Spherical Shallow Water Turbulence: Cyclone-Anticyclone Asymmetry, Potential Vorticity Homogenisation and Jet Formation

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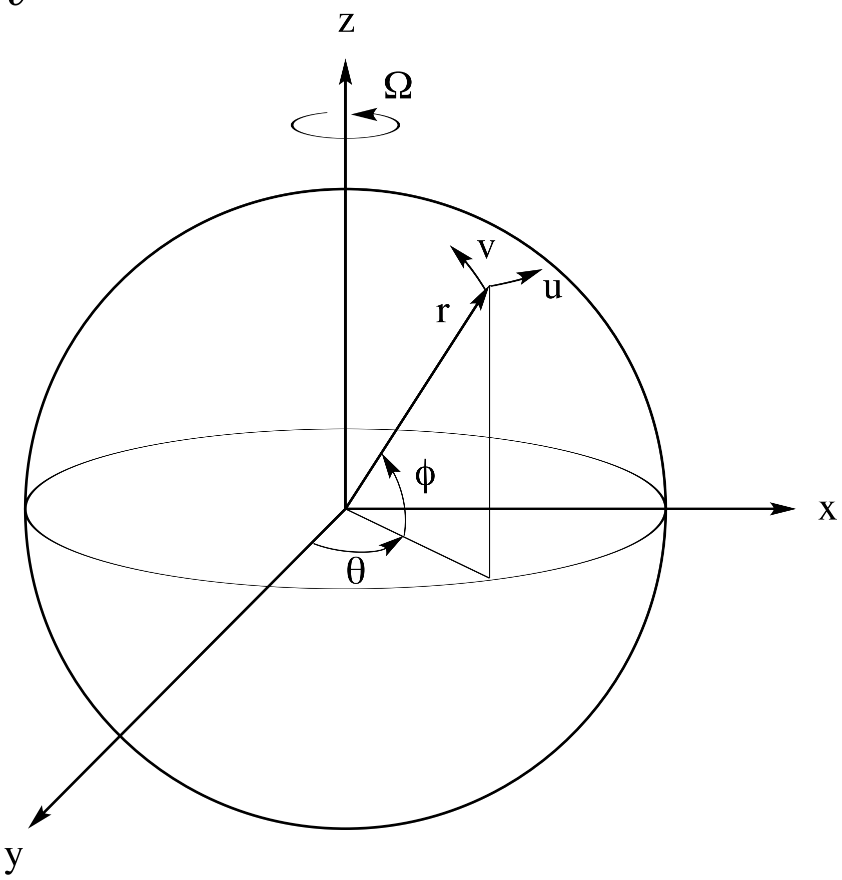
Outline

- Shallow water turbulence
- Contour advective semi-Lagrangian algorithm
- Turbulence simulations
- Cyclone-anticyclone asymmetry
- Potential vorticity (PV) homogenisation and jet formation

Shallow Water Equations

$$\begin{aligned} \frac{\mathrm{D}\boldsymbol{u}}{\mathrm{D}t} + f\boldsymbol{k} \times \boldsymbol{u} &= -g\boldsymbol{\nabla} h \\ \frac{\partial h}{\partial t} + \boldsymbol{\nabla} \cdot (h\boldsymbol{u}) &= 0 \end{aligned}$$



h : fluid depth

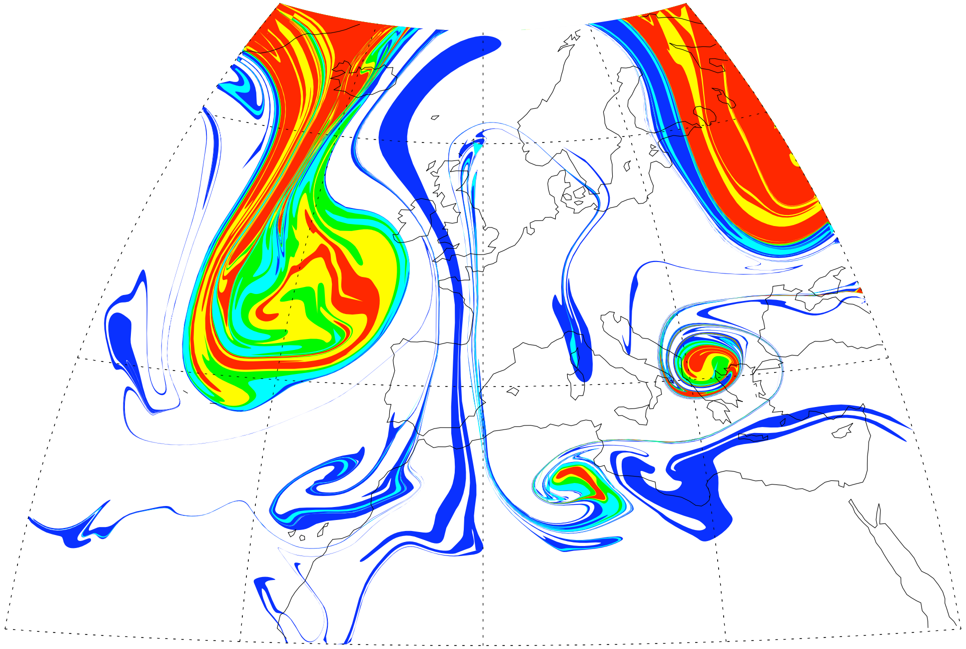
$\boldsymbol{u}$: horizontal velocity

$\boldsymbol{k}$: local vertical

$$f = 2\Omega \sin \phi$$

Shallow Water Equations

- Slow modes *and* fast modes
- Balanced flow
- Potential vorticity
  - materially conserved
  - develops fine scales
  - homogeneous regions separated by sharp gradients



