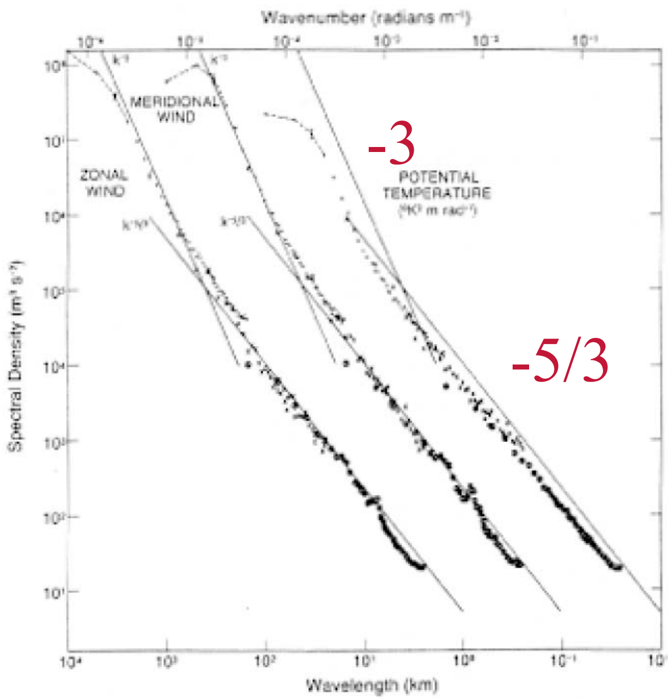
Workshop on Petascale Computing :
Its Impact on Geophysical Modeling and simulation
5-7 May 2008, NCAR Mesa Laboratory

Transition in Energy Spectrum for Forced Stratified Turbulence

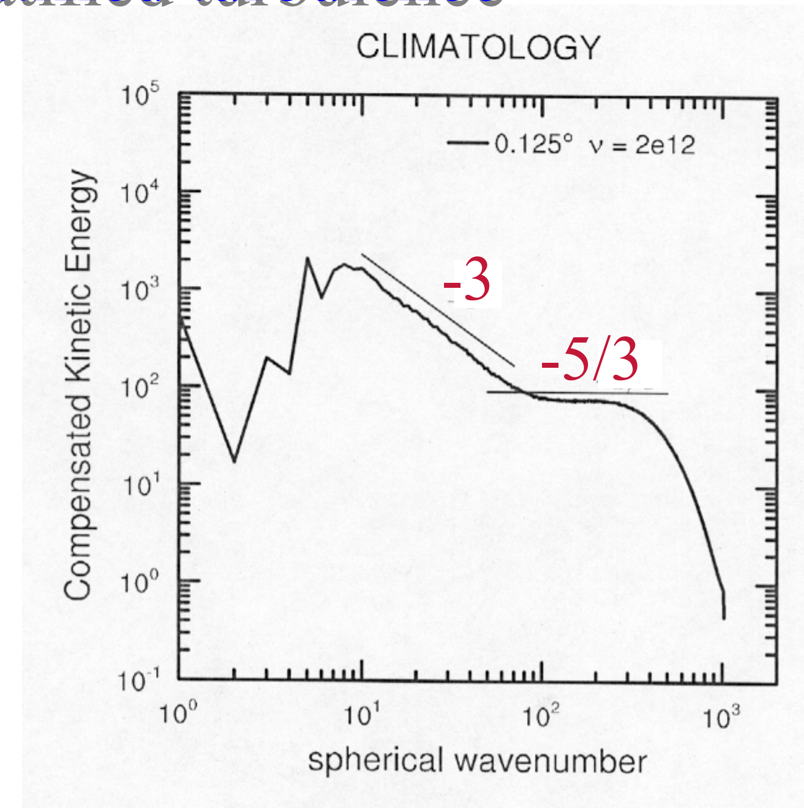
Yoshifumi Kimura
Graduate School of Mathematics, Nagoya University

in collaboration with
Jack Herring (NCAR)

Transition in Energy Spectrum for Rotating and Stratified turbulence



Nastrom-Gage's atmospheric observation (1985) (*JAS* **42** 950-960.)



Mark Taylor's climate model simulation (2008)
(CCSM project at NCAR)

Both stratification and rotation are essential

-3 : enstrophy cascade for Quasi-Geostrophic turbulence (~2D)

-5/3 : Kolmogorov turbulence (3D)

The objective of this talk is

to see if there is also a transition in the energy spectrum for flows with only stratification.

