Dissipation of synoptic-scale flow by small-scale turbulence

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1. Motivation
2. Three-dimensionalisation of 2-D turbulence
3. Rotating stratified turbulence
4. Summary

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Motivation

Subgrid-scale parameterisation

Many atmospheric and oceanic models use large eddy simulation to represent unresolved processes. Common implementation: eddy viscosity represents net effect.

It's desirable to partition the feedback into contributions from different wave modes, i.e. wave drag.

$\Longrightarrow$ This is a problem in balanced dynamics. How can the effect of the unbalanced motion on the balanced flow be parameterised?

Balanced-unbalanced interactions

Conventional view: these interactions are weak for fast gravity waves (e.g. Errico 1981; Majda & Embid & 1998). Numerical simulations indicate minimal transfer (e.g. Farge & Sadourny 1989; Dewar & Killworth 1995).

However, gravity waves can play an important role, e.g., in mesoscale flows and certain synoptic weather systems.

* Timescale separation breaks down for larger Ro and Fr.
* Geostrophic-ageostrophic transfer can be significant even when the ageostrophic motion is weak (e.g. Errico 1982).

Possible outcome: enhanced dissipation of balanced flow.

Theoretical approaches

Breakdown of balance:

* Ford et al. (2000): Lighthill generation
* Vanneste & Yavneh (2004,2007): exponential asymptotics, unbalanced instabilities
* McWilliams et al. (1998,1999): solvability conditions on balance equation

Asymptotic results: relevance to balanced-unbalanced interactions is unclear.

$\Longrightarrow$ But mechanism underlying spontaneous imbalance should be robust: generation of unbalanced motion by balanced flow (cf. Warn 1986, Warn & Menard 1986).

Applicability to turbulent flows

Idea. Focus on specific manifestation: growth via random straining by the balanced flow.

Understand rotating, stratified problem by appealing to the analogy with the $N = 0, f = 0$ case.

* non-rotating, unstratified: 2-D/3-D interactions
* rotating, stratified: geostrophic/ageostrophic interactions

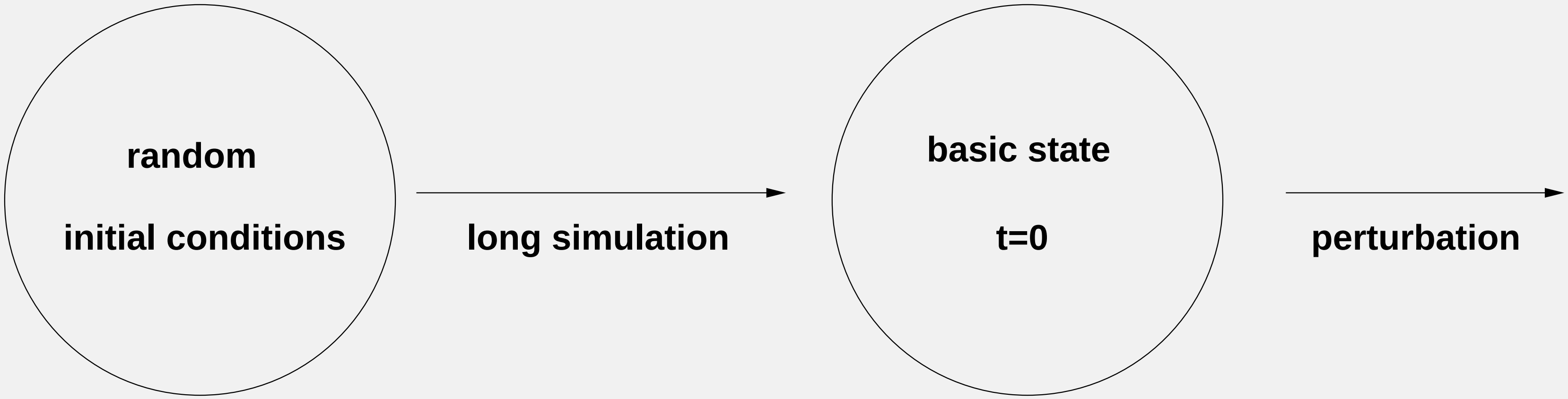
Three-dimensionalisation of 2-D turbulence

References: Phys. Fluids, **16**, 2918 (2004)
Phys. Fluids, **17**, 125102 (2005)

Setup

Study growth of 3-D perturbations to 2-D base flow, i.e., 3-D N-S with initial conditions given by decaying 2-D turbulence.

