Identifying Vortical Structures and Their Impact From Laboratory Studies of Wall Turbulence

K. T. Christensen

Students: Y. Wu and V. K. Natrajan

Laboratory for Turbulence and Complex Flow (LTCF)
Department of Mechanical Science and Engineering
University of Illinois at Urbana-Champaign
Urbana, IL 61801 USA

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Wall turbulence

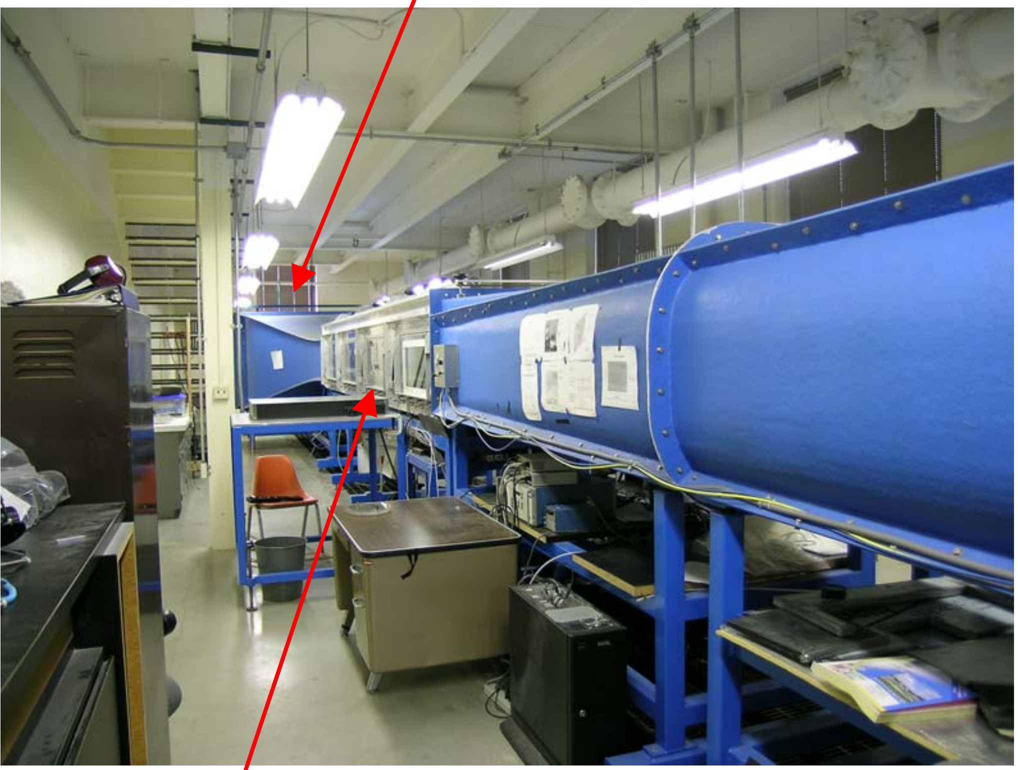
- Refers to a broad class of turbulent flows bounded by a surface:
  - Atmospheric boundary layer
  - Boundary layer on ocean floor
  - Flows over aircraft, ships, submarines, etc.
  - Flows over turbine blades, blades of windmills, etc.

- These flows are extremely difficult to study both experimentally and computationally.
  - As such, the simplest (canonical) cases have received the vast majority of research attention despite most practical flows of interest occurring in the presence of significantly more complexity.
    - High Reynolds numbers (Re)
    - Other influences: Surface roughness, pressure gradients, curvature, free-stream effects, multiple phases, buoyancy, etc.

- Coherent structures play a pivotal role in the evolution of such flows.

