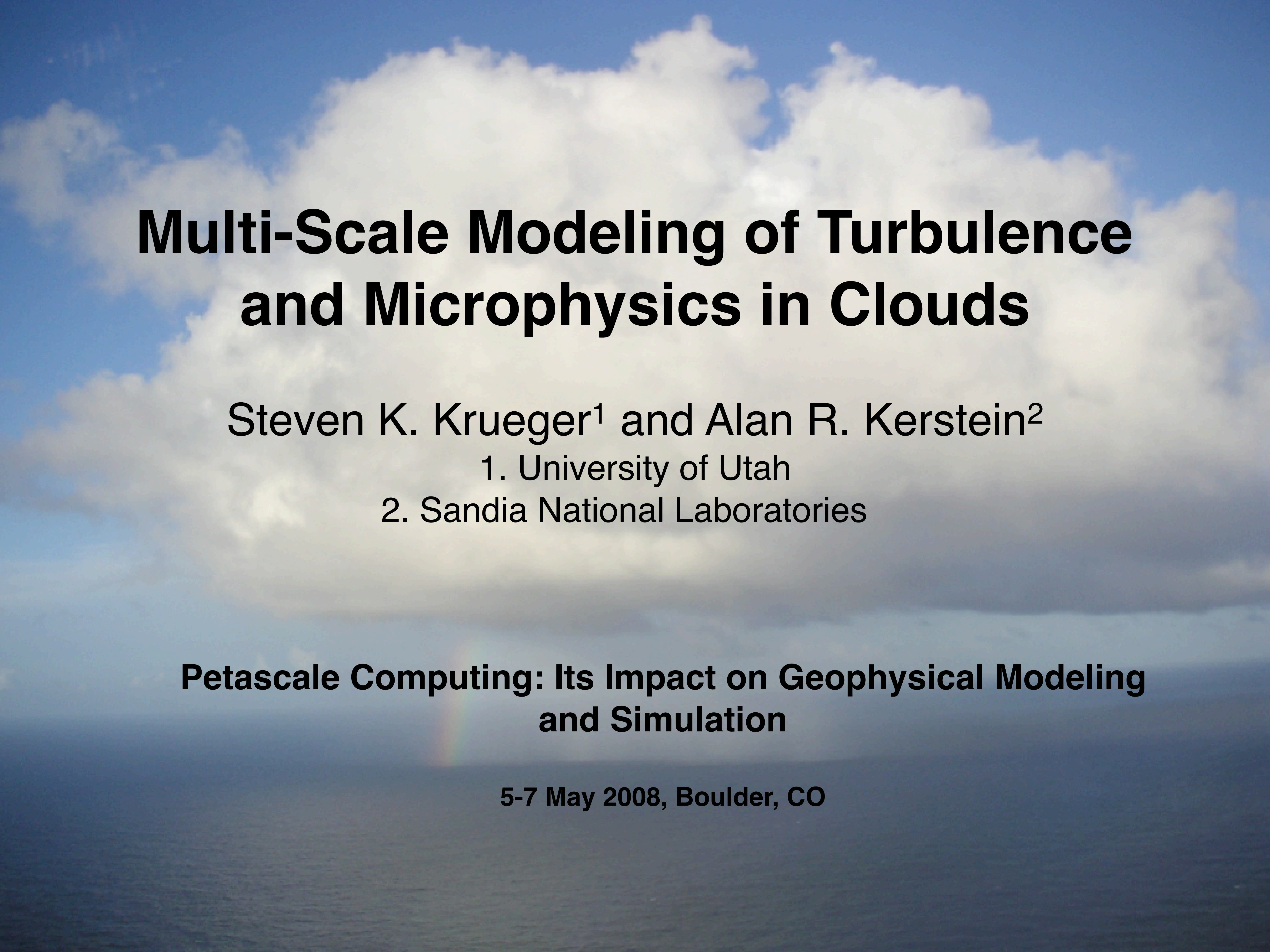

Multi-Scale Modeling of Turbulence and Microphysics in Clouds

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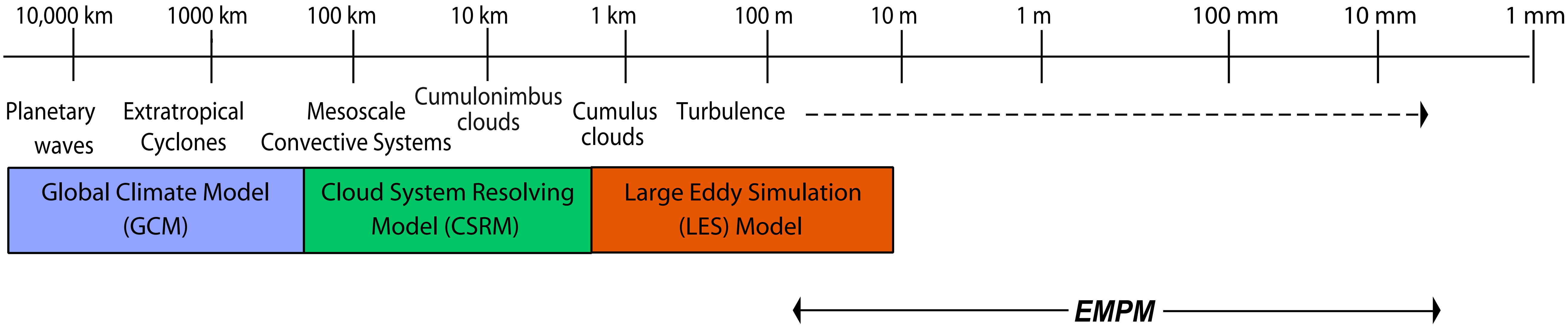
Petascale Computing: Its Impact on Geophysical Modeling and Simulation

5-7 May 2008, Boulder, CO

Scales of Atmospheric Motion



10,000 km
1000 km
100 km
10 km
1 km
100 m
10 m
1 m
100 mm
10 mm
1 mm

Planetary waves
Extratropical Cyclones
Mesoscale Convective Systems
Cumulonimbus clouds
Cumulus clouds
Turbulence

Global Climate Model (GCM)
Cloud System Resolving Model (CSRM)
Large Eddy Simulation (LES) Model

EMPM

The smallest scale of turbulence is the Kolmogorov scale:

$$\eta \equiv (\nu^3/\epsilon)^{1/4}$$

For $\epsilon = 10^{-2}$ m^2 s^{-3} and $\nu = 1.5 \times 10^{-5}$ m^2 s^{-1}, $\eta = 0.7$ mm.

Small-scale variability in Cumulus fractus

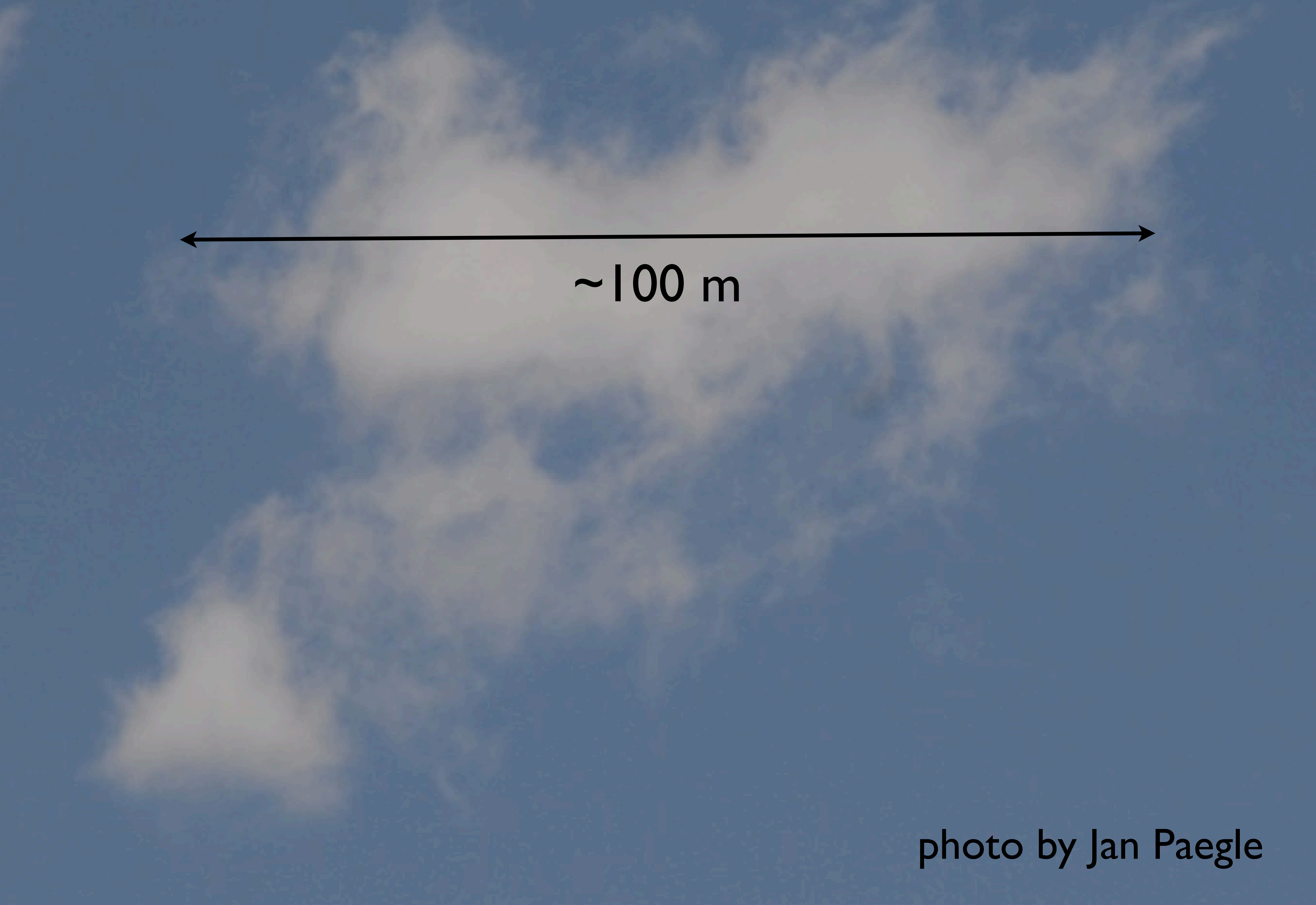
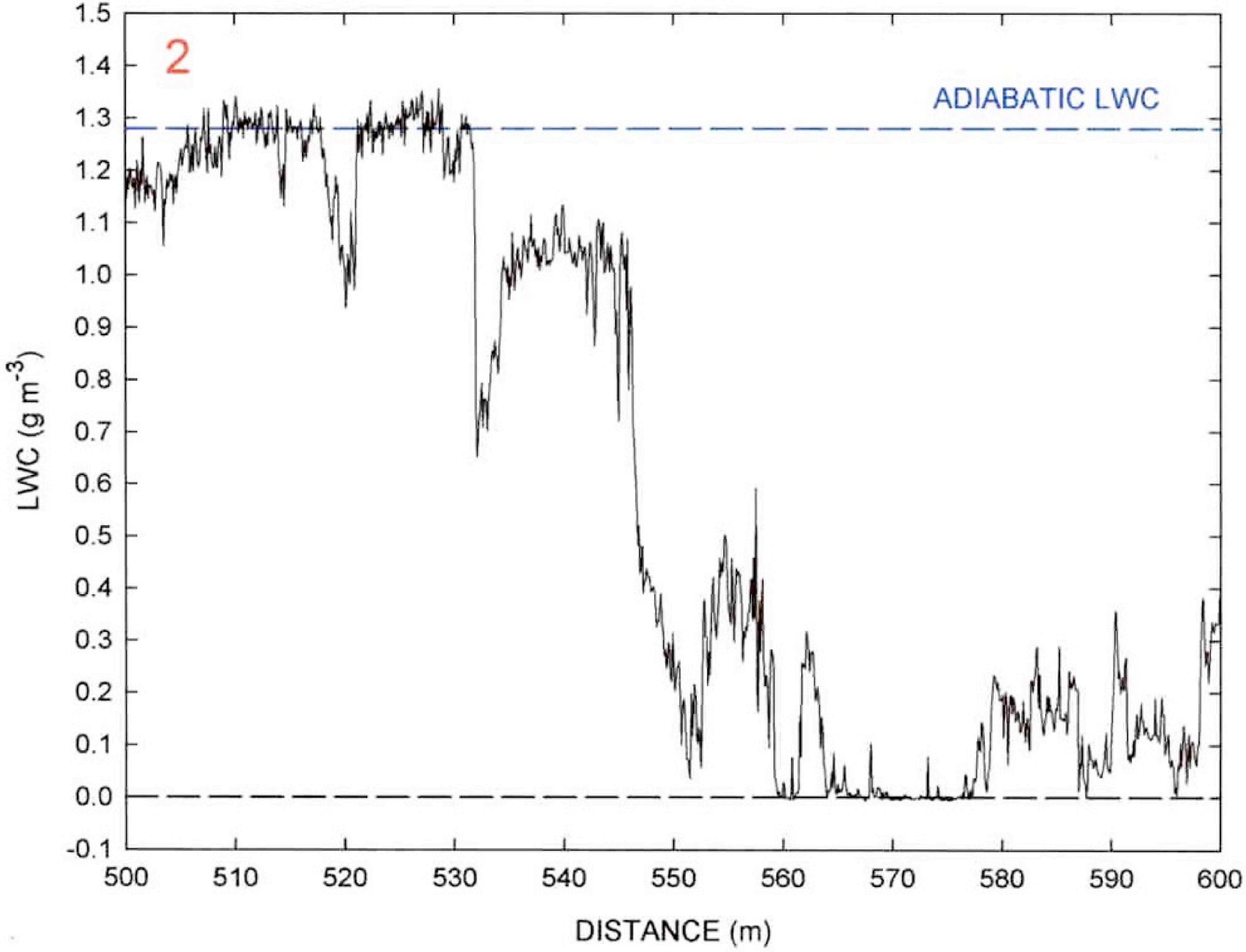
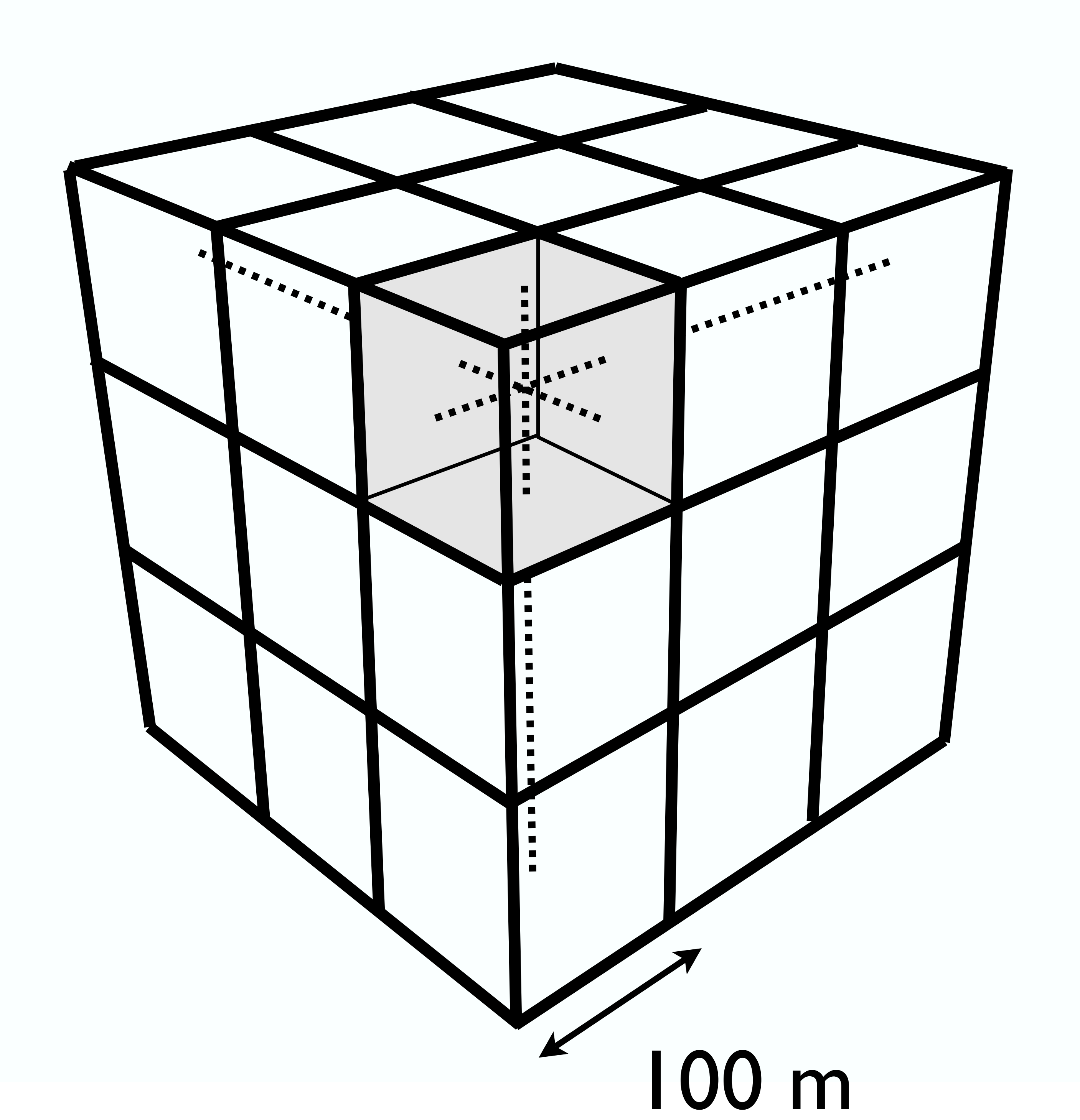


