Turbulence Thermalization and Spectral Bottleneck

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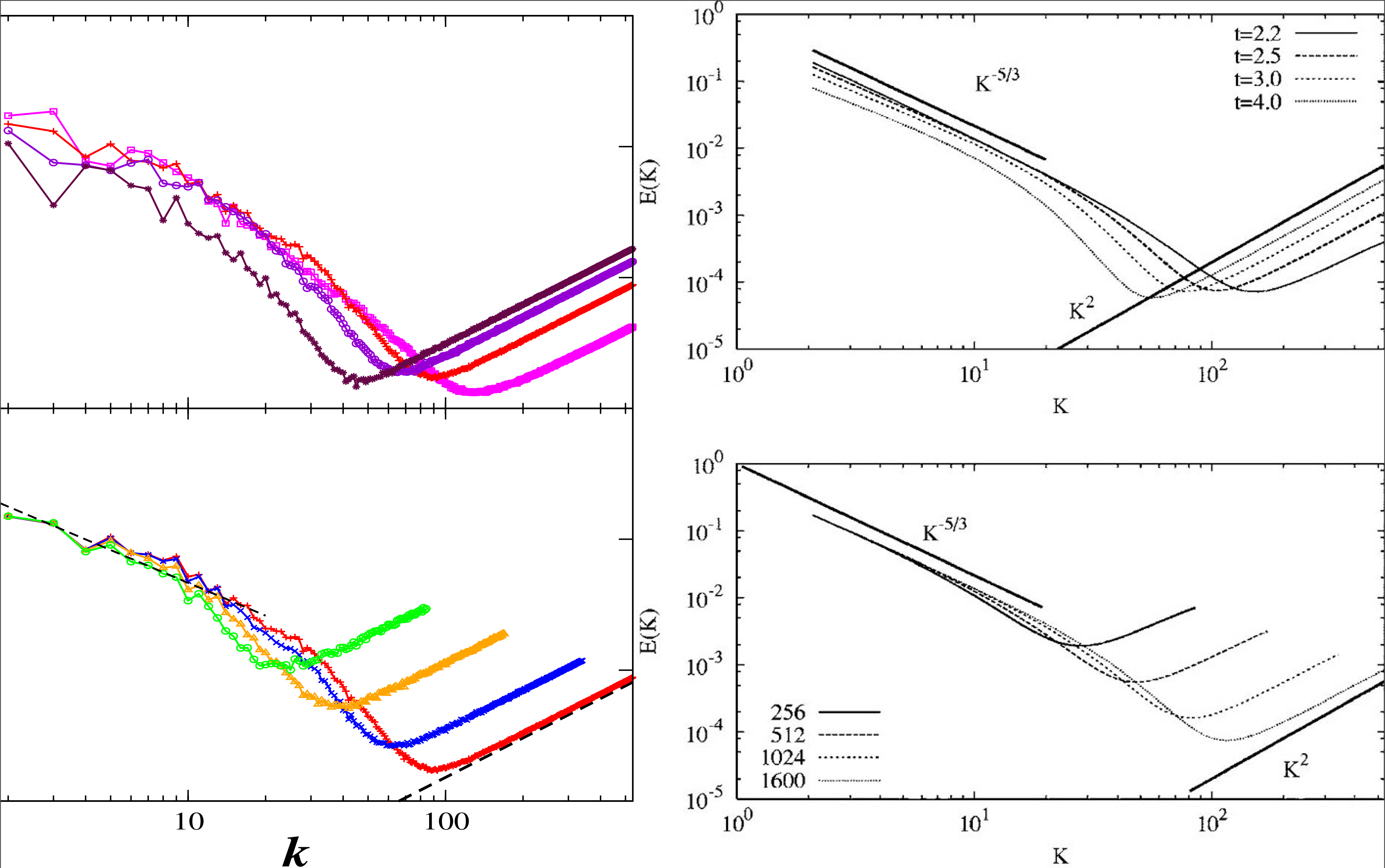


