A Baroclinic Model for the Atmospheric Energy Spectrum

Ross Tulloch & Shafer Smith
CAOS/CIMS/NYU

NCAR TOY 2008

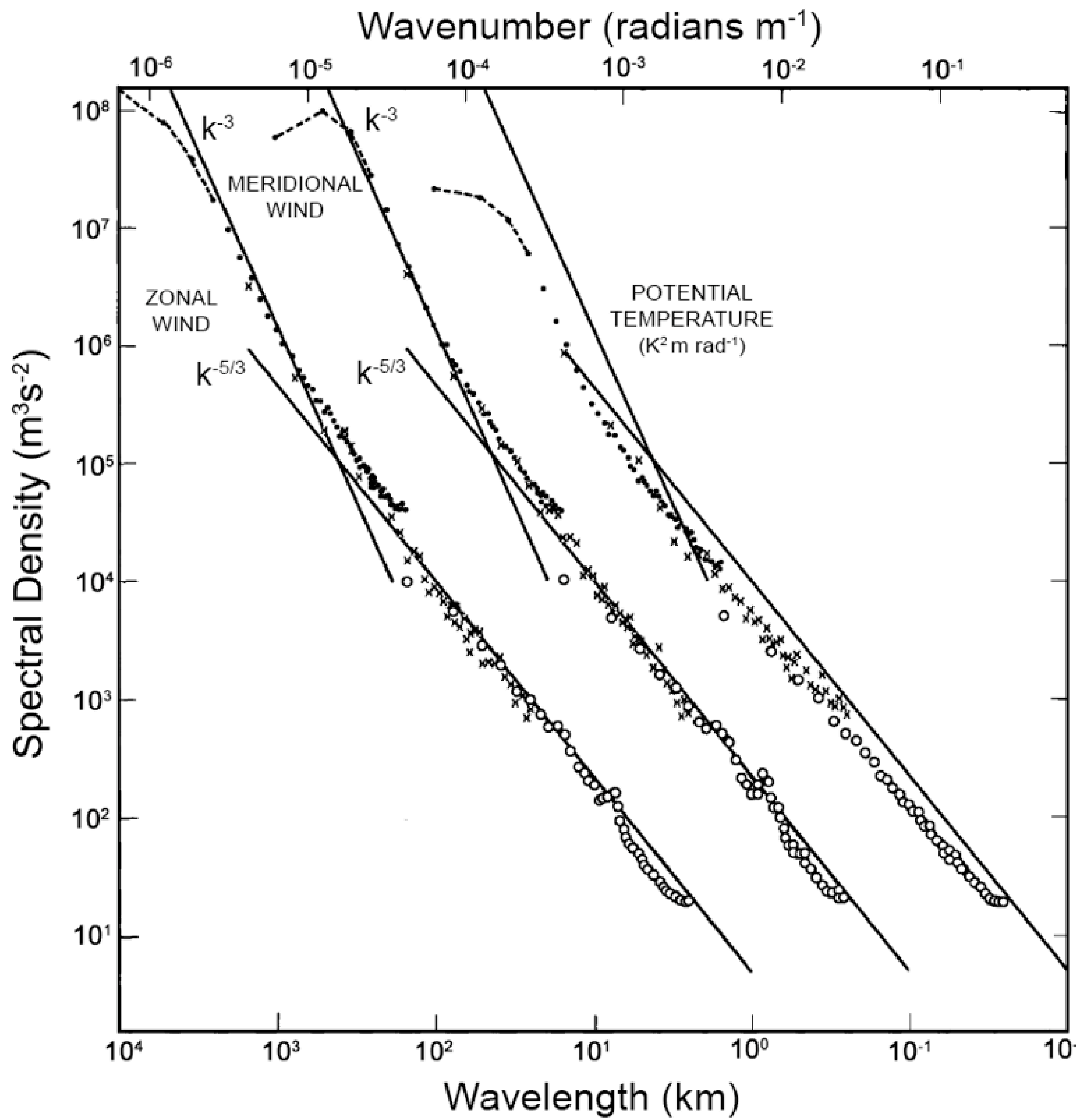
Mesoscale spectra

- In the atmosphere and ocean, the mesoscales are more energetic than balanced theory suggests
- Question: Where does that energy come from? Smaller scales, cascading up? Local instabilities? Larger scales, cascading down?
- Most likely all contribute to some degree

Question addressed here: to what extent can such spectra be generated from balanced large scale flow?

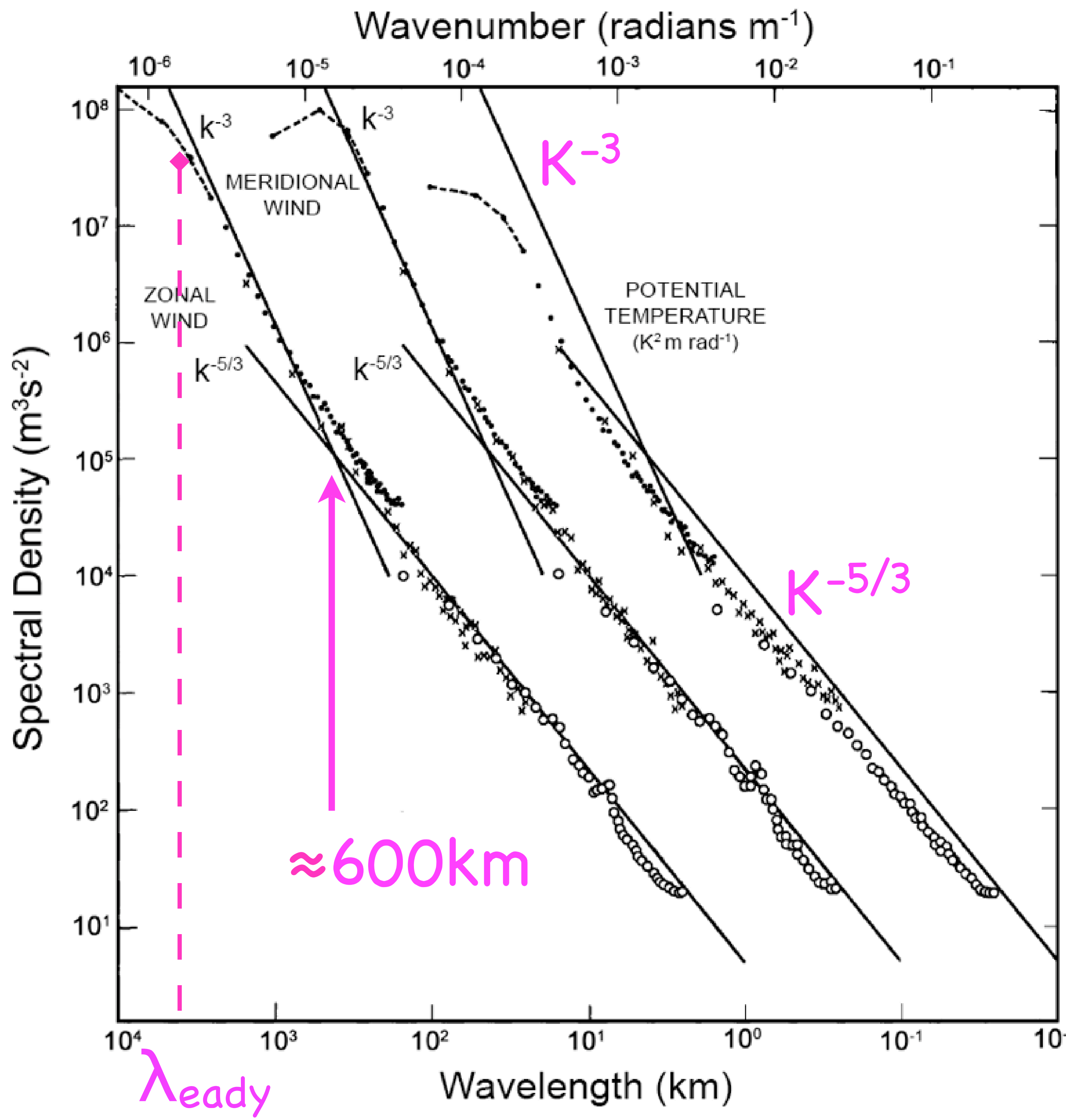
Nastrom & Gage (1985) Spectra

- global dataset collected by commercial 747's near the tropopause, mainly from 30°N to 60°N

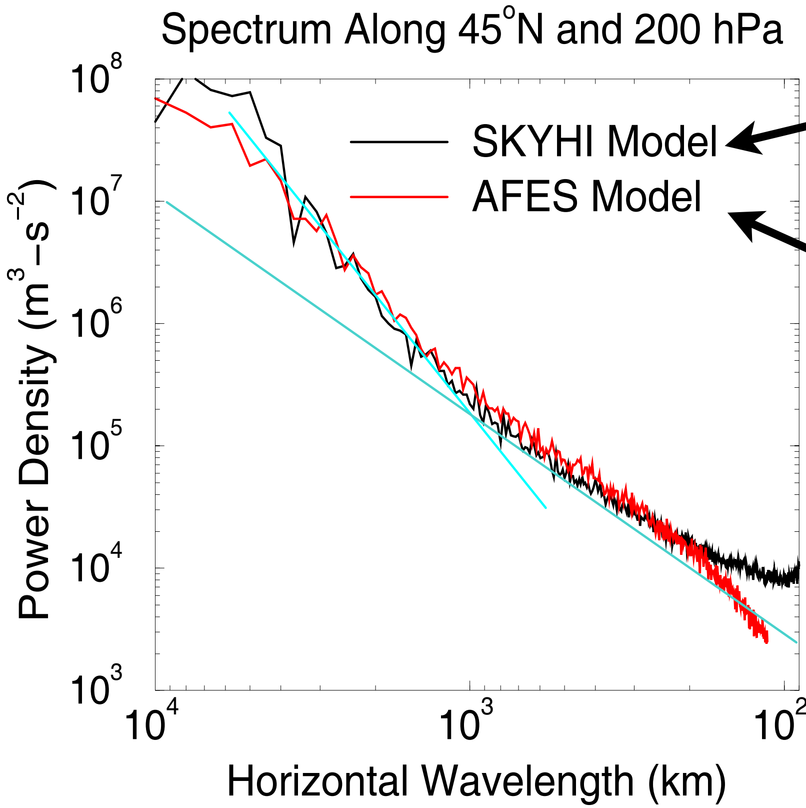


Nastrom & Gage (1985) Spectra

- global dataset collected by commercial 747's near the tropopause, mainly from 30°N to 60°N
- spectra of KE and θ variance have the same universal shape
- transition from K^{-3} slope to $K^{-5/3}$ near 600km
- some variation with latitude and season
- more recent MOZAIC observations give a consistent picture Cho & Lindborg (2001)

