Exploiting symmetries to explore the dynamics of ideal and dissipative MHD flows with wide scale separation

Ed Lee *(NCAR, Columbia Univ.)*
Marc Brachet *(École Normale Supérieure)*
Annick Pouquet *(NCAR)*
Pablo Mininni *(Univ. Buenos Aires)*
Duane Rosenberg *(NCAR)*

Turbulence Numerics Team
Institute for Mathematics Applied to Geoscience (IMAGe)
National Center for Atmospheric Research (NCAR)
Boulder, Colorado

Outline

- I. Recent advances in MHD/turbulence
  - Universal laws in turbulence
  - Well-posedness: singularity vs. regularity
  - Multi-scale interactions
- II. A new type of flow with symmetries
  - Taylor-Green vortex
  - “Magnetic Taylor-Green” flows
  - Results: current sheets and turbulence
- III. Breakthrough prospects with petascale resources
  - Tracing complex space singularities
  - Structures, wave turbulence
  - Computational challenges

I. Recent advances in MHD/turbulence:

Turbulence: old questions, new answers

- Kolmogorov constant (Sreenivasan 1995; Kaneda et al., 2003)
- Scaling of energy spectra (Mininni et al., 2006, Phys. Rev. Lett.)
- Self-similarity (Mininni et al., 2006, Phys. Rev. E)
- Energy dissipation (Biskamp and Schwarz, 2001)
- Cascades of invariants (Onsager 1949 and onwards)

I. Recent advances in MHD/turbulence:

Search for singularity

- Euler singularity
  - Beale-Kato-Majda (1984):

    $\limsup_{(t \uparrow T^*)} \|\omega(t)\|_\infty = \infty$ OR $\int \|\omega(t)\|_\infty \, dt < \infty$

- MHD
  - Caflisch-Klapper-Steele (1997):

    $\limsup_{(t \uparrow T^*)} (\|\omega(t)\|_\infty + \|j(t)\|_\infty) = \infty$ if singularity exists

$\omega^i\,(x^i, t)$ = vorticity

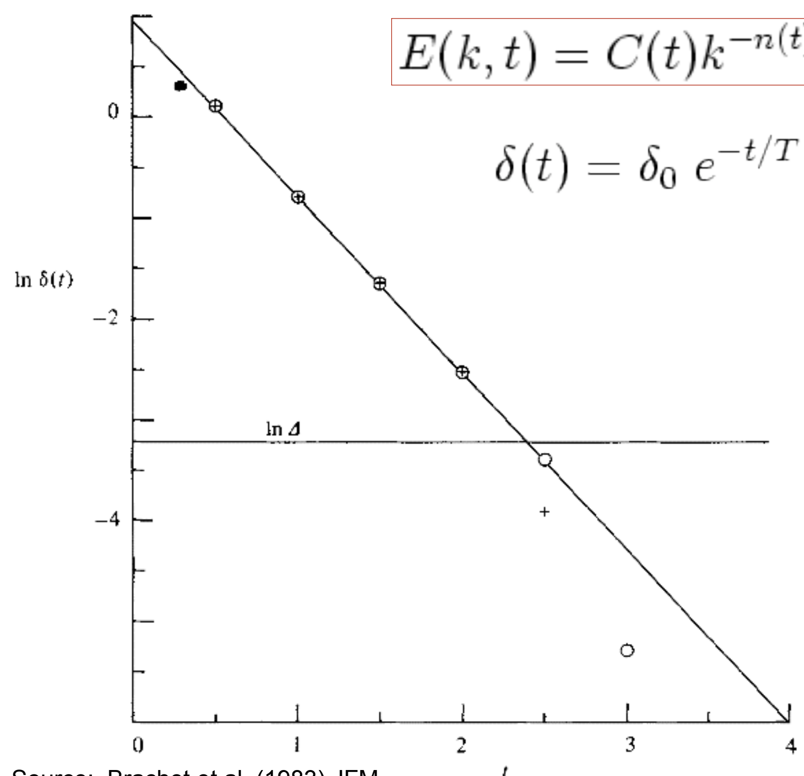
$j^i(x^i, t)$ = current density

Analyticity strip

- Sulem, Sulem, Frisch (1983):
  - Also Frisch, Pouquet, Sulem, Meneguzzi (1983) - 2D MHD
  - Also Brachet et al. (1983) - 3D Euler

$$E(k,t) = C(t)k^{-n(t)} \exp[-2\delta(t)k]$$

$$\delta(t) = \delta_0 \, e^{-t/T}$$



Source: Brachet et al. (1983) JFM

Multi-scale interactions

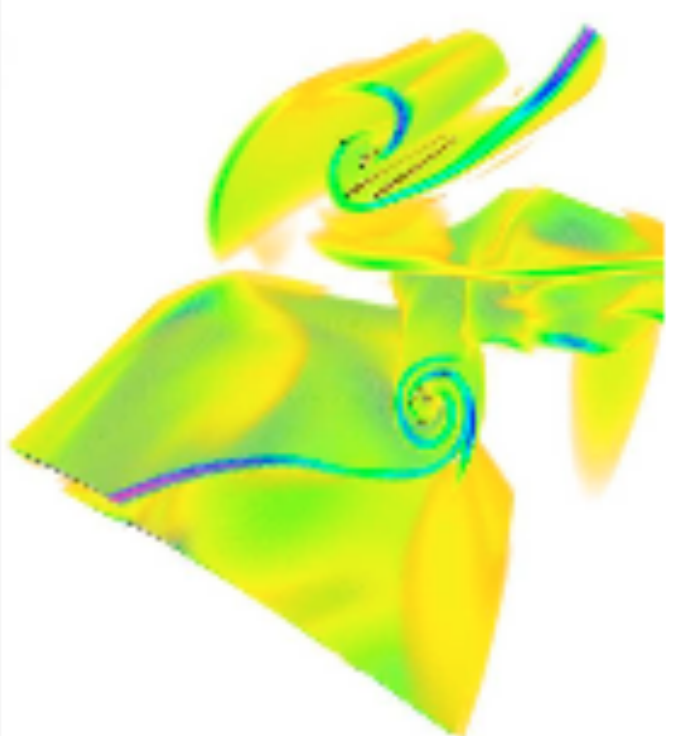
- Physics:
  - Current sheets (Mininni et al.)
  - Dynamo (Bourgoin et al.; Pinton et al.?)