Navier-Stokes equation with the Boussinesq approximation

$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u}\cdot\nabla)\mathbf{u} = -\nabla p + \nu\nabla^2\mathbf{u} + \theta\hat{\mathbf{z}}$$

$$\frac{\partial \theta}{\partial t} + (\mathbf{u}\cdot\nabla)\theta = \kappa\nabla^2\theta - N^2 w$$

$$\nabla\cdot\mathbf{u} = 0$$

where

$\mathbf{u} = (u,\ v,\ w)$: velocity

θ : temperature fluctuations

$N^2 = \dfrac{g\alpha}{T_0}\dfrac{\partial \overline{T}}{\partial z}$: Brunt - Väisälä frequency

Numerical Methods

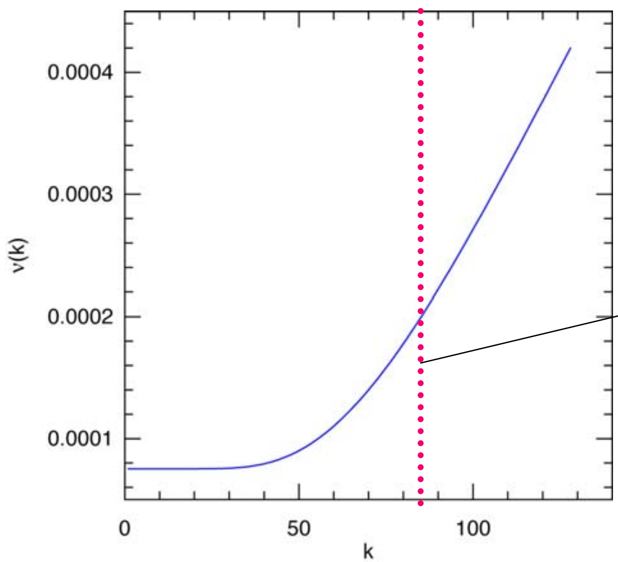
- forced simulations
- 2π-periodic box with up to 512^3 grid points
- 3rd order time-marching scheme
- Initial energy spectrum : $E(k) = 0$
- Force horizontal velocity components
- Add red noise to modes within a wave number band

Forced turbulence simulation

- Starting from energy 0.
- Putting stochastic noise (O-U process) at low wave number modes.
- Using spectral eddy viscosity,

$$\nu(k) = \left[A + B\exp(-Ck_c/k)\right]\left(E(k_c)/k_c\right)^{1/2}$$

k_c : *cutoff wave number* $\quad (A, B, C) = (0.267, 9.21, 3.03)$



Lesieur, M. & Rogallo, R. *Phys. Fluids* (1989) A1, 718-723.

cutoff wave number for 256^3 simulation

“Craya-Herring” decomposition

$$\mathbf{e}_1(\mathbf{k}) = \frac{\mathbf{k}\times\mathbf{g}}{|\mathbf{k}\times\mathbf{g}|} \quad \mathbf{e}_2(\mathbf{k}) = \frac{\mathbf{k}\times(\mathbf{k}\times\mathbf{g})}{|\mathbf{k}\times(\mathbf{k}\times\mathbf{g})|} \quad \mathbf{e}_3(\mathbf{k}) = \frac{\mathbf{k}}{|\mathbf{k}|}$$

orthnormal coordinates

$$\tilde{\mathbf{u}}(\mathbf{k}) = \phi_1\mathbf{e}_1(\mathbf{k}) + \phi_2\mathbf{e}_2(\mathbf{k}) \qquad \tilde{\theta}(\mathbf{k}) = N\ \phi_3\mathbf{e}_3(\mathbf{k})$$

new variables

$$\partial_t\tilde{\mathbf{u}} = -\mathrm{i}\,\mathbf{k}\tilde{p} - \mathbf{g}\tilde{\theta} - 2\Omega\times\tilde{\mathbf{u}}$$
$$\partial_t\tilde{\theta} = N^2\tilde{u}_3$$

linear Navier-Stokes eqs. in Fourier space

$$\boxed{\partial_t\begin{pmatrix}\phi_1\\\phi_2\\\phi_3\end{pmatrix} = \begin{pmatrix}0 & 2\Omega\cos\xi & 0\\ -2\Omega\cos\xi & 0 & N\sin\xi\\ 0 & -N\sin\xi & 0\end{pmatrix}\begin{pmatrix}\phi_1\\\phi_2\\\phi_3\end{pmatrix}}$$

ϕ_1 : horizontal, vortical

ϕ_2 : wavy

Enstrophy contours for 512^3 simulation side view (forced turbulence)