photo by Jan Paegle

Aircraft Measurements of Liquid Water Content



Large-Eddy Simulation (LES) model



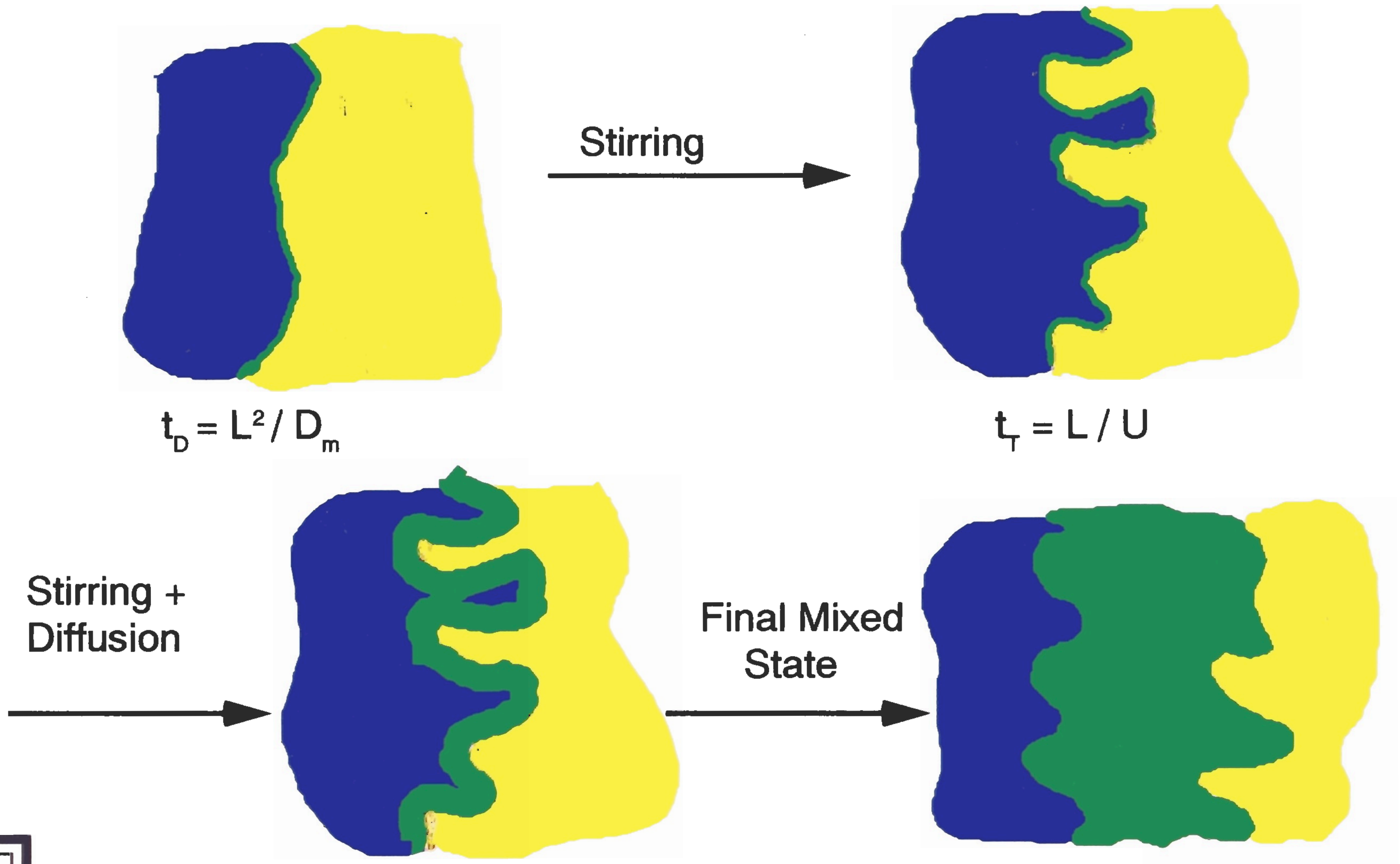
LES Limitations

- The premise of LES is that only the large eddies need to be resolved.
- Why resolve any finer scales? Why resolve the finest scales?
- LES is appropriate if the important small-scale processes can be parameterized.
- Many cloud processes are subgrid-scale, yet can’t (yet) be adequately parameterized.

Subgrid-scale Cloud Processes

- SGS finite-rate mixing of clear and cloudy air slows evaporative cooling and affects buoyancy and cloud dynamics.
- SGS variability of due to entrainment and mixing broadens droplet size distribution (DSD) and increases droplet collision rates.
- SGS turbulence increases droplet collision rates.

Turbulent Mixing: Process by which a fluid with two initially segregated scalar properties mix at the molecular level



Stirring

$t_D = L^2 / D_m$

$t_T = L / U$

Stirring + Diffusion

Final Mixed State

LES of passive scalar in a convective boundary layer
(grid size = 20 m)

Mixing Time Scale

$$\tau = \left(\frac{d^2}{\epsilon}\right)^{1/3},$$

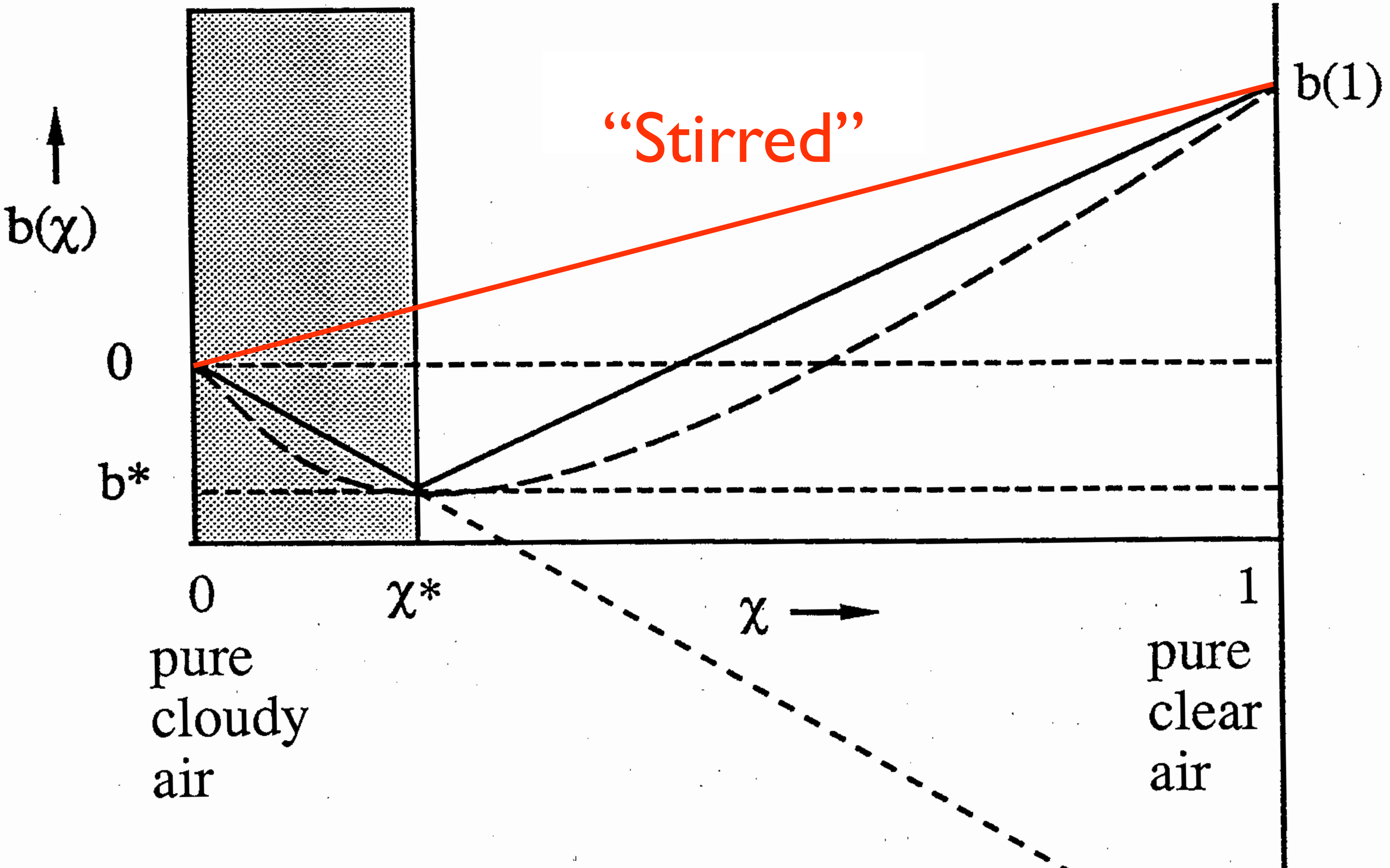
d is entrained blob size, ϵ is dissipation rate of turbulence kinetic energy.

For a **cumulus cloud,** $U \sim 2$ m/s, $L \sim 1000$ m, so $\epsilon \sim U^3/L = 10^{-2}$ m^2/s^3. For $d = 100$ m, $\tau \sim 100$ s.

Classic (instant mixing) parcel model is recovered when

- Entrained blob size, $d \rightarrow 0$
- Turbulence intensity, $\epsilon \rightarrow \infty$

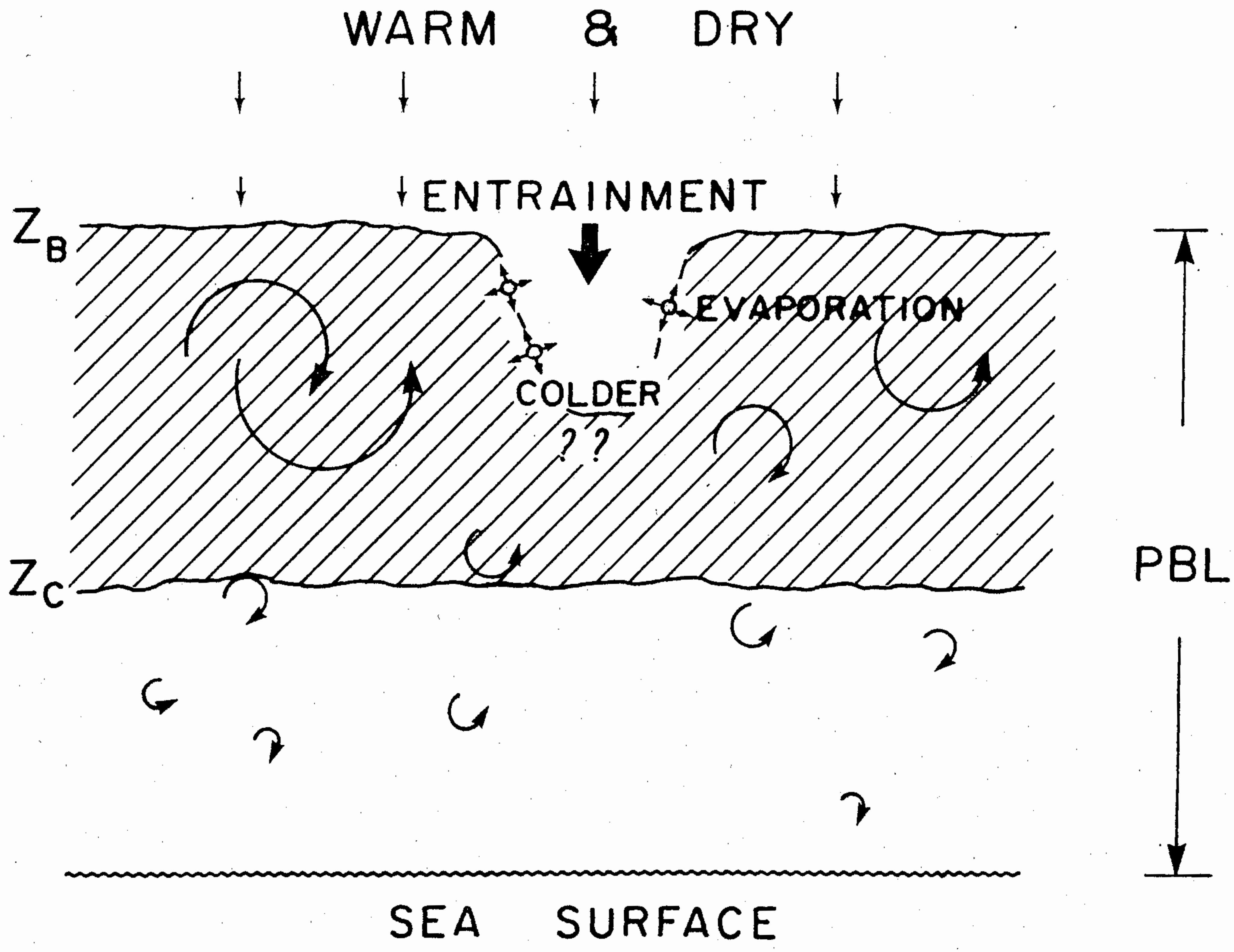
Buoyancy vs Mixture Fraction



χ: mixing fraction of clear air

$b(\chi)$: buoyancy of air with mixing fraction χ

Cloud-top Entrainment Instability (CEI)



Modeling Mixing

- Small-scale turbulence is important for mixing, unlike for dynamics where it provides dissipation for large-scale structures.