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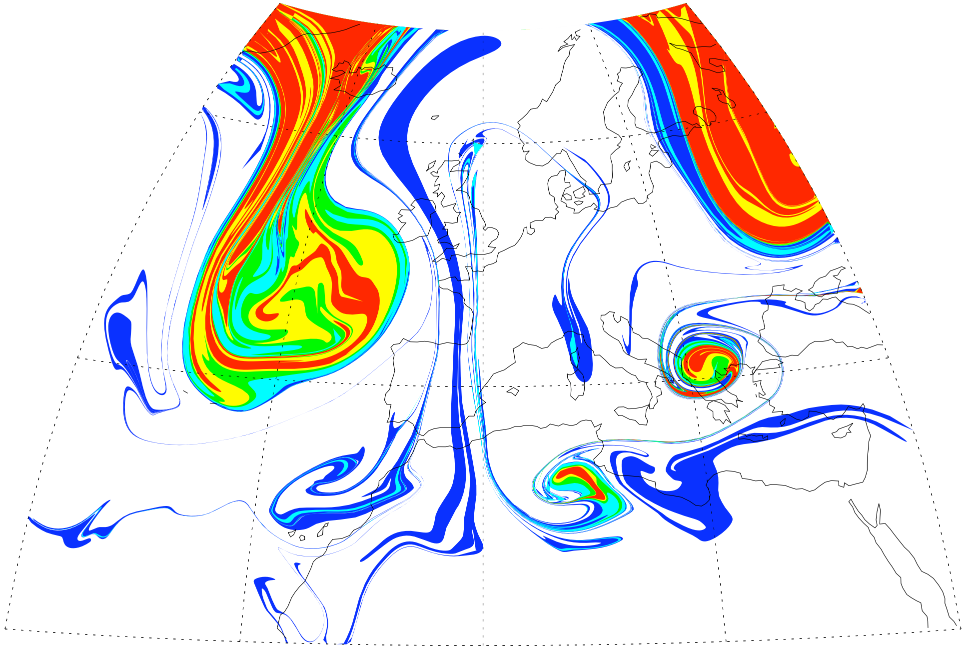
Motivates choice of variables



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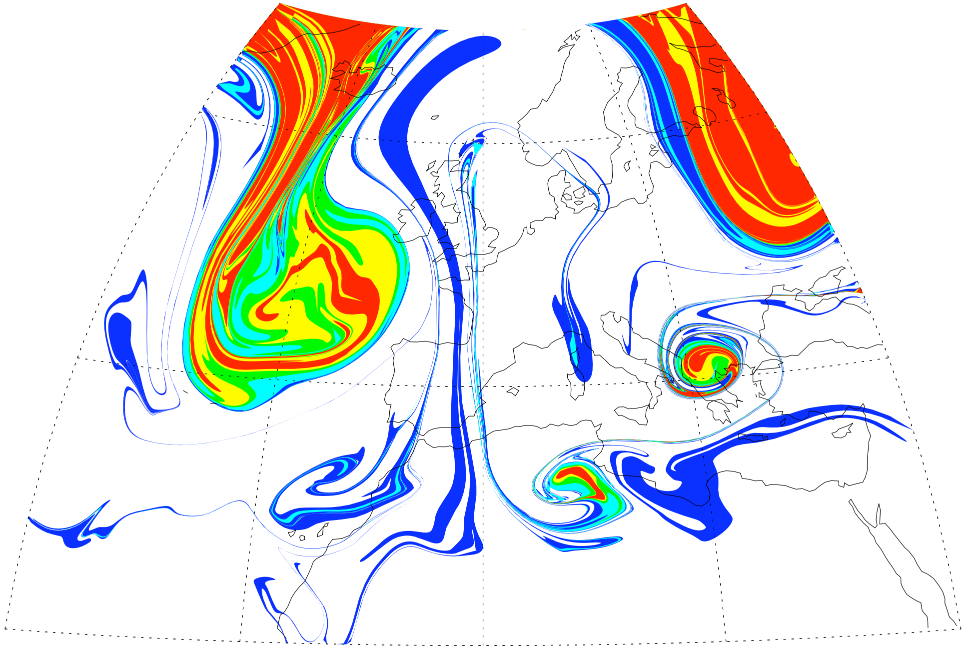
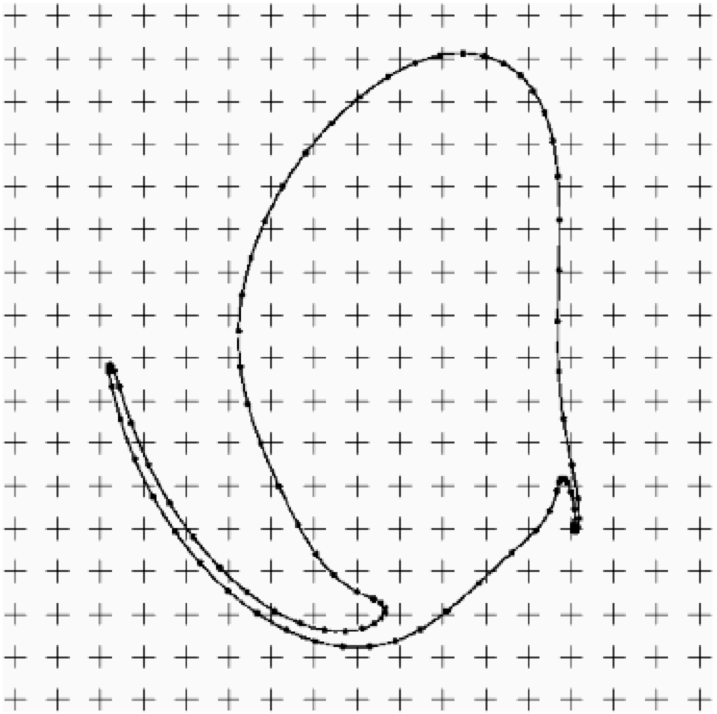
Motivates contour representation for PV



Potential Vorticity Conservation

$$\frac{D}{Dt}\left(\frac{\zeta + f}{h}\right) \equiv \frac{\mathrm{D}\Pi}{\mathrm{D}t} = 0$$

Motivates contour representation for PV



Dritschel and Ambaum (1997)

Dritschel, Polvani and Mohebalhojeh (1999)

Choice of Variables

- PV controls balanced motion
- Choose other two variables that represent (to leading order) unbalanced motion
  - vanish in the limit of vanishing Fr and Ro
  - hierarchy of such variables:

$$\delta = \nabla \cdot \boldsymbol{u} \qquad \begin{aligned} \gamma &= \nabla \cdot \boldsymbol{a} \\ &= \nabla \cdot \frac{D\boldsymbol{u}}{Dt} \end{aligned}$$

    and their time derivatives
  - see Mohebalhojeh and Dritschel 2000

Turbulence Simulations

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1. generate random, iostropic perturbation with defined length scale and PV anomaly