Cichowlas et al. (2005) "reproduced" by Bos and Bertoglio(2006) with **EDQNM**

Eddy-Damped Quasi-Normal Markovian spectrum

$$\left(\frac{\partial}{\partial t} + 2\nu k^2\right) E(k,t) =$$
$$\iint_{\triangle_k} dp dq \theta_{kpq} b(k,p,q) \frac{k}{pq} E(q,t) \left[k^2 E(p,t) - p^2 E(k,t)\right]$$

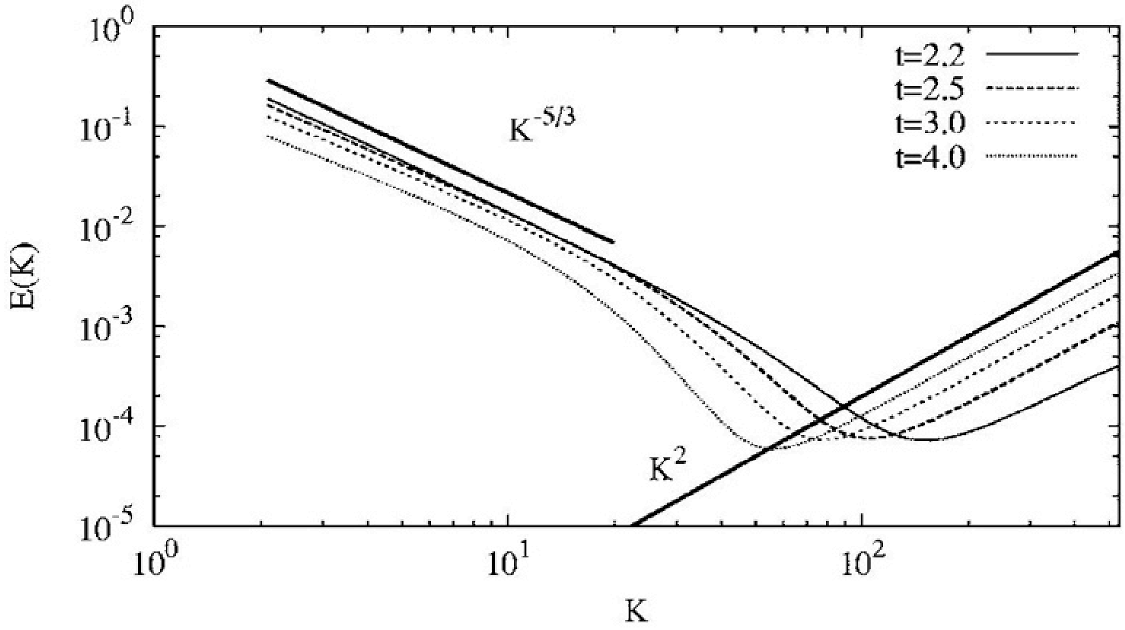
"QN" --- Chou(1940), Millionshtchikov(1941): realizability problem