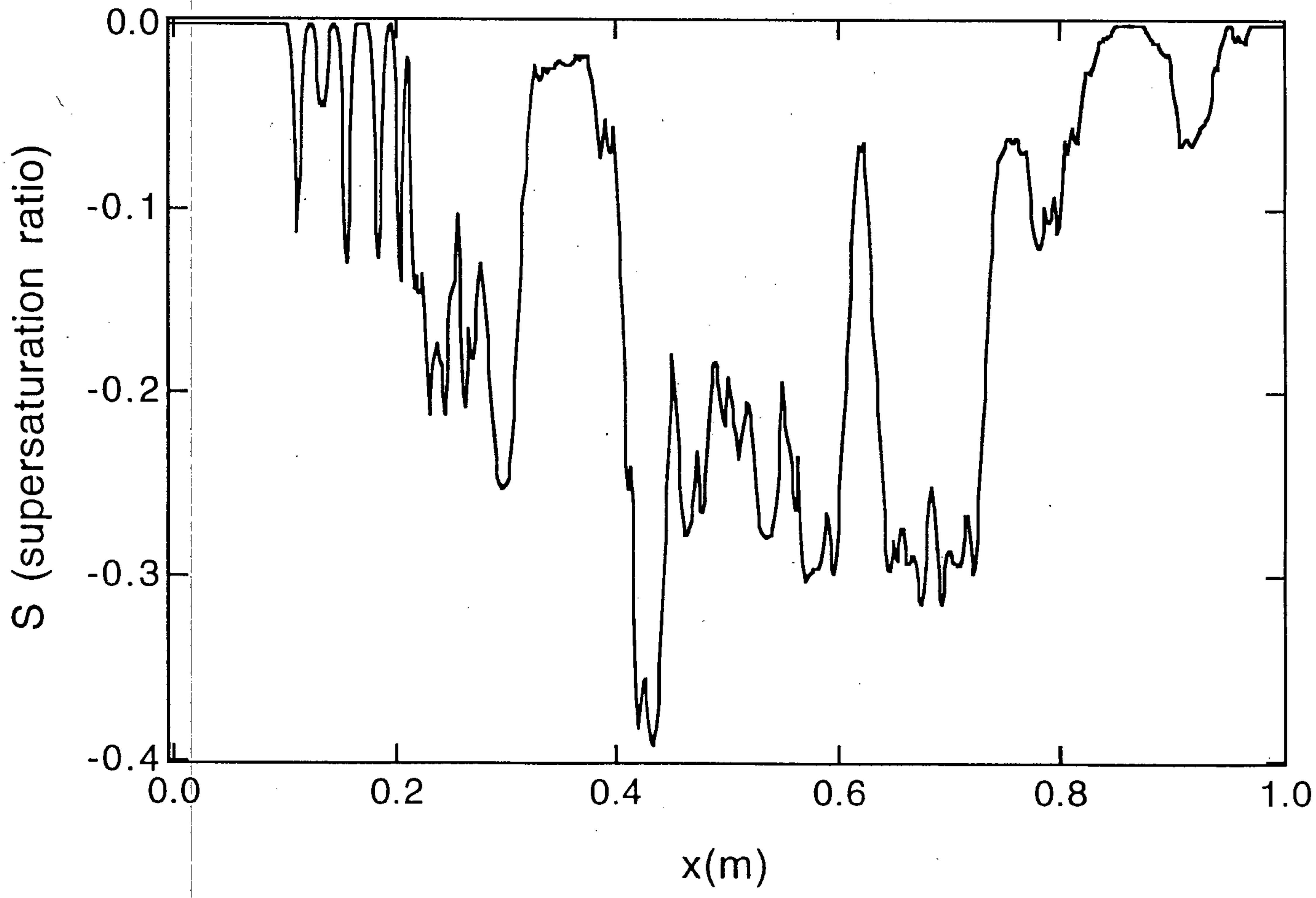
- Most parameterizations of mixing can't accurately predict molecular mixing.

  Most make no distinction between turbulent advection and molecular diffusion.

  This distinction is crucial for an accurate representation of mixing.

- In the atmosphere, the extreme range of length scales present magnifies the difficulties.

Snapshot of supersaturation ratio during mixing



An unsaturated blob is entrained

individual droplet radii

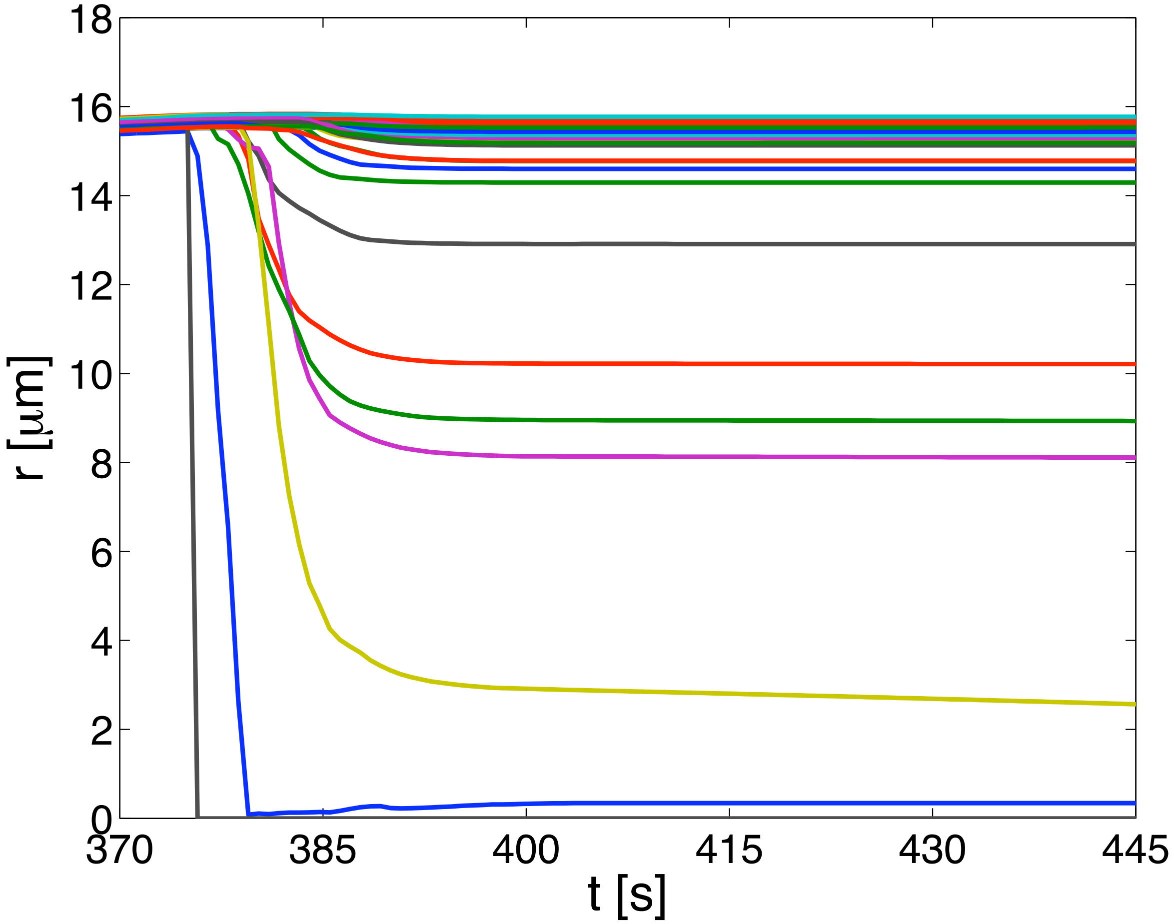


Figure 4.10: Radius histories of 30 droplets for $f = 0.1$ and $RH_e = 0.219$.

width of droplet size distribution

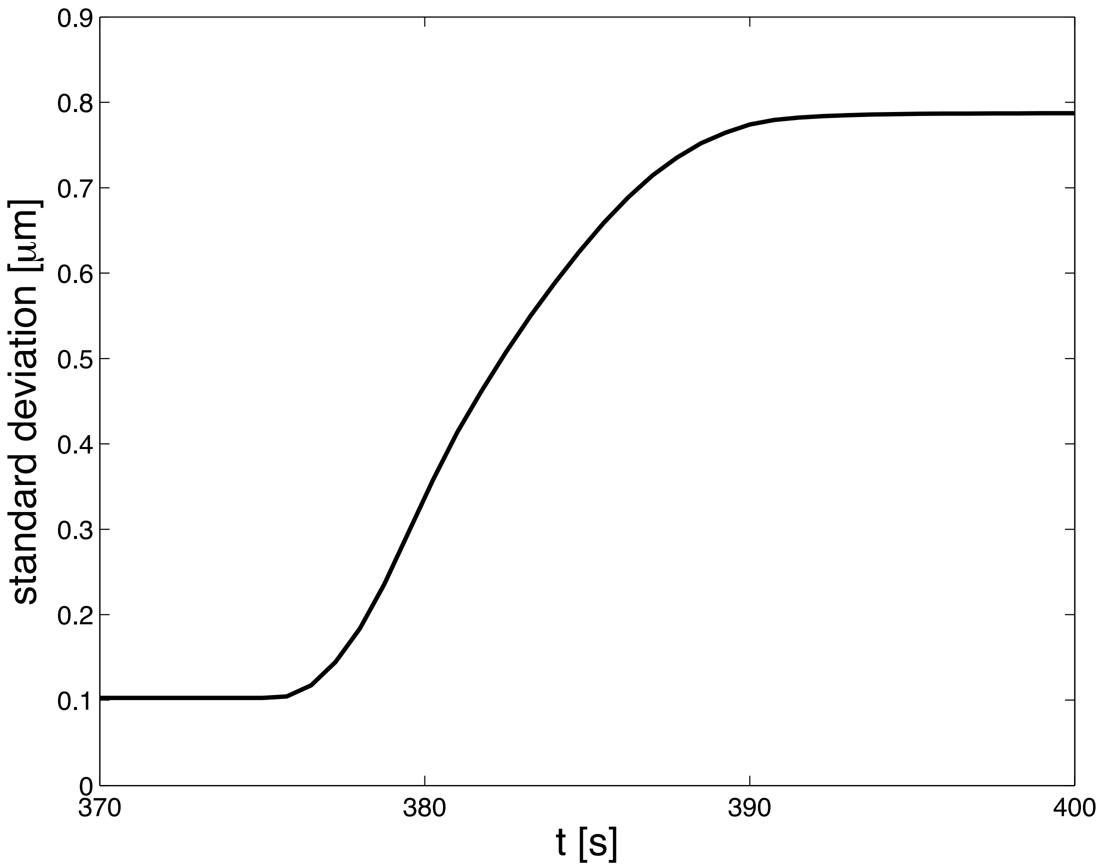


Figure 4.6: Standard deviation of the droplet radii just before entrainment until homogenization for entrainment fraction $f = 0.2$ for the control case.

Large droplets are needed to initiate collision-coalescence growth

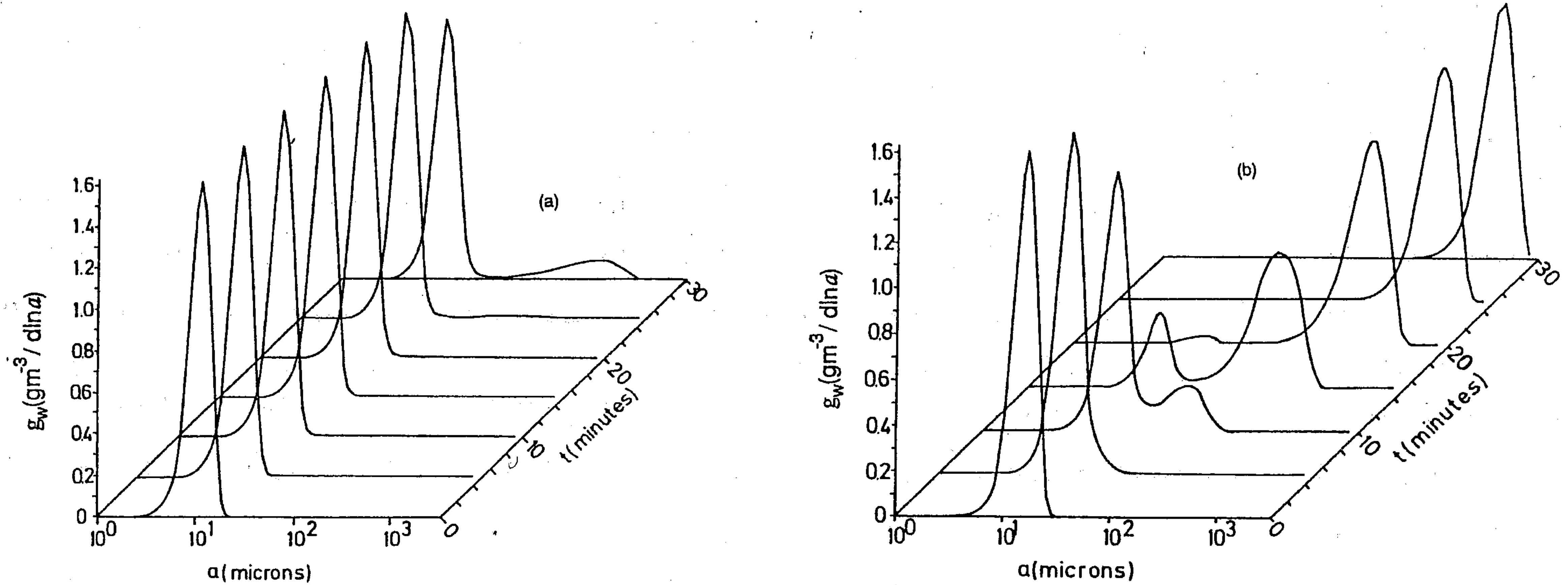
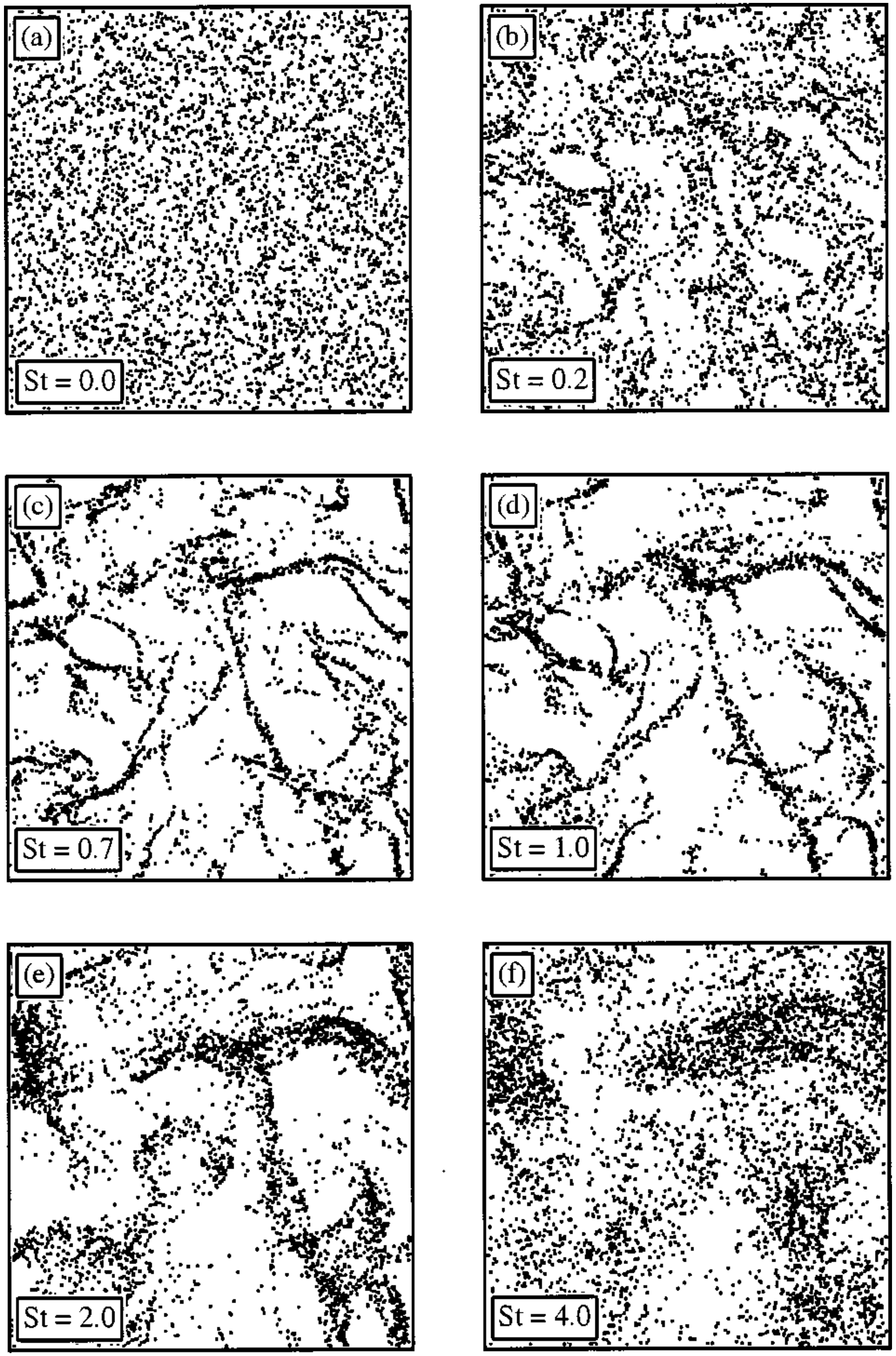


