Transition in energy spectrum for forced stratified turbulence

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Energy spectrum for forced stably stratified turbulence is investigated numerically. The 3D Navier-Stokes equations under the Boussinesq approximation,

$$(\partial_t - \nu\nabla^2)\mathbf{u} = -\mathbf{u}\cdot\nabla\mathbf{u} - \nabla p + \theta\hat{\mathbf{z}} \tag{1}$$

$$(\partial_t - \kappa\nabla^2)\theta = -N^2 w - \mathbf{u}\cdot\nabla\theta \tag{2}$$

$$\nabla\cdot\mathbf{u} = 0 \tag{3}$$

are solved pseudo-spectrally with stochastic forcing applied to the largest velocity scales. Following Lesieur & Rogallo (1989) and Carnevale *et. al.*(2001), spectral eddy viscosity, $\nu_t(k) = (a_1 + a_2\exp(-a_3 k_c/k))\sqrt{E(k_c)/k_c}$, is used for small scale dissipation. Using toroidal-poloidal decomposition (Craya-Herring decomposition), the velocity field is divided into the vortex mode (ϕ_1) and the wave mode (ϕ_2). With the initial kinetic energy being zero, the ϕ_1 spectra as a function of horizontal wave numbers, $k_\perp$, first develops a $k_\perp^{-3}$ spectra for the whole $k_\perp$ range, and then $k_\perp^{-5/3}$ part appears at large $k_\perp$ with rather a sharp transition wave number. Meanwhile the ϕ_2 spectra shows $k_\perp^{-2}$ first, and then $k_\perp^{-5/3}$ part appears with the same transition wave number.

Figure 1 shows ϕ_1(left) and ϕ_2(right) spectra for $N^2 = 1, 10, 50, 100$ (from the top to the bottom). The transition is clearly observed for both cases, and the transition point depends on the value of N. According to Carnevale *et. al.*, the transition wave number is understood as the Ozmidov scale with a correction by the coefficients of the buoyancy spectrum, $E(k) = \alpha N^2 k^{-3}$, and the Kolmogorov spectrum, $E(k) = C_K\epsilon^{2/3}k^{-5/3}$. By equating these spectra, $k_b \sim (\alpha/C_K)^{3/4}\sqrt{N^3/\epsilon}$ is obtained for the transition wavenumber.

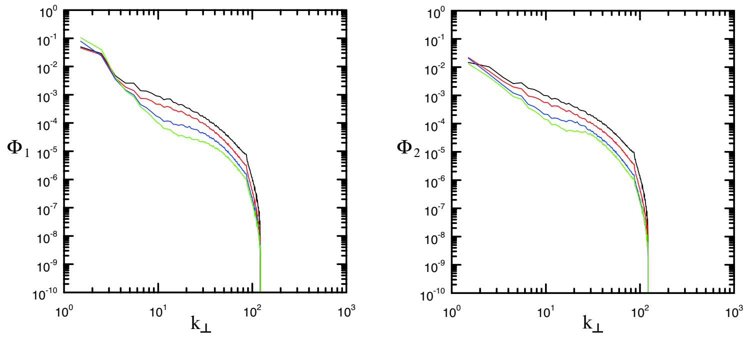

Figure 1: ϕ_1 (left) and ϕ_2 (right) spectra as a function of $k_\perp$ for $N^2 = 1, 10, 50, 100$

Our calculation shows, however, that the ϕ_1 spectra at small $k_\perp$ seem to have the same slope irrespective of N, which implies a possibility of a different mechanism for producing the $k_\perp^{-3}$ spectrum, perhaps the $k_\perp^{-3}$ spectrum of two-dimensional turbulence.

References

[1] Carnevale,G.F. *et. al*: 2001 J. Fluid Mech. **427** 205–239.

[2] Lesieur, M. & Rogallo, R. 1989 Phys. Fluids A**1** 718–722.