Procedure:



Mechanism

Problem is related to the three-dimensionalisation of mixing layers.

Extension to decaying 2-D turbulence: time-dependent hyperbolic instability (cf. Leblanc & Cambon 1997; Caulfield & Kerswell 2000).

Physical idea: Growth of 3-D perturbation, $\boldsymbol{u}$, via random straining by the 2-D base flow, $\boldsymbol{U}$, with $\epsilon := k_h/k_z \to 0$,

$$\frac{\partial \boldsymbol{u}}{\partial t} + \boldsymbol{U} \cdot \boldsymbol{\nabla} \boldsymbol{u} = -\boldsymbol{\nabla} p - \boldsymbol{u} \cdot \boldsymbol{\nabla} \boldsymbol{U}.$$

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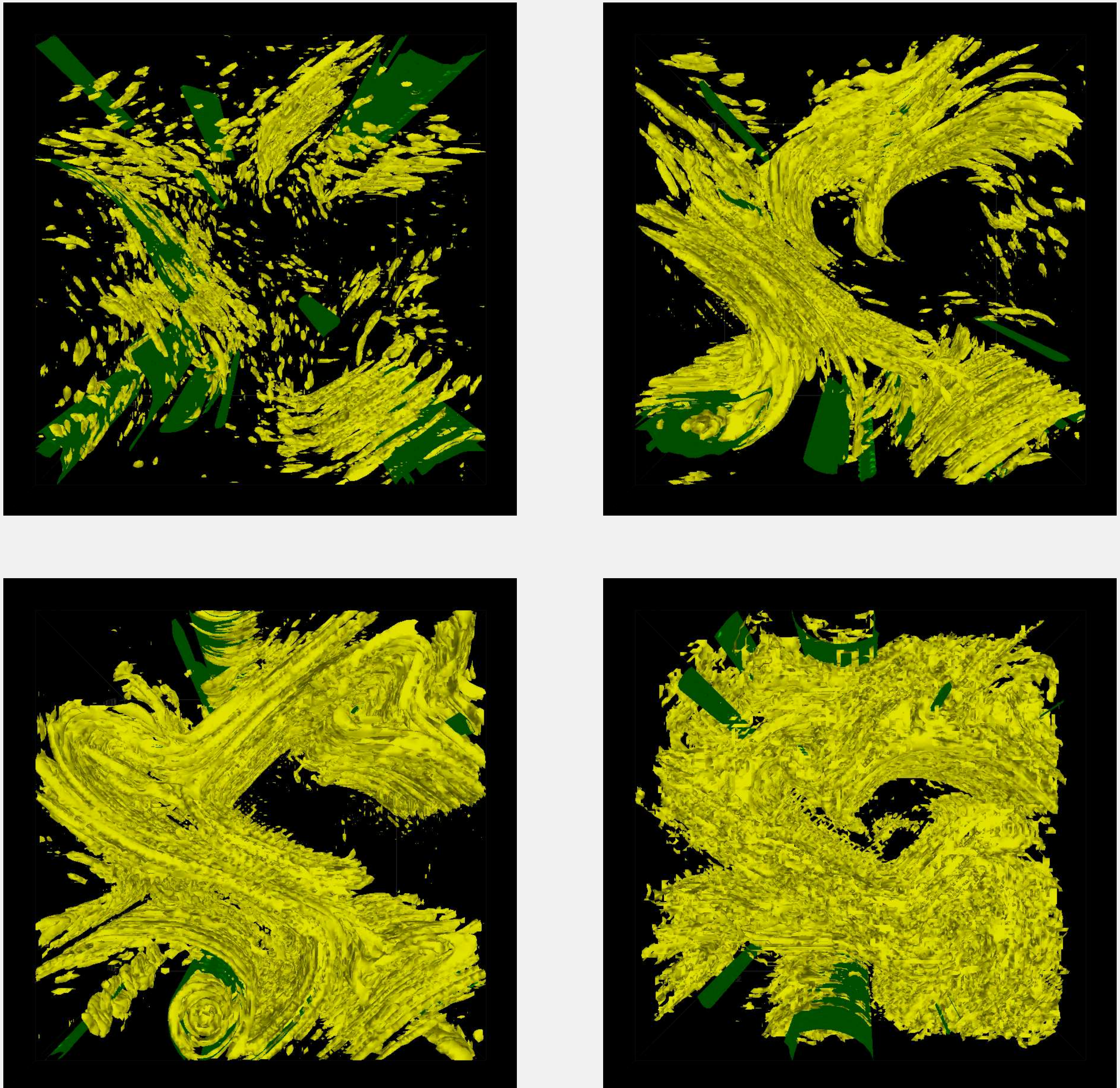
Implications