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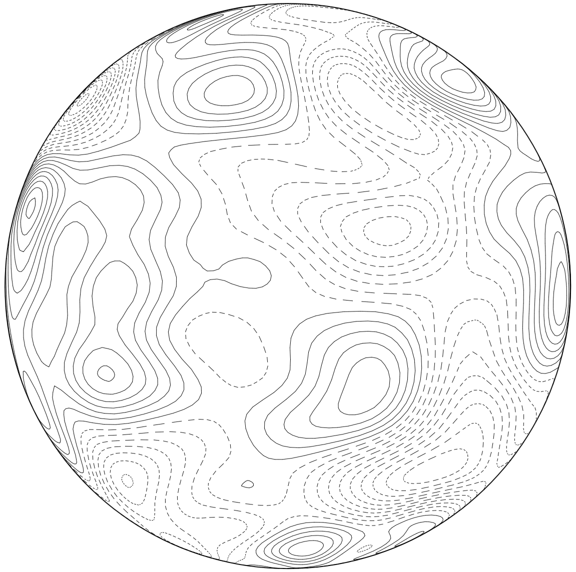

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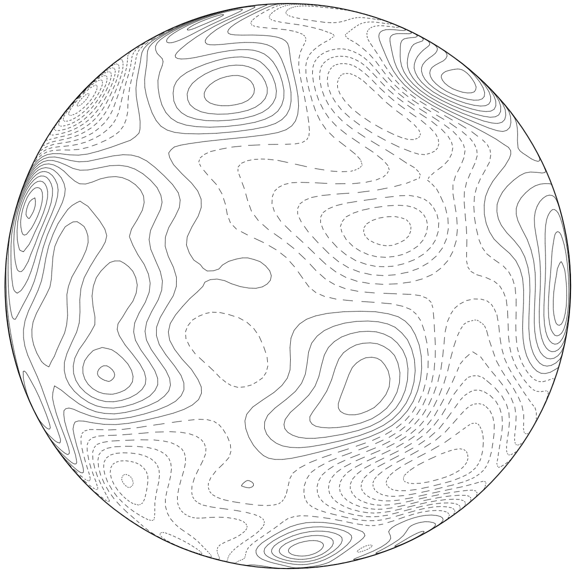
1. generate random, iostropic perturbation with defined length scale and PV anomaly
2. add this to zonal PV distribution



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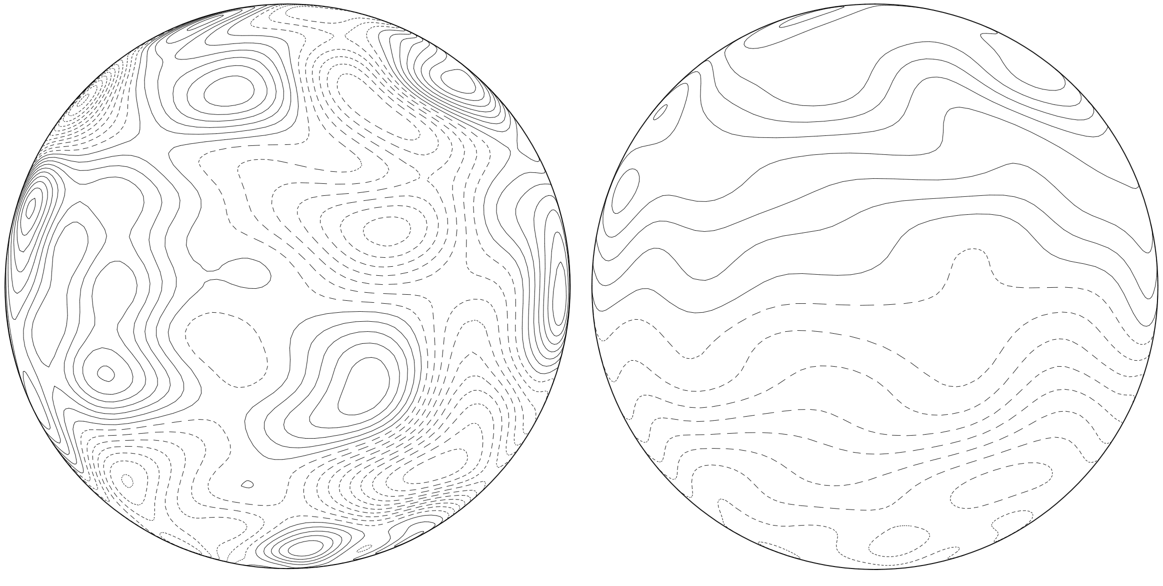

Turbulence Simulations

1. generate random, iostropic perturbation with defined length scale and PV anomaly
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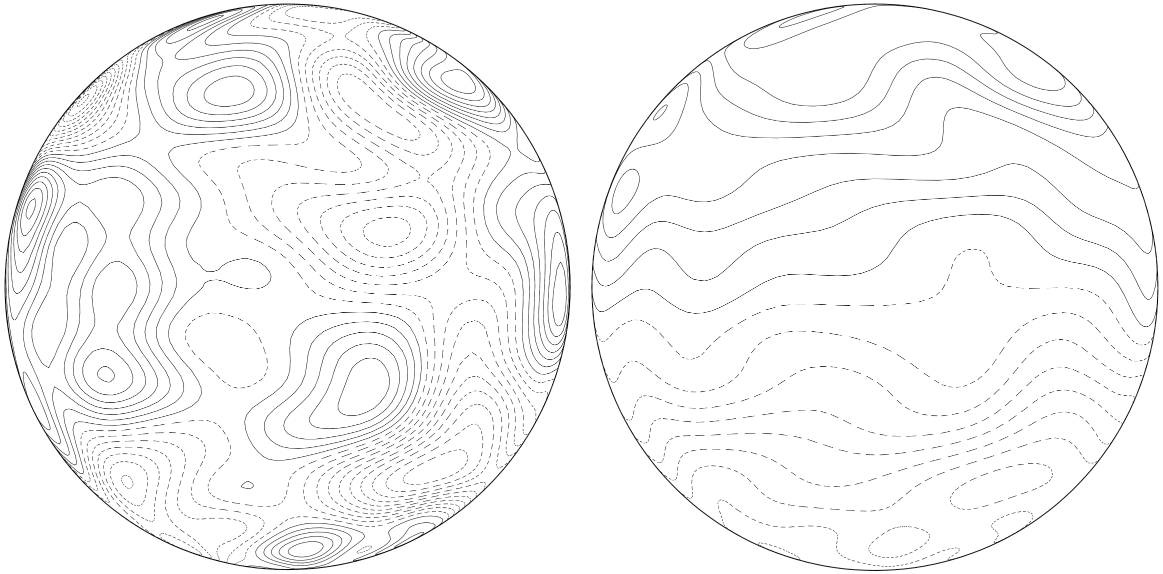
3. ramp PV up from zero amplitude by multiplying by a smooth ramp function, allowing contours to deform