Fig. 15-8: Three-dimensional display of the time evolution of the drop mass distribution function as a function of drop radius, for an assumed initial spectrum of drops growing by collision and coalescence: (a) $\bar{a} = 9\ \mu m$, $N_d = 237\ cm^{-3}$, $w_L = 1\ g\ m^{-3}$; (b) $\bar{a} = 13\ \mu m$, $N_d = 108\ cm^{-3}$, $w_L = 1\ g\ m^{-3}$. Based on the Berry Reinhardt method. (From Flossmann *et al.*, 1985, with changes.)

Large droplets are needed to initiate collision-coalescence growth

- *Processes that may contribute to large droplet production*
  - Entrainment and mixing of unsaturated air
  - Droplet clustering due to turbulence
  - Giant aerosols

Clustering of inertial particles in turbulence increases collision rates



Direct numerical simulation results from Reade & Collins (2000)

Parameterization of SGS Cloud Processes in LES

- SGS mixing is instantaneous in most LES.
- SGS variability does not affect DSD in any LES.
- SGS turbulence affects droplet collision rates in very few LES.

Parameterization of SGS Cloud Processes in LES

(and how to improve, v. 1)

- SGS mixing is instantaneous in most LES. (Decrease grid size or estimate SGS PDF.)
- SGS variability does not affect DSD in any LES. (Decrease grid size.)
- SGS turbulence affects droplet collision rates in very few LES. (Modify collision kernel.)

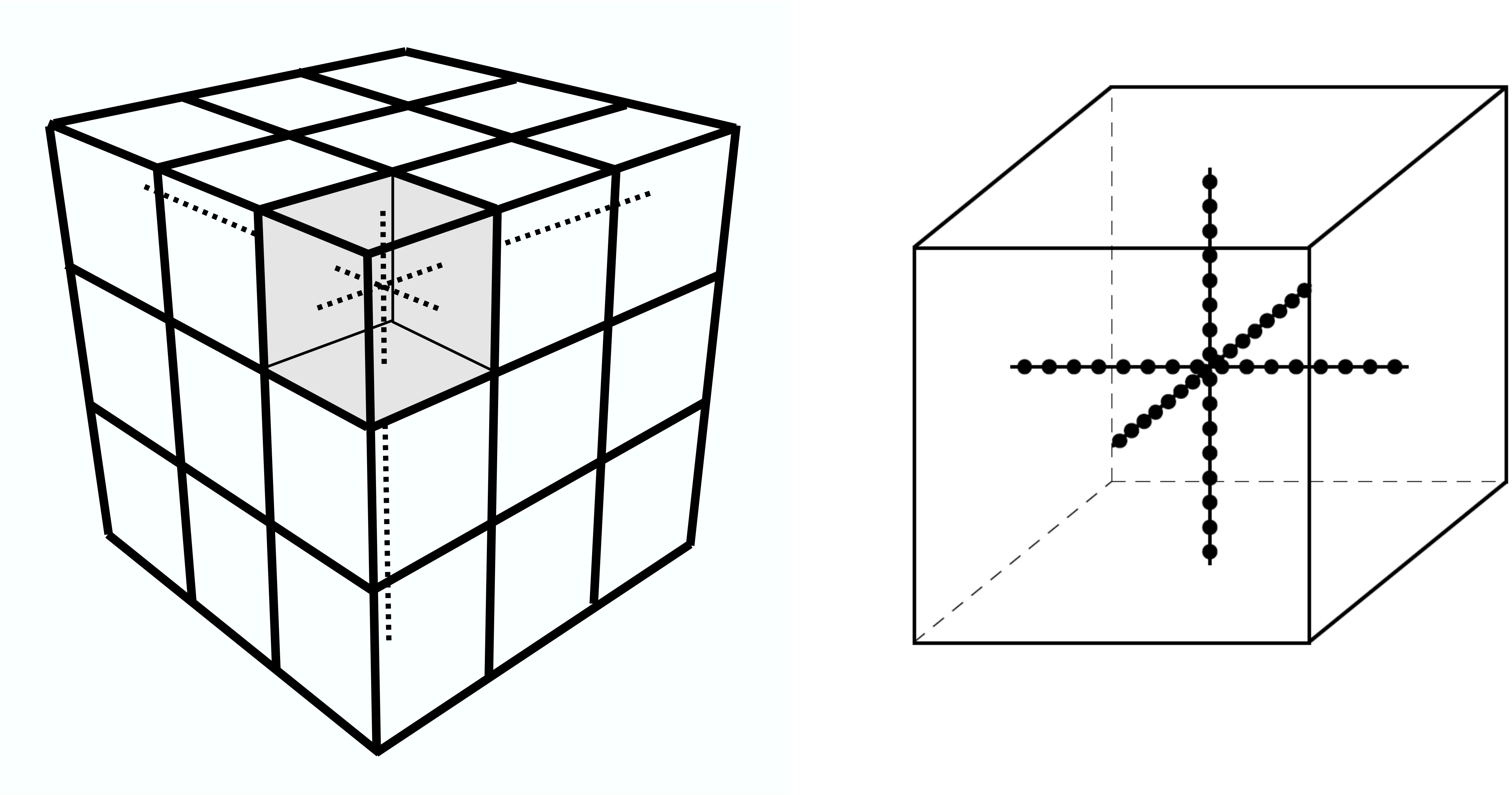
How to resolve the small-scale variability?

- Decrease LES grid size?
  - To decrease LES grid size from 100 m to 1 cm would require 10^{12} grid points per $(100\ \text{m})^3$ and an increase in CPU time of 10^{16}.
  - *This is not possible now or in the forseeable future.*

How to resolve the small-scale variability?

- Decrease dimensionality from 3D to 1D?
  - To decrease grid size from 100 m to 1 cm would require only 10^4 grid points points per $(100\text{ m})^3$.
  - *This is feasible now.*

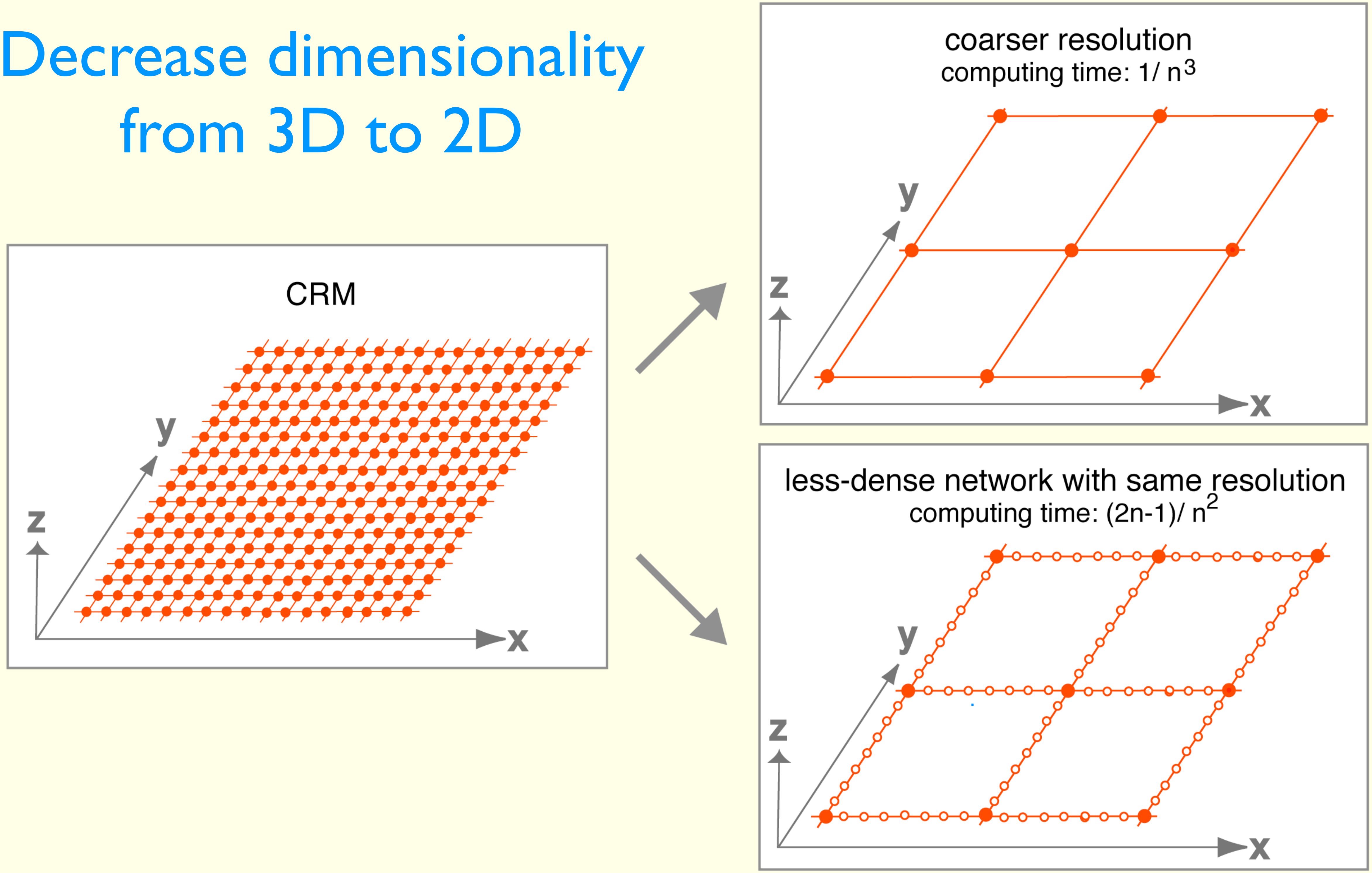
LES with 1D
subgrid-scale model



GENERATION OF LESS-EXPENSIVE MODELS FROM A CRM

(Example: interval factor: n = 8)

Decrease dimensionality from 3D to 2D

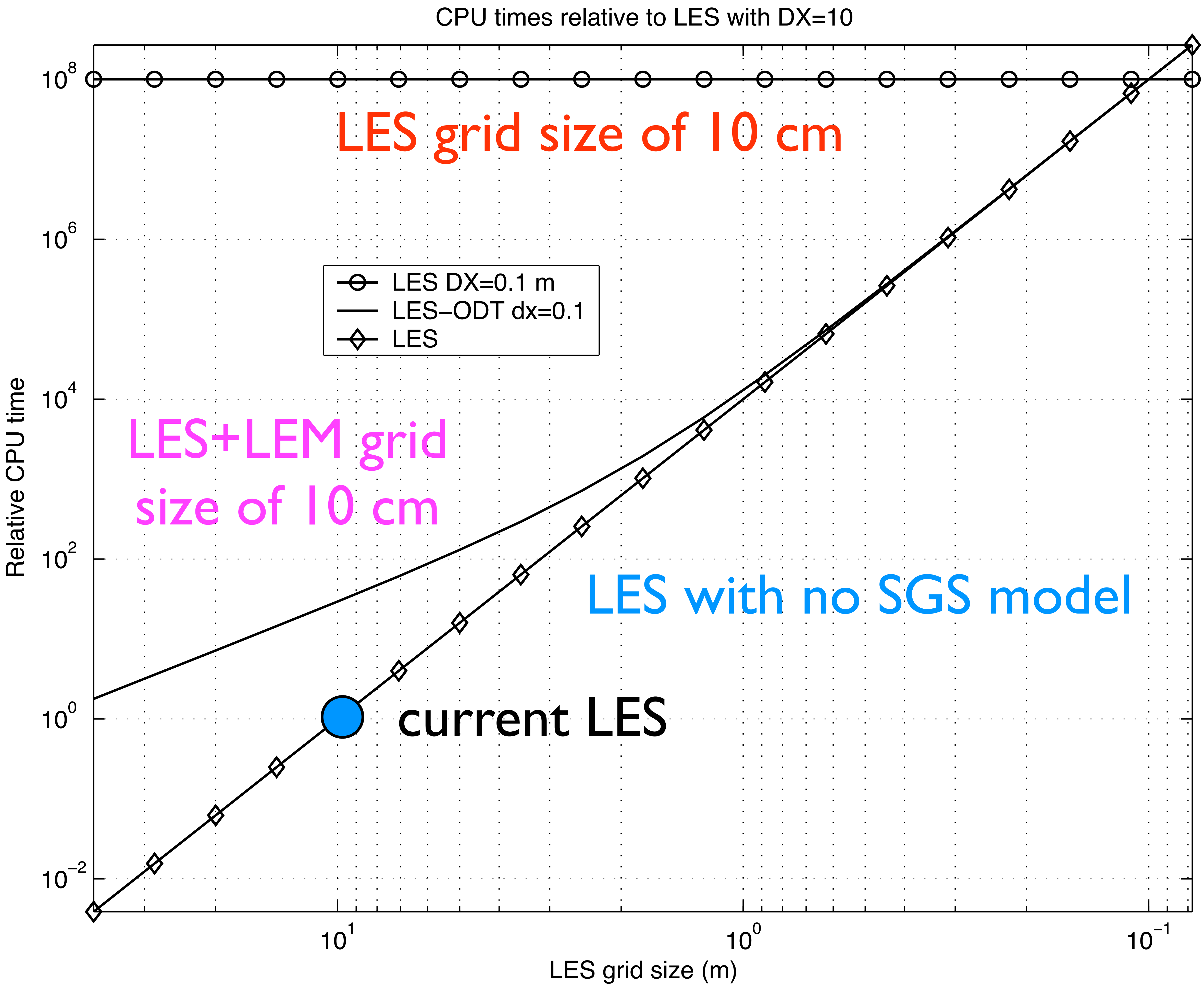


CRM

coarser resolution
computing time: $1/n^3$

less-dense network with same resolution
computing time: $(2n-1)/n^2$

(from Akio Arakawa)