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Sorenson Video 3 êLí£ÉvÉçÉOÉâÉÄ
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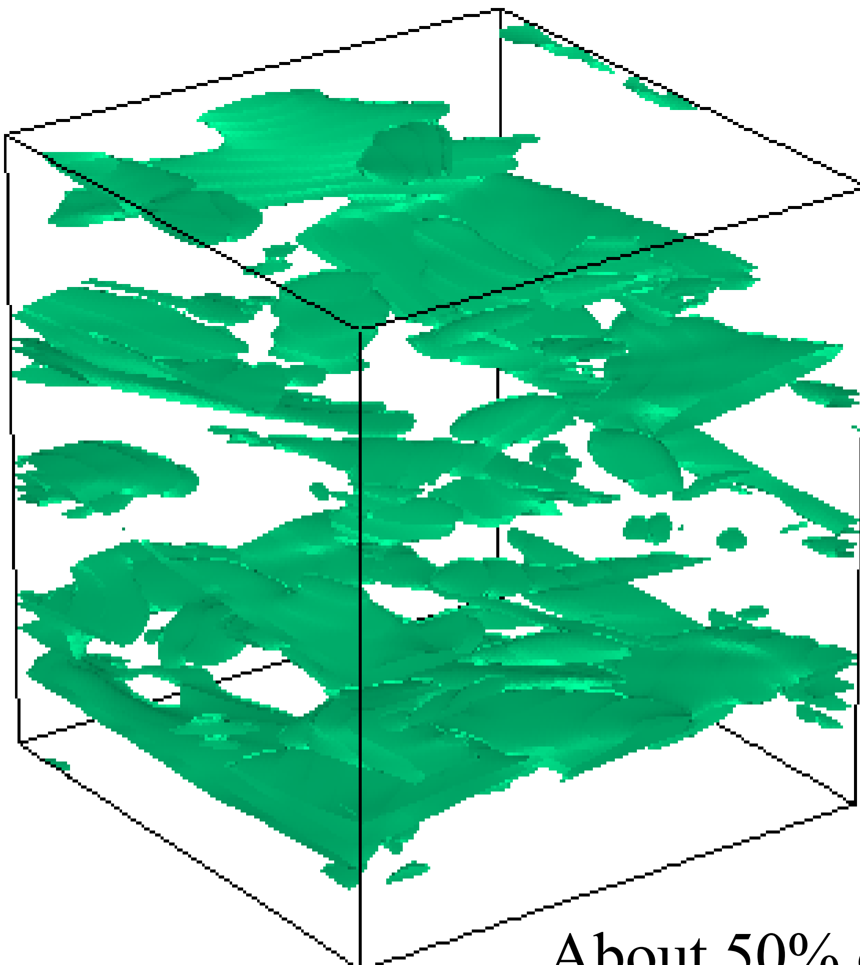
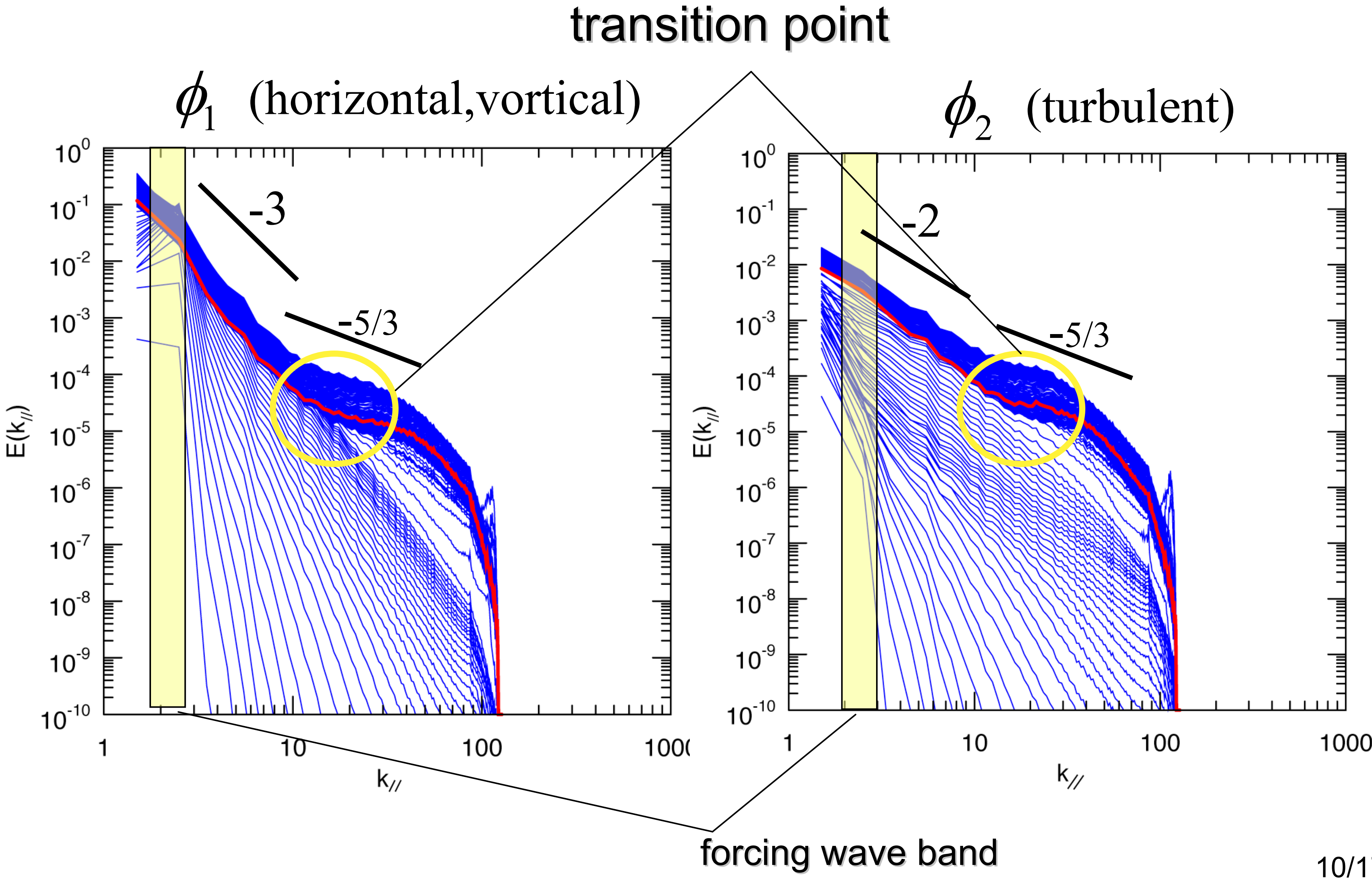
Thanks to John Clyne (NCAR)