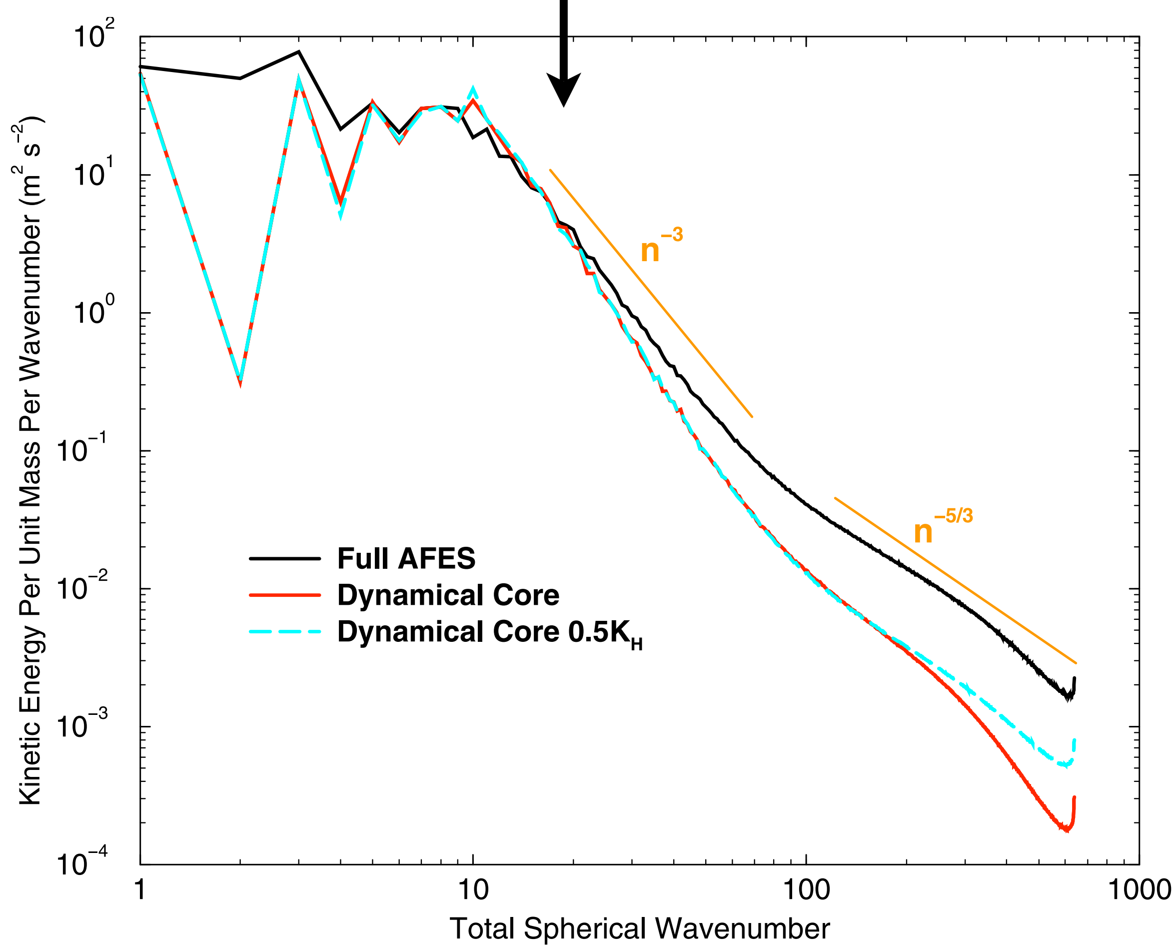


GCM simulations



Gridpoint N270–Koshyk and Hamilton (2001)

Spectral T639 – Takahashi et al. (2006)



- requires high horizontal and vertical resolution
- used hyperdiffusion
- convection scheme not necessary for transition

Review of geostrophic turbulence

- Charney (1971) noted that quasigeostrohpic flow with constant stratification is isomorphic to two-dimensional turbulence when boundaries are neglected:

  conserves $E = -\overline{\psi q}$ and $Z = \overline{q^2}$ where $q = \tilde{\nabla}^2 \psi$

  $\Rightarrow$ inverse cascade of energy $\mathcal{E}(K) = C\epsilon^{2/3} K^{-5/3}$

  & direct cascade of potential enstrophy $\mathcal{Z}(K) = C_z \eta^{2/3} K^{-1}$

  $\Rightarrow \boxed{\mathcal{E}(K) = C_z \eta^{2/3} K^{-3}}$

- Blumen (1978) considered uniform PV flow driven by potential temperature on a rigid lid (i.e. SQG):

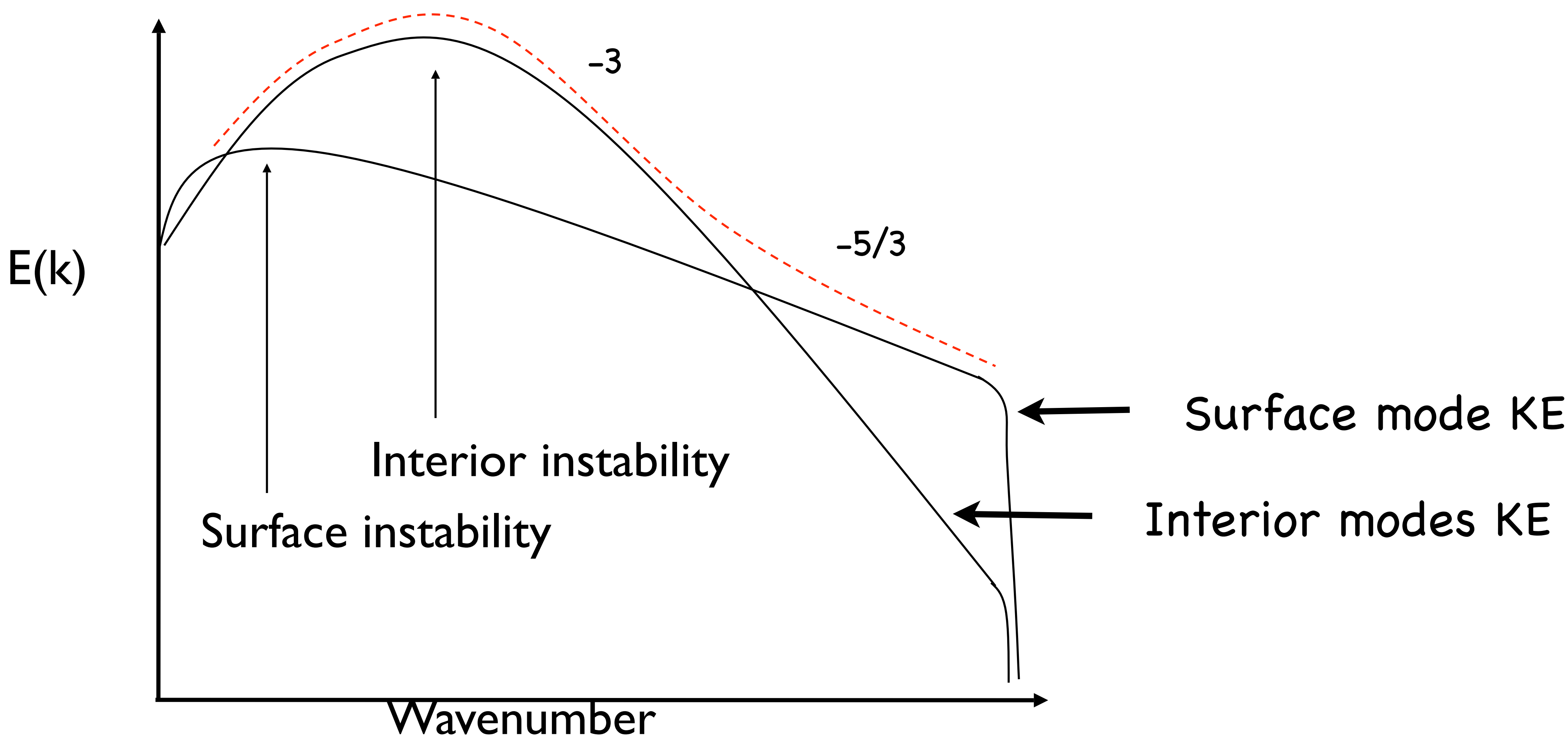
  conserves 3D energy $E = \overline{\psi \theta_s}$ and θ variance $T = \overline{\theta_s^2}$

  $\Rightarrow$ direct cascade of energy at surface $\boxed{\mathcal{T}(K) = C_T \epsilon^{2/3} K^{-5/3}}$

- Juckes (1994) argued for the relevance of SQG in the atmosphere

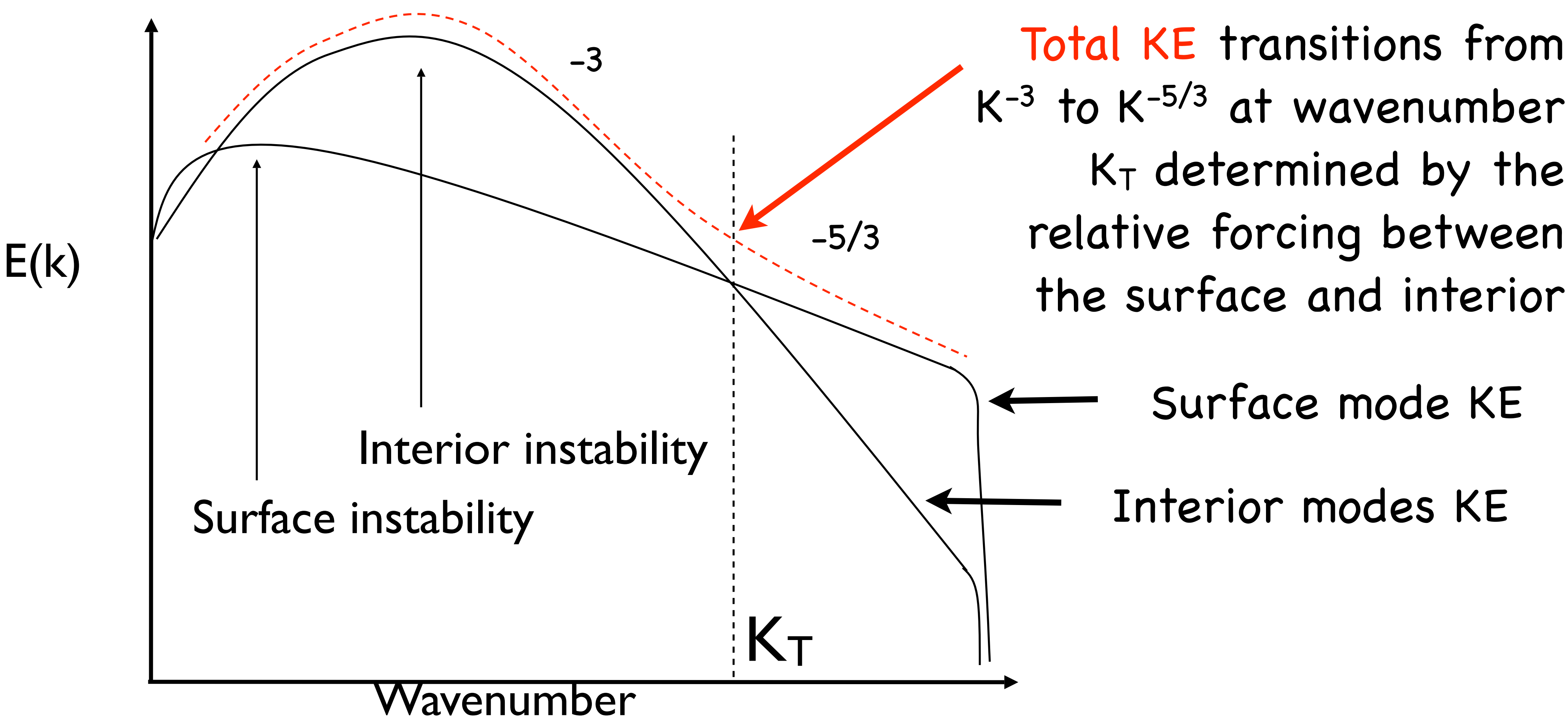
Surface-interior interaction

- The two seemingly opposing flows co-exist, and may be excited by a single baroclinic shear