Boundary-layer wind tunnel

Inlet and flow conditioning



test section

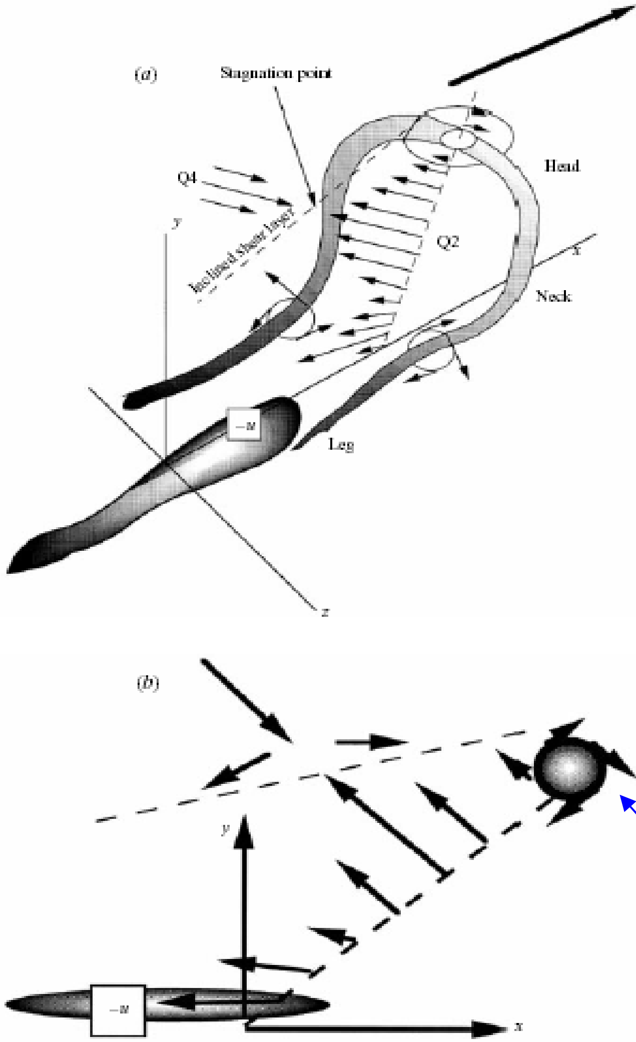
- Low-speed suction wind tunnel
- Test section: 1m × 1m cross-section; 6m streamwise fetch
- Boundary-layer thickness: ~100 mm
- Free-stream velocities: $3 < U_\infty < 40$ m/s
- Reynolds-number range: $1000 < Re_\theta < 15000$
 $300 < \delta^+ < 5000$

Particle image velocimetry (PIV)

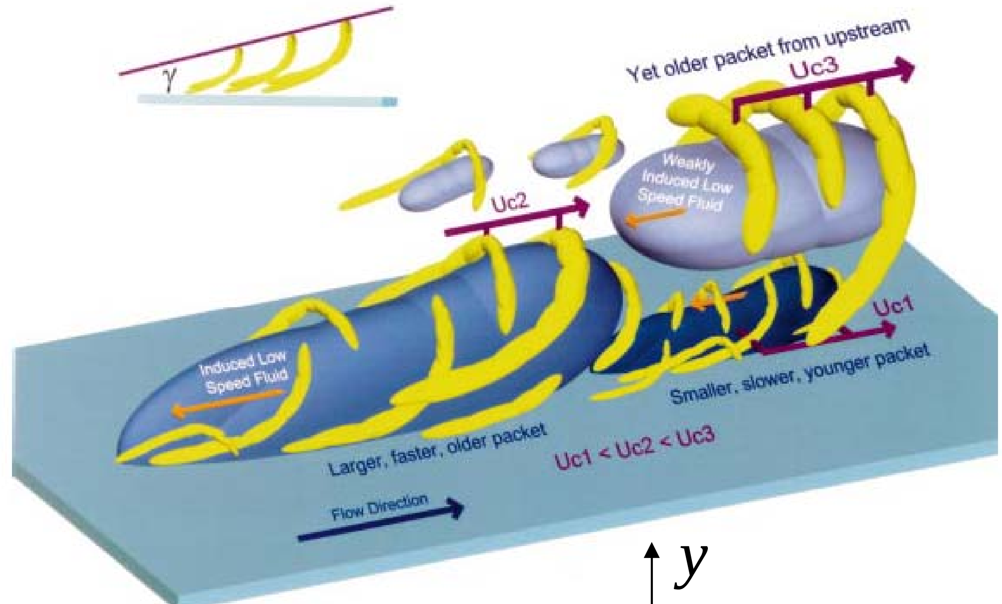
- Illumination: Pulsed laser (Nd:YAG)
- Imaging: Highly sensitive CCD cameras
- Tracer particles: sub-micron olive oil droplets

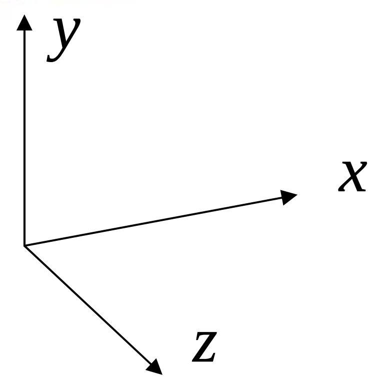
- RESULT: Velocity resolved *instantaneously* with high spatial resolution (10–20y_*) over planar domain comparable to the outer length scale in moderate Reynolds-number wall-bounded turbulence.