Enstrophy contours for forced turbulence



About 50% of the maximum

History of energy spectra (ϕ_1 & ϕ_2, N^2=100)

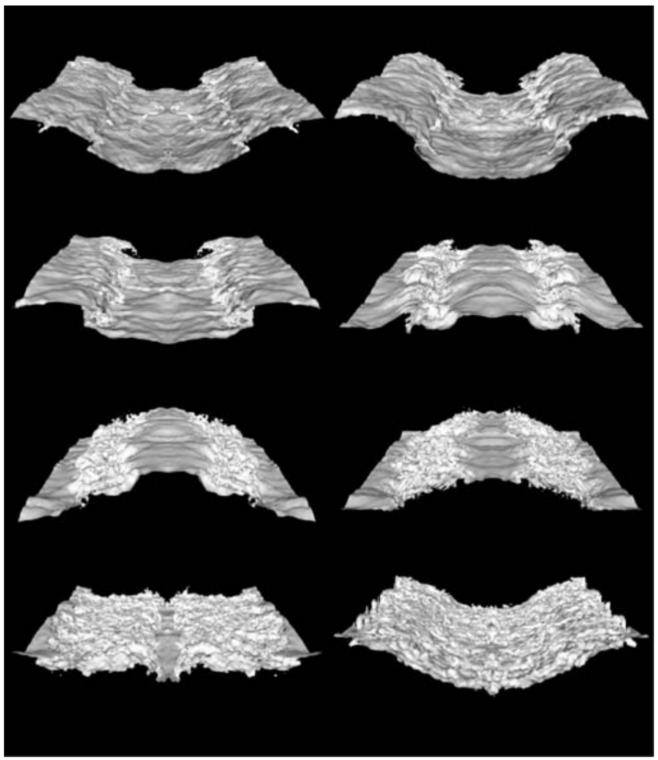


Transition in stratified turbulence spectra

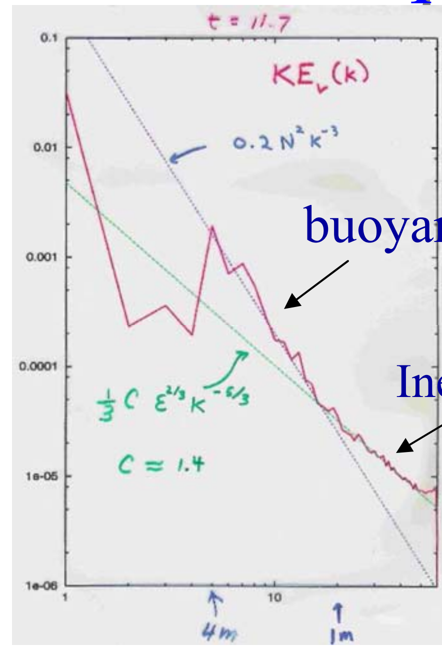
Buoyancy- to inertial range transition in forced stratified turbulence