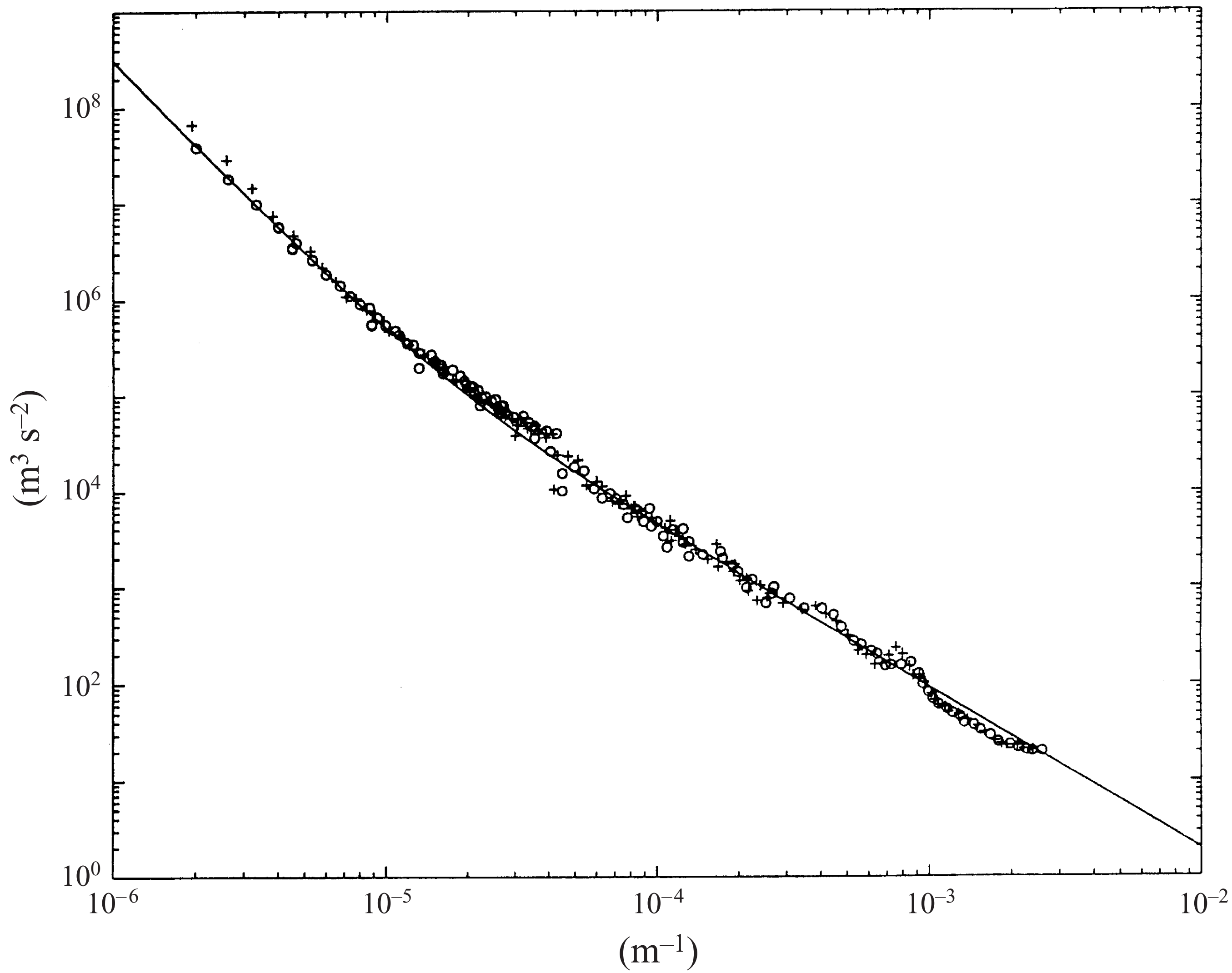


Surface-interior interaction

- The two seemingly opposing flows co-exist, and may be excited by a single baroclinic shear



$$E_1(K) = d_1 K^{-5/3} + d_2 K^{-3}$$



$d_1 \approx 9.1 \times 10^{-4}$

$d_2 \approx 3.0 \times 10^{-10}$

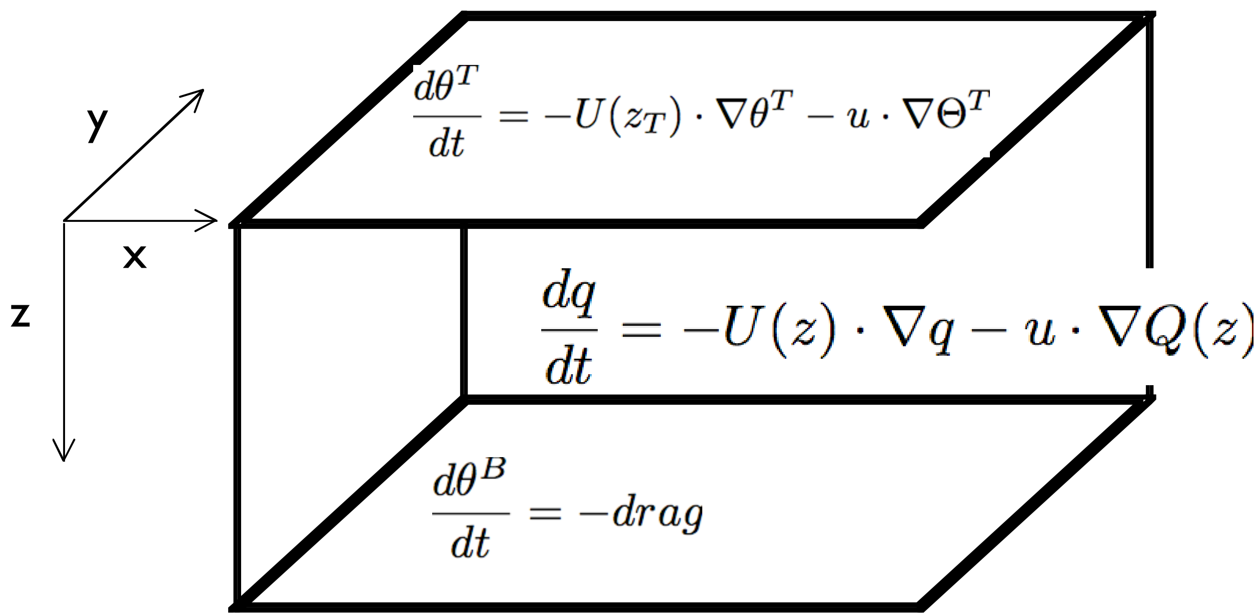
FIGURE 7. Energy spectrum according to (71), together with the data points given by Nastrom *et al.* (1984). Circles: zonal wind power spectrum. Crosses: meridional wind power spectrum.

Lindborg (1999)

Baroclinic model

- Modal QG model like Flierl 1978 but with explicit interior and surface dynamics, with mean velocity U(z), constant stratification N^2 and Ekman drag at the lower boundary

Advection



Inversion

$$\left.\frac{\partial\psi^T}{\partial z}\right|_{z_T} = \theta^T$$

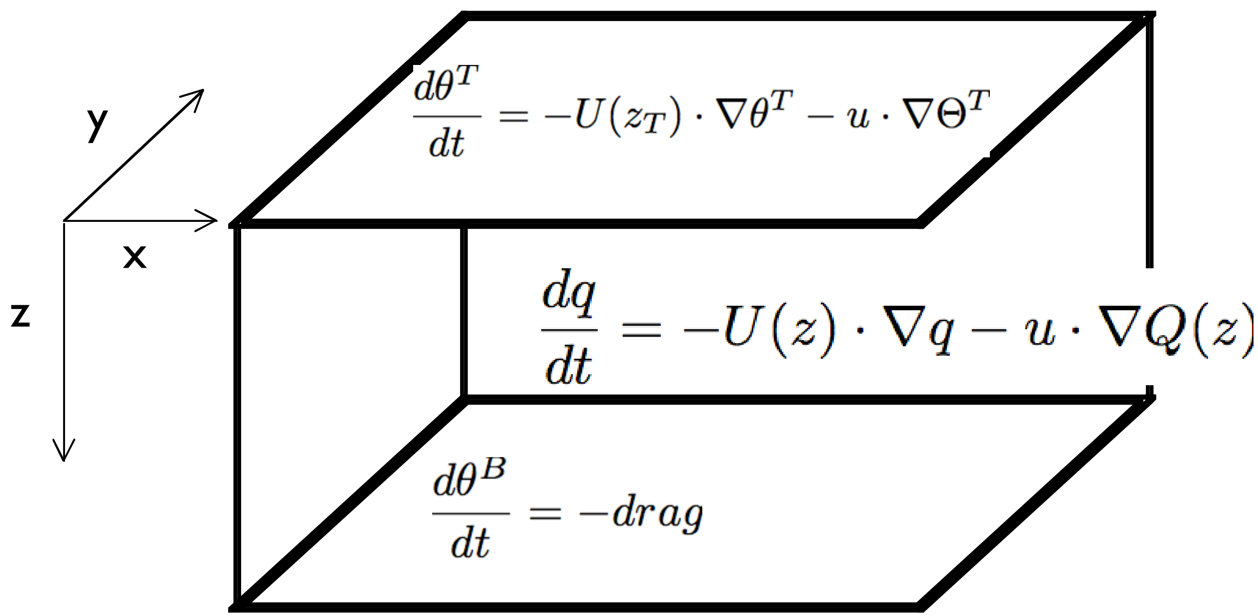
$$(\nabla^2 + \Gamma)\psi^I = q$$

$$\left.\frac{\partial\psi^B}{\partial z}\right|_{z_B} = \theta^B$$

Baroclinic model

- Modal QG model like Flierl 1978 but with explicit interior and surface dynamics, with mean velocity U(z), constant stratification N^2 and Ekman drag at the lower boundary

Advection



Inversion

$$\left.\frac{\partial\psi^T}{\partial z}\right|_{z_T} = \theta^T$$

$$(\nabla^2 + \Gamma)\psi^I = q$$

$$\left.\frac{\partial\psi^B}{\partial z}\right|_{z_B} = \theta^B$$

- Mean velocity forcing becomes: $\boldsymbol{U}(z) = \boldsymbol{U}^S(z) + \sum_{j=0}^{N-1} \boldsymbol{U}_j\phi_j(z)$

where U_j is the projection of (U-U^S) onto mode j, and U^S solves

$$\Gamma\boldsymbol{U}^S = \frac{f^2}{H}\left[\frac{\nabla^\perp\Theta(z)}{N^2(z)}\right]\Bigg|_{z_B}^{z_T}, \quad \frac{d\boldsymbol{U^S}}{dz}(z_T) = \nabla^\perp\Theta^T, \quad \frac{d\boldsymbol{U^S}}{dz}(z_B) = \nabla^\perp\Theta^B$$

Truncated Modal Model

- doubly periodic ⇒ switch to Fourier domain (~) in horizontal
- expand $\tilde{\psi}^I$ into vertical modes and then truncate, for simplicity we truncated at BC1: $\tilde{\psi}^I = \tilde{\psi}_0 + \tilde{\psi}_1 \phi_1(z)$
- the vertical structure of $\tilde{\psi}^T$ and $\tilde{\psi}^B$ can similarly be expressed in terms of 'surface modes' $\phi^T(K, z)$ and $\phi^B(K, z)$
- shear forcing U(z) is decomposed into a quadratic surface component and a sinusoidal interior component: $U(z) = U^S(z) + \tilde{U}_1 \sqrt{2} \cos(\pi z / H)$