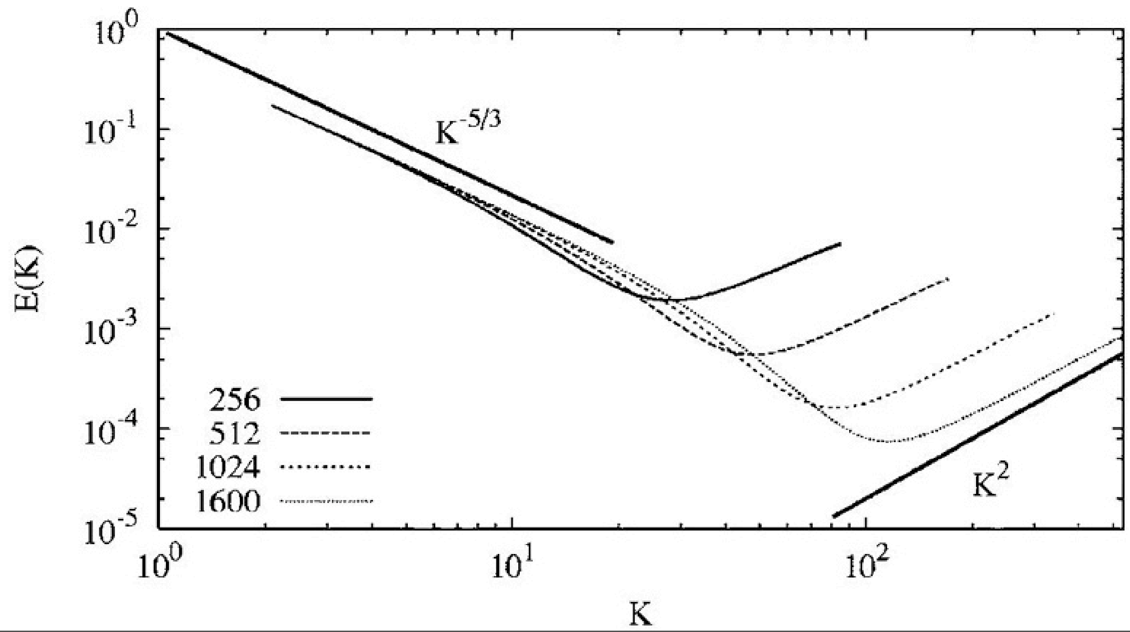
"N" --- Lee (1952), Hopf(1952): statistics of absolute equilibria of truncated Euler

DIA (Kraichnan): tractability problem

"ED","M" --- Orszag(1970, 1977)