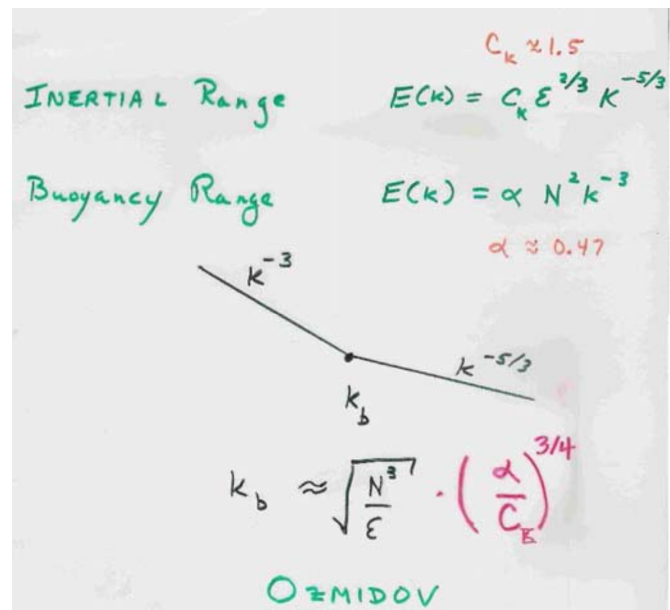
Carnevale, Briscolini & Orlandi

J. Fluid Mech. (2001) **427** pp. 205-239.

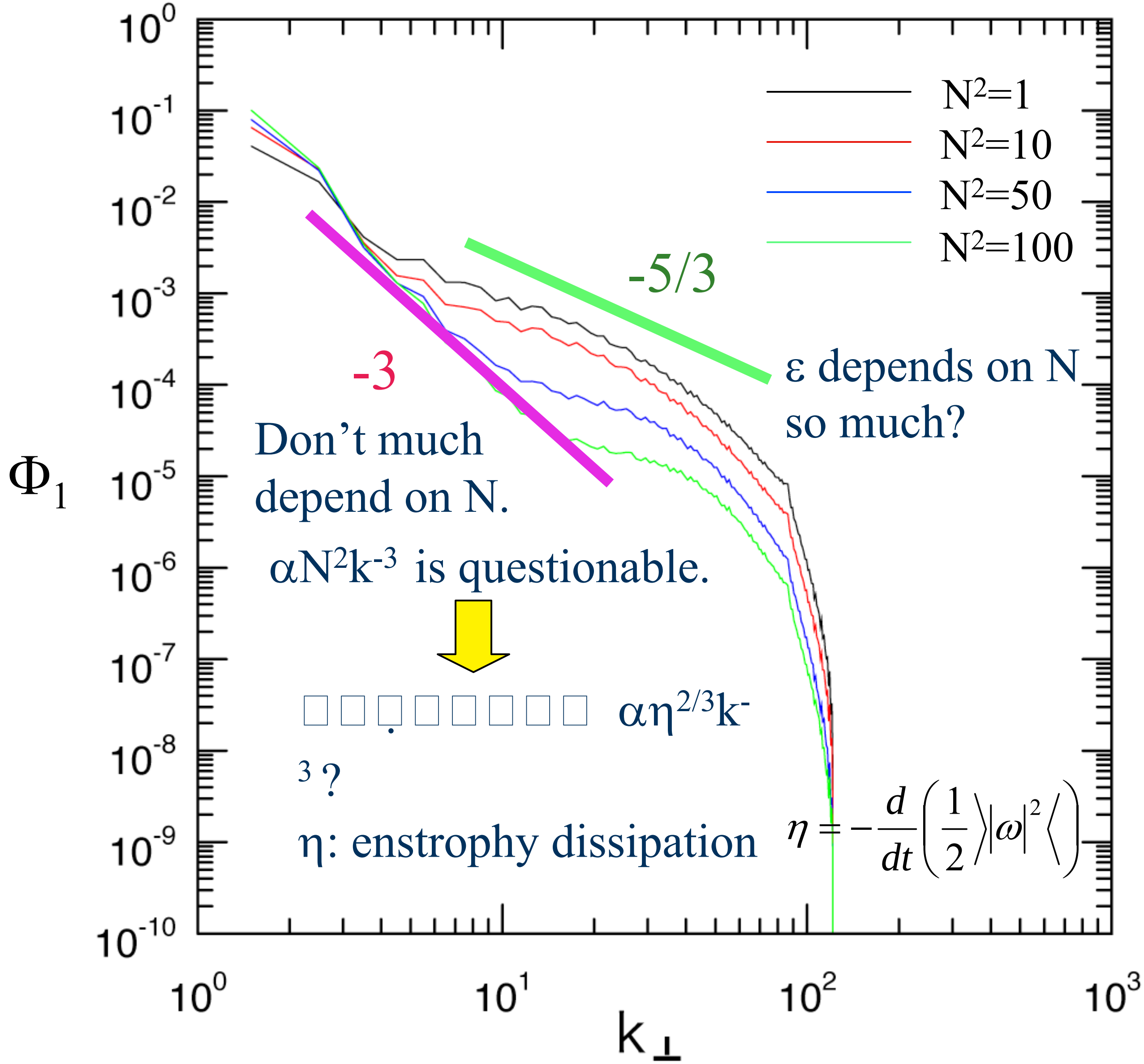


equi-density surface





Energy spectra for various N