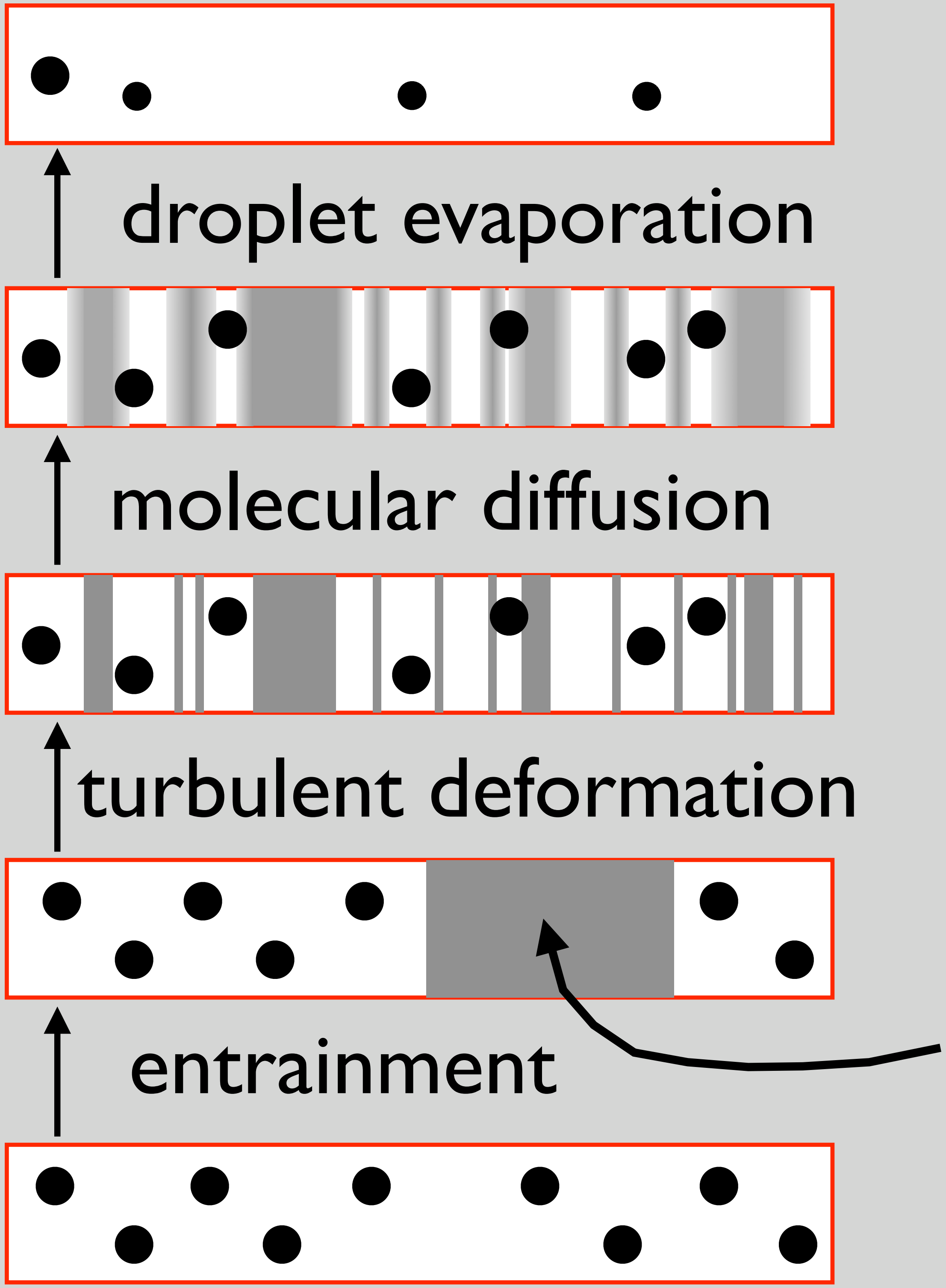
LES grid size of 10 cm

LES+LEM grid size of 10 cm

LES with no SGS model

current LES

EMPM with droplets and entrainment



droplet evaporation

molecular diffusion

turbulent deformation

entrainment

saturated parcel

Linear Eddy Model

(Kerstein 1988)

- Distinguishes turbulent deformation and molecular diffusion.
- The mixing process:

1. turbulent deformation (increases gradients)
2. molecular diffusion (reduces gradients)

- The linear eddy model:

  1-D so all relevant scales can be represented.

  Molecular diffusion is explicit.

  Turbulent deformation is represented by rearrangement events:

    Size of event represents the eddy size.

    Distribution of eddy sizes is obtained from Kolmogorov scaling laws for high *Re* turbulence.

    Process is consistent with turbulent kinetic energy cascade.

Turbulent Motion is Represented by Applying Maps

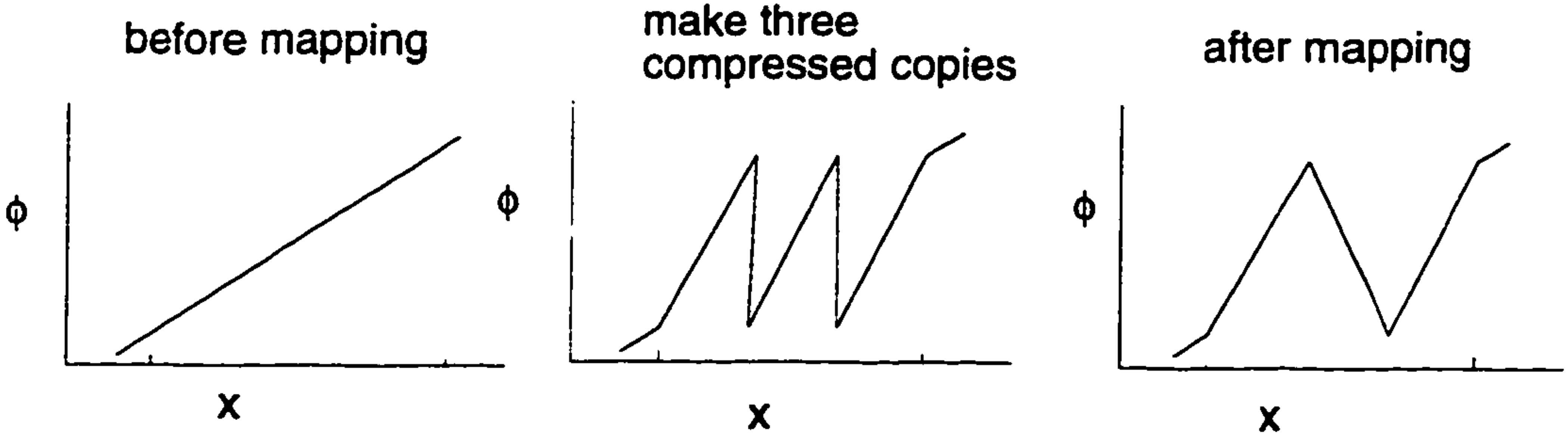


Figure 2.1. Schematic diagram of a triplet mapping event.

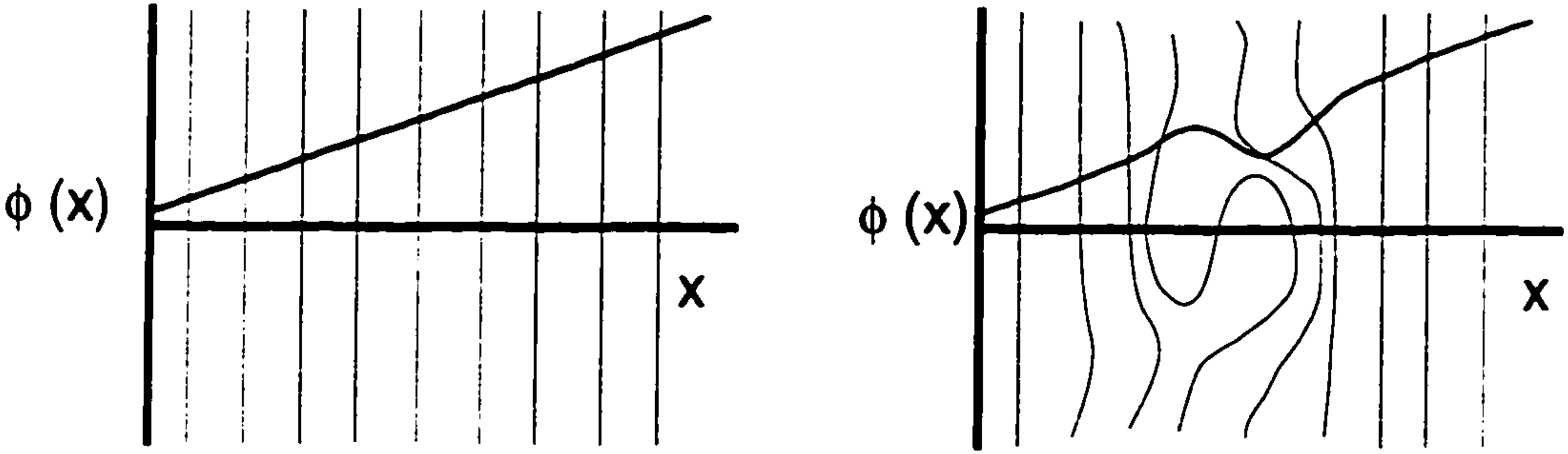


Figure 2.2. Effect of a single counterclockwise eddy on a scalar field with linear gradient.

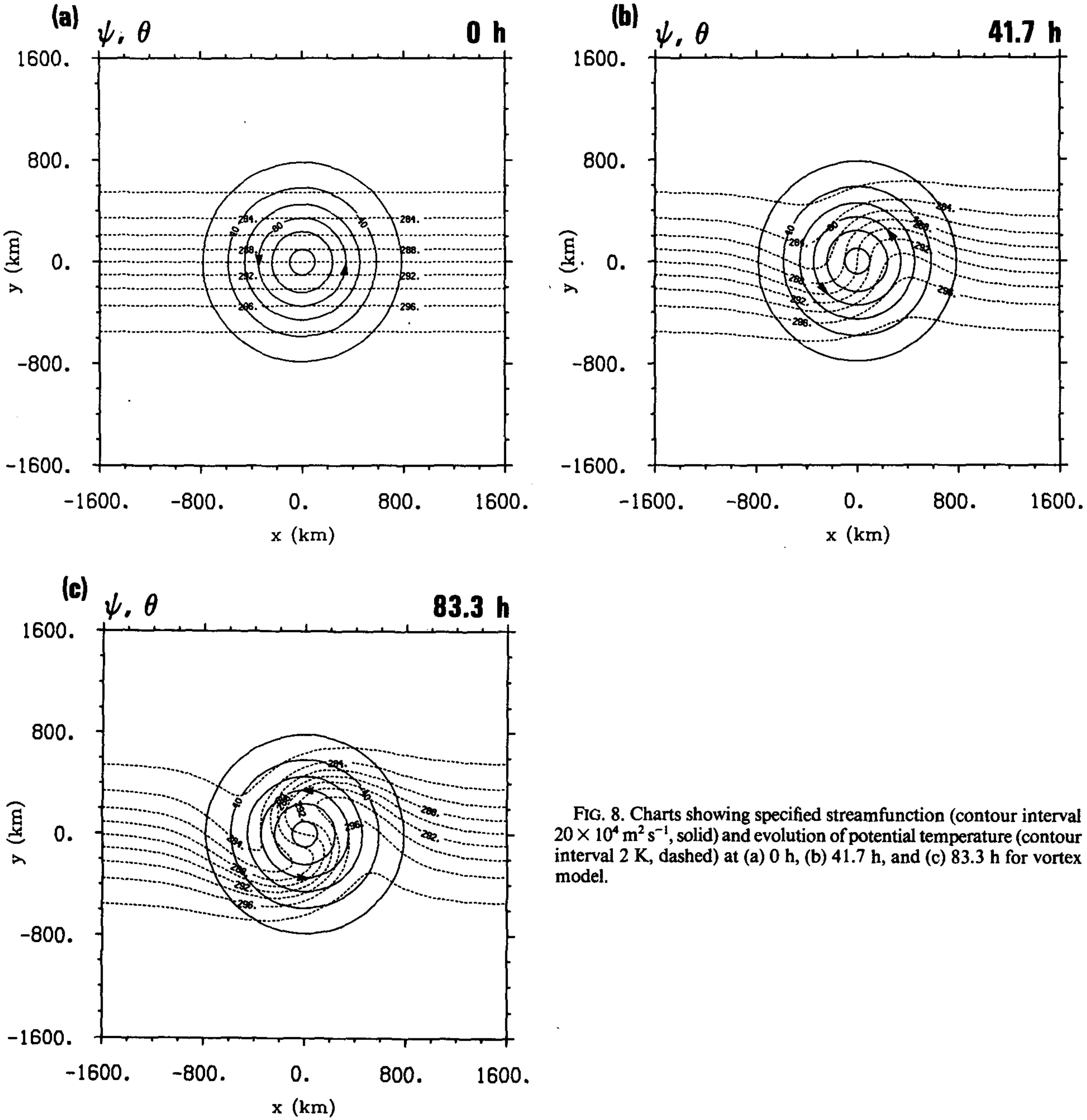


FIG. 8. Charts showing specified streamfunction (contour interval 20×10^4 m^2 s^{-1}, solid) and evolution of potential temperature (contour interval 2 K, dashed) at (a) 0 h, (b) 41.7 h, and (c) 83.3 h for vortex model.

Triplet Map

Each triplet map has a location, size, and time.

- Location is randomly chosen.
- Size is randomly chosen from a distribution that matches inertial range scalings.
  - Smallest map (eddy) is Kolmogorov scale.
  - Largest eddy is L, usually domain size.
- Eddies occur at a rate determined by the large eddy time scale and eddy size range.