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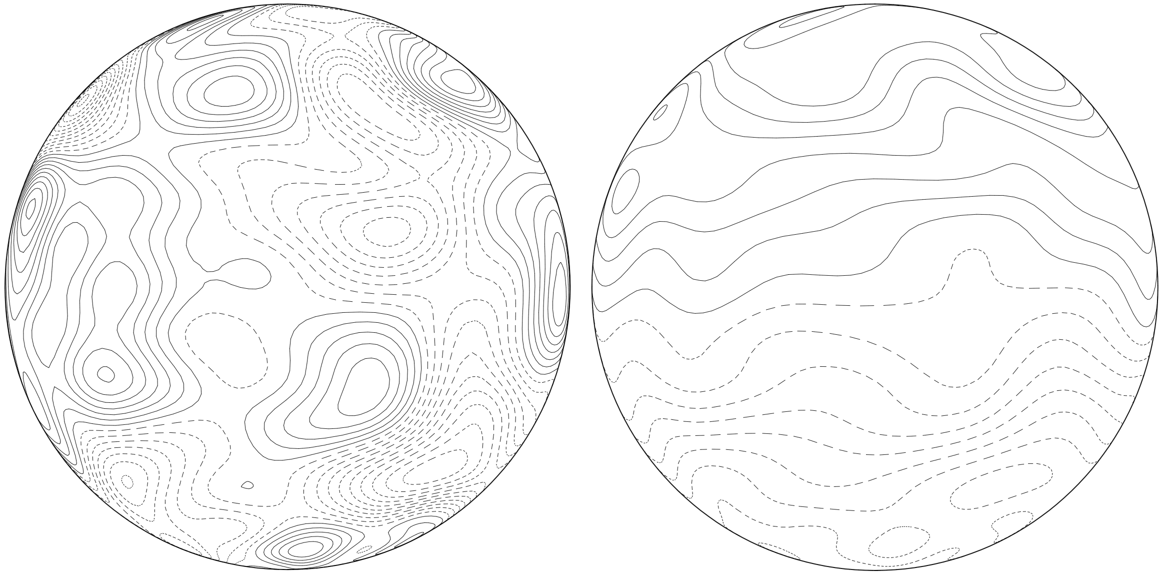
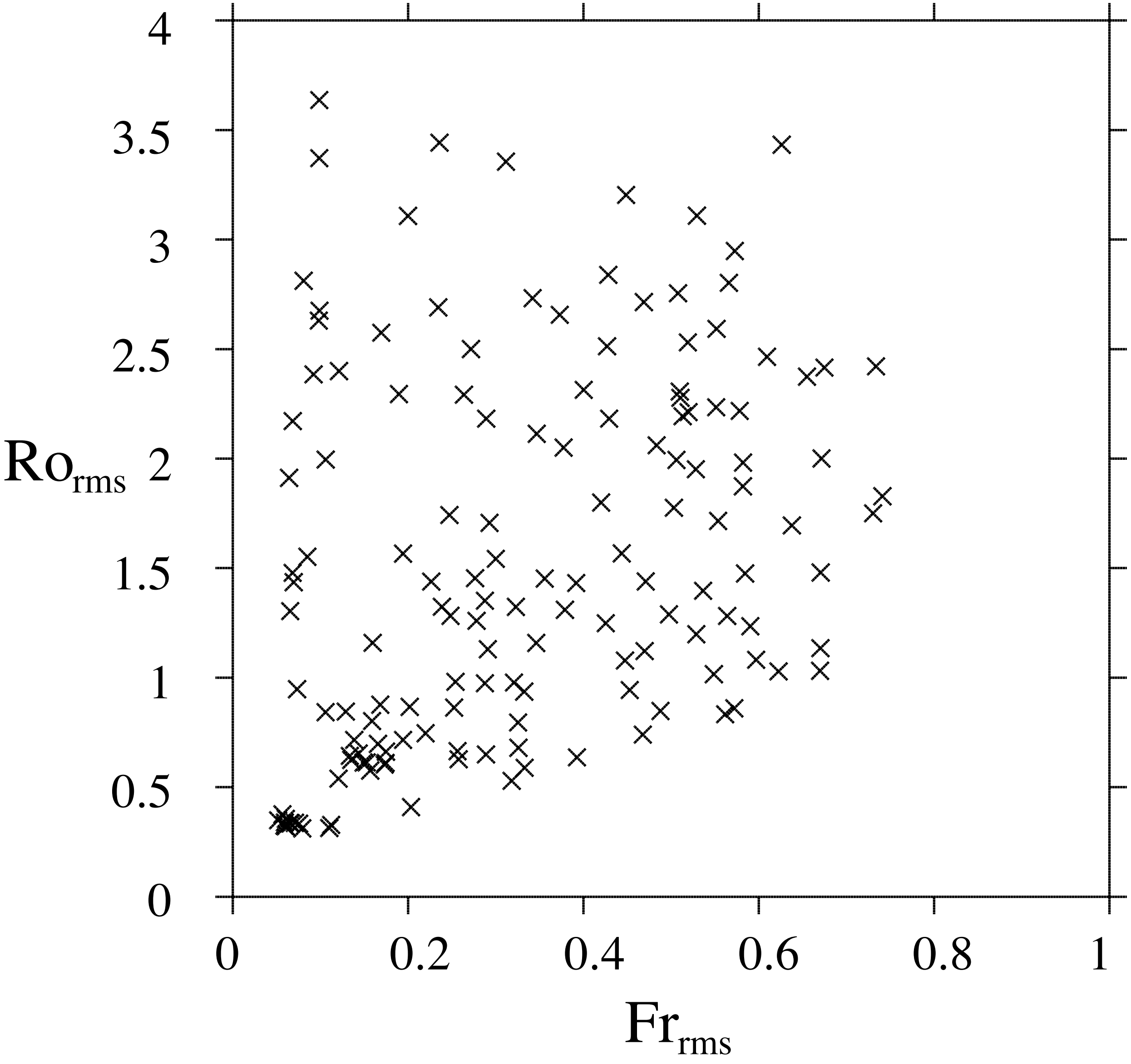