Energy and Enstrophy dissipations

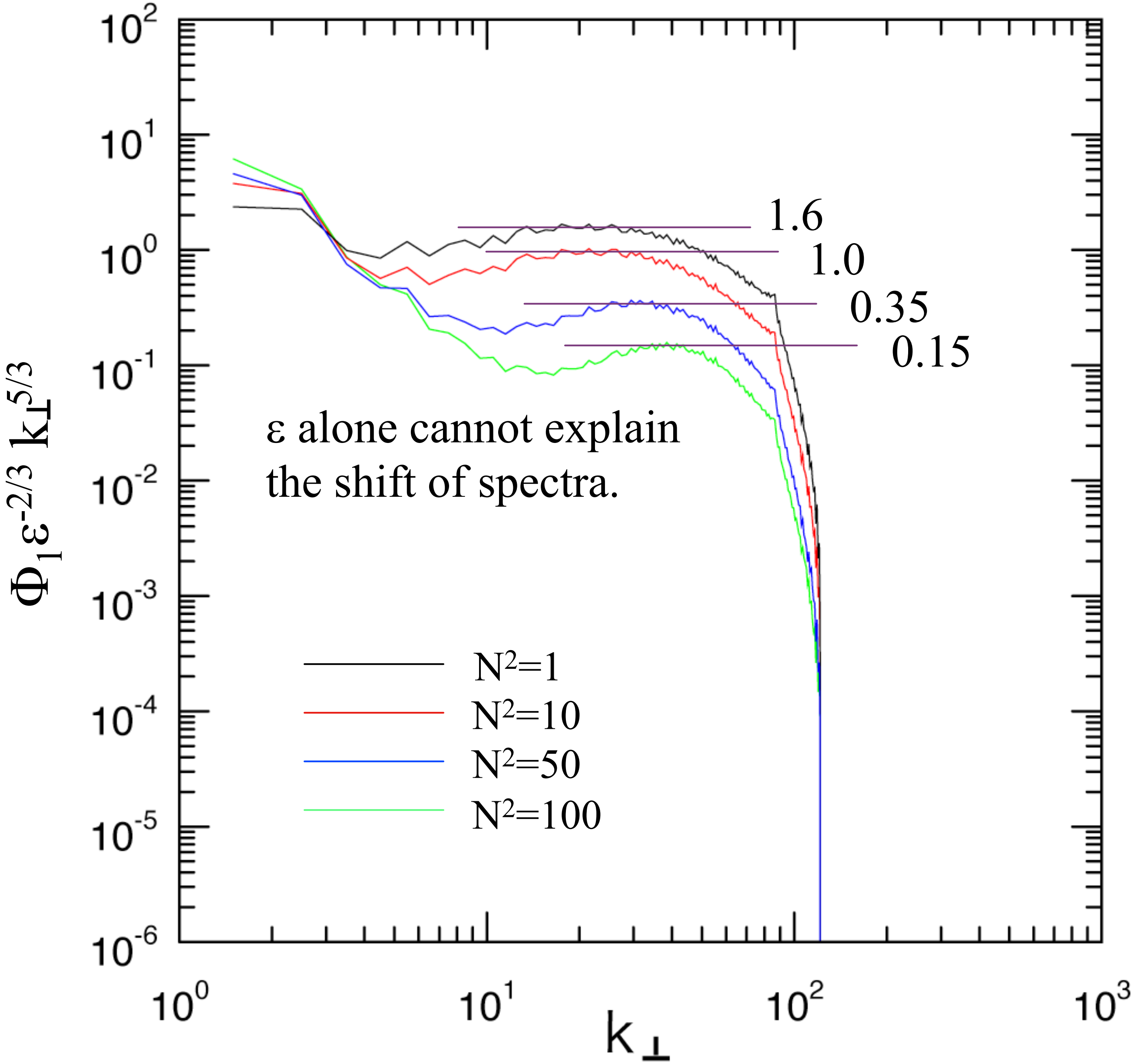
$$\varepsilon = 2\int_0^\infty \nu_{\mathrm{eddy}}(k)k^2 E(k)dk \qquad \eta = 2\int_0^\infty \nu_{\mathrm{eddy}}(k)k^4 E(k)dk$$