- CFD:
  - Direct Numerical Simulation
  - Adaptive Mesh Refinement
  - Large-Eddy Simulation

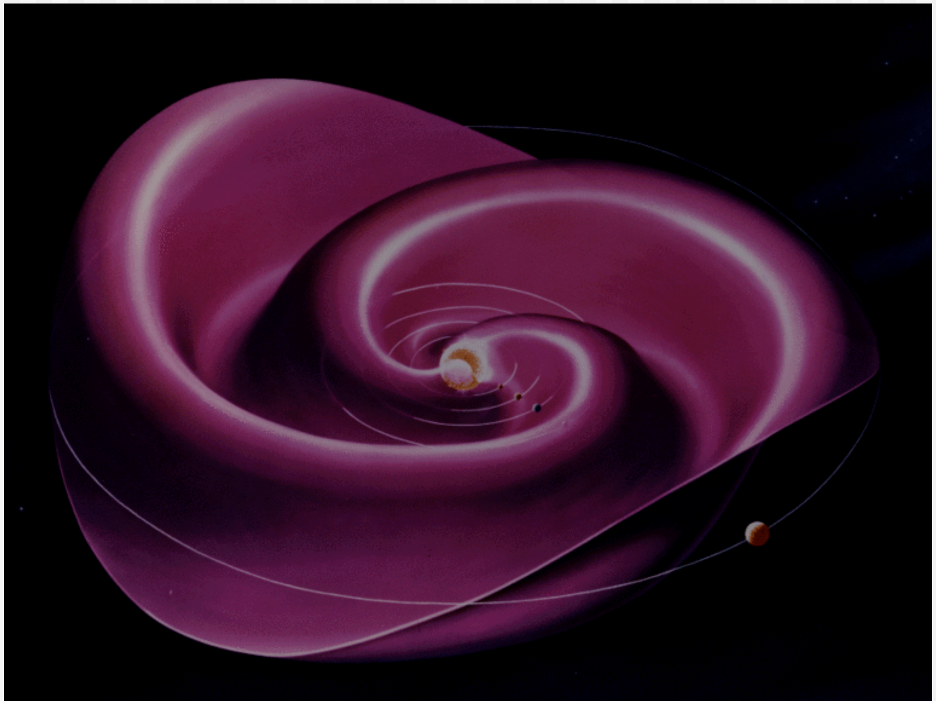
I. Recent advances in MHD/turbulence:

Multi-scale interactions

- Current sheets



Source: Mininni et al. (2006) Phys. Rev. Lett. 97

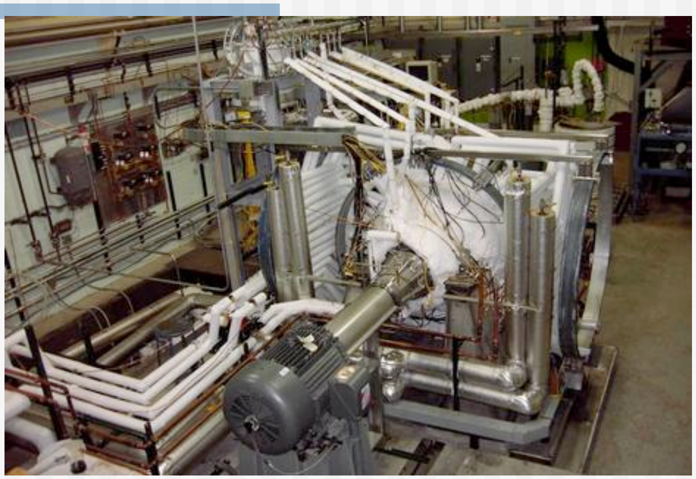


Source: Kenneth R. Lang, *The Sun from Space* (2000) Springer

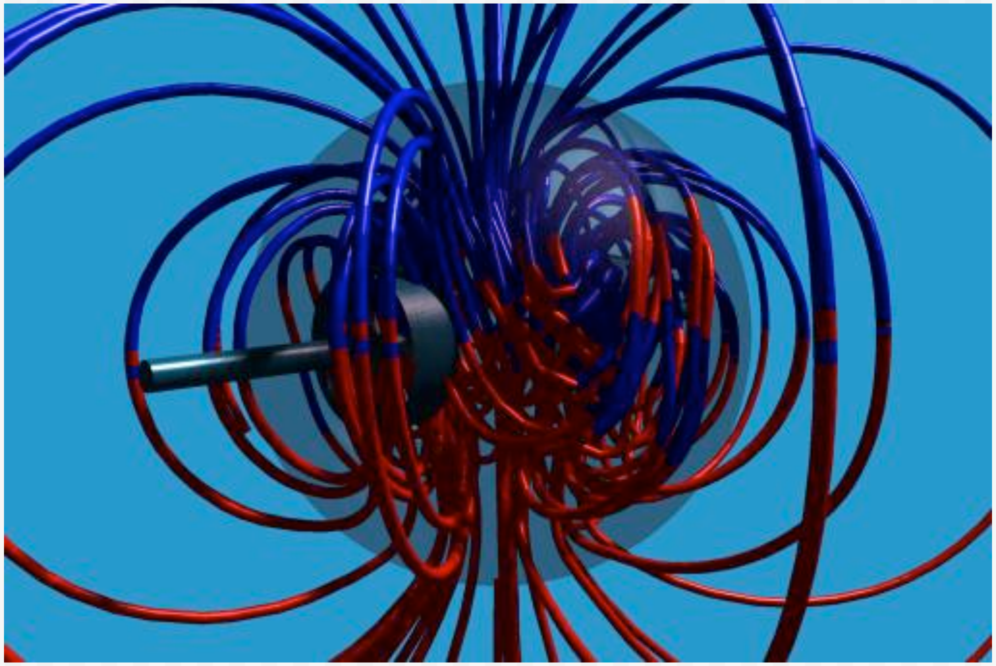
I. Recent advances in MHD/turbulence:

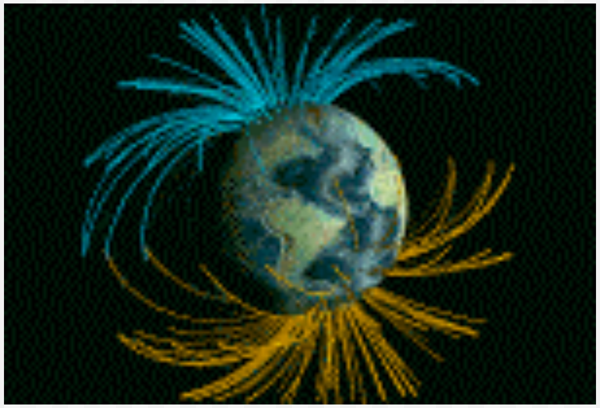
Multi-scale interactions

- Dynamo



Source (below, right): http://www.epm.ethz.ch/research/experimental_studies/exp_dyn