Three linear instability types

Surface-surface ('Eady-Green')

$$\Theta_y^T = \Theta_y^B = -1, \quad \tilde{U}_1 = 0$$

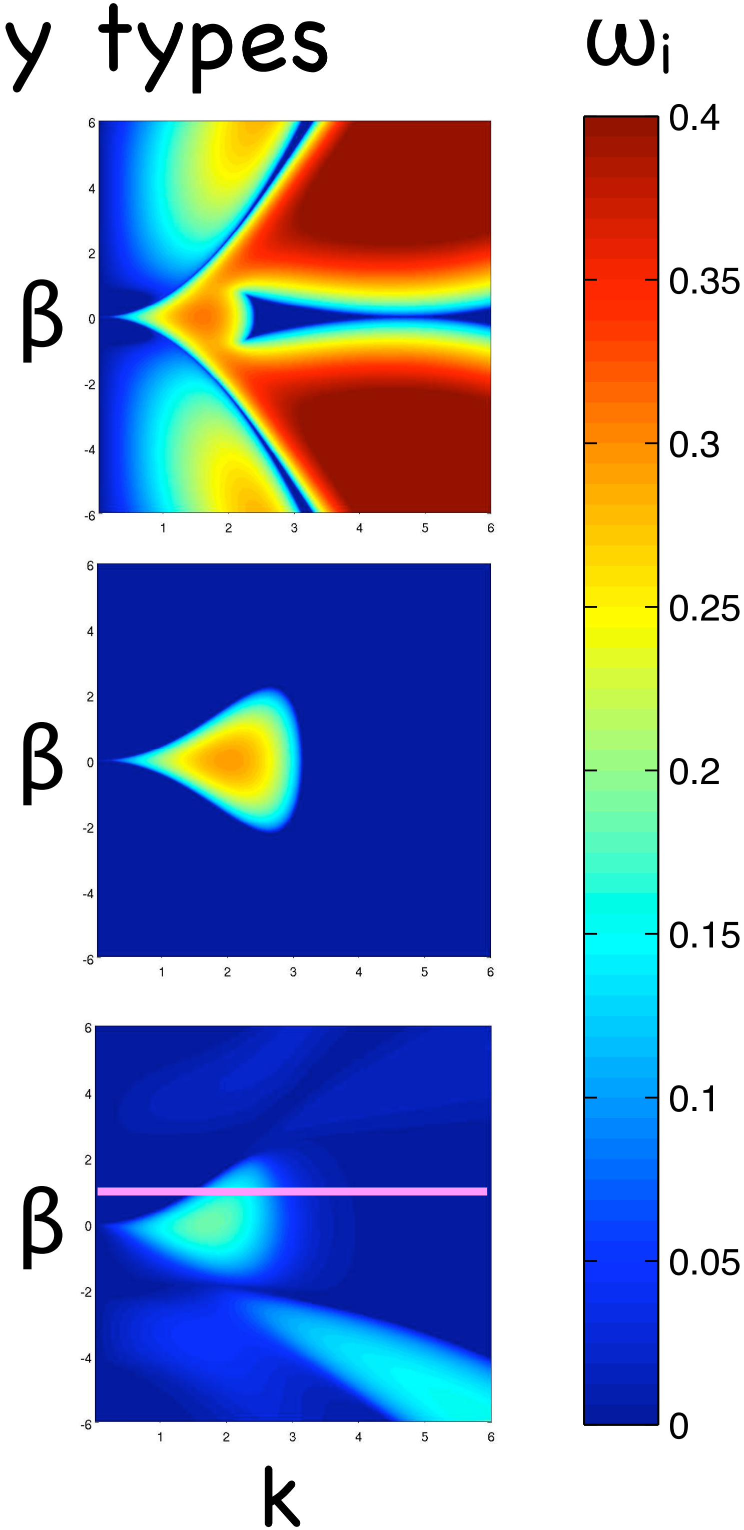
Interior-interior ('Phillips')

$$\Theta_y^T = \Theta_y^B = 0, \; \tilde{U}_1 = \frac{1}{\sqrt{2\pi}}$$

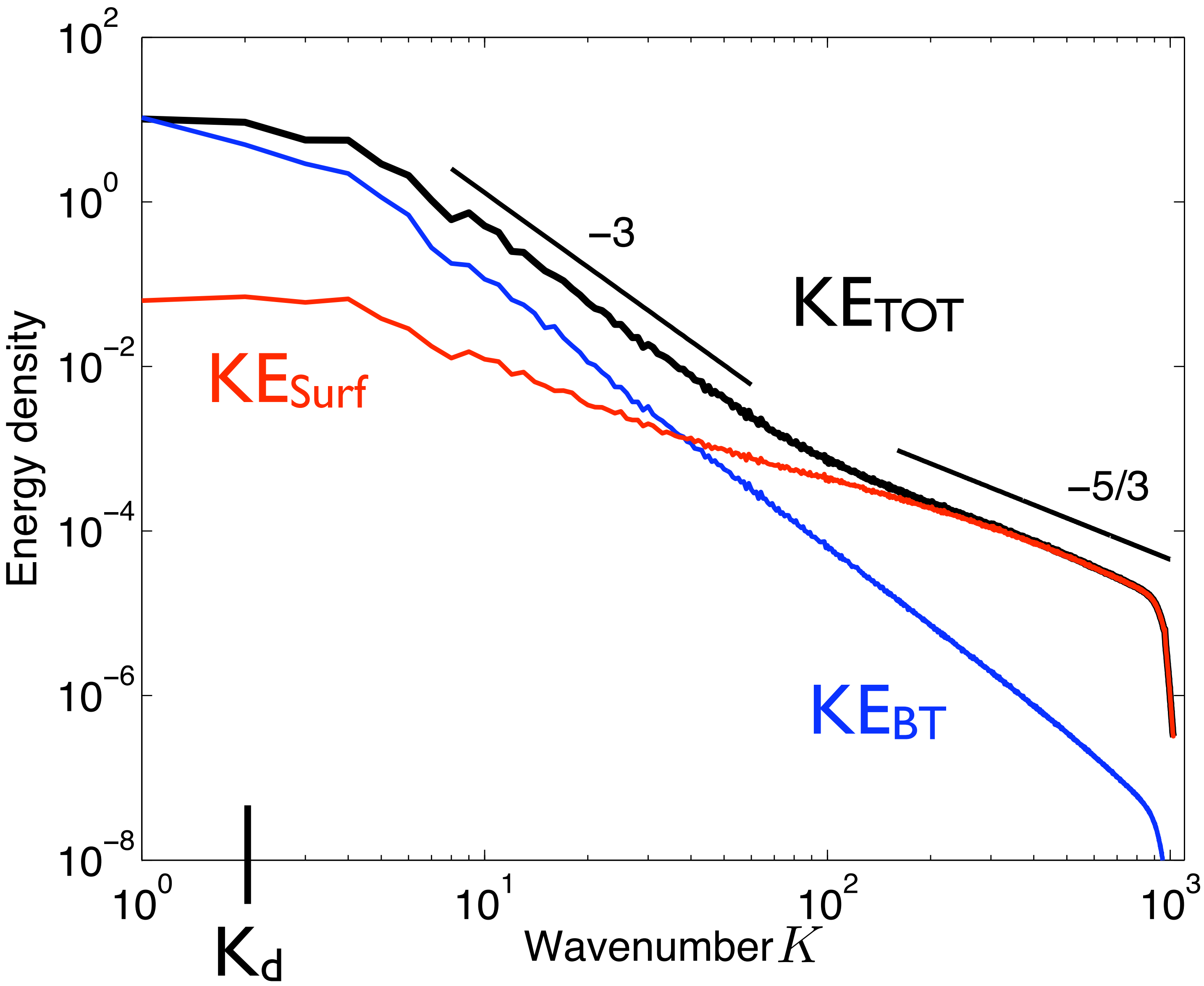
Surface-interior ('Charney')

$$\Theta_y^T = -0.5, \; \Theta_y^B = 0, \; \tilde{U}_1 = \frac{-4}{\sqrt{2\pi}}$$

$\left\{ \text{includes Ekman drag at bottom} \right\}$



KE spectra at the surface



Theory for transition wavenumber

- Assume forward enstrophy cascade of the form:

$$\mathcal{E}(K) = \mathcal{C}_E \eta^{2/3} K^{-3}$$

with enstrophy cascade rate:

$$\eta = -Q_y(H)\ \overline{vq}|_{z=H} \equiv \kappa_q Q_y(H)^2$$

- Equate the enstrophy cascade with a temperature cascade at the surface:

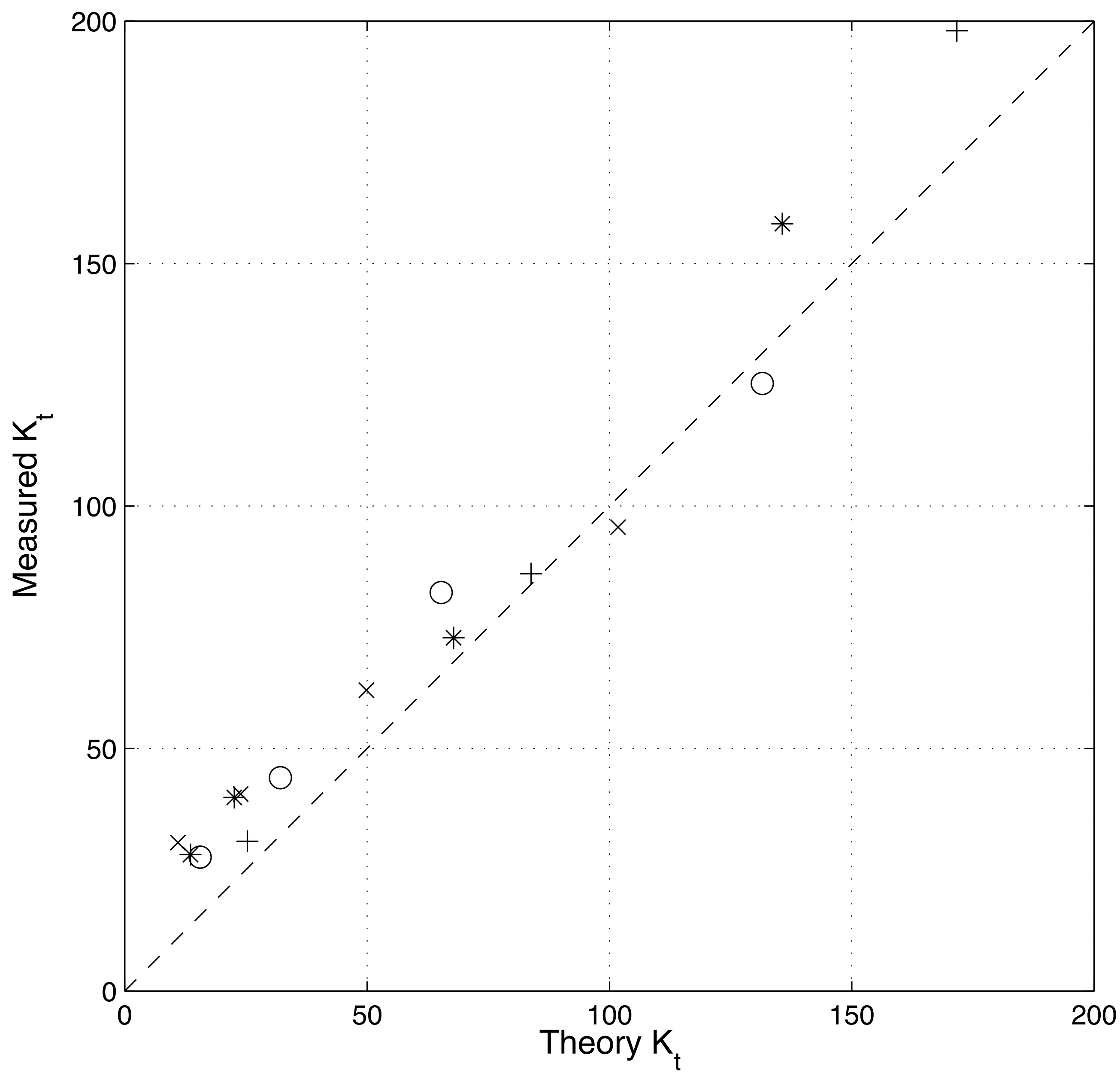
$$\mathcal{A}(K) = \mathcal{C}_A \epsilon^{2/3} K^{-5/3}$$

that has energy cascade rate:

$$\epsilon = -\frac{f^2 \Theta_y^T}{N^2}\ \overline{v\theta^T}|_{z=H} \equiv \kappa_\theta \left(\frac{f\Theta_y^T}{N}\right)^2$$