Galerkin truncation, hyperviscosity and bottleneck for EDQNM

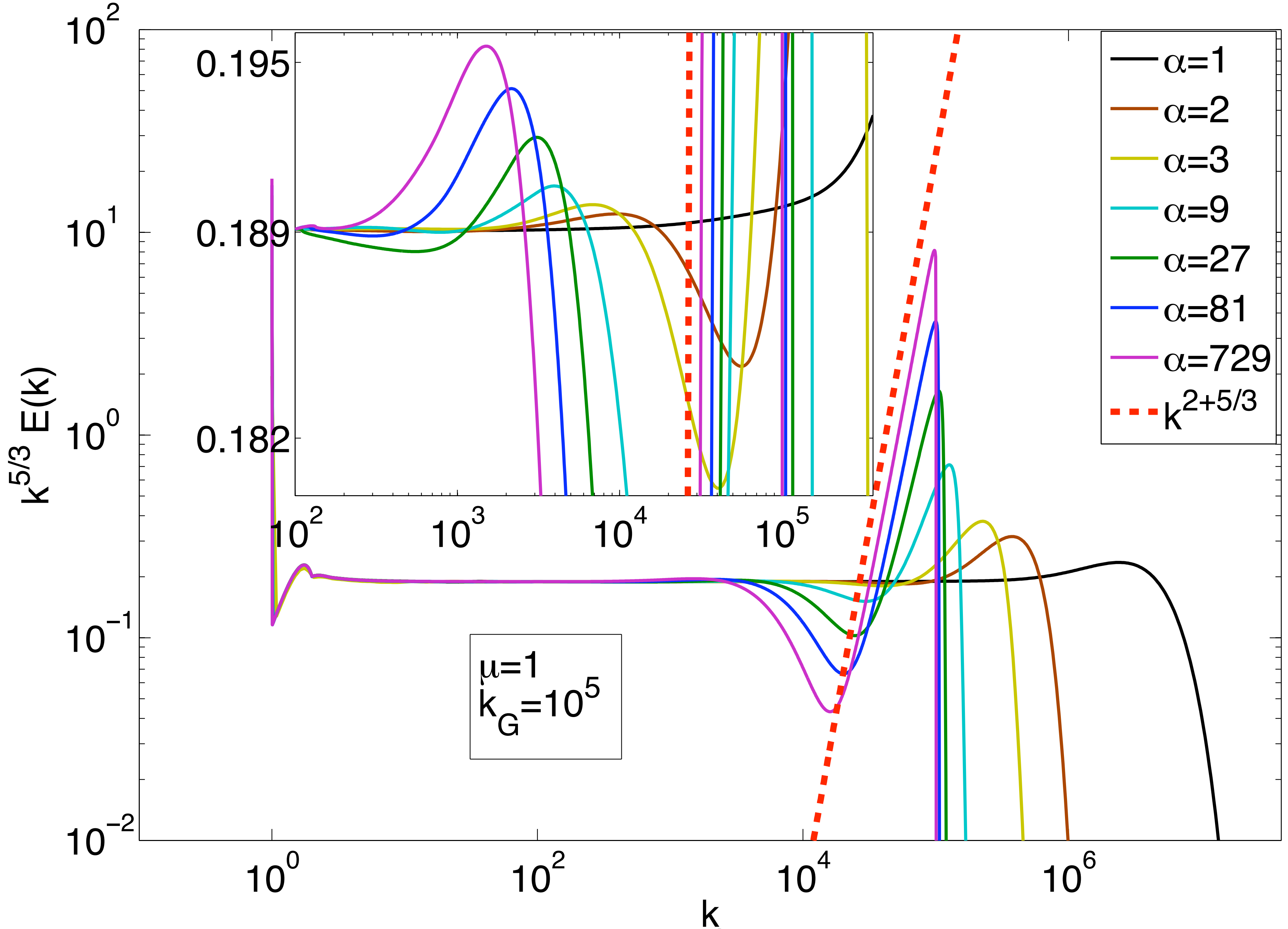




Bos and Bertoglio (2006)

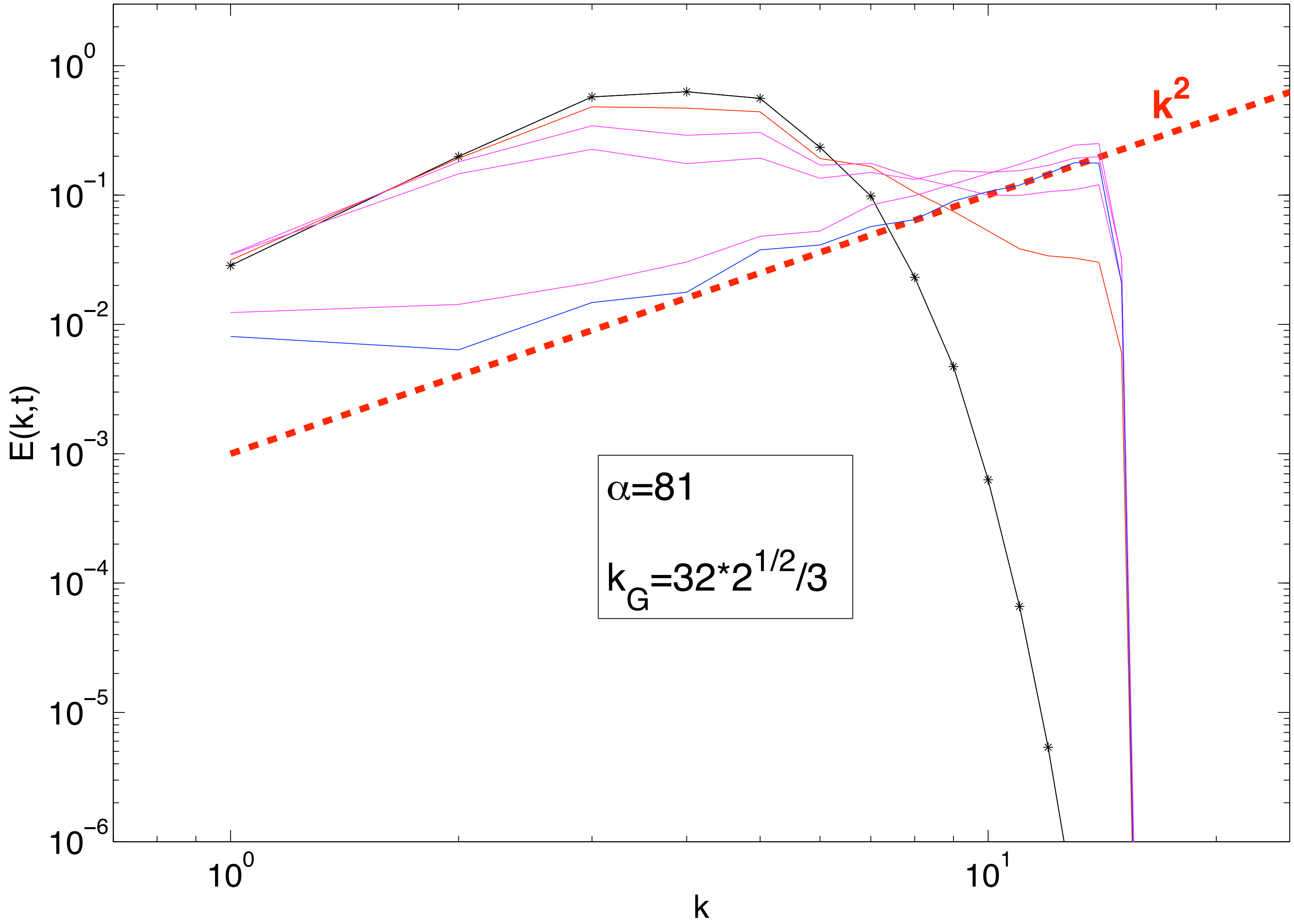
Anything else?