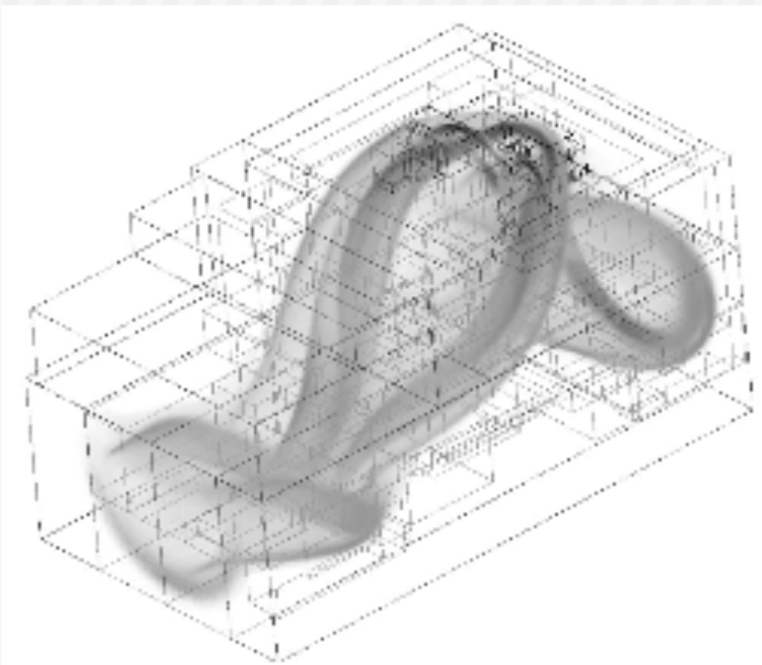




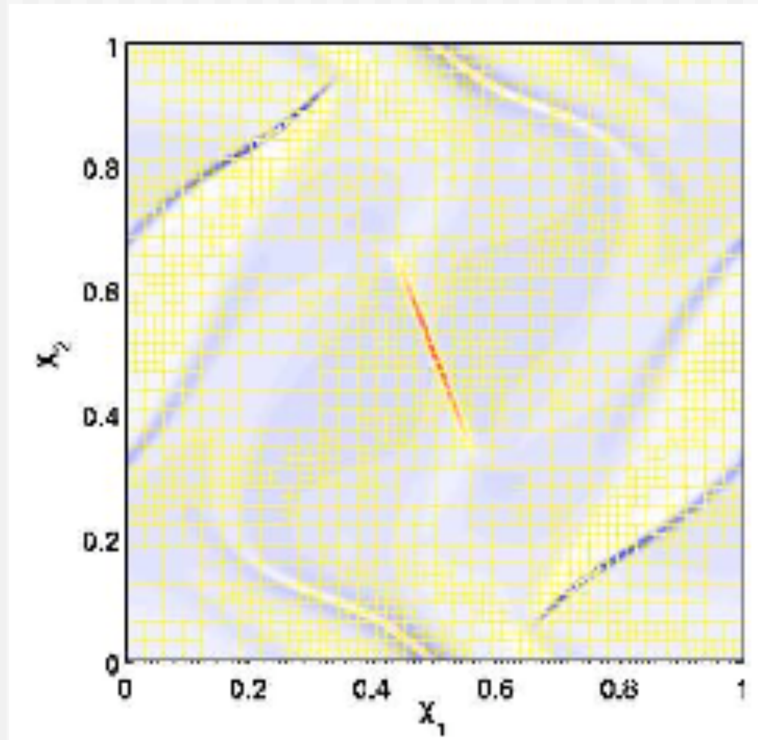
Source: http://www.physics.ucsb.edu/~airboy/physics_links.html

Multi-scale interactions

- Adaptive mesh refinement (AMR)



Source: Grauer et al. (1998) Phys. Rev. Lett.