$N^2 = 1$	0.625 Ξ 10^{-2}	0.143 Ξ 10^{2}
$N^2 = 10$	0.626 Ξ 10^{-2}	0.124 Ξ 10^{2}
$N^2 = 50$	0.629 Ξ 10^{-2}	0.815 Ξ 10^{1}
$N^2 = 100$	0.575 Ξ 10^{-2}	0.615 Ξ 10^{1}

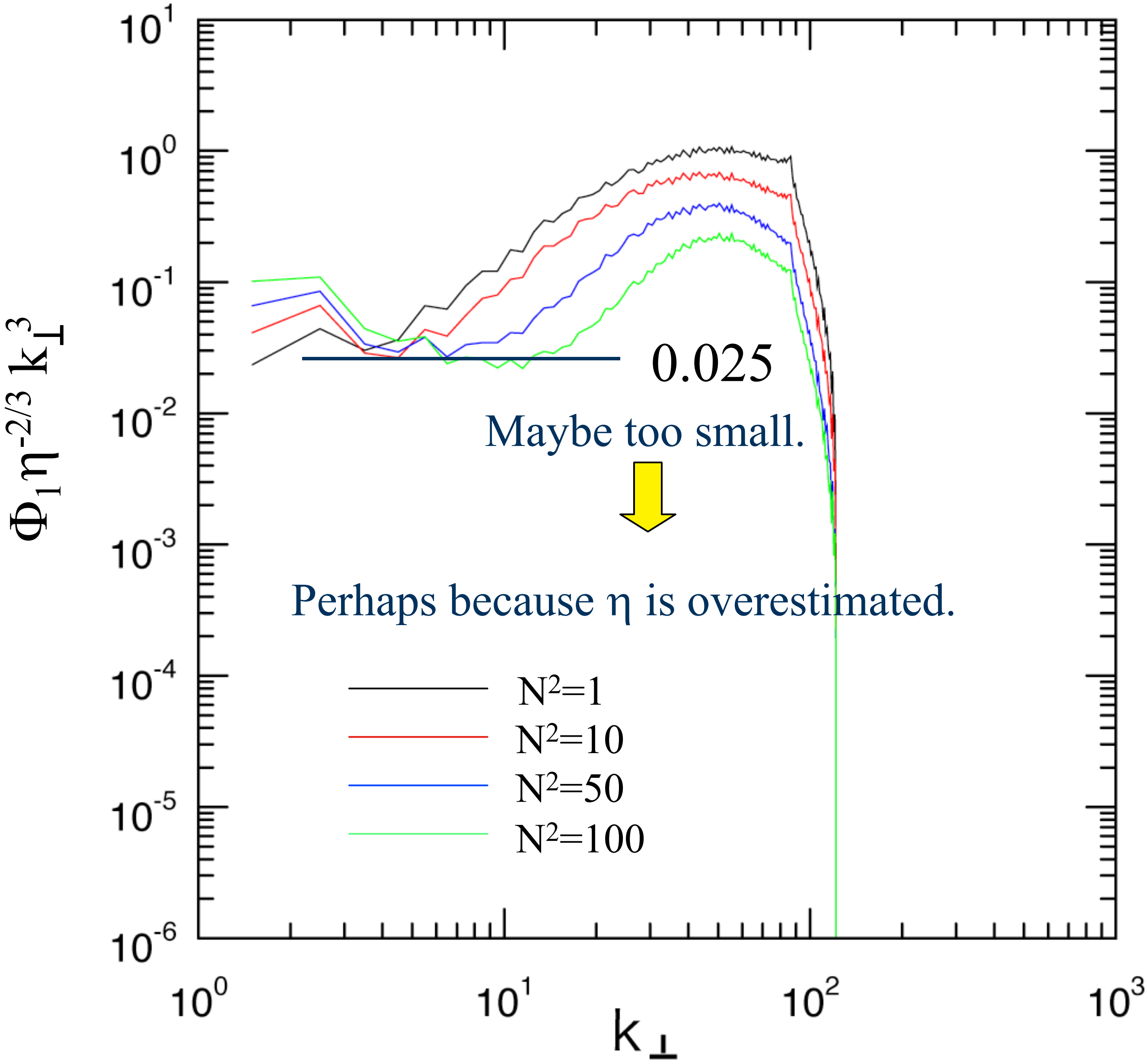
Not big change!