Outer-layer structure of wall turbulence



Head of hairpin vortex
("prograde" spanwise vortex)





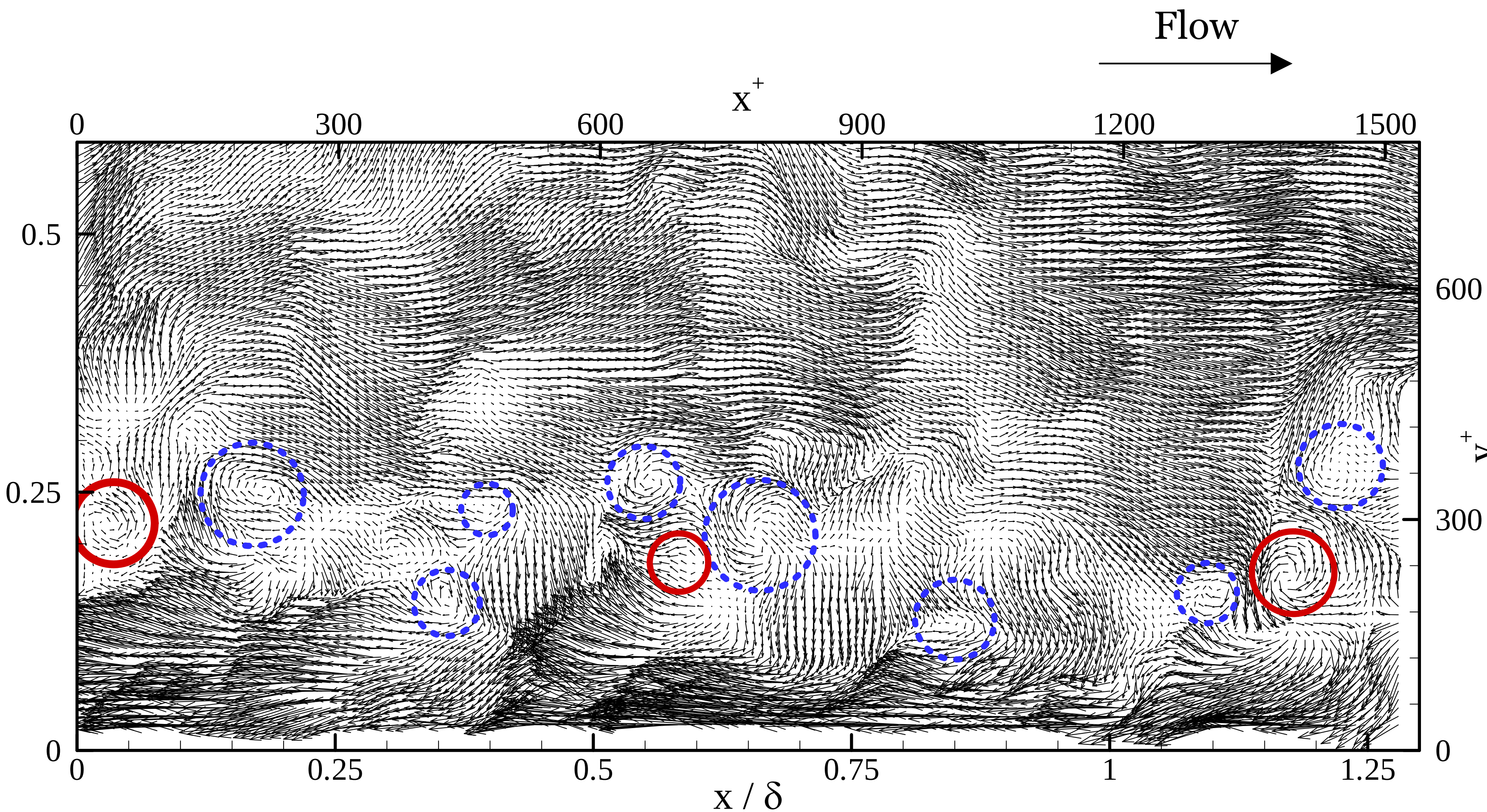
Adrian, Meinhart and Tomkins (2000), JFM

Assessing the importance of underlying structure

- It is now well established that an underlying structural foundation exists in wall-bounded turbulent flows.
  - What role do these structures play in the turbulence statistics (single- as well as multi-point)?
  - What are the basic characteristics of this organization?
  - What role might this structural foundation play in turbulence modeling and control?