Triplet Map

Eddy Size Distribution

$$f(l) = \begin{cases} \dfrac{5}{3}\,\dfrac{l^{-8/3}}{\eta^{-5/3} - L^{-5/3}}, & \text{if } \eta \le l \le L \\ 0, & \text{otherwise} \end{cases}$$

Eddy Rate

$$\Lambda = \frac{54}{5}\,\frac{D_T}{L^3}\left(\frac{L}{\eta}\right)^{5/3}.$$

Molecular Diffusion

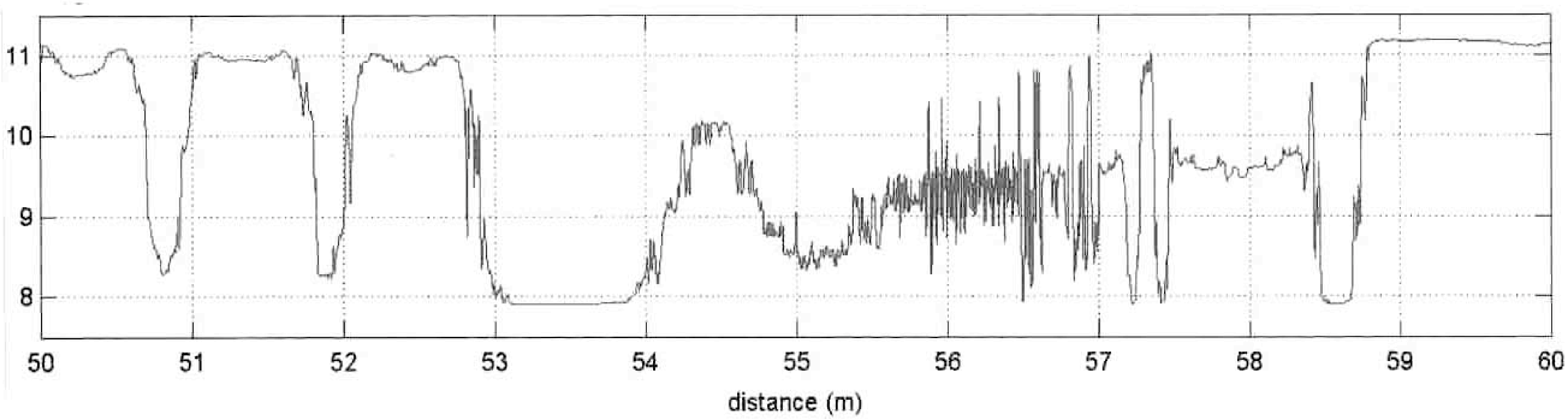
$$\frac{\partial \phi_i}{\partial t} = D_M \frac{\partial^2 \phi_i}{\partial x^2}$$

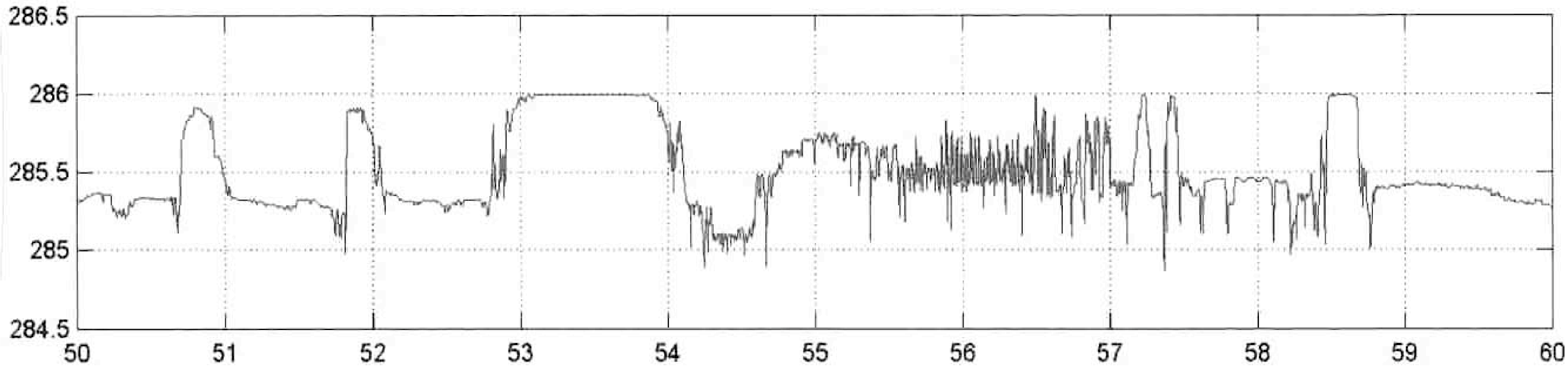
D_M is the *molecular* diffusivity of the scalar ϕ_i.

EMPM Variables

- Bulk microphysics:
  - Liquid water static energy
  - Total water mixing ratio
- Droplet microphysics:
  - Temperature
  - Water vapor mixing ratio

EMPM water vapor and temperature fields





Droplet growth by diffusion of water vapor

$$r_j \frac{dr_j}{dt} = \frac{S - A_1 + A_2}{A_3 + A_4}$$

r_j is the radius of the jth droplet, A_1 and A_2 are the correction factors for droplet curvature and solute effects, A_3 and A_4 are the heat conduction and vapor diffusion terms, and S is the supersaturation.

Droplets move relative to the fluid at their terminal velocities.

A 3D triplet map for inertial droplets

The droplet trajectory model idealizes droplet response to continuum flow (dashed curves: notional continuum fluid streamline and droplet trajectory)

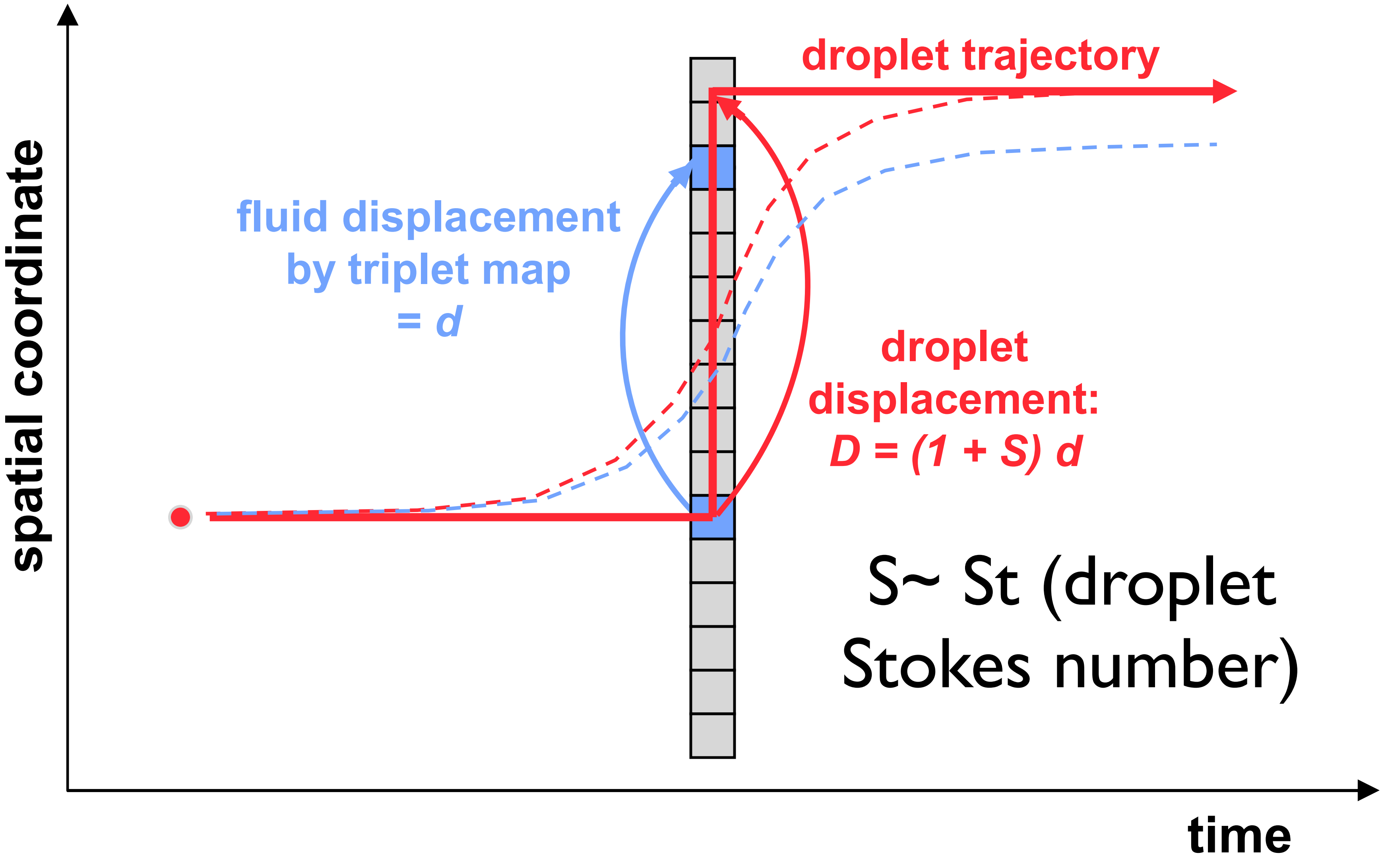
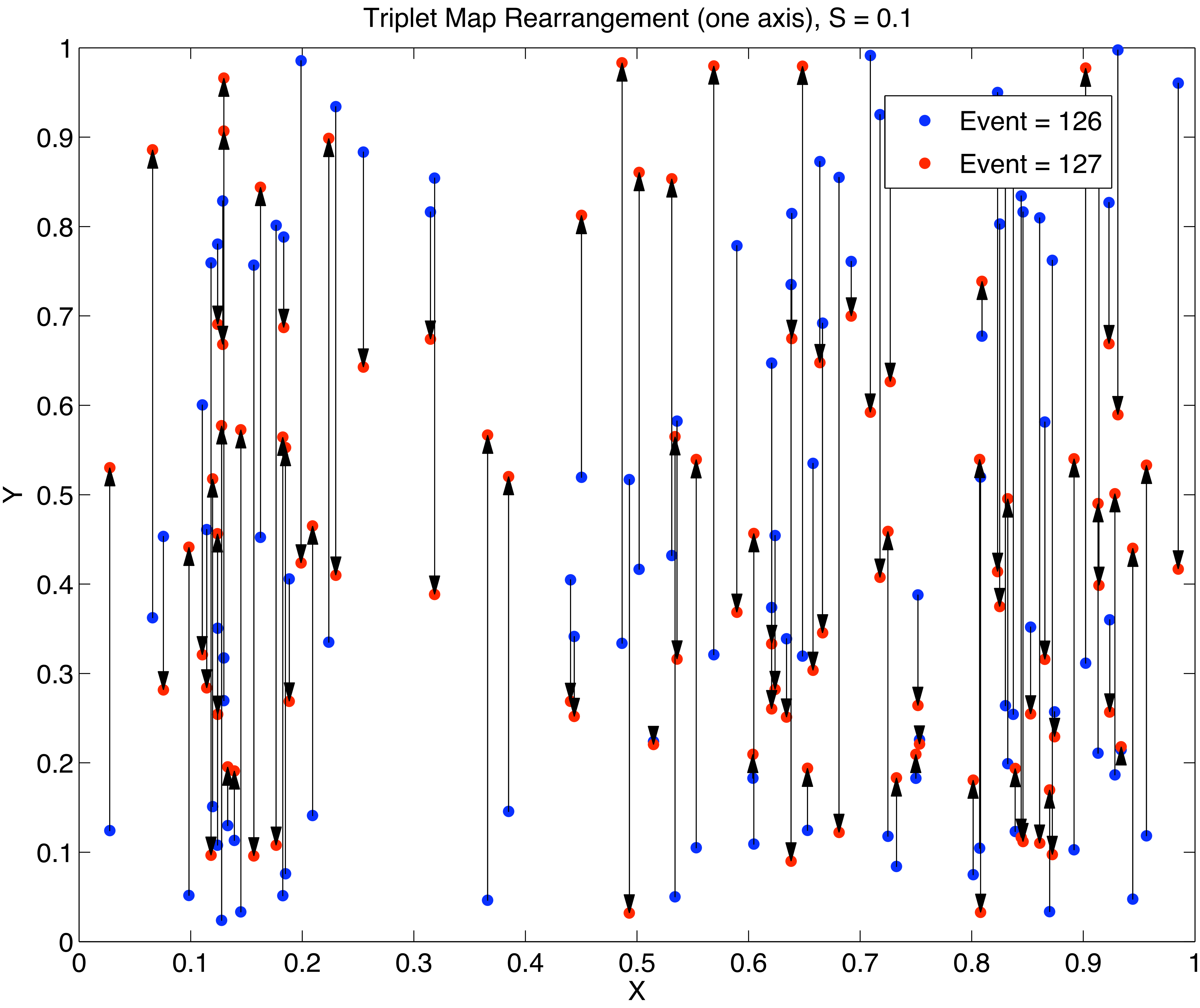
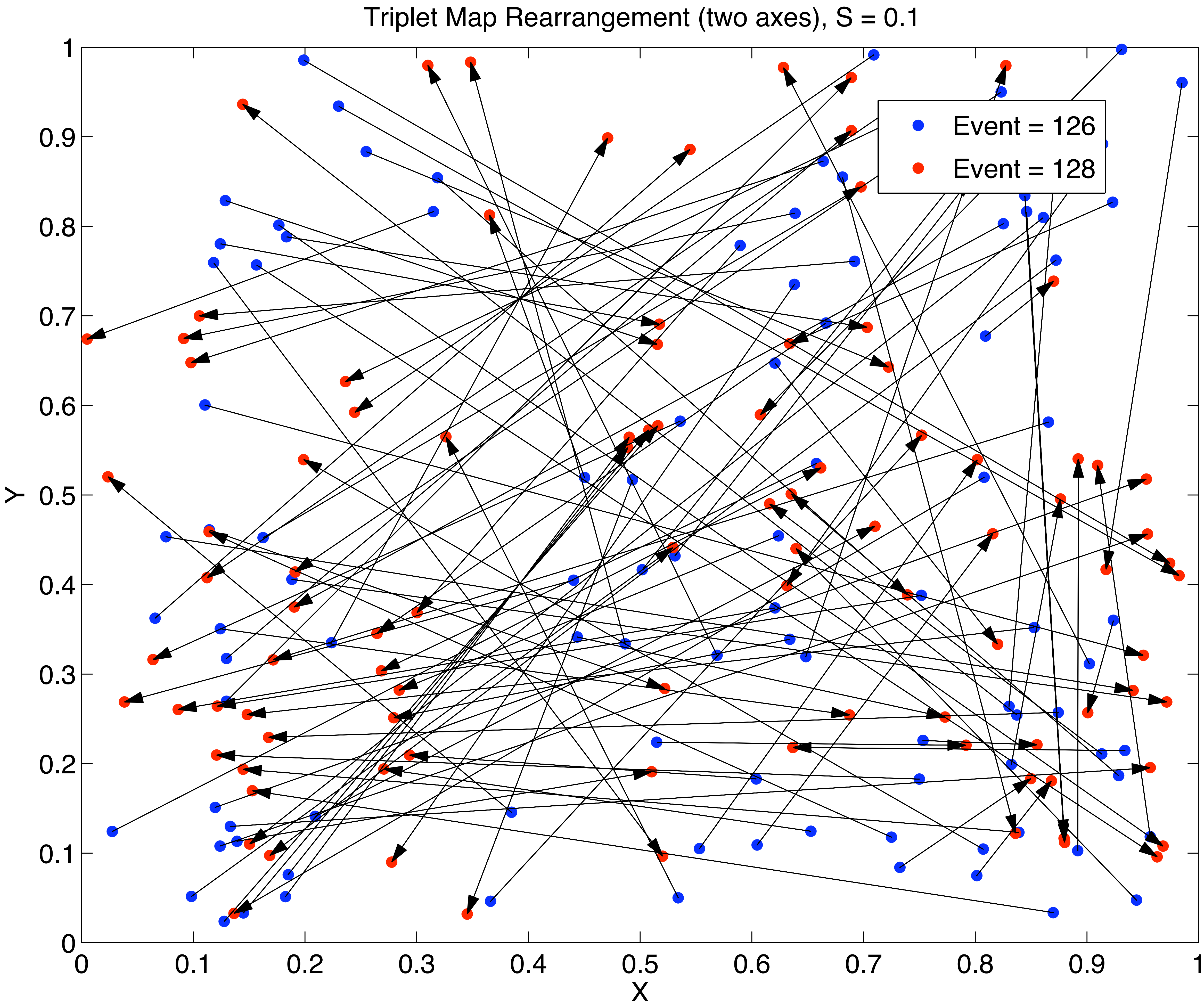
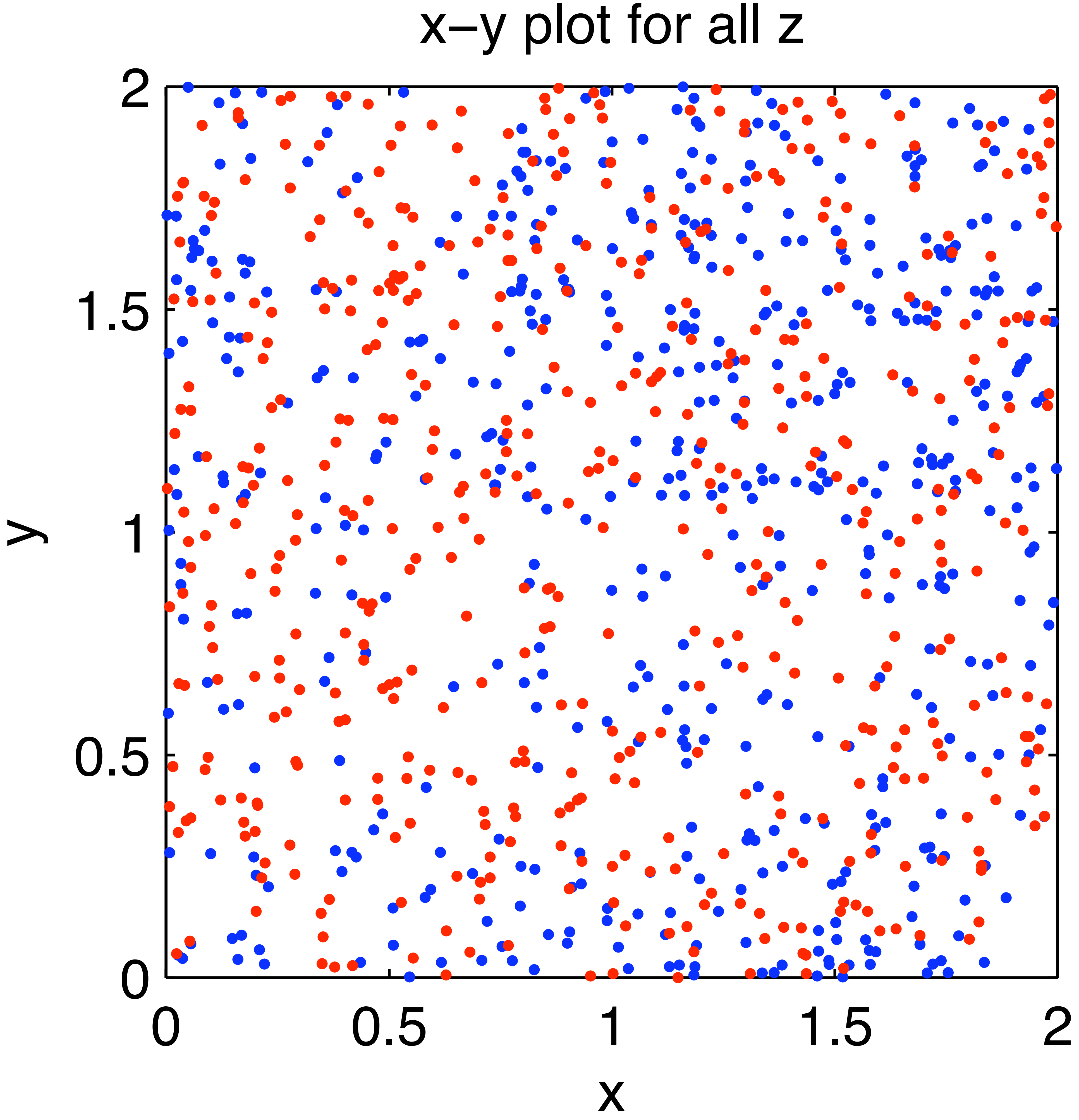