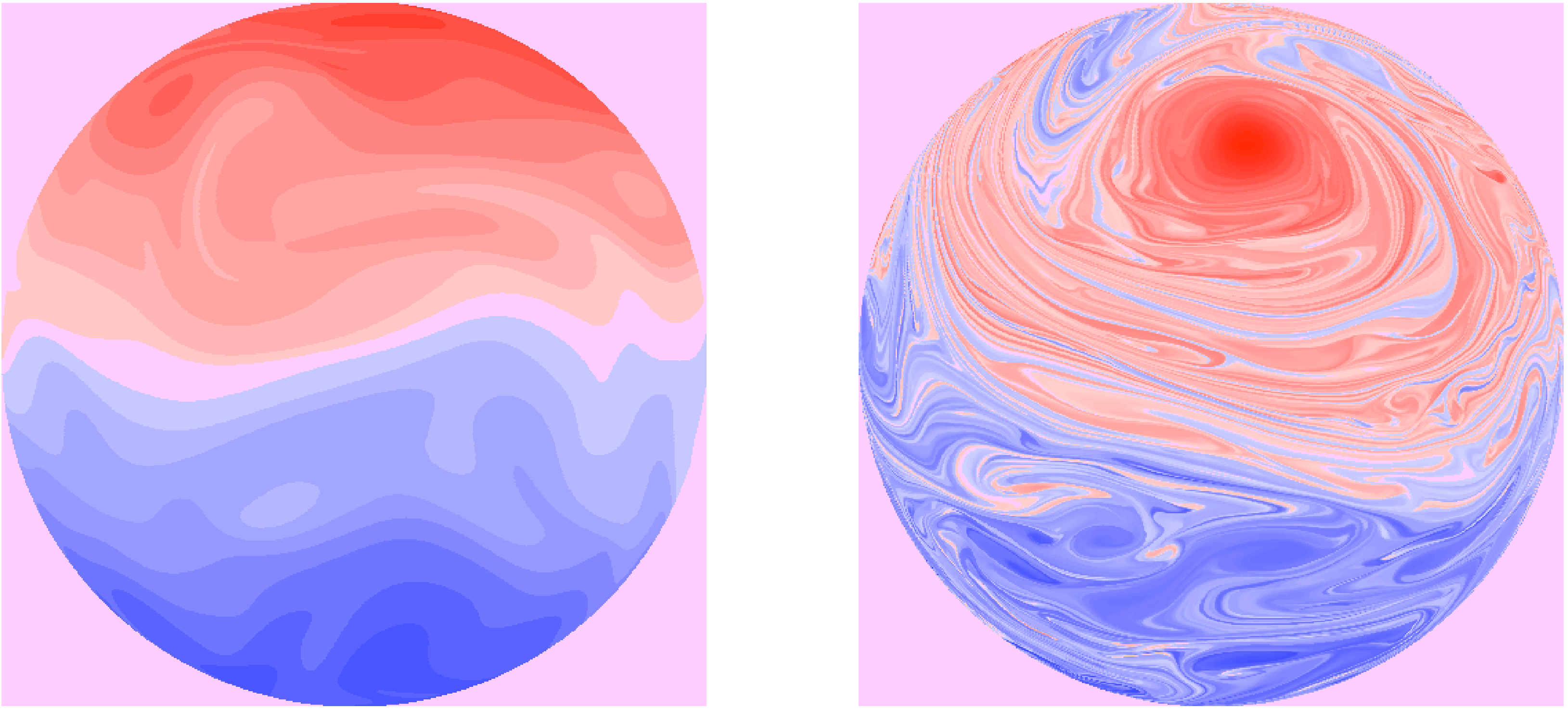
Parameter Space



$$\mathrm{Fr} = \frac{U}{\sqrt{g\tilde{h}}}$$

$$\mathrm{Ro} = \frac{U}{fL}$$

Turbulence Simulations

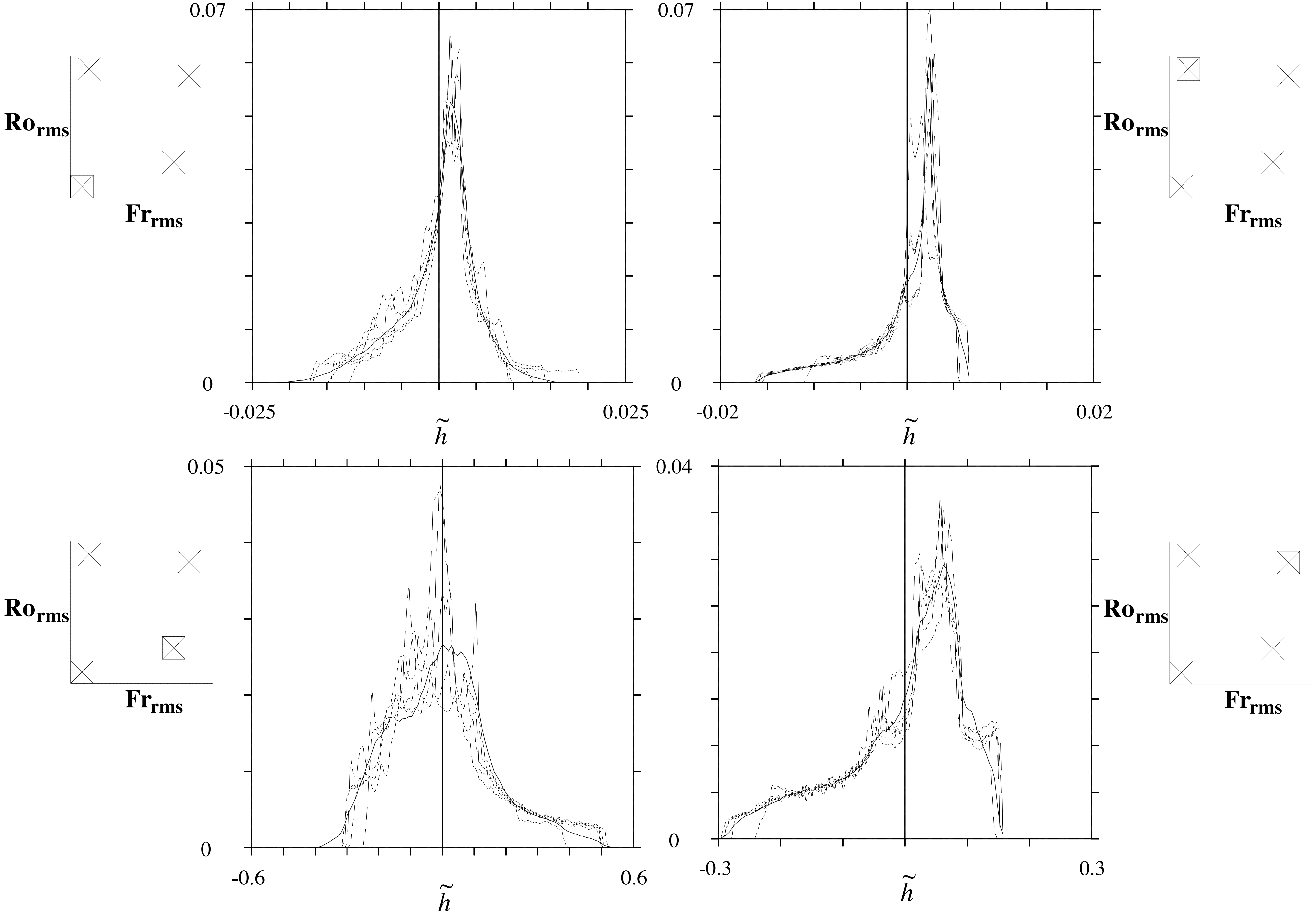


low Fr, low Ro

high Fr, high Ro

PV field: red indicates positive, blue indicates negative

Cyclone-Anticyclone Asymmetry



Cyclone-Anticyclone Asymmetry

- asymmetry generally favours anticyclones
- distribution increasingly asymmetric with increasing Froude and Rossby numbers
- significant tail of strong cyclones in most cases

In general: there is a greater area of anticyclonic vorticity but cyclones are more extreme.