* Initial growth rate can be predicted from properties of 2-D flow (Lapeyre et al. 2000; Straub 2003).
* Anisotropic interaction between large horizontal scales and small vertical scales.

Real space picture



(top view)

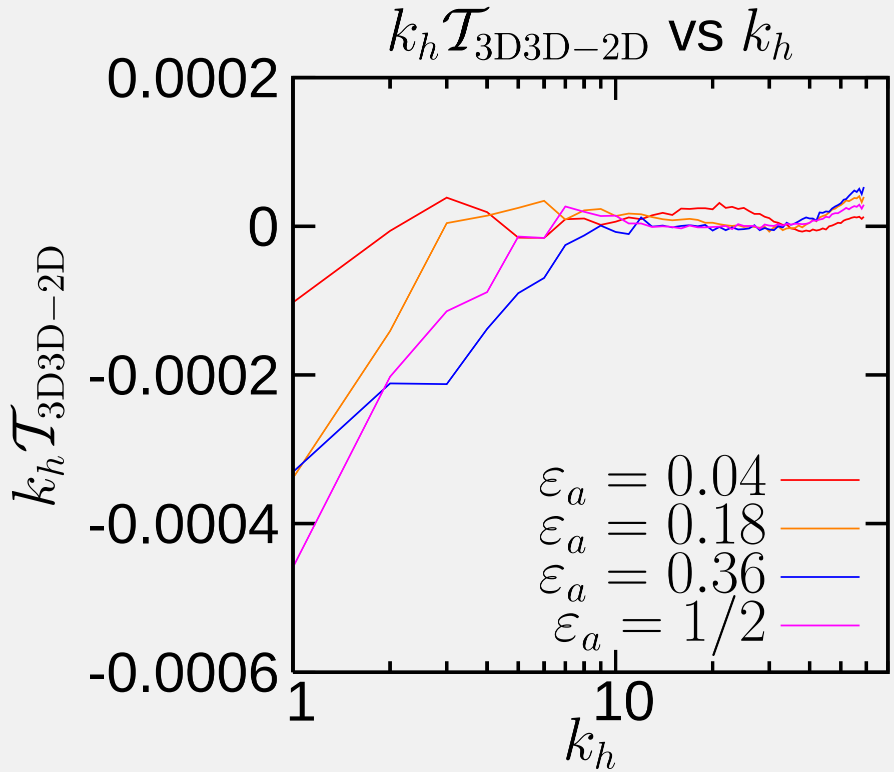
yellow: $|\boldsymbol{\omega}_h|$ isosurfaces