Fig 3. Difference between dispacement due to triplet map and the total displacement. (A. R. Kerstein)


Triplet Map Rearrangement (one axis), S = 0.1
● Event = 126
● Event = 127
Y
X
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1


Triplet Map Rearrangement (two axes), S = 0.1
Event = 126
Event = 128
Y
X
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1



St = 0

St = 0.025

length unit is
triplet map
eddy size =
20 x
Kolmogorov
scale

1-D SGS models for LES

Turbulence	Physics	Microphysics	Dimension	Domain	Grid size	Grid points (Droplets)
LEM	mixing & buoyancy	bulk condensation & evaporation	1	10 m	1 m	10 -
LEM	mixing & DSD	droplet condensation & evaporation	1	10 m	1 mm	10^4 (10^3)
3D droplet triplet map	turbulence & collisions	droplet collision & coalescence	1 (3)	10 m × (1 cm)^2	-	- (10^3)
LEM	all	bin model: droplet cond/evap & coll/coal	1	10 m	1 cm	10^2 -

Summary

- Reducing the dimensionality is an established method.
- Removes or reduces the need for SGS parameterizations.
- It is very well suited for high-Reynolds number turbulent flows when small-scale mixing processes are important.

EXTRA SLIDES

Triplet map vs. 3D turbulence

Strengths

- Transport: map frequency is set so that fluid transport matches turbulent eddy diffusivity.
- Length scale reduction: by matching the inertial-range size-vs.-frequency distribution of eddy motions, the rate of length scale reduction as a function of fluid parcel size is consistent with 3D turbulence.
- Intermittency: Random sampling of triplet map occurrences and sizes reproduces, qualitatively and to some degree quantitatively, intermittency properties of 3D turbulence.
- Mixing: In conjunction with molecular diffusion, the map sequence reproduces mixing features.

Triplet map vs. 3D turbulence

Weaknesses

- Omits effects of time persistence of turbulent motions.
- When diffusive time scales are shorter than turbulent time scales, diffusion can suppress scalar fluctuations faster than they are generated in 3D turbulence.
- In some cases, turbulence spreads a slow-diffusing scalar faster than a fast-diffusing scalar. This is a multi-dimensional effect that 1D advection cannot capture.

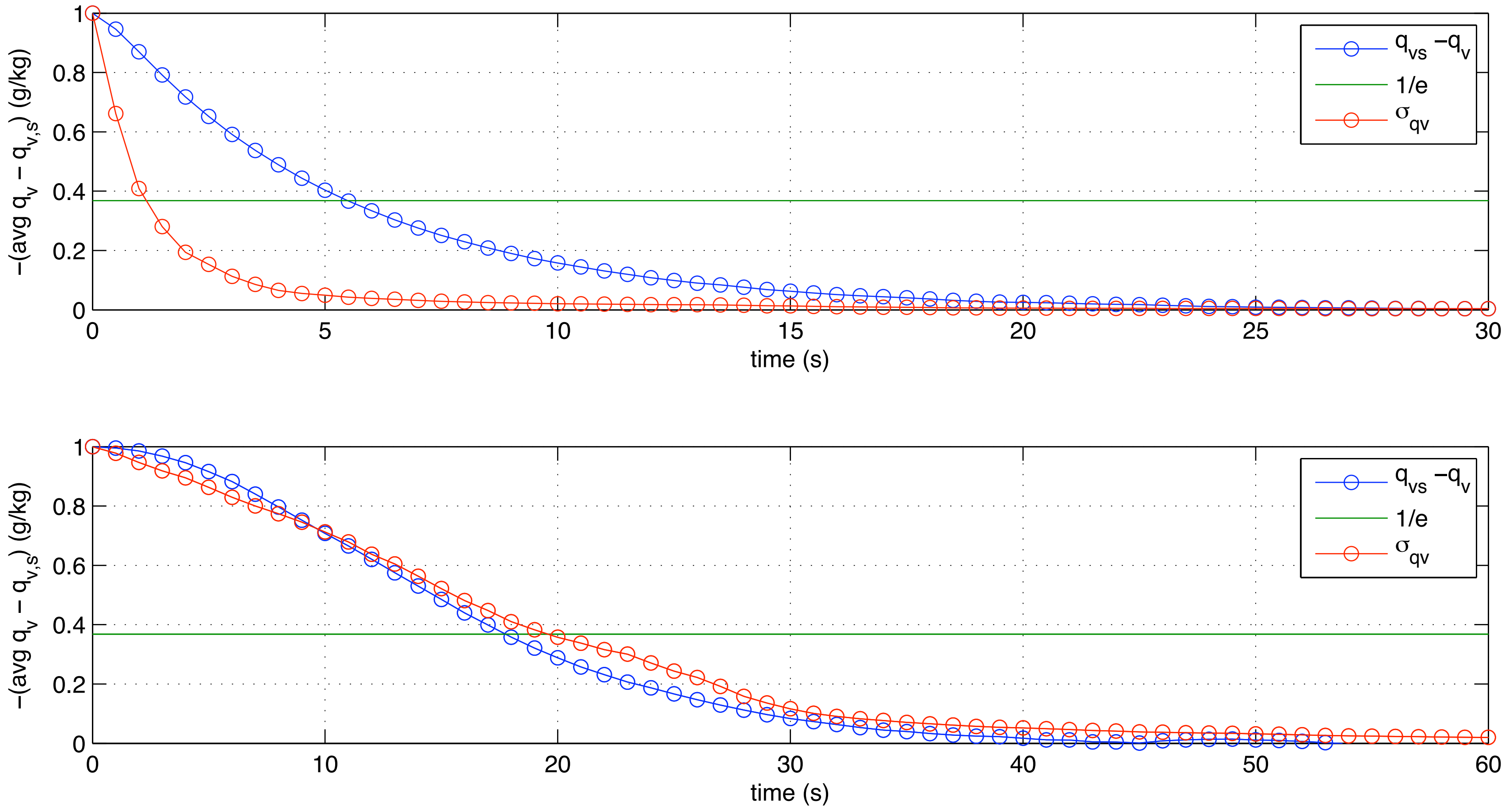
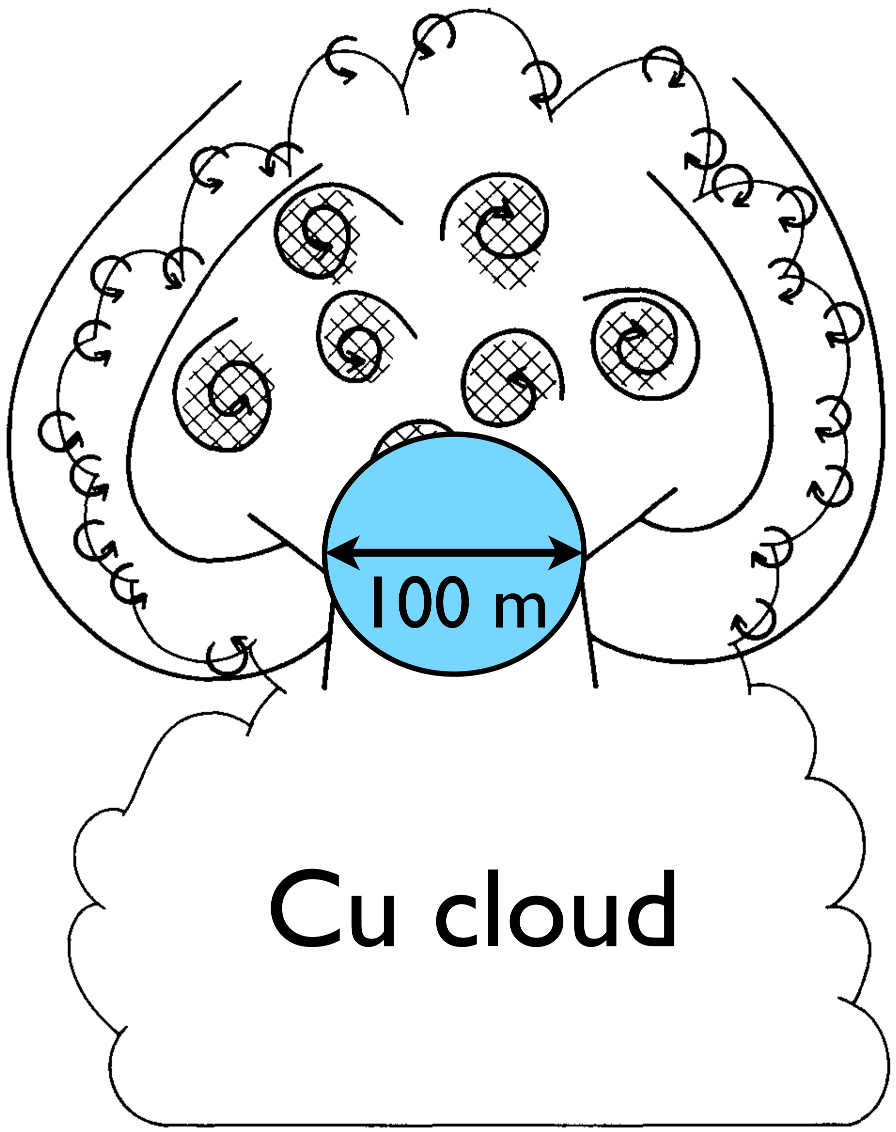


Figure 1: (top) Explicit Mixing Parcel Model simulation of isobaric mixing of saturated air (containing 100 cloud droplets per cm^{-3} of radius 15 μm) with 1 segment of subsaturated air 0.25 m in length in a 1D domain 20 m in length, with a dissipation rate of 10^{-2} m^2 s^{-3}. The blue curve is the average subsaturation normalized by its initial value, the red curve is the std dev of the water vapor mixing ratio, and the green line is $1/e$. The e-folding times for saturation adjustment and decay of water vapor std dev obtained from the plot are 5.5 s and 1.2 s, respectively. The calculated evaporation and mixing timescales are 4 s and 1.8 s, respectively. (bottom) Same as top except for mixing of 5 segments of subsaturated air (each 10 m in length) in a 1D domain 100 m in length. The e-folding times for saturation adjustment and decay of water vapor std dev obtained from the plot are 18 s and 20 s, respectively. The calculated evaporation and mixing timescales are 4 s and 22 s, respectively.