Both ε and η are small scale quantities !

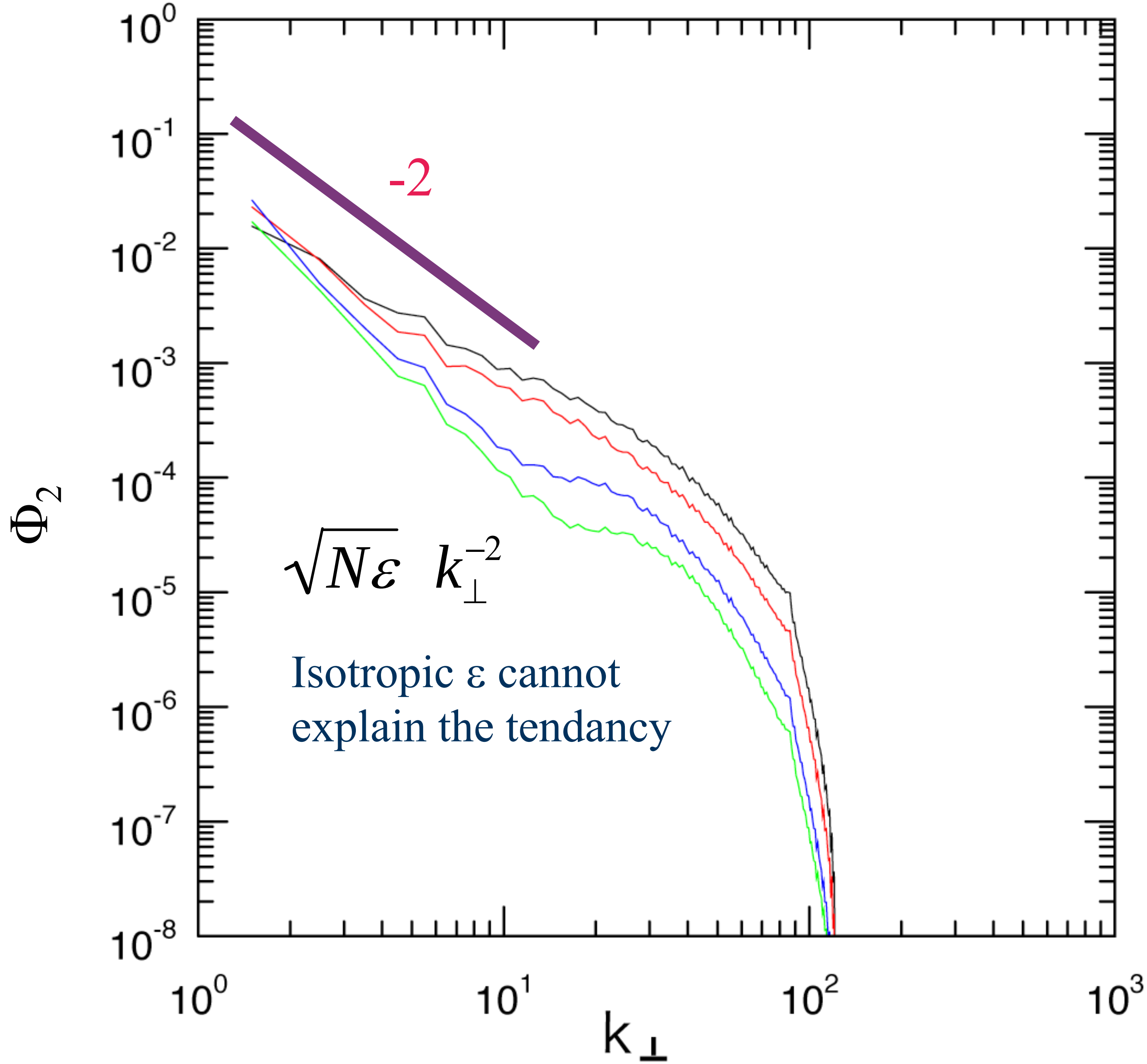
Compensated Φ_1 spectra (high wave number)



Compensated Φ_1 spectra (low wave number)



Φ_2 spectra



Difficulties in the problem

- The spectra are extended in the wide range of wave number space which have different features due to different physical (or instability) mechanisms.

- Parametrization of the large scale spectra needs small scale information (on dissipation.)

- Slow processes (such as spectrum building, or large scale structure formation) exist in the calculation.

Those are some reasons we need “petascale computing”