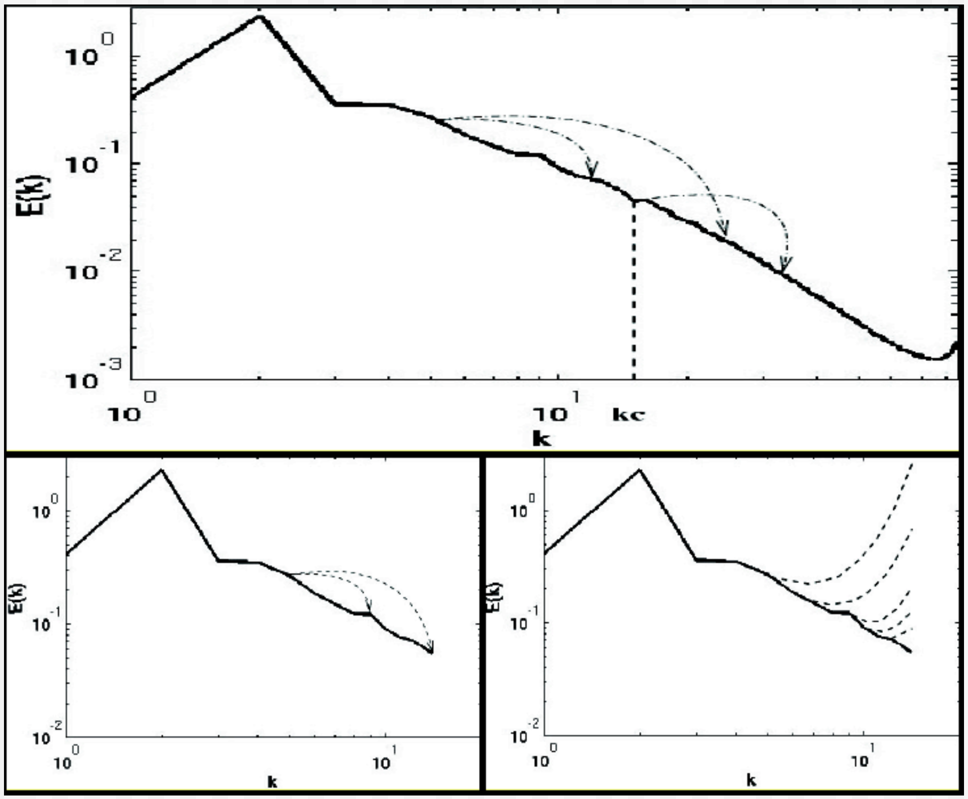


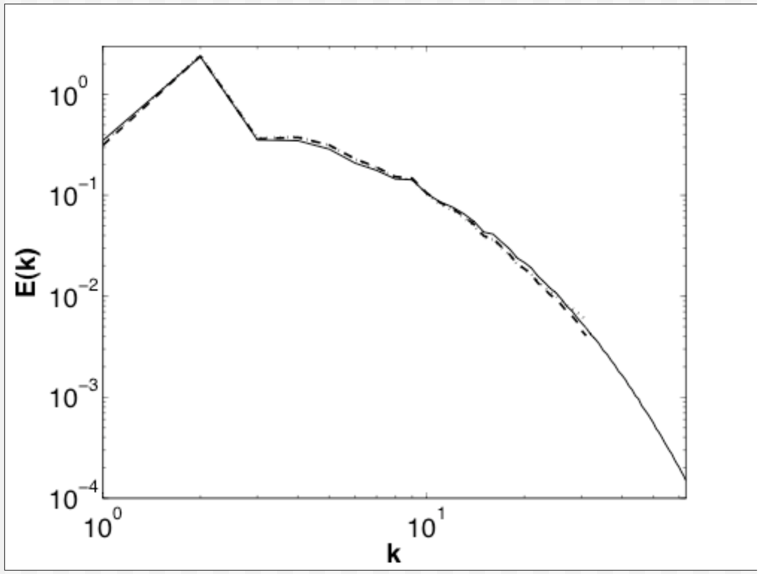
Source: Rosenberg et al. (2007) New J. Phys.

I. Recent advances in MHD/turbulence:

Multi-scale interactions

- Large-eddy simulation





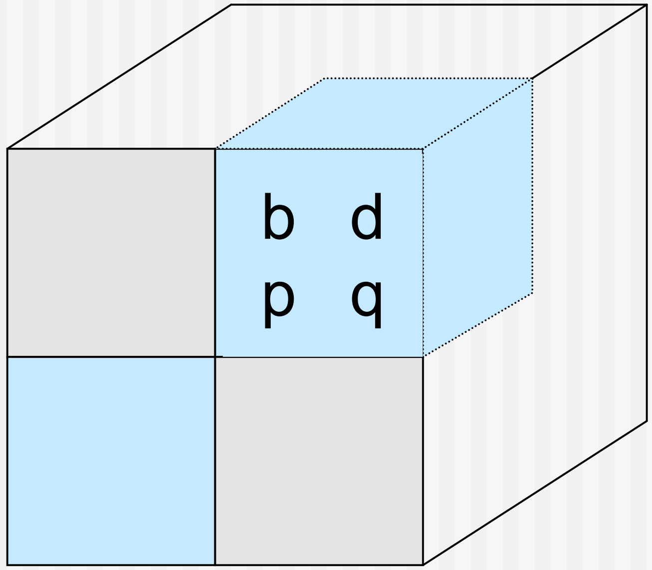
Source: Baerenzung et al. (2008) Phys. Rev. E (submitted)

Part II:

Exploiting symmetries

II. A flow with symmetries:

Taylor-Green vortex

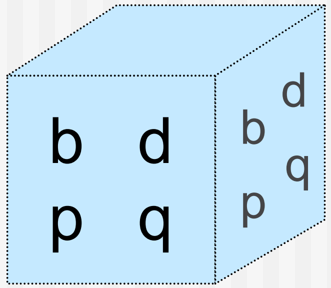


$$v_x = v_0 \sin(x)\cos(y)\cos(z)$$

$$v_y = -v_0 \cos(x)\sin(y)\cos(z)$$

$$v_z = 0$$

- TG vortex:
  - Brachet et al., 1983; Brachet 1991
- Analyticity strip:
  - Brachet et al., 1992;
  - Cichowlas et al., 2005;
- Dynamo:
  - Nore et al., 1997



II. A flow with symmetries:

Magnetic Taylor-Green

- MHD equations

$$\partial_t \mathbf{v} + \boxed{\mathbf{v}\cdot\nabla\mathbf{v}} + \nabla p = \boxed{\mathbf{j}\times\mathbf{b}} + \boxed{\nu\Delta\mathbf{v}}$$

$$\partial_t \mathbf{b} = \boxed{\nabla\times(\mathbf{v}\times\mathbf{b})} + \boxed{\eta\Delta b}$$

$$\nabla\cdot\mathbf{v} = 0 = \nabla\cdot\mathbf{b}$$

Nondimensional Alfvén units ($v \propto b$):

v = velocity (momentum)
b = magnetic field
j = current density = curl(**b**)

"IDEAL" ⇔ $\nu = 0 = \eta$

- Initial velocity field

$$v_x = v_0 \sin(x)\cos(y)\cos(z)$$
$$v_y = -v_0 \cos(x)\sin(y)\cos(z)$$
$$v_z = 0$$

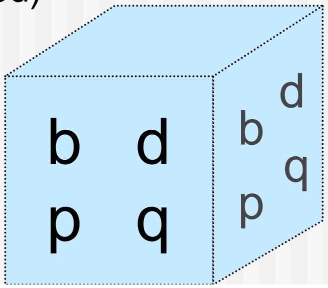
- Initial magnetic field

$$b_x = b_0 \cos(x)\sin(y)\sin(z)$$
$$b_y = b_0 \sin(x)\cos(y)\sin(z)$$
$$b_z = -2b_0 \sin(x)\sin(y)\cos(z)$$