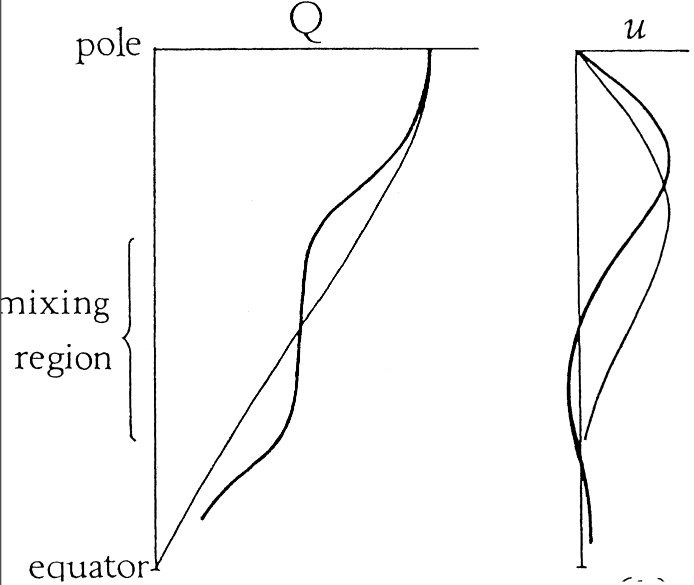
Potential Vorticity Homogenisation

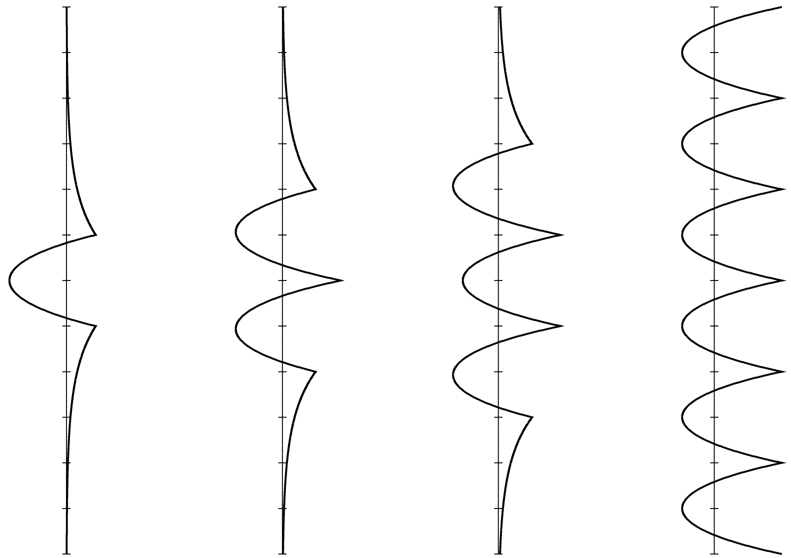
t = 0 t = 5 t = 10

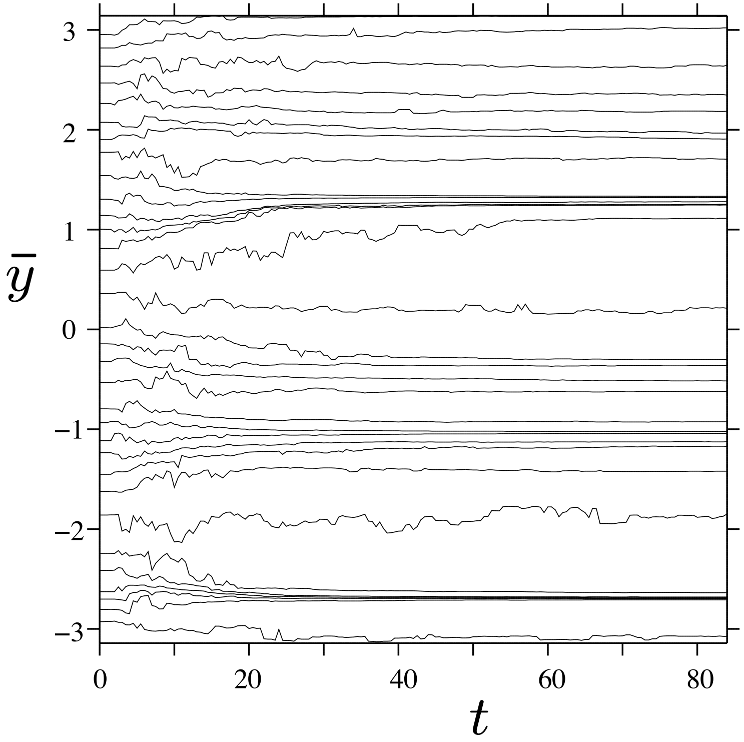
low Fr, low Ro

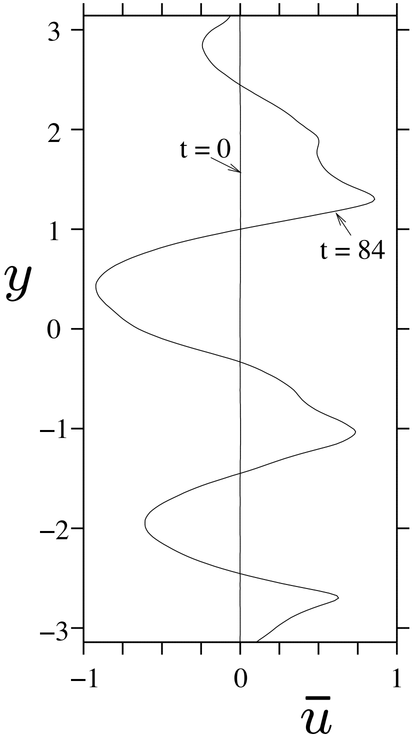
high Fr, high Ro

Potential Vorticity Homogenisation