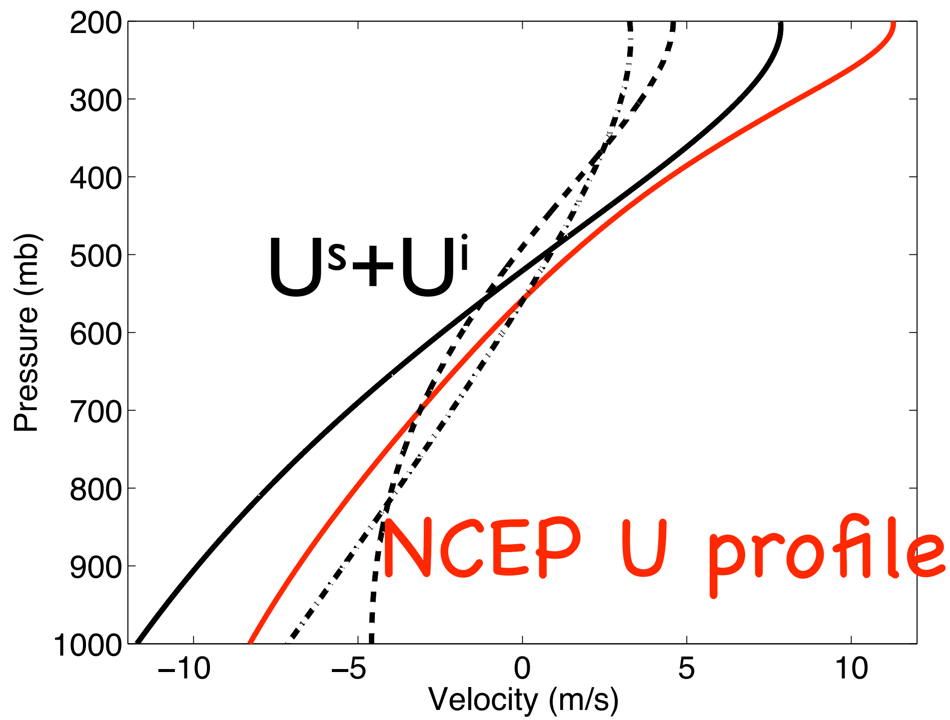
- Then assuming $\mathcal{C}_E \simeq \mathcal{C}_A$ and $\kappa_q \simeq \kappa_\theta$ $\Rightarrow$ $\boxed{K_t \simeq \frac{N}{f}\left|\frac{Q_y(H)}{\Theta_y^T}\right|}$

Theory for transition wavenumber



Atmospheric Parameters

- NCEP LTM zonally averaged zonal velocity profile U(z) at 45°N
  - compute $U^s(z)$ from U_z(top) and U_z(bottom) estimated from U values at 200mb&250mb and 1000mb&925mb
  - residual $U(z)-U^s(z)$ put into first baroclinic mode



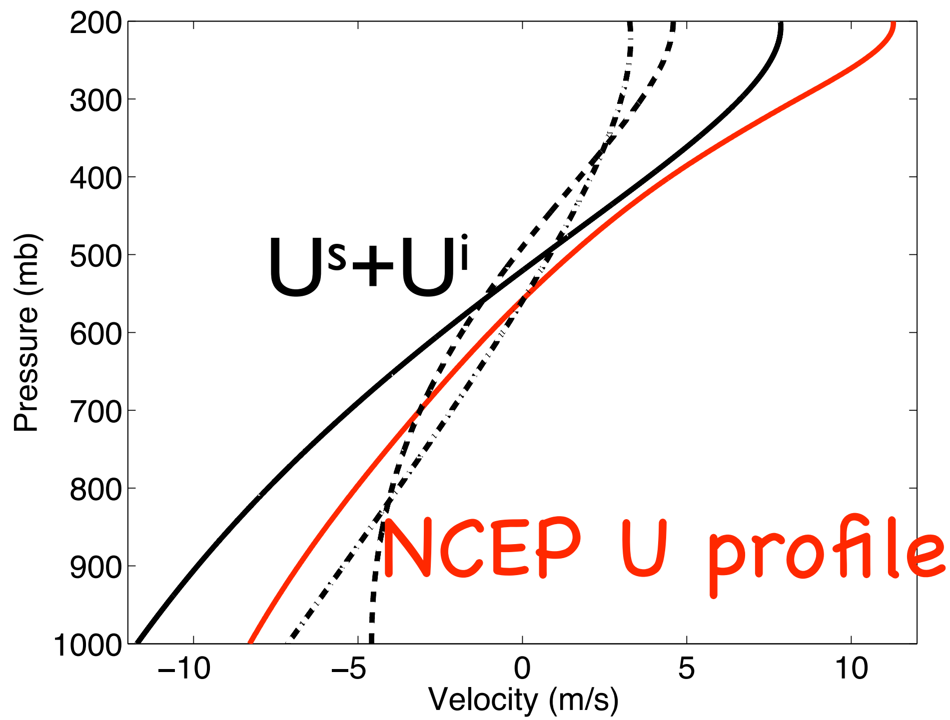
Atmospheric Parameters

- NCEP LTM zonally averaged zonal velocity profile U(z) at 45°N
  - compute $U^s(z)$ from $U_z(\text{top})$ and $U_z(\text{bottom})$ estimated from U values at 200mb&250mb and 1000mb&925mb
  - residual $U(z)-U^s(z)$ put into first baroclinic mode
- pseudo-height vertical coordinate (Hoskins '72)

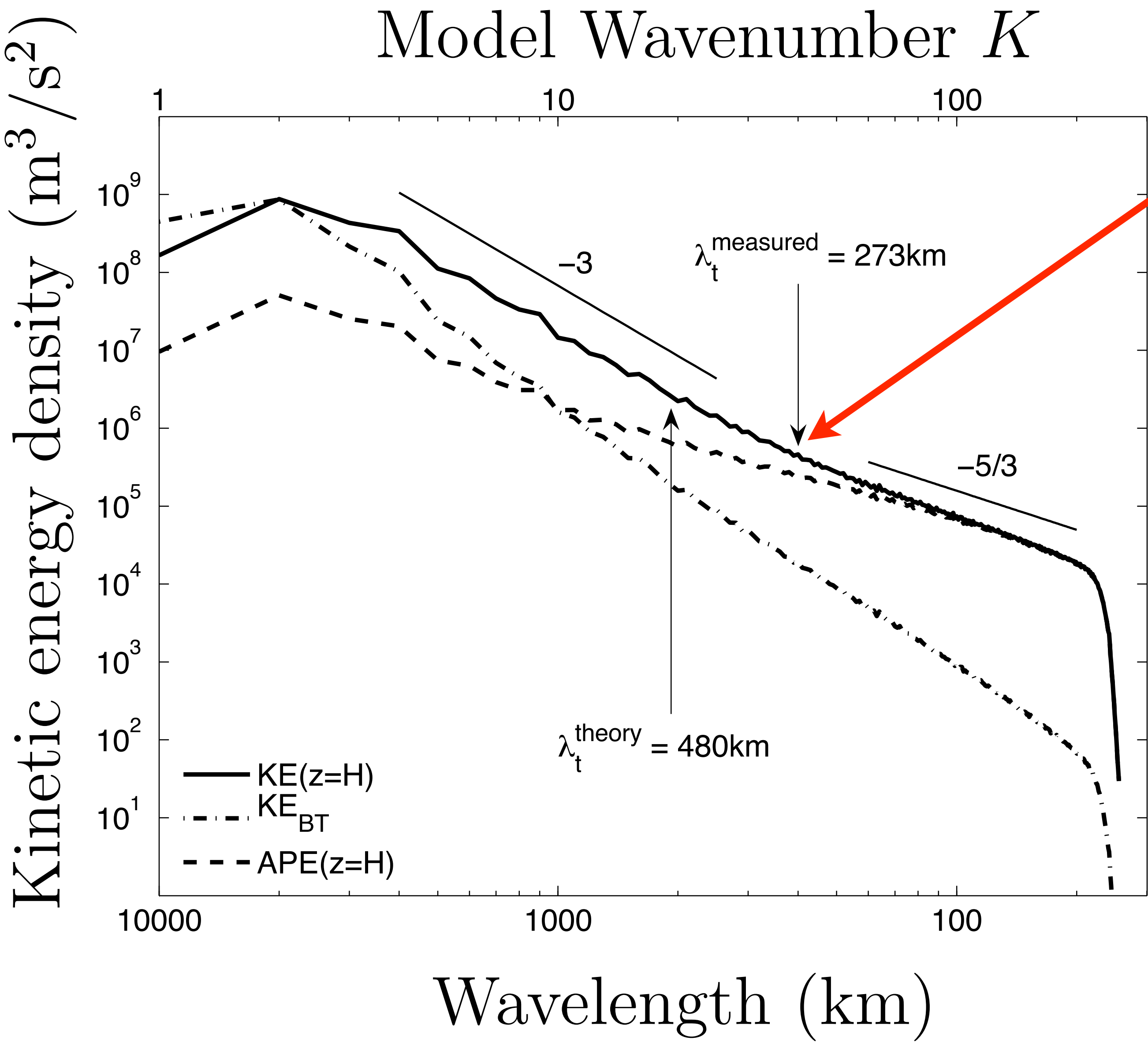
$$z = \left[1 - \left(\frac{p}{p_0}\right)^{2/7}\right] \cdot 28\text{km}$$

- Parameters: H≈10km, $N=10^{-2}s^{-1}$ (L_d≈1000km), $U_z(\text{top})=5.6\times10^{-4}s^{-1}$, $U_z(\text{bot})=2.1\times10^{-3}s^{-1}$, $U_1=-2.6ms^{-1}$

=> $\boxed{K_t \approx 1/77 \text{ rad/km } (\approx 480\text{km})}$



Atmospheric Parameters: surface spectra



Transition scale ≈ 270km

Forward flux of
$\theta(k)^2 \approx 8\times10^{-5} m^2 s^{-3}$

Cho & Lindborg (2001):
$\epsilon = 6\times10^{-5} m^2 s^{-3}$

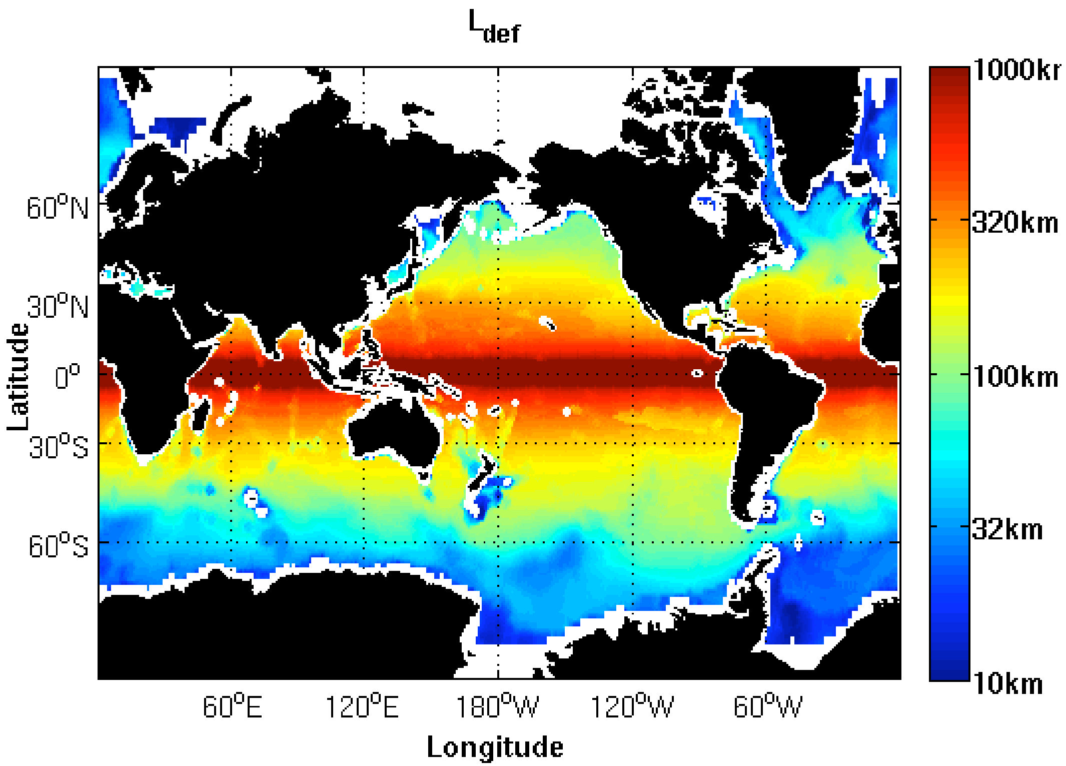
Dewan (1997):
$\epsilon \sim 10^{-6} - 10^{-4}\ m^2 s^{-3}$