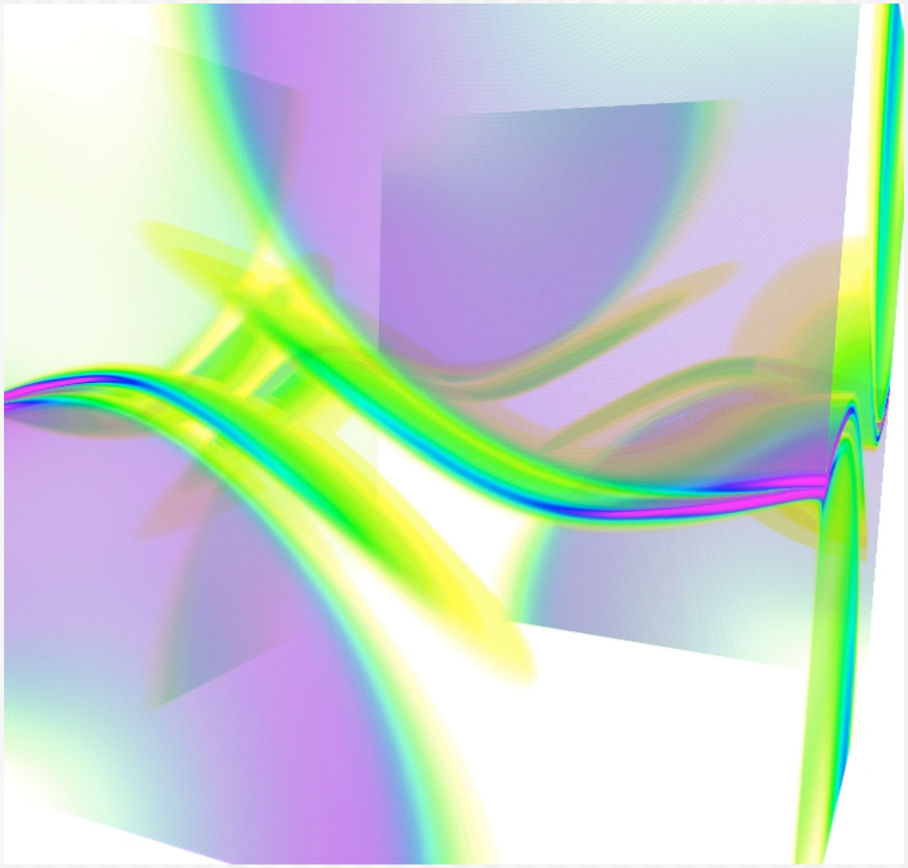
Lee, Brachet, Pouquet, Mininni, Rosenberg (2008) arXiv:0802:1550

High-res simulation results

- IDEAL CASE ($\nu, \eta = 0$)
  - 2084^3 resolution
- Integration:
  - NCAR IBM BlueGene/L ("Frost")
  - 80K CPU hrs (to t=3)
  - Pseudospectral, periodic BC, w/ symmetries implemented in code
    - Also code without imposed symmetries
  - 2nd-order RK timestepping
    - Also 4th-order
- Visualization:
  - VAPOR (Clyne et al., 2007; Mininni et al., 2008 submitted)



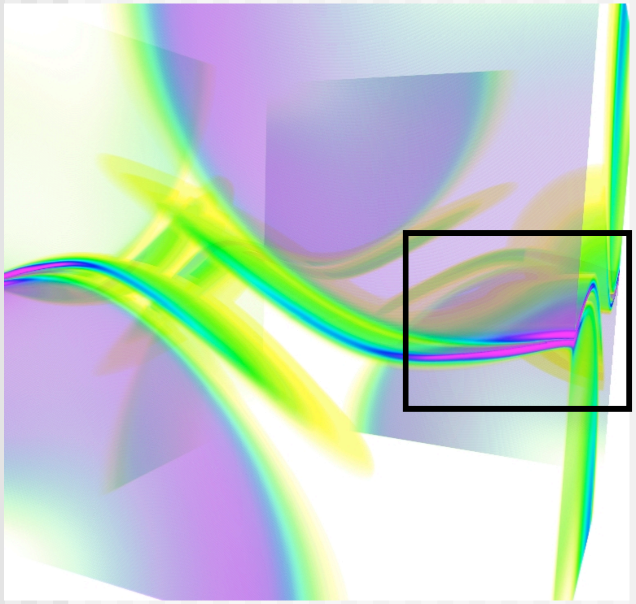
- Current sheets

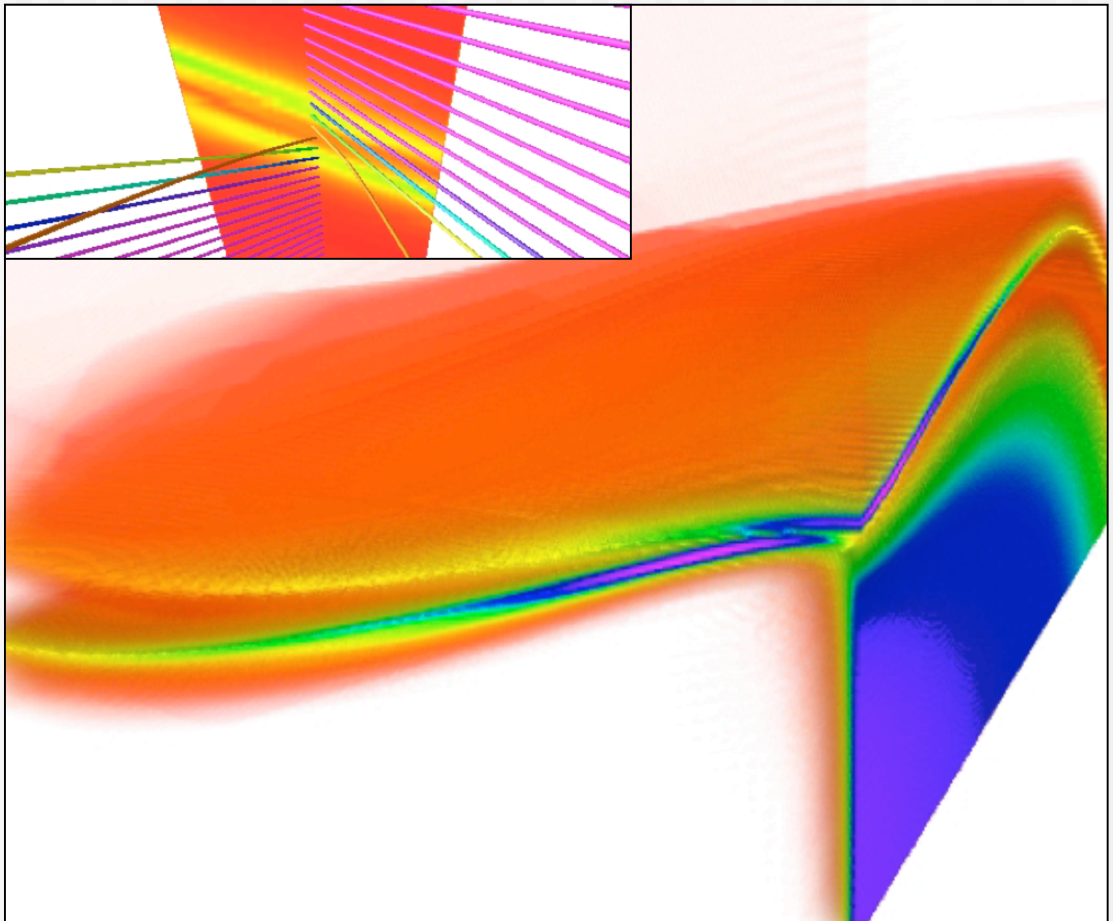


II. A flow with symmetries:

High-res simulation results

- Current sheets
  - Thinning, merging
  - Accompanied by rotational “discontinuity” of **B** (cf. Whang et al., 2004)