YES!:
secondary bottleneck
and
even more:



Hyperviscous EDQNM:
convergence to Galerkin truncation and secondary bottleneck ...

Hyperviscous Navier-Stokes



64^3 DNS

Conclusions and perspectives

- Hyperviscous EDQNM converges to Galerkin truncation;
- Secondary bottleneck.

- Dynamics/Mechanics, Flow Structures ......
- Statistics: intermittency, Fermi-Pasta-Ulam-*Tsingou* problem and all that ......

Acknowledgements

Bos and Bertoglio for sharing EDQNM codes
and
many other people for helpful communications.

Thank You !

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Irreversible Statistical Mechanics of Incompressible Hydromagnetic Turbulence*

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(Received April 8, 1957)

The irreversible statistical mechanics of incompressible hydromagnetic turbulence driven by external forces is treated by methods which do not require that the system be close to a state of detailed balance. The equations of motion are expressed in terms of linearly independent modes formed from the wave-vector components of velocity and magnetic fields, and the nonlinear interaction is exhibited as the sum of individually conservative three-mode interactions. A fundamental statistical equation is constructed giving necessary and sufficient conditions for all members of a distribution of time-functions to satisfy the equations of motion; it involves only second-, third-, and fourth-order distribution moments. A variational criterion is proposed for specifying a distribution consistent with the fundamental equation under physically appropriate constraints. It leads to a complete formal solution of the statistical problem. This solution is not exploited. Instead, two statistical hypotheses based on the assumption of high mode density are advanced. With their aid, each three-mode interaction is treated as a small perturbation on the motion due to all the three-mode interactions and the external forces. The moments in the fundamental equation for the stationary case thereby are expressed in terms of the diagonal elements of the time-covariance matrix and distribution-averaged infinitesimal-impulse-response matrix of the system. Closed equations are obtained which fix these matrix elements in terms of the covariance matrix of the external forces. If the statistical hypotheses are sound, this provides a theory of unbounded turbulence (infinite mode density) driven by Gaussian-distributed homogeneous forces which is exact at all Reynolds numbers based on rms velocity and the macroscale determined by the driving forces. The general theory is specialized to obtain integro-differential equations determining the covariance scalars and modal impulse-response functions for stationary, isotropic hydromagnetic turbulence. In the nonmagnetic case, the asymptotic inertial-range solution yields the wave-number spectrum $E(k)=2\pi c(\epsilon v_0)^{\frac{1}{2}}k^{-\frac{3}{2}}$ and the modal time-autocorrelation function $J_1(2v_0k\tau)/(v_0k\tau)$, where v_0 is the rms velocity in any direction, ϵ is the mean rate of energy-cascade/unit-mass, and c is a universal number fixed by the theory. This contradicts the Kolmogorov similarity hypotheses; independent arguments are advanced against the latter.

Comparison of Truncated 3D Euler Energy Spectra