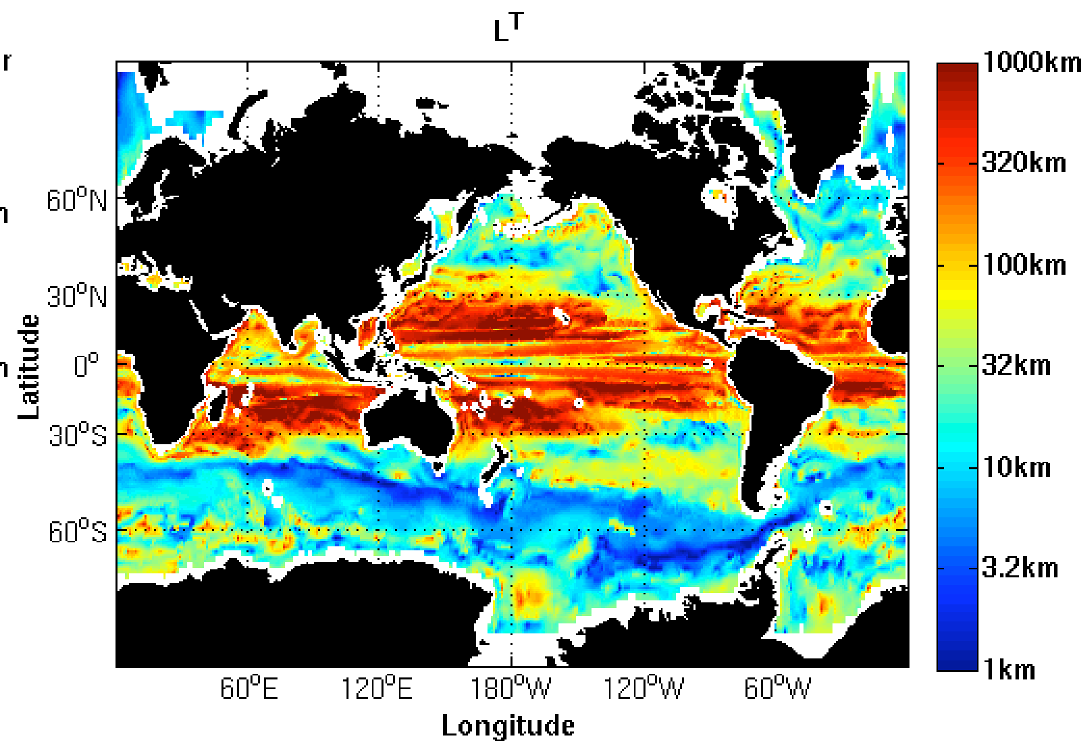
Conclusions

- shallow mesoscale spectra near surfaces can be explained by balanced dynamics alone (although other processes are surely also active)
- QG models typically don't show this effect due to insufficient vertical resolution (in layered models) or exclusion of surface signals (in modal models)
- caveats: perhaps too little interior energy at small scale since SQG signal rapidly decays away from surface, eventual breakdown as Rossby number increases

What about the ocean?

Forget and Wunsch (2007) data set, computing $U_z = \nabla^{\perp}\Theta$ via thermal wind at the base of the mixed layer