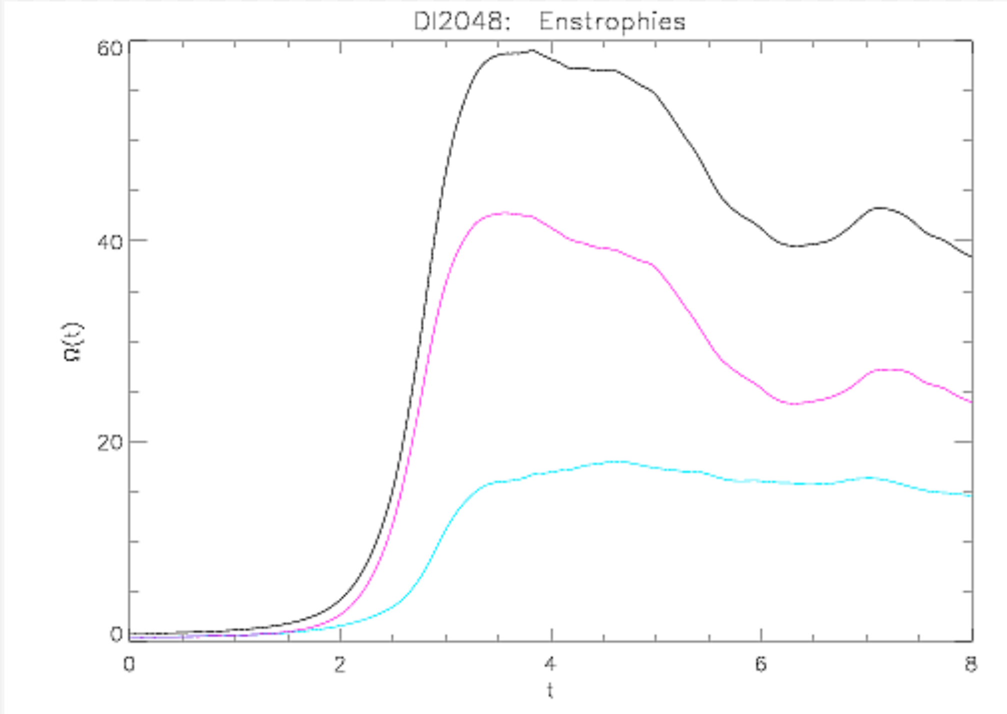




Lee, Brachet, Pouquet, Mininni, Rosenberg (2008) arXiv:0802:1550

High-res simulation results

- DISSIPATIVE CASE
  - 2048^3 resolution
- Integration:
  - NCAR IBM POWER5+ (“Blueice”)
  - 10K CPU hrs (to t=8)
  - Pseudospectral, periodic BC, w/ symmetries implemented in code
    - Also code without imposed symmetries
  - 2nd-order RK timestepping
- Visualization:
  - VAPOR (to follow)
- Dissipation



Part III:

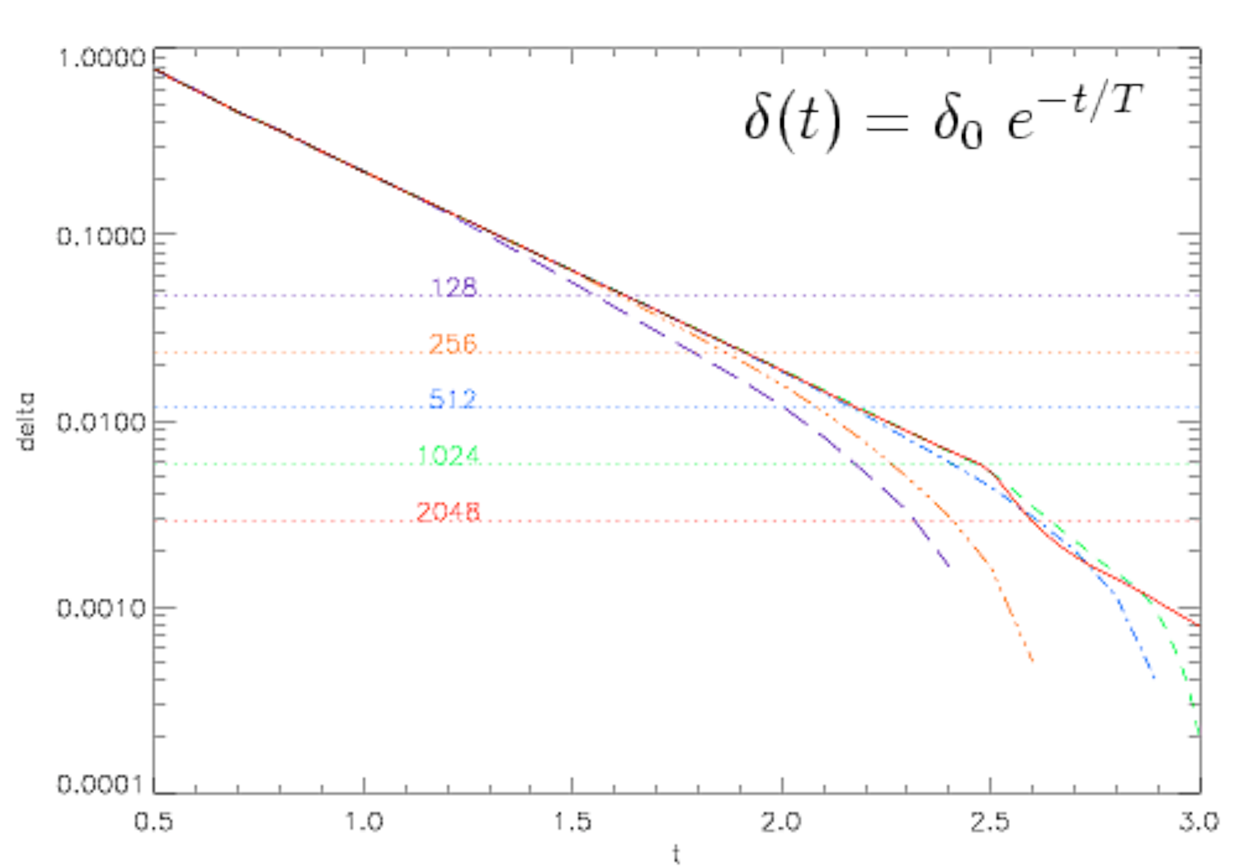
Why we need bigger and better computers

(and why we think we deserve them)