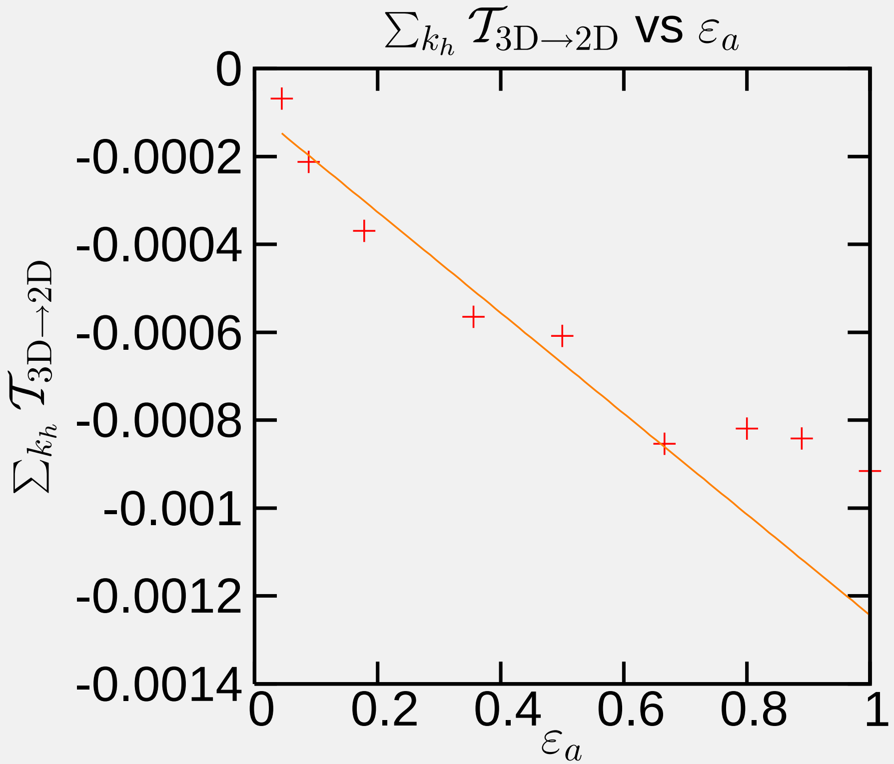
green: ω_z isosurfaces

Random straining by the base flow causes the perturbation vorticity to grow

2D-3D interactions: spectral energy transfers

Perturbation extracts 2D energy at large horizontal scales:

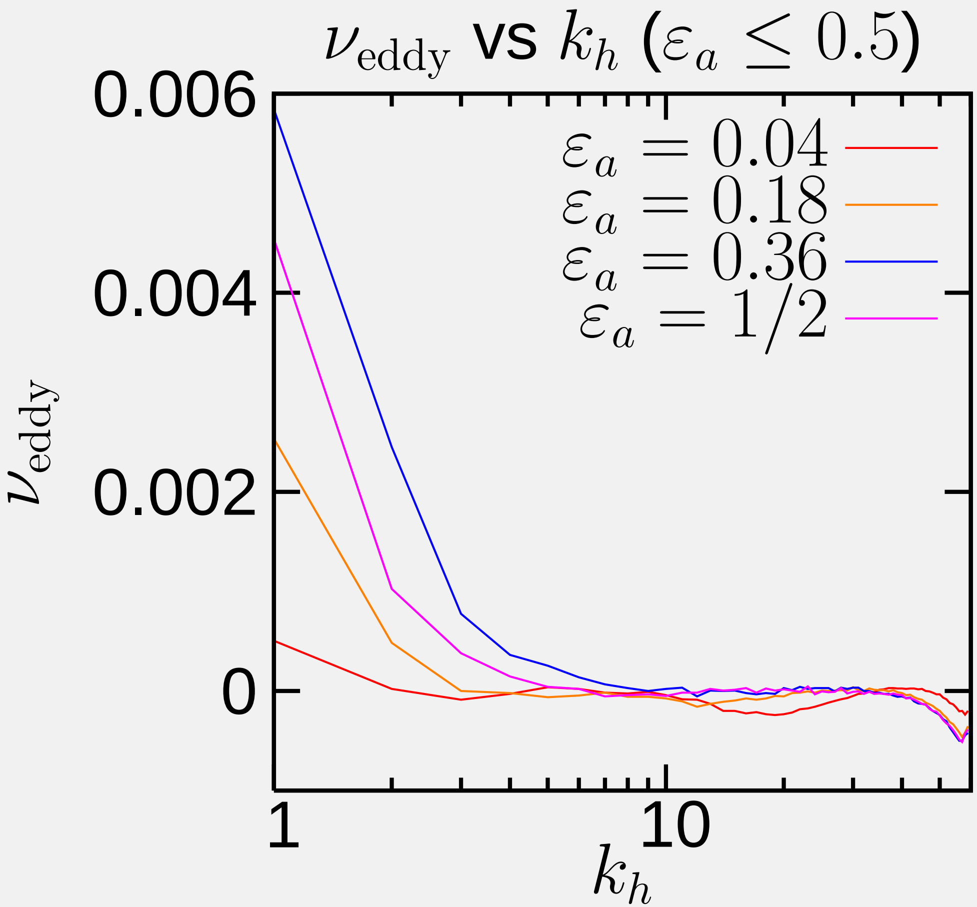




$$\mathcal{T}_{\rm 3D3D-2D}(k_h) = \Re \sum_{|\boldsymbol{k}_h| = k_h} -U_l(\boldsymbol{u}\cdot\boldsymbol{\nabla}\boldsymbol{u})_l^*(\boldsymbol{k})$$