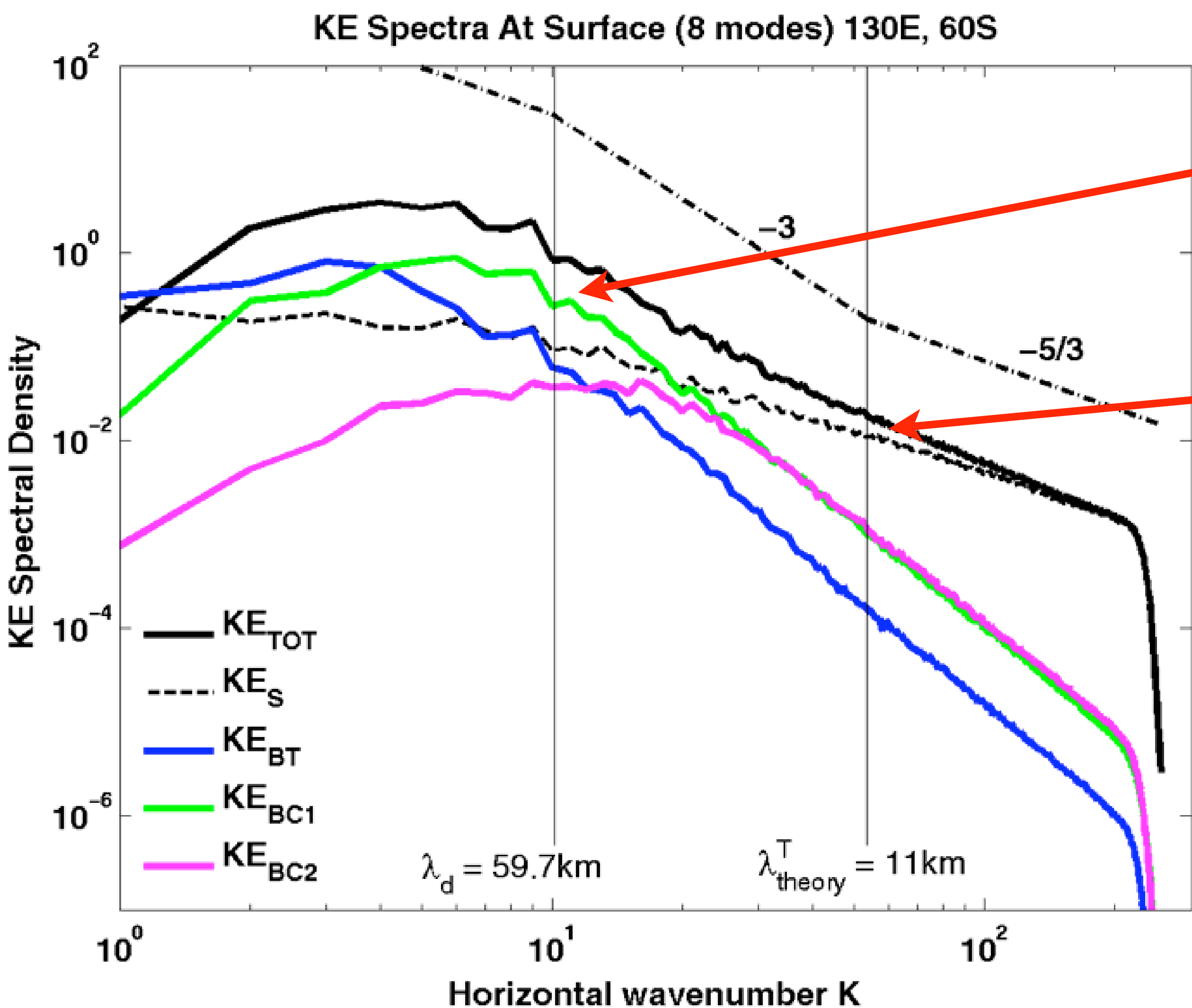


Deformation wavelength

Transition wavelength

using $K_t = \frac{N}{f}\left|\frac{\nabla Q(H)}{\nabla \Theta^T}\right|$

Example Calculation (130E, 60S): Surface KE spectra

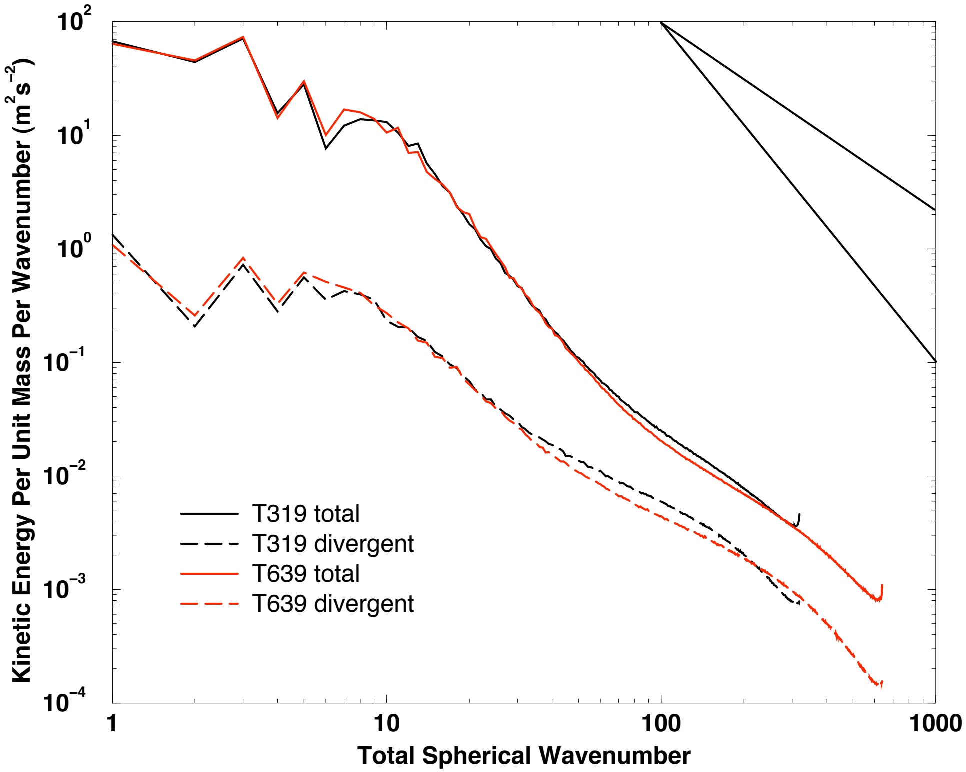


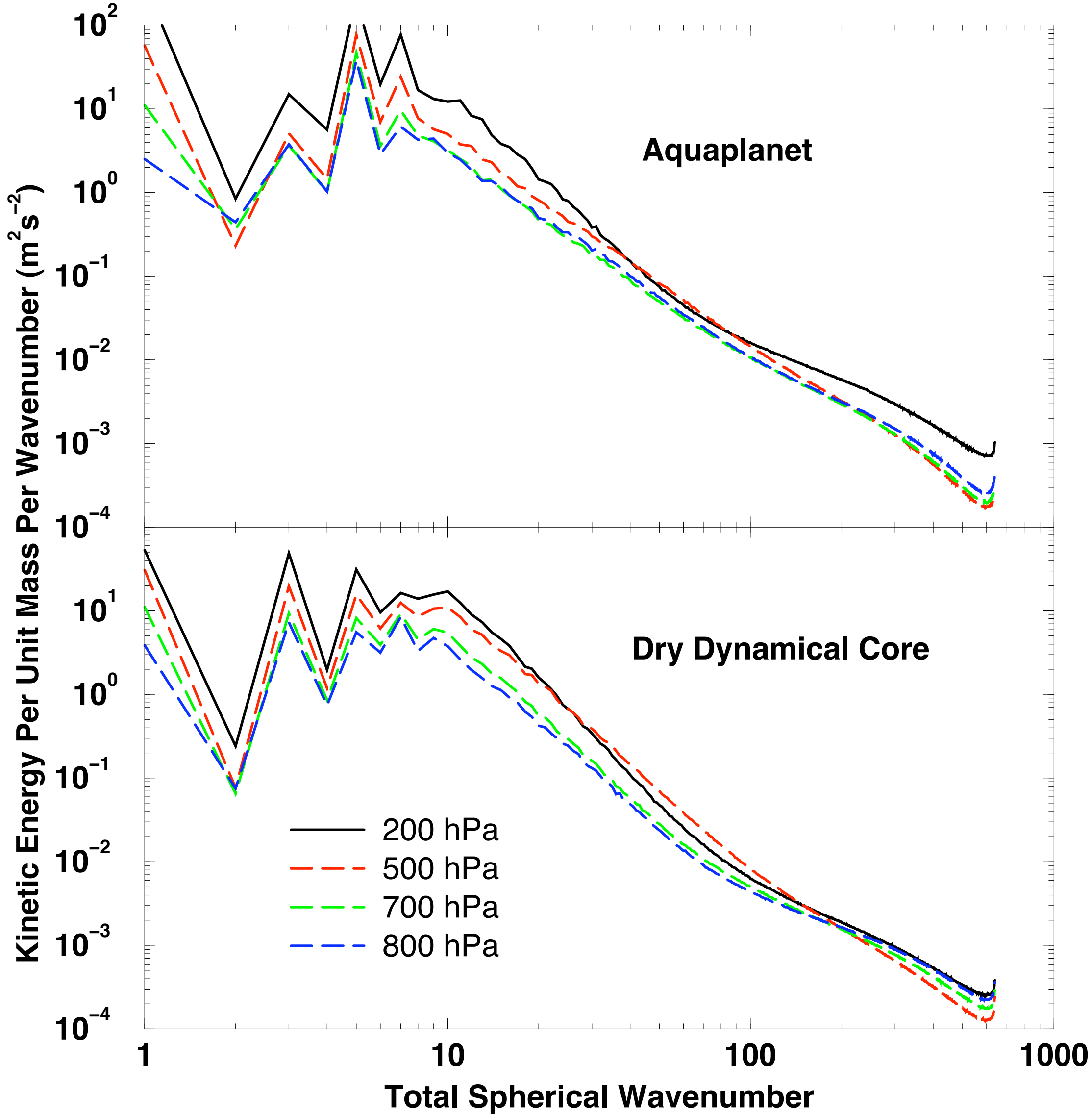
BC1 most dominant at deformation scale

SURF dominates small scales

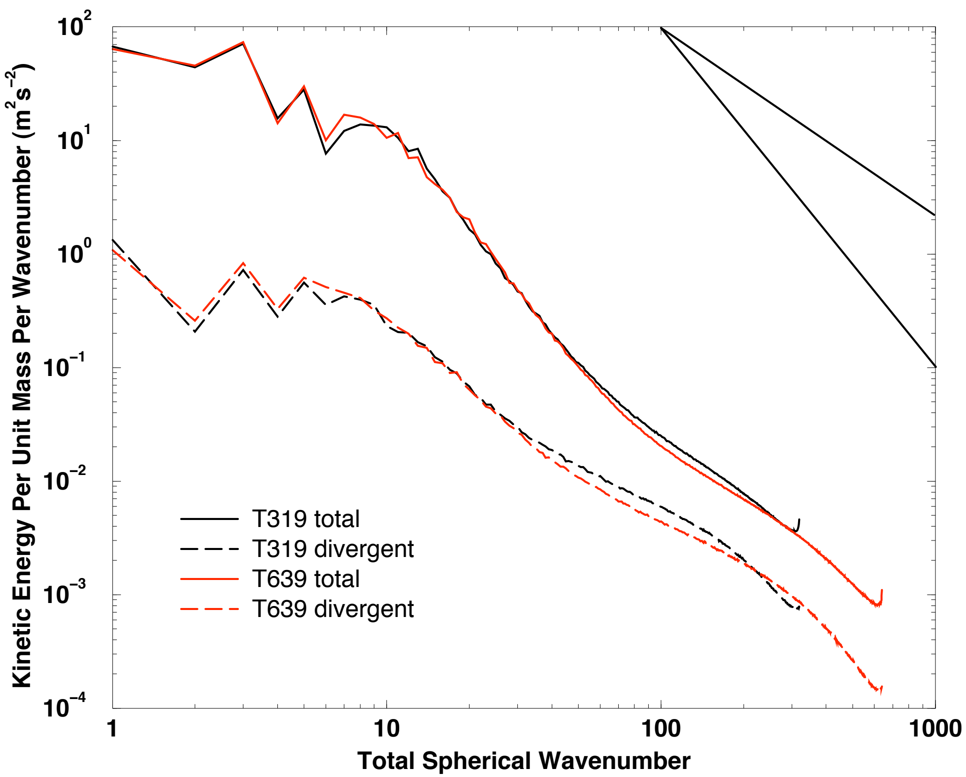
Note: in a layered model, some of the SURF mode would be present in BC1 at deformation scales.

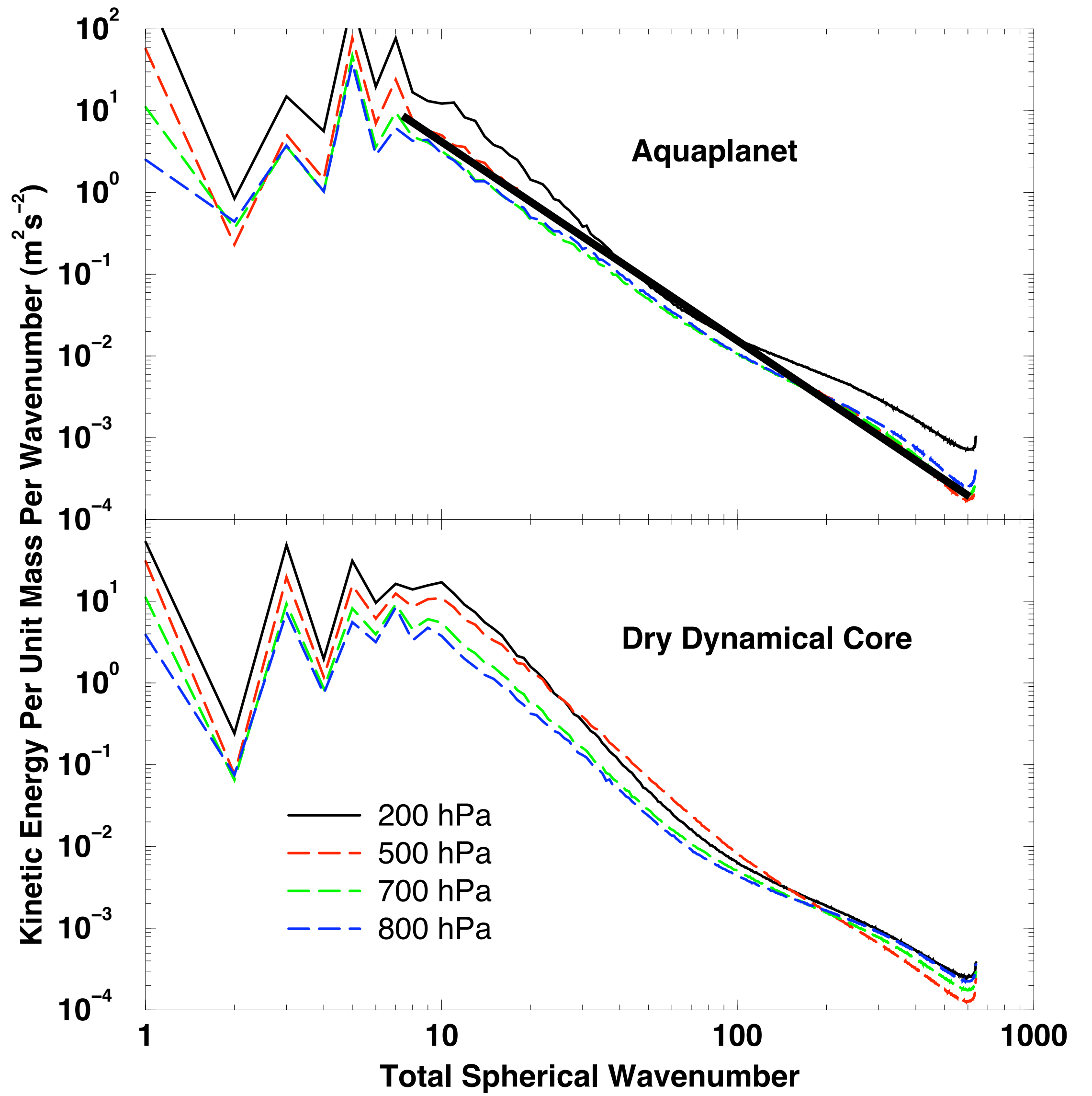
Hamilton et al (2008 submitted)