Ideal MTG

- Evolution of delta

$$E(k,t) = C(t)k^{-n(t)}\exp[-2\delta(t)k]$$



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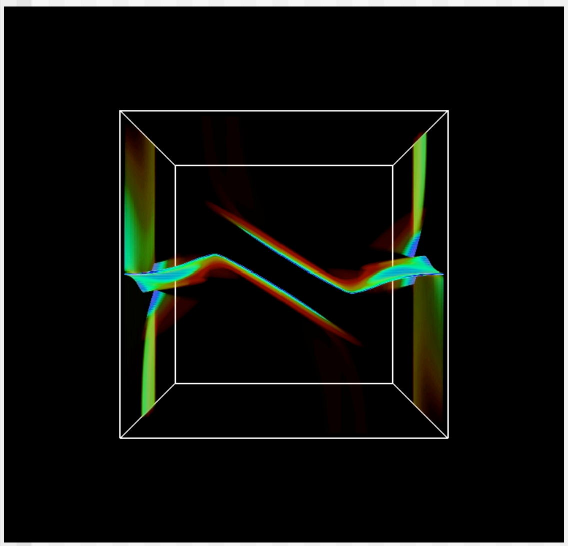
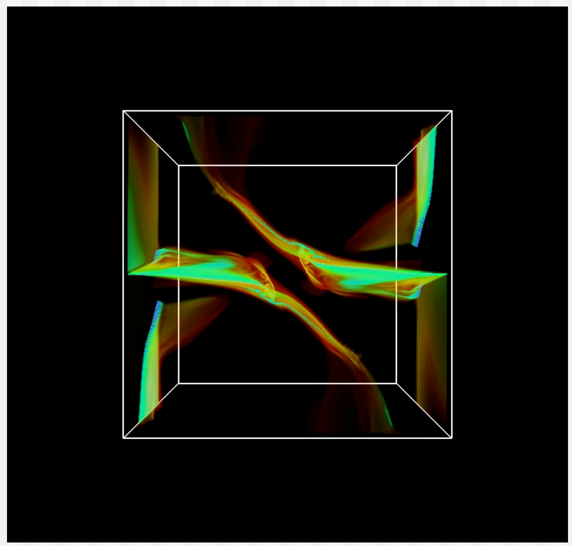
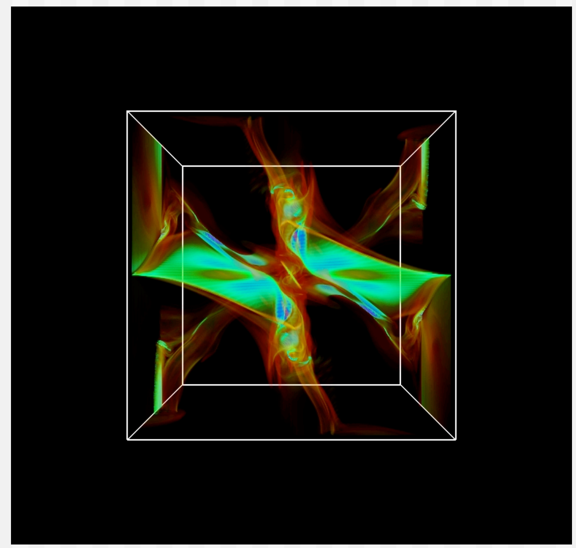
III. Breakthrough prospects with petascale resources:

Dissipative MTG

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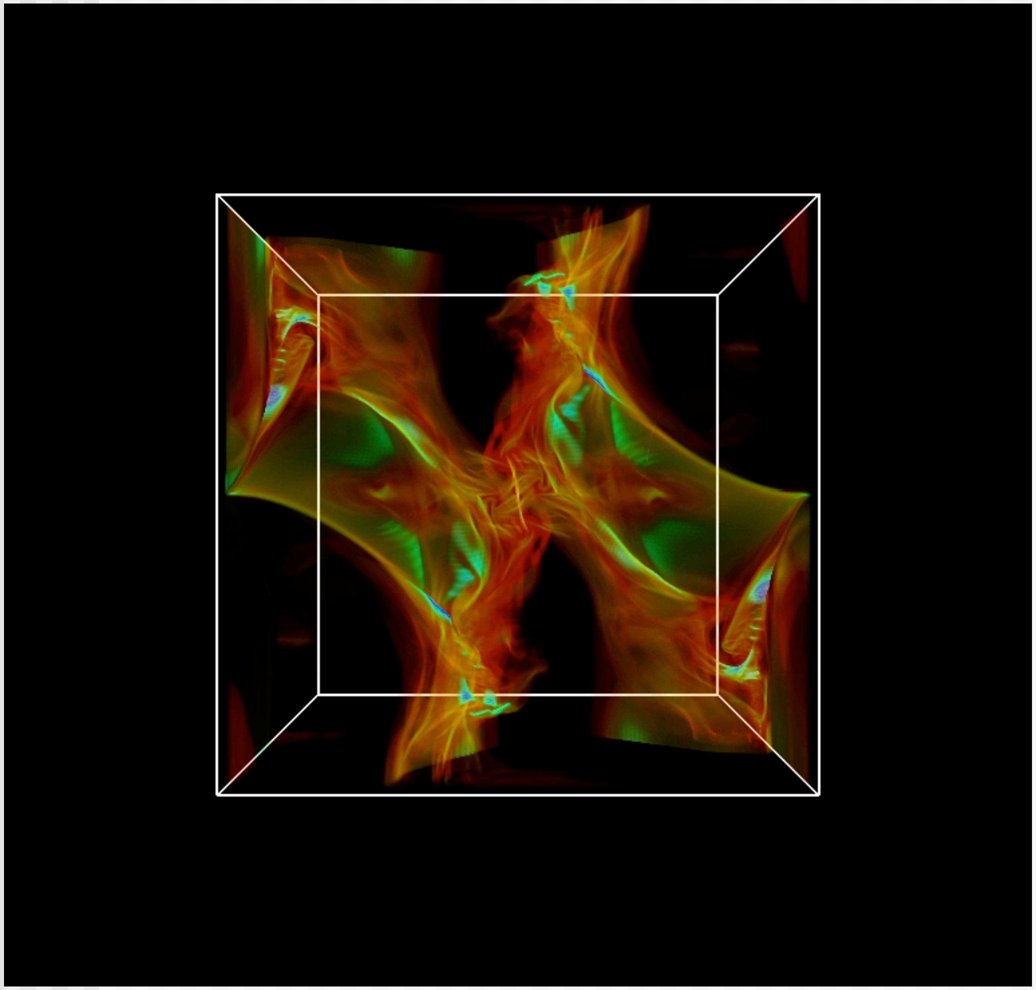
Dissipative MTG