2D-3D interactions: eddy viscosity

Eddy viscosity models effect of perturbation as $\nu_{\text{eddy}} \nabla^2 \boldsymbol{U}$ (e.g. Domaradzki et al. 1993).



$$\nu_{\text{eddy}} = -\sum_{k_h} \frac{-U_l(\boldsymbol{u} \cdot \boldsymbol{\nabla} \boldsymbol{u})_l^*}{k_h^2 E_{2D}}$$

ν_{eddy} parameterises perturbation rather than subgrid-scale processes.

Simple structure implies straightforward parameterisation.

Rotating stratified turbulence

Reference: J. Atmos. Sci., **65**, 766 (2008)

Governing equations for rotating stratified flow

We consider the non-hydrostatic Boussinesq equations for buoyancy fluctuations about a mean profile:

$$\begin{aligned}
\frac{\partial \boldsymbol{u}}{\partial t} + \boldsymbol{u}\cdot\boldsymbol{\nabla}\boldsymbol{u} + 2\boldsymbol{\Omega}\times\boldsymbol{u} &= -\frac{1}{\rho_0}\boldsymbol{\nabla} p + b + \nu\mathcal{D}(\boldsymbol{u}) \\
\frac{\partial b}{\partial t} + \boldsymbol{u}\cdot\boldsymbol{\nabla} b &= -N^2 w + \nu\mathcal{D}(b) \\
\boldsymbol{\nabla}\cdot\boldsymbol{u} &= 0,
\end{aligned}$$

where

$b := g\theta'/\theta_0$ is the (perturbation) buoyancy

$\theta = \theta_0 + (d\bar{\theta}/dz)z + \theta'$ is the potential temperature

$N = (\frac{g}{\theta_0}\frac{d\bar{\theta}}{dz})^{\frac{1}{2}}$ is the Brunt-Vaisala frequency

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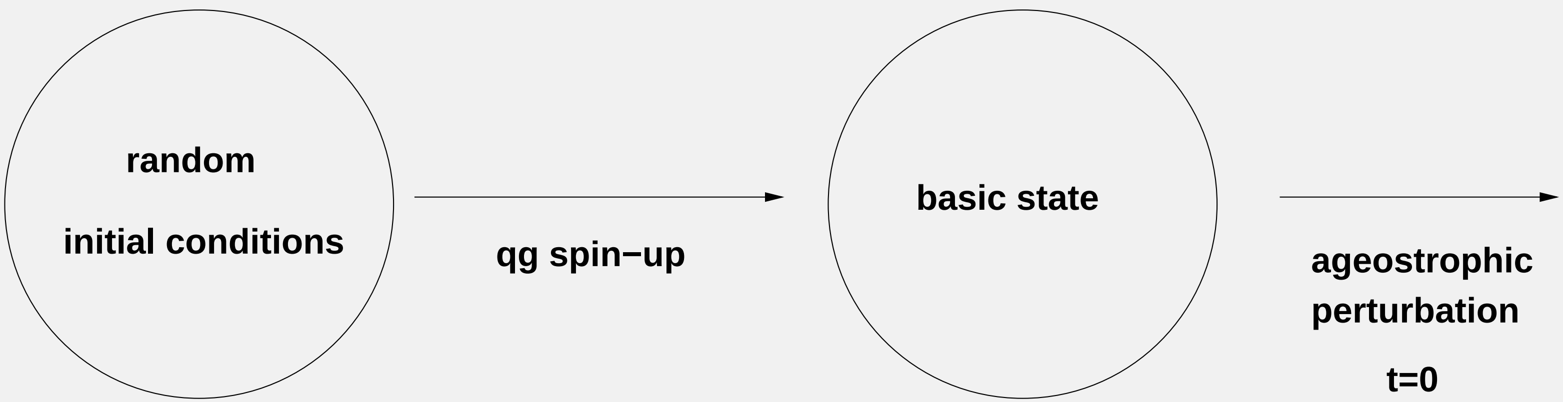


Geostrophic-ageostrophic interactions

Geostrophic (balanced) and ageostrophic (unbalanced) motion are defined using normal modes (Bartello 1995).