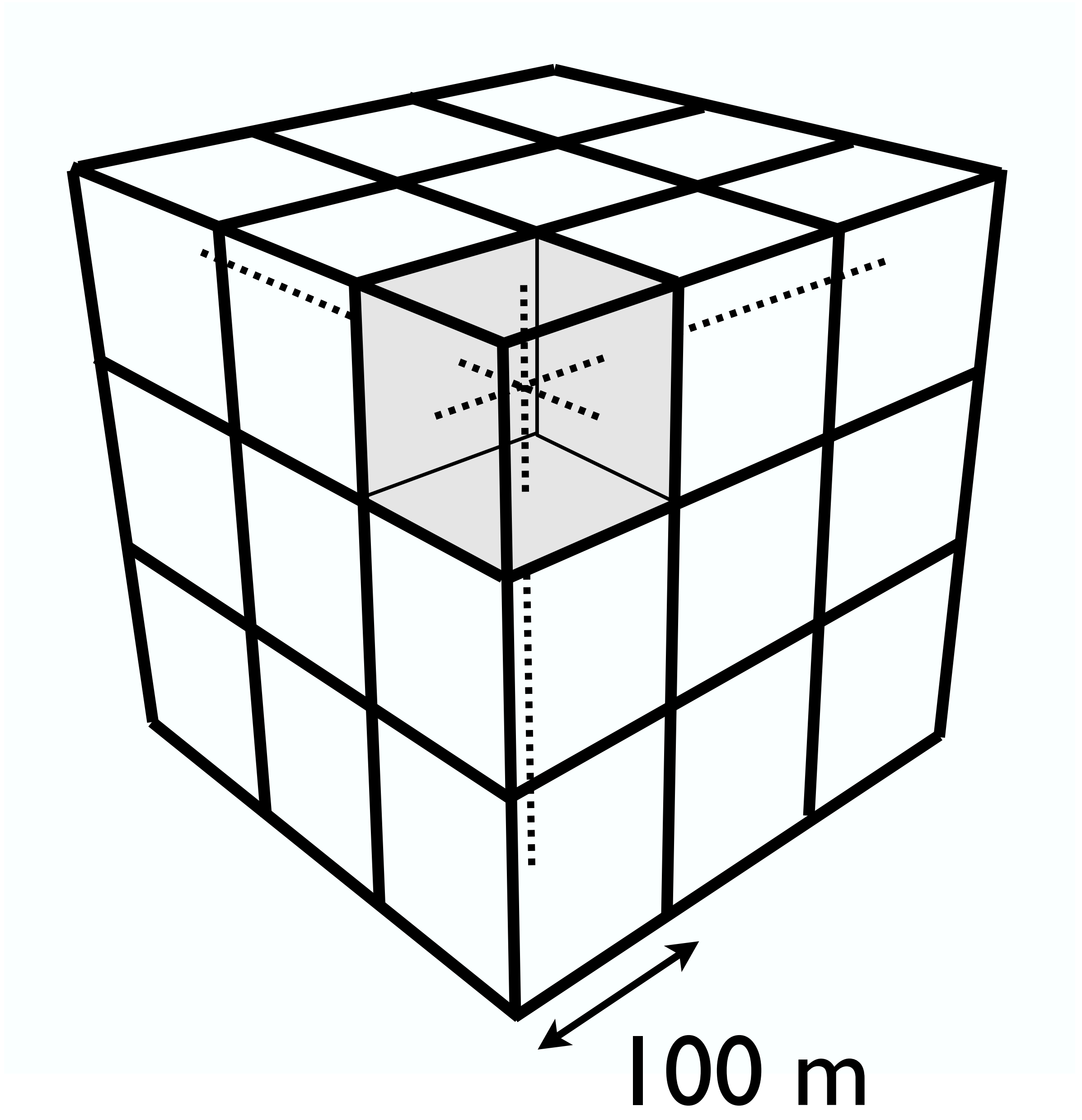
Parameterization of SGS Cloud Processes in LES

- SGS mixing is instantaneous in most LES.
- SGS variability does not affect DSD in any LES.
- SGS turbulence affects droplet collision rates in very few LES.

Parcel model

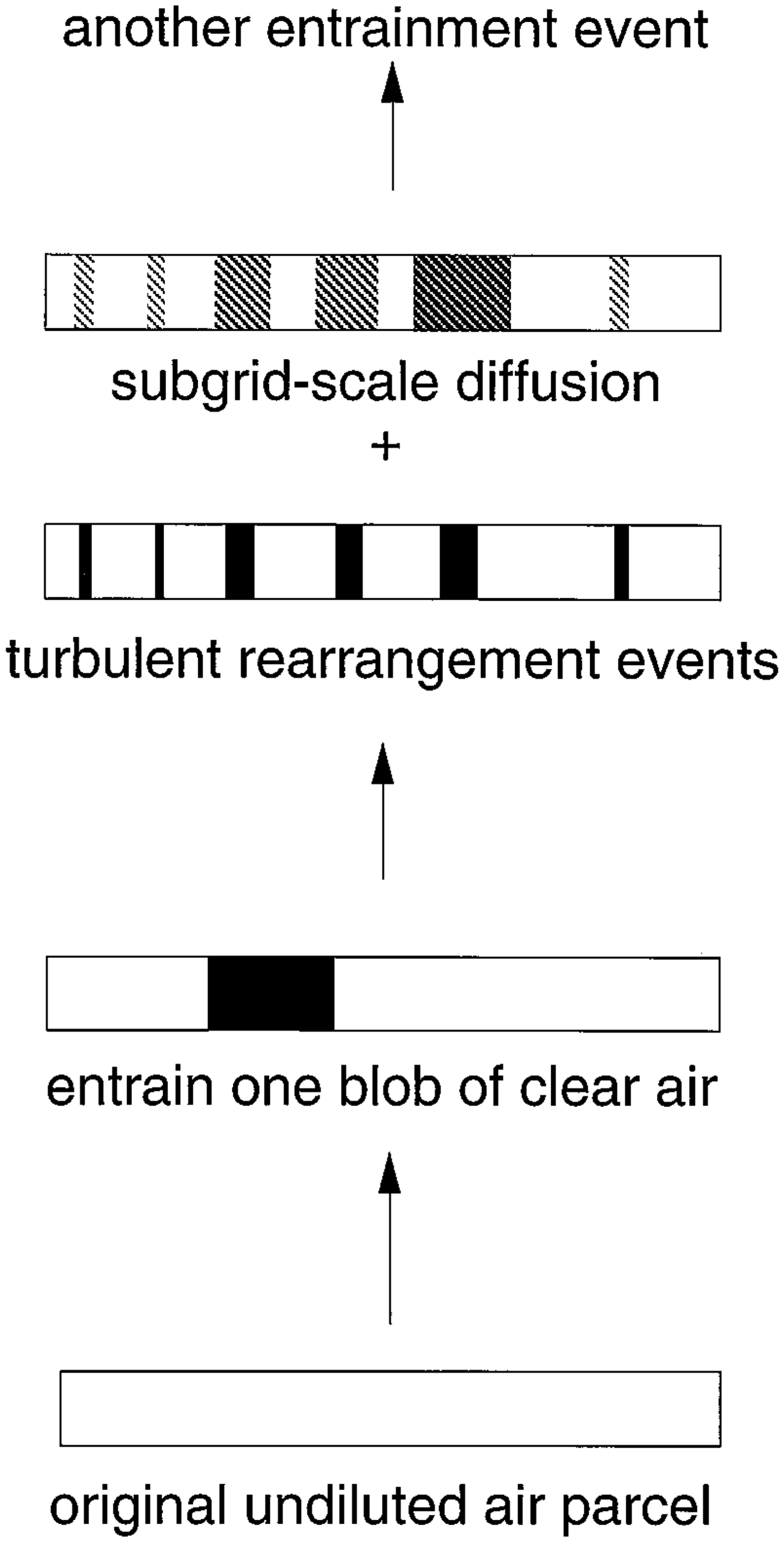


Large-Eddy Simulation (LES) model



no sub-parcel or subgrid-scale variability

Explicit Mixing Parcel Model (EMPM)



another entrainment event

subgrid-scale diffusion

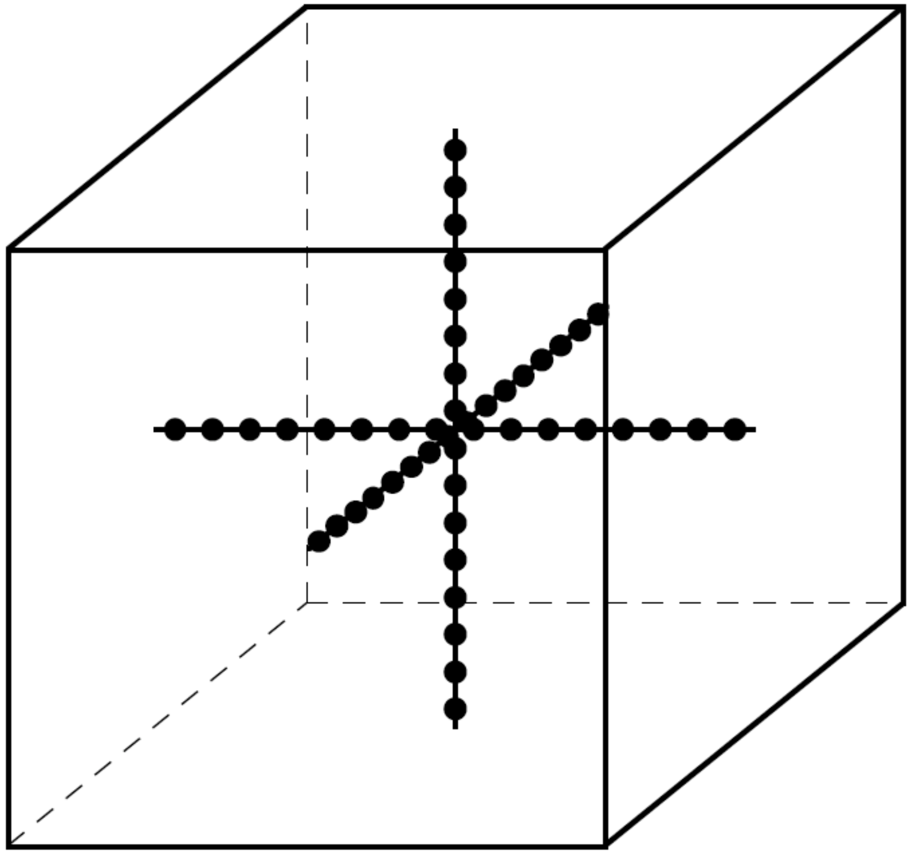
+

turbulent rearrangement events

entrain one blob of clear air

original undiluted air parcel

LES with 1D subgrid-scale model



Explicit Mixing Parcel Model (EMPM)

- The EMPM predicts the evolving in-cloud variability due to entrainment and finite-rate turbulent mixing using a 1D representation of a rising cloudy parcel.

- The 1D formulation allows the model to resolve fine-scale variability down to the smallest turbulent scales ($\sim$ 1 mm).

- The EMPM can calculate the growth of several thousand individual cloud droplets based on each droplet's local environment.

EMPM Required Inputs

- Required for a classical (instant mixing) parcel model calculation:

  Thermodynamic properties of cloud-base air

  Updraft speed

  Entrainment rate

  Thermodynamic properties of entrained air

  Aerosol properties

- In addition, the EMPM requires:

  Parcel size

  Entrained blob size, d

  Turbulence intensity (e.g., dissipation rate, ϵ)

The droplet Stokes number is

$$\mathsf{St} = t_d \gamma$$

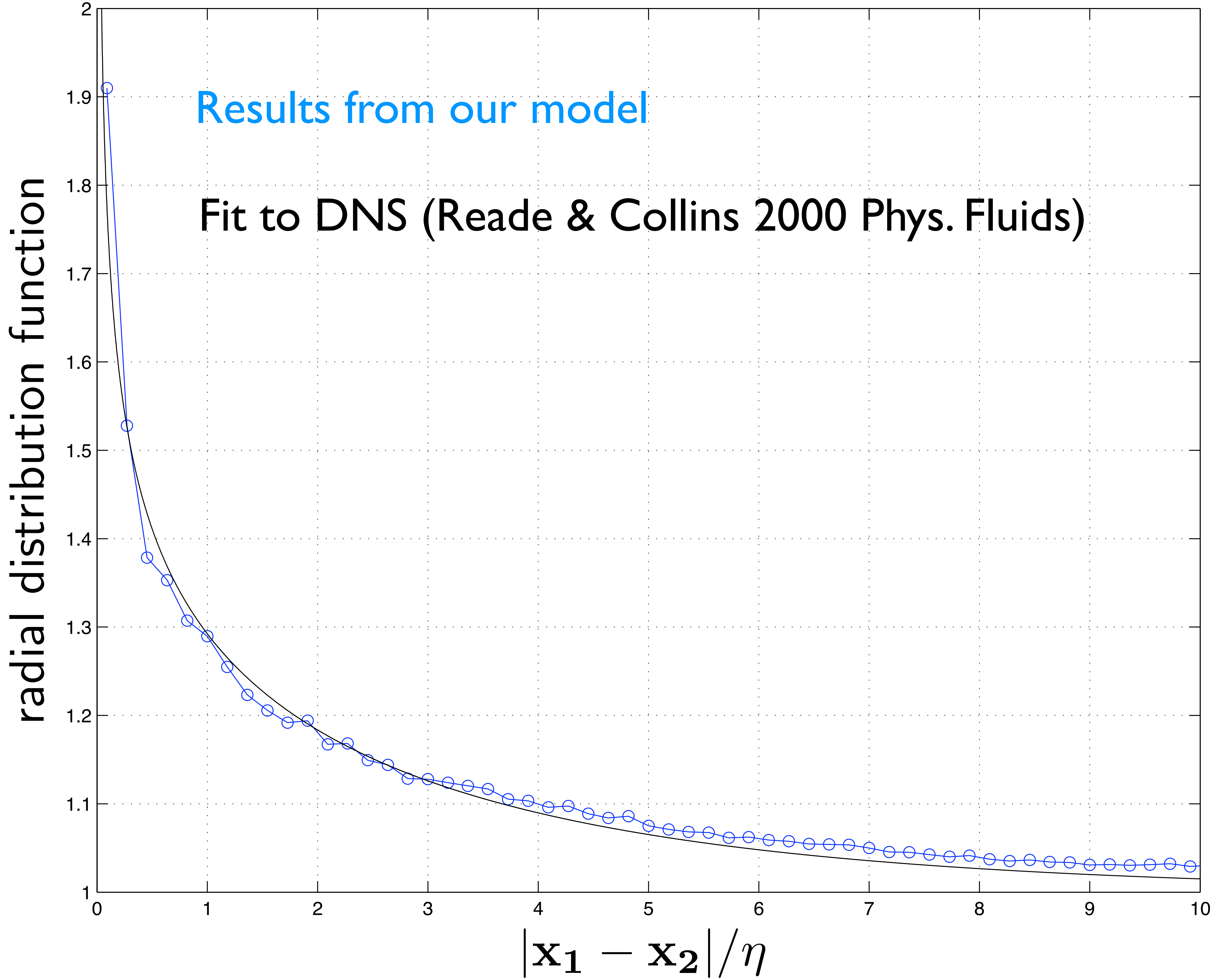
where

$$t_d = \frac{m_p}{6\pi r \mu} = \frac{2\rho_p r^2}{9\mu}$$

is the droplet response time, m_p is the droplet mass, ρ_p the droplet density, r the droplet radius, and μ the dynamic viscosity, and

$$\gamma = (\epsilon/\nu)^{1/2} = 1/\tau_K$$

is a global measure of strain, and τ_K is the Kolmogorov time scale.


Results from our model
Fit to DNS (Reade & Collins 2000 Phys. Fluids)
radial distribution function
$|\mathbf{x_1} - \mathbf{x_2}|/\eta$
2
1.9
1.8
1.7
1.6
1.5
1.4
1.3
1.2
1.1
1
0
1
2
3
4
5
6
7
8
9
10