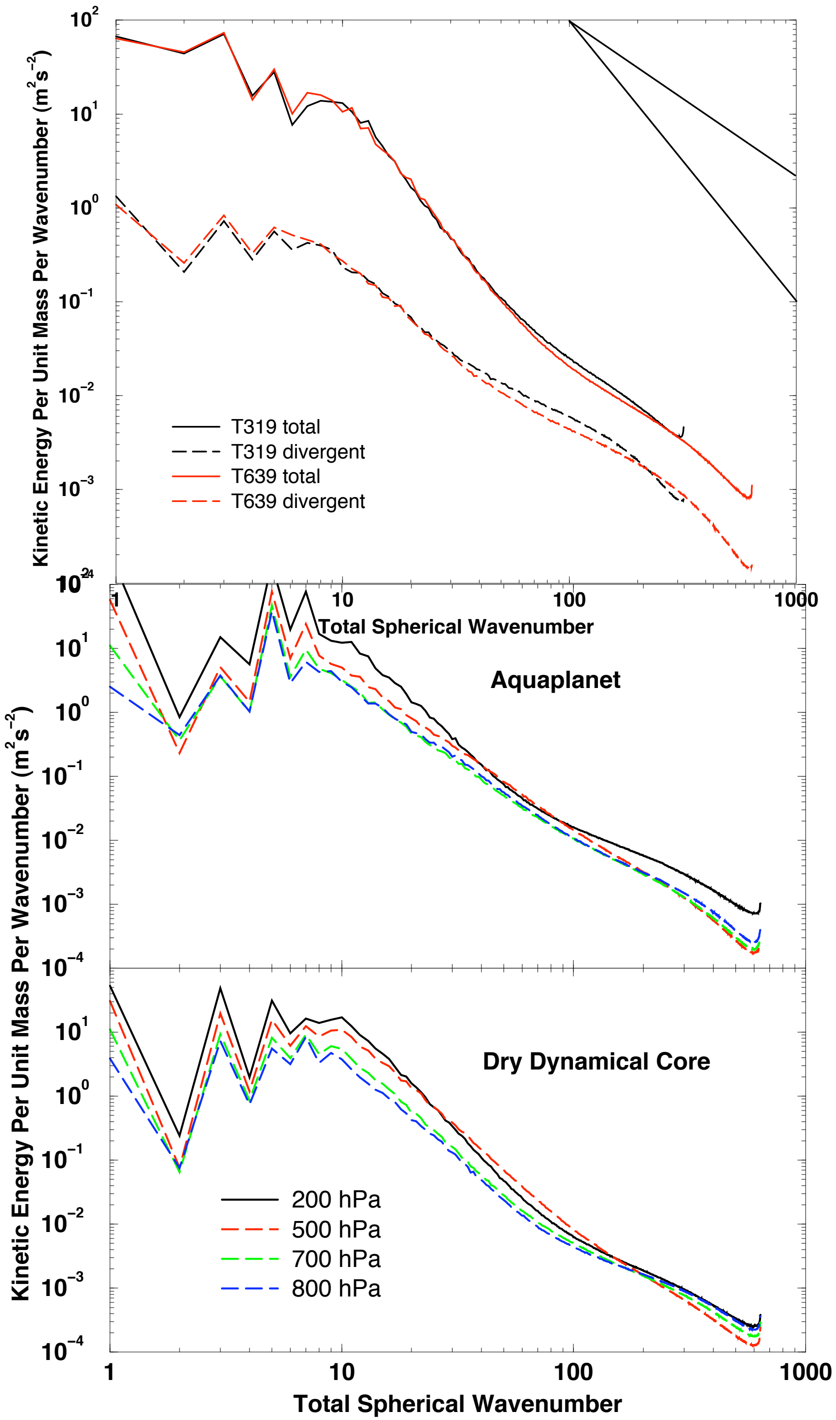


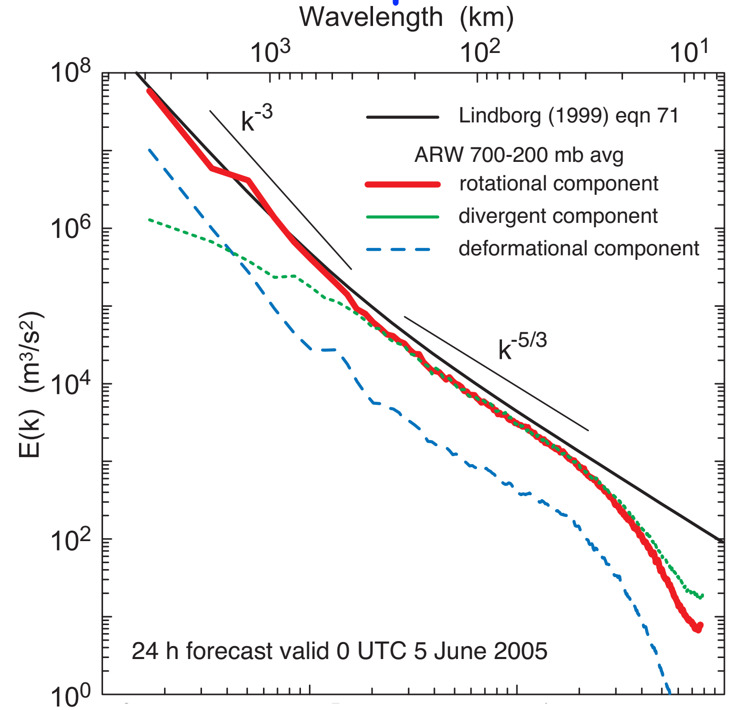
Hamilton et al (2008 submitted)

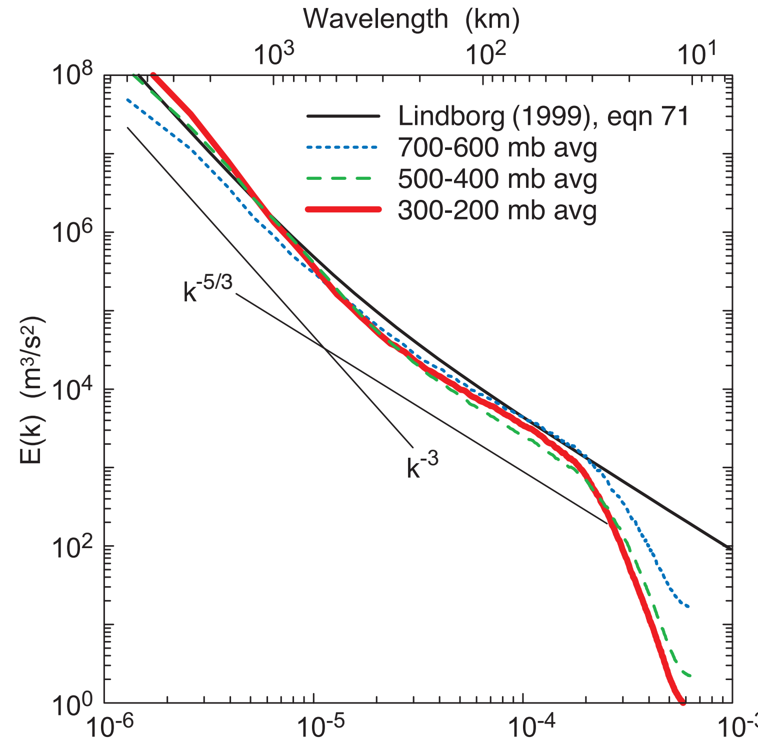




Hamilton et al (2008 submitted) Skamarock & Klemp (2007 submitted)







Lindborg 2007

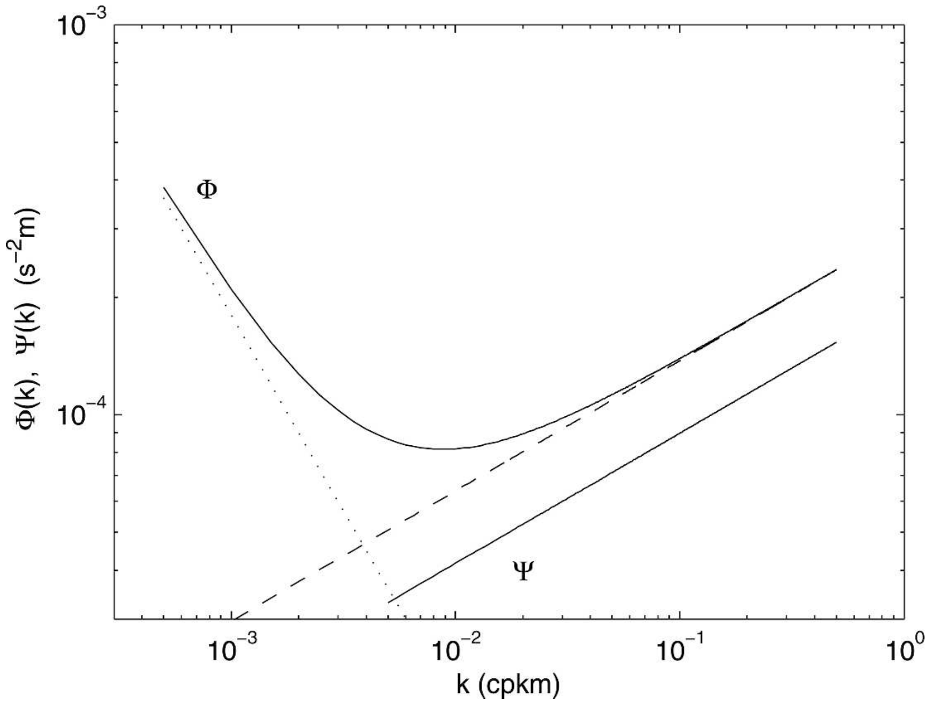
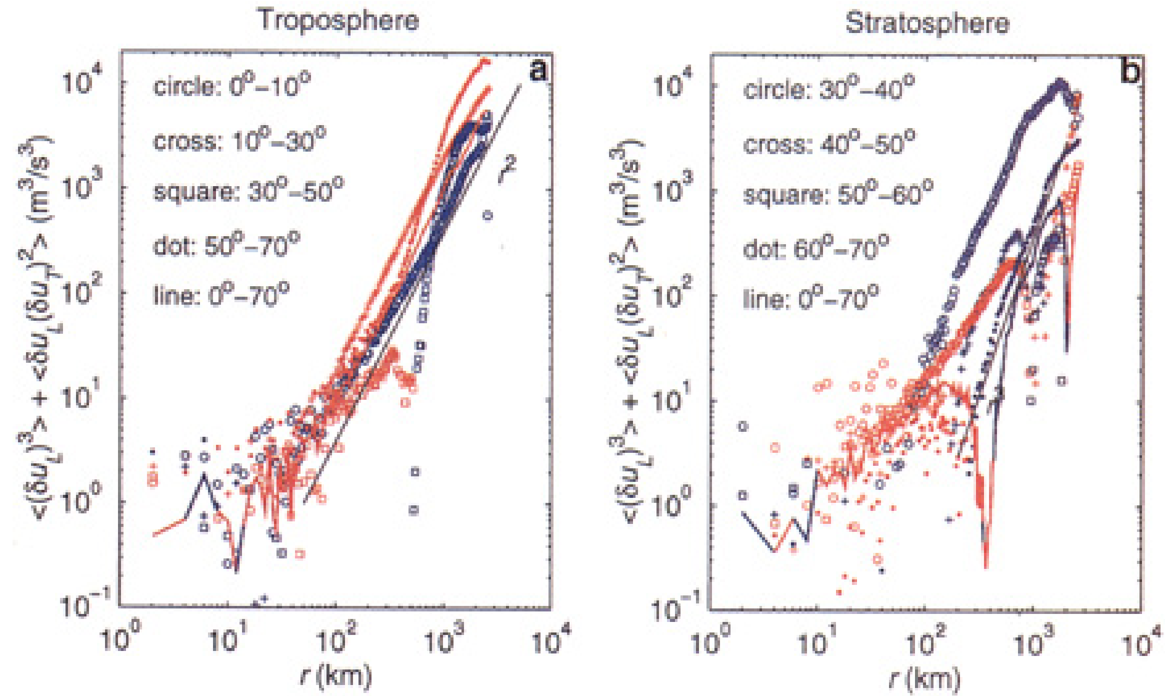
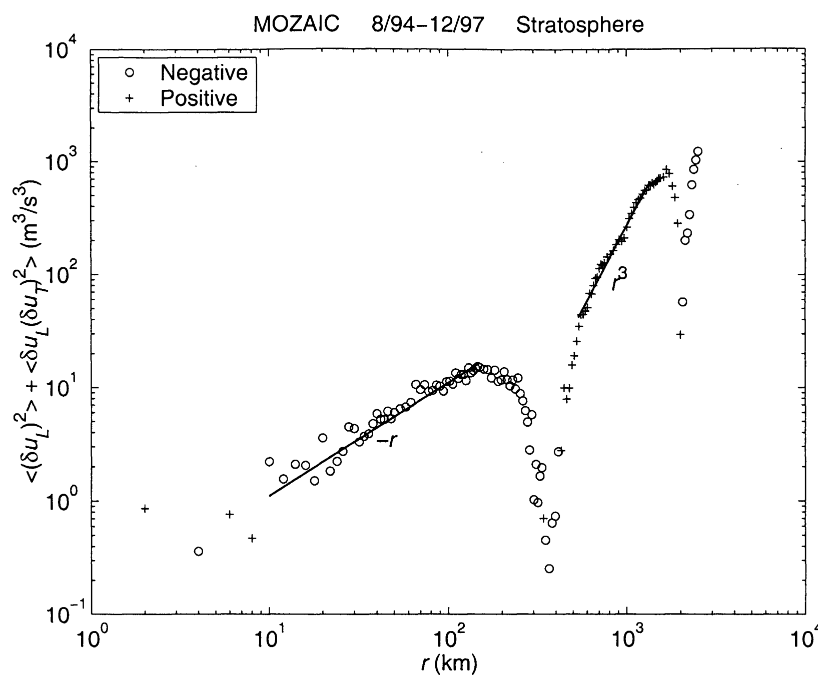


FIG. 3. The spectra of vertical vorticity and horizontal divergence in the upper troposphere and lower stratosphere. The dashed line represents the curve $1.57q_1k^{1/3}$ and the dotted line represents the curve q_2k^{-1}; k is here measured in cpkm, which means that $1/k$ is the corresponding wavelength measured in kilometers.

ABSTRACT

The author shows that the horizontal two-point correlations of vertical vorticity and the associated vorticity wavenumber spectrum can be constructed from previously measured velocity structure functions in the upper troposphere and lower stratosphere. The spectrum has a minimum around $k = 10^{-2}$ cycles per kilometer (cpkm) corresponding to wavelengths of 100 km. For smaller wavenumbers it displays a k^{-1} range and for higher wavenumbers, corresponding to mesoscale motions, it grows as $k^{1/3}$. The two-point correlation of horizontal divergence of horizontal velocity and the associated horizontal spectrum is also constructed. The horizontal divergence spectrum is of the same order of magnitude as the vorticity spectrum in the mesoscale range and show similar inertial range scaling. It is argued that these results show that the mesoscale motions are not dominated by internal gravity waves. Instead, the author suggests that the dynamic origin of the $k^{1/3}$ range is stratified turbulence. However, in contrast to Lilly, the author finds that stratified turbulence is not a phenomenon associated with an upscale energy cascade, but with a downscale energy cascade.





Cho and Lindborg 2001