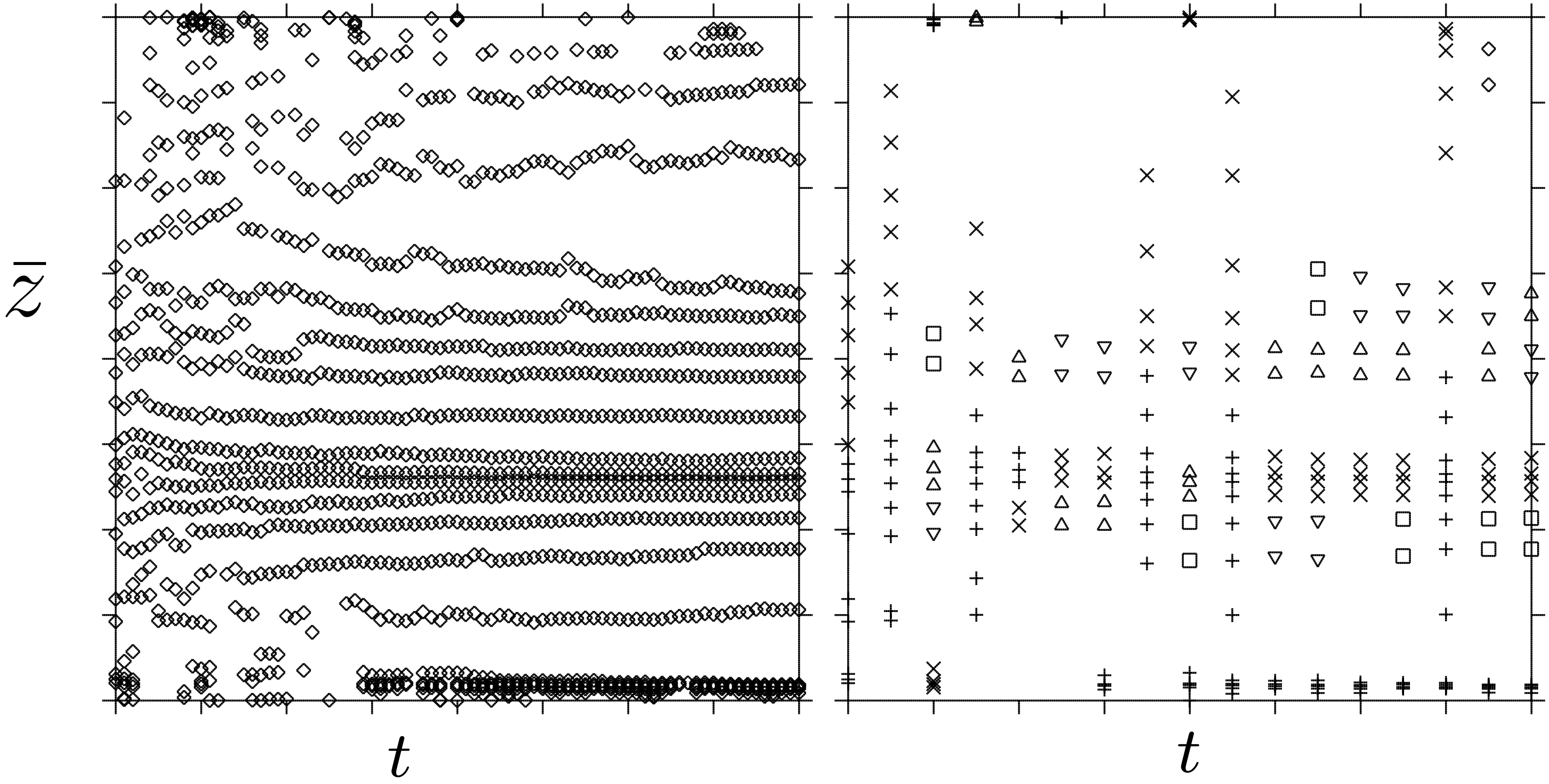




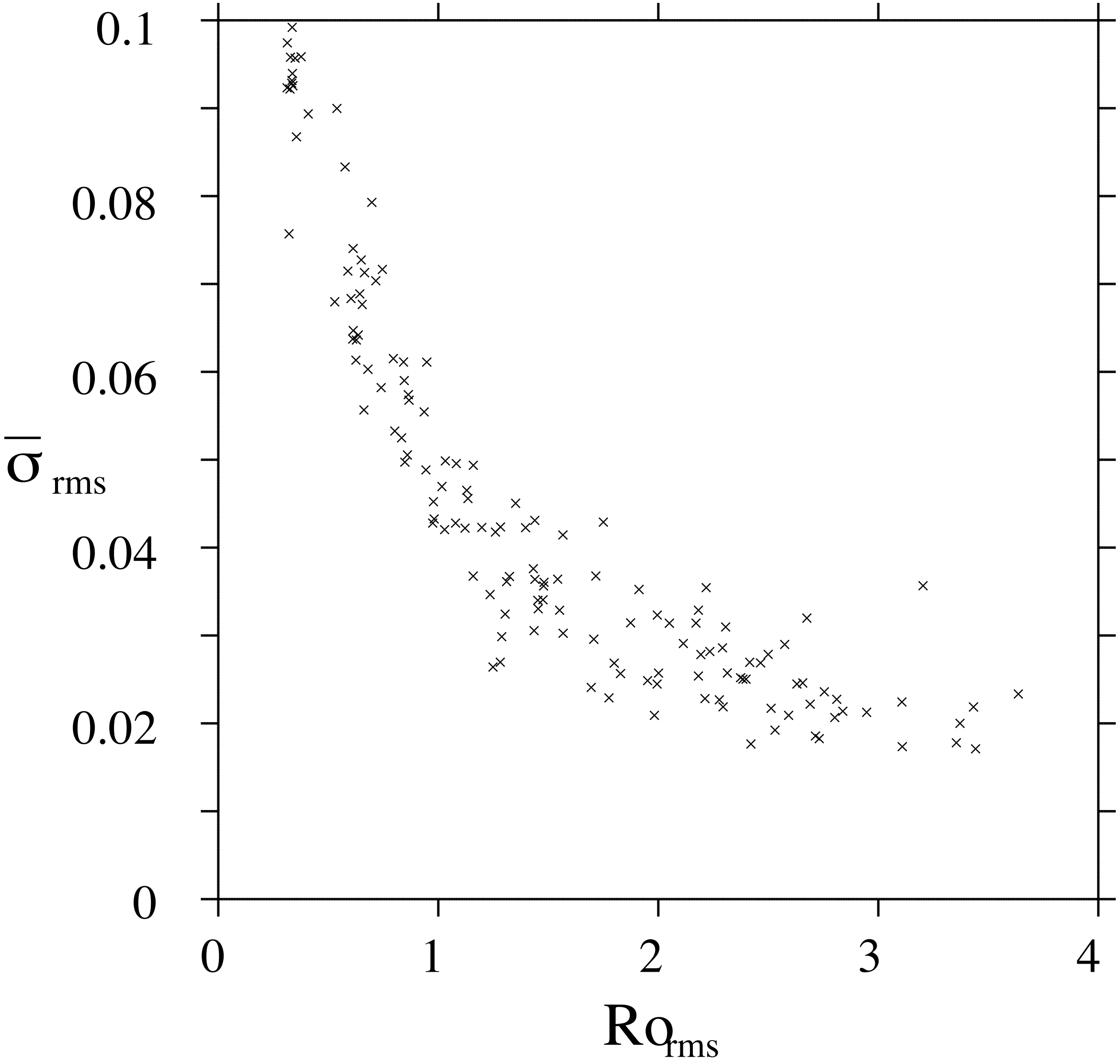
Multiple jets as PV staircases: the Phillips effect and the resilience of eddy-transport barriers.
Dritschel and McIntyre
J. Atmos. Sci. special issue from AGU Chapman conference "Jets and annular structures in geophysical fluids.

