implementation: eddy viscosity represents net effect.

It’s desirable to partition the feedback into contributions from different wave modes, i.e. wave drag.

This is a problem in balanced dynamics. How can the effect of the unbalanced motion on the balanced flow be parameterised?
Balanced-unbalanced interactions

Conventional view: these interactions are weak for fast gravity waves (e.g. Errico 1981; Majda & Embid & 1998). Numerical simulations indicate minimal transfer (e.g. Farge & Sadourny 1989; Dewar & Killworth 1995).

However, gravity waves can play an important role, e.g., in mesoscale flows and certain synoptic weather systems.

- Timescale separation breaks down for larger Ro and Fr.
- Geostrophic-ageostrophic transfer can be significant even when the ageostrophic motion is weak (e.g. Errico 1982).

Possible outcome: enhanced dissipation of balanced flow.
Theoretical approaches

Breakdown of balance:

* Ford et al. (2000): Lighthill generation
* McWilliams et al. (1998, 1999): solvability conditions on balance equation

Asymptotic results: relevance to balanced-unbalanced interactions is unclear.

Applicability to turbulent flows

Idea. Focus on specific manifestation: growth via random straining by the balanced flow.

Understand rotating, stratified problem by appealing to the analogy with the $N = 0, f = 0$ case.

- non-rotating, unstratified: 2-D/3-D interactions
- rotating, stratified: geostrophic/ageostrophic interactions
Three-dimensionalisation of 2-D turbulence

Setup

Study growth of 3-D perturbations to 2-D base flow, i.e., 3-D N-S with initial conditions given by decaying 2-D turbulence.

Procedure:
Mechanism

Problem is related to the three-dimensionalisation of mixing layers.

Extension to decaying 2-D turbulence: time-dependent hyperbolic instability (cf. Leblanc & Cambon 1997; Caulfield & Kerswell 2000).

*Physical idea:* Growth of 3-D perturbation, \( u \), via random straining by the 2-D base flow, \( U \), with \( \epsilon := k_h/k_z \to 0 \),

\[
\frac{\partial u}{\partial t} + U \cdot \nabla u = -\nabla p - u \cdot \nabla U.
\]
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Extension to decaying 2-D turbulence: time-dependent hyperbolic instability (cf. Leblanc & Cambon 1997; Caulfield & Kerswell 2000).

**Physical idea**: Growth of 3-D perturbation, $u$, via random straining by the 2-D base flow, $U$, with $\epsilon := \frac{k_h}{k_z} \to 0$, 

$$\frac{\partial u}{\partial t} + U \cdot \nabla u = -\nabla p - u \cdot \nabla U.\,$$

**Implications**

* Initial growth rate can be predicted from properties of 2-D flow (Lapeyre et al. 2000; Straub 2003).

* Anisotropic interaction between large horizontal scales and small vertical scales.
Real space picture

(top view)

yellow: $|\omega_h|$

isosurfaces

green: $\omega_z$

isosurfaces

Random straining by the base flow causes the perturbation vorticity to grow
Perturbation extracts 2D energy at large horizontal scales:

\[ k_h \mathcal{T}_{3D3D \rightarrow 2D} \text{ vs } k_h \]

\[ \varepsilon_a = 0.04 \quad \text{red} \]
\[ \varepsilon_a = 0.18 \quad \text{orange} \]
\[ \varepsilon_a = 0.36 \quad \text{blue} \]
\[ \varepsilon_a = 1/2 \quad \text{pink} \]

\[ \sum_{k_h} \mathcal{T}_{3D \rightarrow 2D} \text{ vs } \varepsilon_a \]

\[ \mathcal{T}_{3D3D \rightarrow 2D}(k_h) = \Re \sum_{|k_h| = k_h} -U_l(u \cdot \nabla u)_l^*(k) \]
Eddy viscosity models effect of perturbation as $\nu_{\text{eddy}} \nabla^2 U$ (e.g. Domaradzki et al. 1993).

\[
\nu_{\text{eddy}} = - \sum k_h \frac{-U_l (\mathbf{u} \cdot \nabla \mathbf{u})^*_{l}}{k_h^2 E_{2D}}
\]

$\nu_{\text{eddy}}$ parameterises perturbation rather than subgrid-scale processes.

Simple structure implies straightforward parameterisation.
Rotating stratified turbulence

We consider the non-hydrostatic Boussinesq equations for buoyancy fluctuations about a mean profile:

\[
\begin{align*}
\frac{\partial u}{\partial t} + u \cdot \nabla u + 2\Omega \times u &= -\frac{1}{\rho_0} \nabla p + b + \nu \mathcal{D}(u) \\
\frac{\partial b}{\partial t} + u \cdot \nabla b &= -N^2 w + \nu \mathcal{D}(b) \\
\nabla \cdot u &= 0,
\end{align*}
\]

where

\[b := g\theta' / \theta_0\] is the (perturbation) buoyancy

\[
\theta = \theta_0 + (d\bar{\theta}/dz)z + \theta'
\] is the potential temperature

\[
N = \left(\frac{g}{\theta_0} \frac{d\bar{\theta}}{dz}\right)^{\frac{1}{2}}
\] is the Brunt-Vaisala frequency
Governing equations for rotating stratified flow

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\nabla \cdot u &= 0,
\end{align*}
\]

Setup

- random initial conditions
- qg spin-up
- basic state
- ageostrophic perturbation
  \[t=0\]
Geostrophic (balanced) and ageostrophic (unbalanced) motion are defined using normal modes (Bartello 1995).

Defining Rossby and Froude numbers as $Ro := \frac{U}{fL}$, $Fr := \frac{U}{NH}$, one expects analogous results for:

* Synoptic-scale flow (rotation dominated) $L_0 > L_d$, where $L_d = (Ro/Fr)L_0$ is the deformation radius

* Weak stratification (gravity waves negligible) $H_0 < H_b$ where $H_b = U/N$ is the buoyancy scale

i.e. instability of ageostrophic modes due to random straining by geostrophic flow. Particularly interested in GAG interaction.

⇒ What happens more generally?
Perturbation extracts **geostrophic energy** at large horizontal scales:

\[ \frac{L_d}{L_0} = 0.21 \]
\[ \frac{L_d}{L_0} = 0.33 \]
\[ \frac{L_d}{L_0} = 0.67 \]
\[ \frac{L_d}{L_0} = 1.33 \]
\[ \frac{L_d}{L_0} = 2.67 \]
\[ \frac{L_d}{L_0} = 3.33 \]

\[ T_{\text{net}} := T_{GA-G} + T_{AA-G} \]
For synoptic flow, there is preferential damping of large-scale geostrophic modes:

Nature of the “wave drag” depends on the structure of the basic state.
Balanced-unbalanced interactions in rotating stratified turbulence can be analogous to 2D-3D interactions in homogeneous turbulence.

**Requirements:**

- synoptic flow \((L_0 > L_d)\)
- adequate resolution of small-scale modes \((H_b > \Delta z)\)
**Summary**

**Balanced-unbalanced** interactions in rotating stratified turbulence can be analogous to 2D-3D interactions in homogeneous turbulence.

**Requirements:**

- synoptic flow \((L_0 > L_d)\)
- adequate resolution of small-scale modes \((H_b > \Delta z)\)

**Implications**

- Anisotropy needs to be taken into account; vertical resolution is crucial.
- Multiscale parameterisation may be useful.
- Classical picture of predictability may need to be updated.