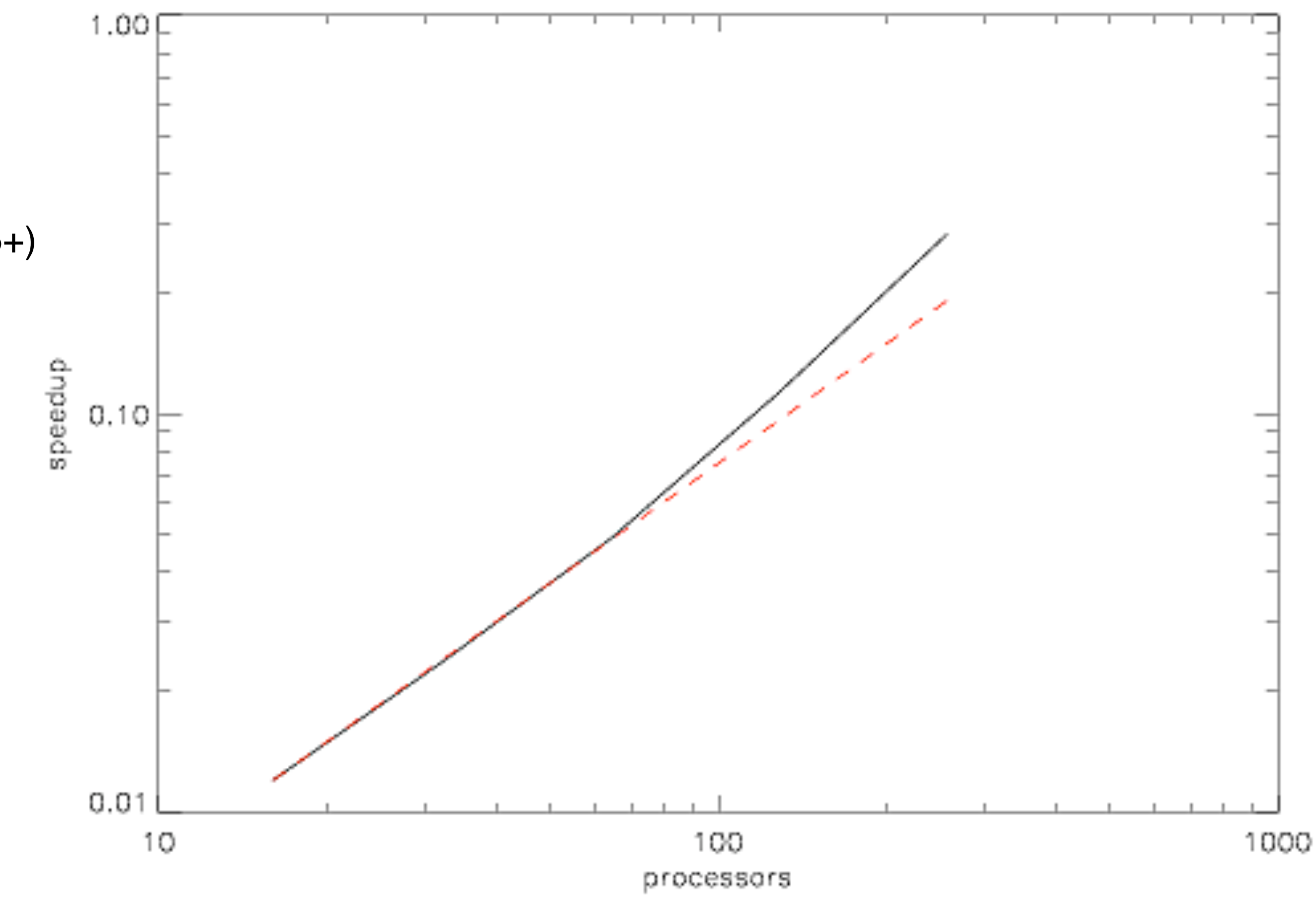
- Structures
  - Current sheets
  - Reconnection
  - Instabilities
- Wave turbulence
  - Spectra
  - Structure functions
  - Time scales

III. Breakthrough prospects with petascale resources:

Advantages of MTG

- Scaling

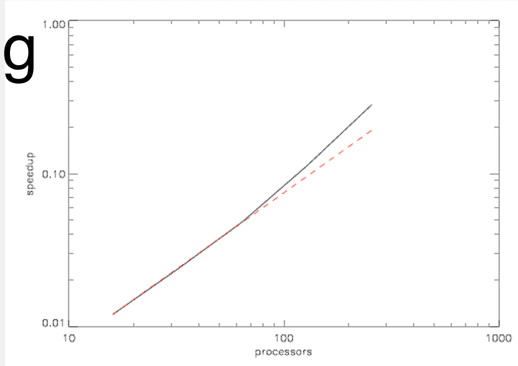
2048^3 TG
On Blueice (Power 5+)
using 16-256
processors



III. Breakthrough prospects with petascale resources:

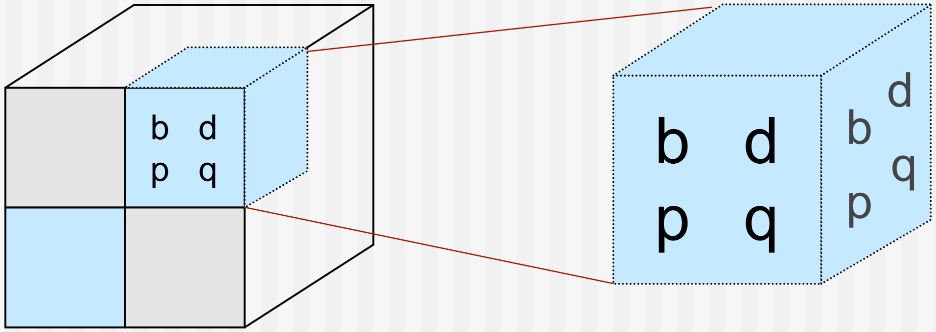
Advantages of MTG

- Scaling



- Symmetries
  - 32x fewer computations
  - 64x savings in memory

- Visualization (can visualize $(1/2)^3$ of the box)



Conclusion

- Taylor-Green symmetries
  - "Fully" resolved
- Ideal MTG
  - Development of smallest scales, complex-space singularities
- Dissipative MTG
  - Multiscale interactions, structures, scaling laws, wave turbulence
- Need for more powerful resources:
  - Integration: scale separation
  - Analysis and visualization