Potential Vorticity Homogenisation

- Calculate mean latitude of all PV contours that wrap the pole
- At each time, perform a cluster analysis on these positions



Potential Vorticity Homogenisation



- Low value of $\overline{\sigma}_{\mathrm{rms}}$ indicates better clusters
- Suggests that increasing importance of rotation (decreasing Rossby number) inhibits clustering
  - contrary to predictions based on Rhines scale
  - similar results to Cho & Polvani 1996
  - importance of forcing and dissipation

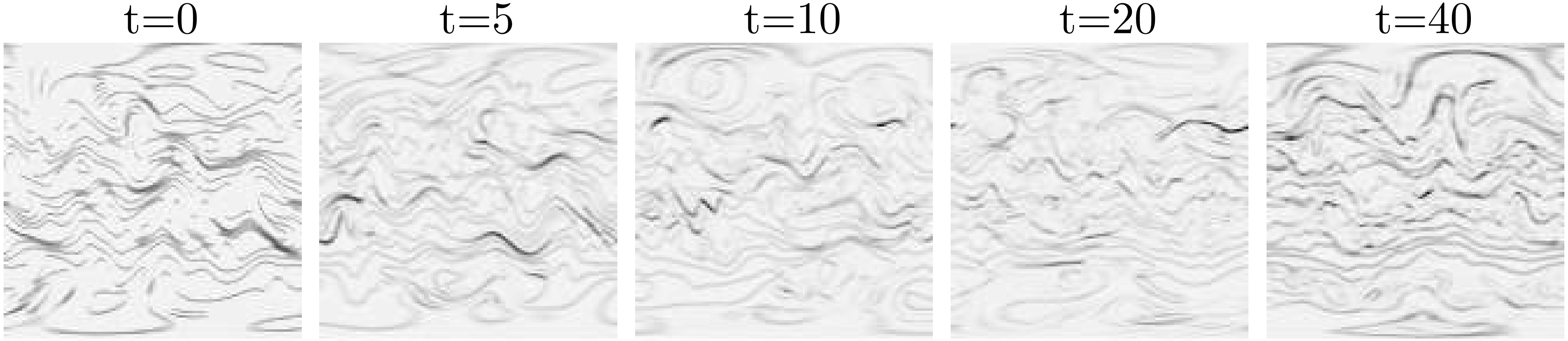
Cluster Analysis Issues

- input parameters for clustering algorithms
- constraint that PV contours must wrap sphere
  - precession of polar vortices
- latitudinal averaging
  - meandering of jets

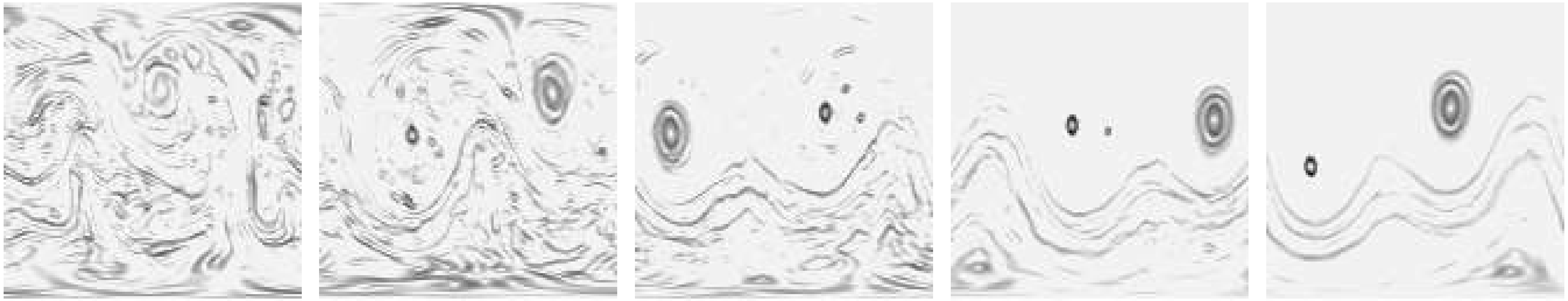
Complex structure of jets defies simple classification - instead a more local examination is required.

Palinstrophy

$P = \frac{1}{2}|\nabla q|^2$ measures strength of gradient of PV



low Fr, low Ro



high Fr, high Ro

Conclusions

- Explored new ways of looking at cyclone-anticyclone asymmetry and jet formation.
- Cyclone-anticyclone asymmetry
  - favours anticyclones
  - asymmetry increases with both Froude and Rossby number
  - significant tail of extreme cyclones
- Jet formation
  - cluster analysis misses complexity of jets
  - palinstrophy field reveals jet structure