Defining Rossby and Froude numbers as $Ro := \frac{U}{fL}$, $Fr := \frac{U}{NH}$, one expects analogous results for:

* *Synoptic-scale flow* (rotation dominated)
  $L_0 > L_d$, where $L_d = (Ro/Fr)L_o$ is the *deformation radius*

* *Weak stratification* (gravity waves negligible)
  $H_0 < H_{\mathrm{b}}$ where $H_{\mathrm{b}} = U/N$ is the *buoyancy scale*

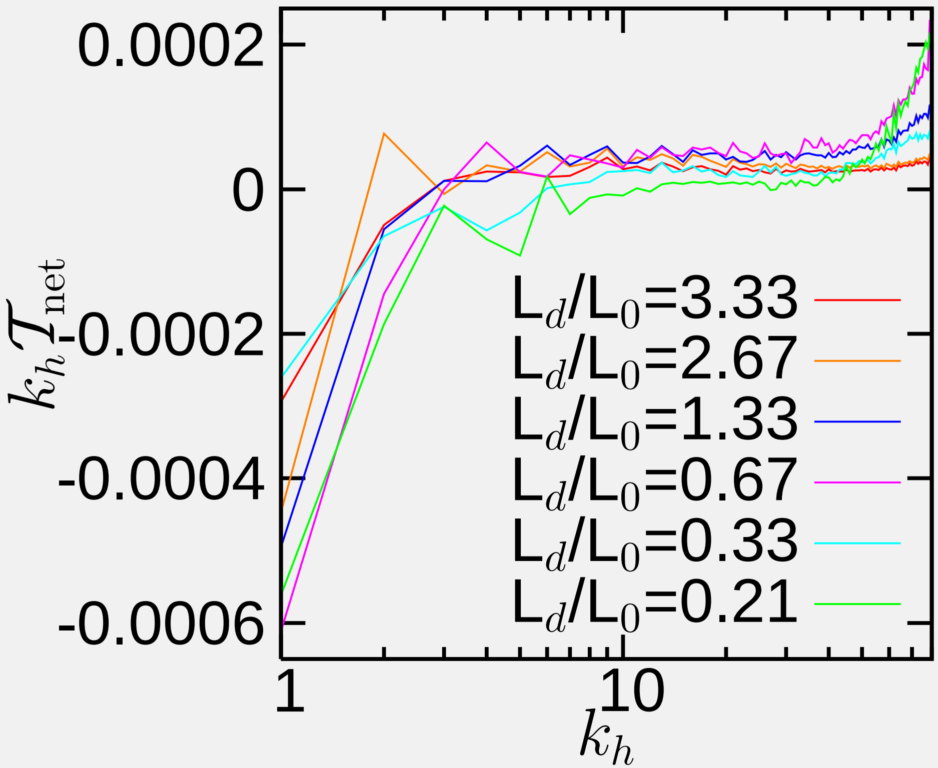
i.e. instability of ageostrophic modes due to random straining by geostrophic flow. Particularly interested in GAG interaction.

$\Longrightarrow$ What happens more generally?

Spectral energy transfers

Perturbation extracts geostrophic energy at large horizontal scales:

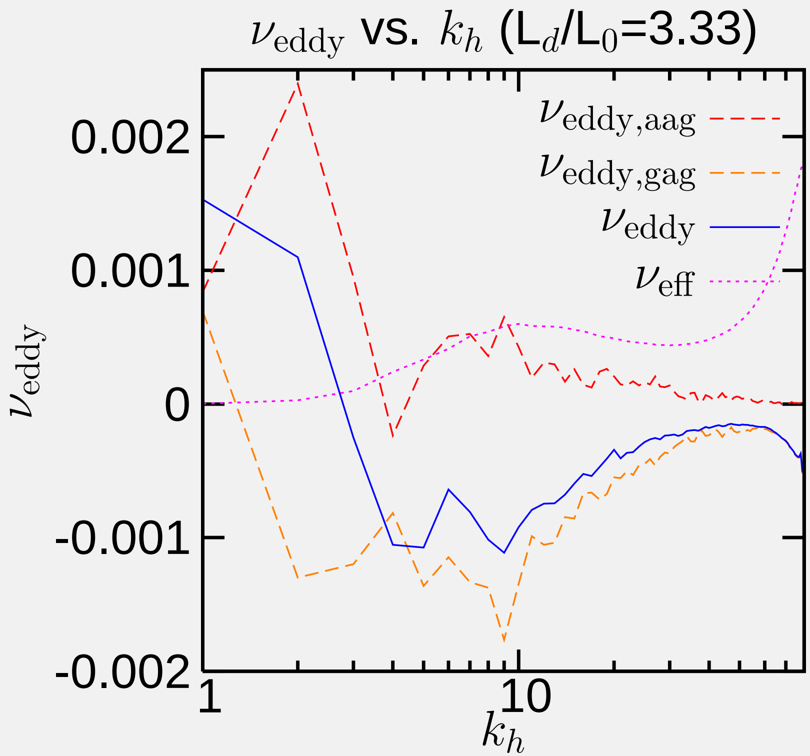
$k_h \mathcal{T}_{\rm net}$ vs k_h (strong strat, $\varepsilon_a < 1$)

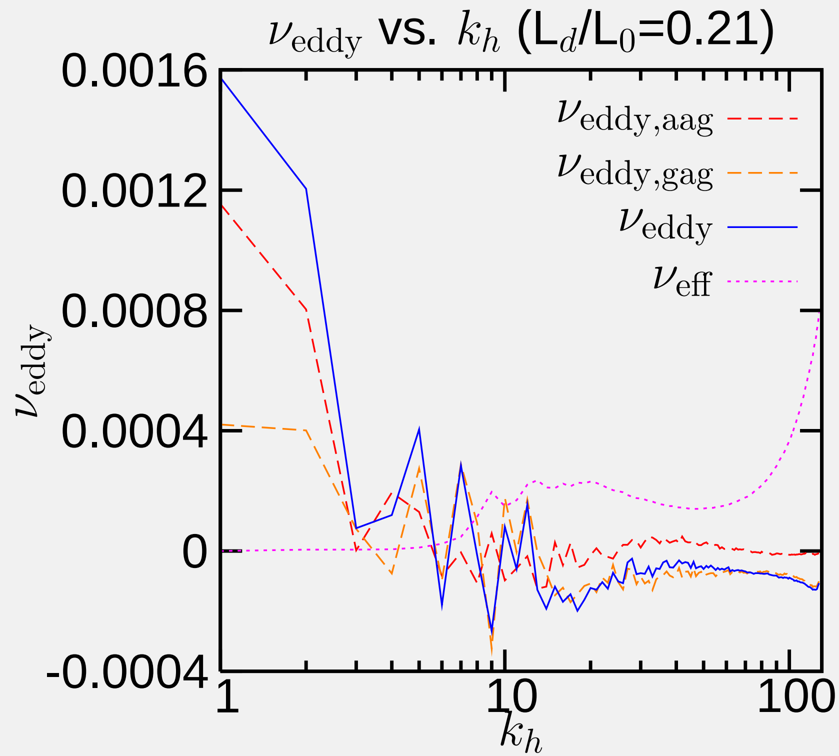


$$\mathcal{T}_{\rm net} := \mathcal{T}_{\rm GA-G} + \mathcal{T}_{\rm AA-G}$$

Eddy viscosity

For synoptic flow, there is preferential damping of large-scale geostrophic modes:





Nature of the “wave drag” depends on the structure of the basic state.

Summary

Balanced-unbalanced interactions in rotating stratified turbulence can be analogous to 2D-3D interactions in homogeneous turbulence.

Requirements:

* synoptic flow ($L_0 > L_d$)
* adequate resolution of small-scale modes ($H_\mathrm{b} > \Delta z$)

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- synoptic flow ($L_0 > L_d$)
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Implications

- Anisotropy needs to be taken into account; vertical resolution is crucial.
- Multiscale parameterisation may be useful.
- Classical picture of predictability may need to be updated.