- Challenges
  - Structures must be effectively extracted from the background turbulence.
    - Galilean decomposition (visualization in the reference frame of structure)
    - Local vortex markers
  - Analysis methodologies must be devised to study their importance and impact on the overall flow.
    - Spatial correlations
    - Conditional averaging

Galilean decomposition of representative instantaneous PIV velocity field



Galilean decomposition reveals only those vortices traveling at the chosen advection velocity

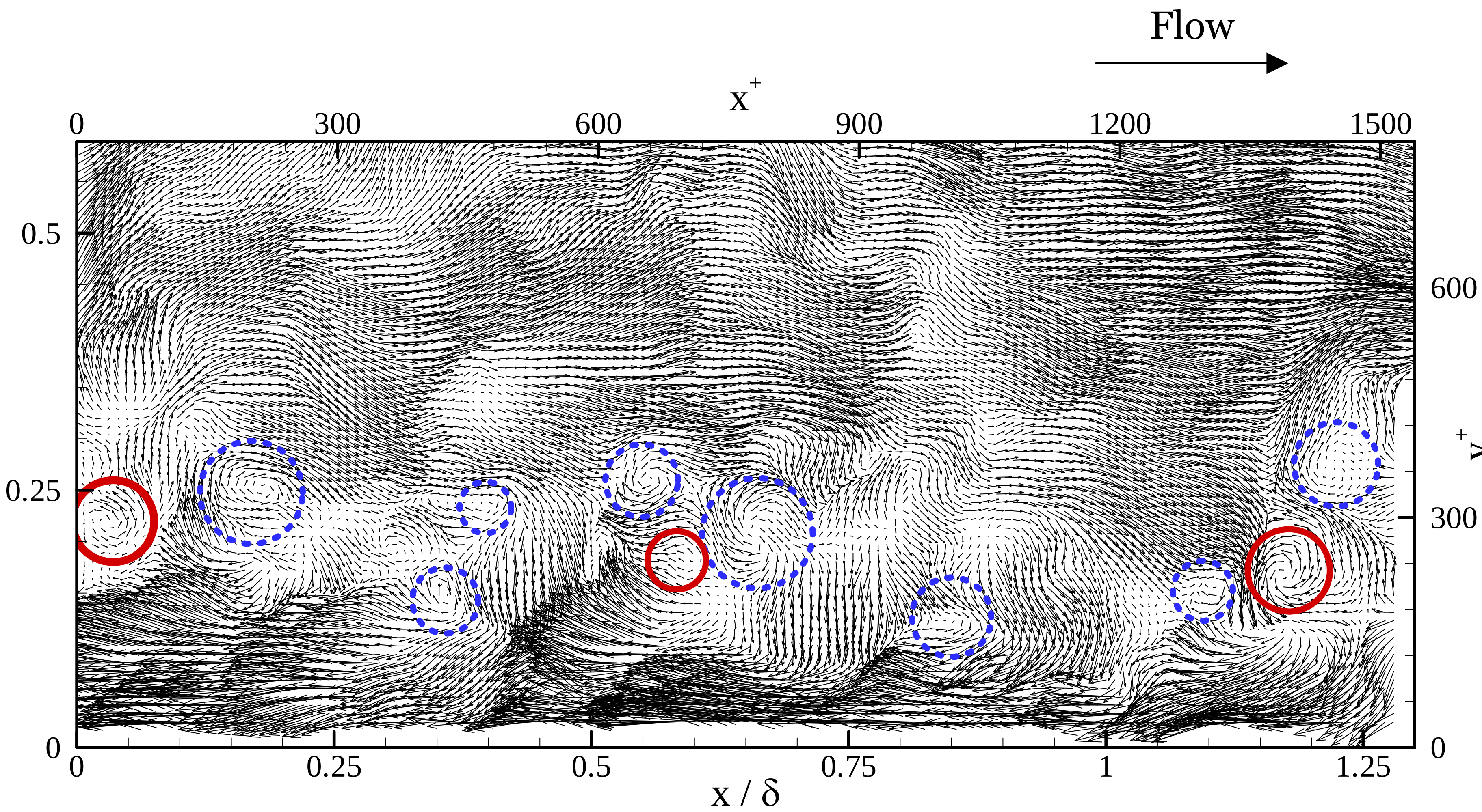
Vortex identification: Swirling strength (λ_{ci})

- Swirling strength (λ_{ci}) is the imaginary portion of the complex conjugate eigenvalues of the local velocity gradient tensor (Zhou *et al.*, 1999; Chakraborty *et al.*, 2005).
  - Unambiguous measure of rotation
  - Frame independent
  - Unlike vorticity, does not identify regions of intense shear

- For planar velocity data, one must employ a 2D version of the local velocity gradient tensor.
  - Will have either 2 real or a complex-conjugate pair of eigenvalues

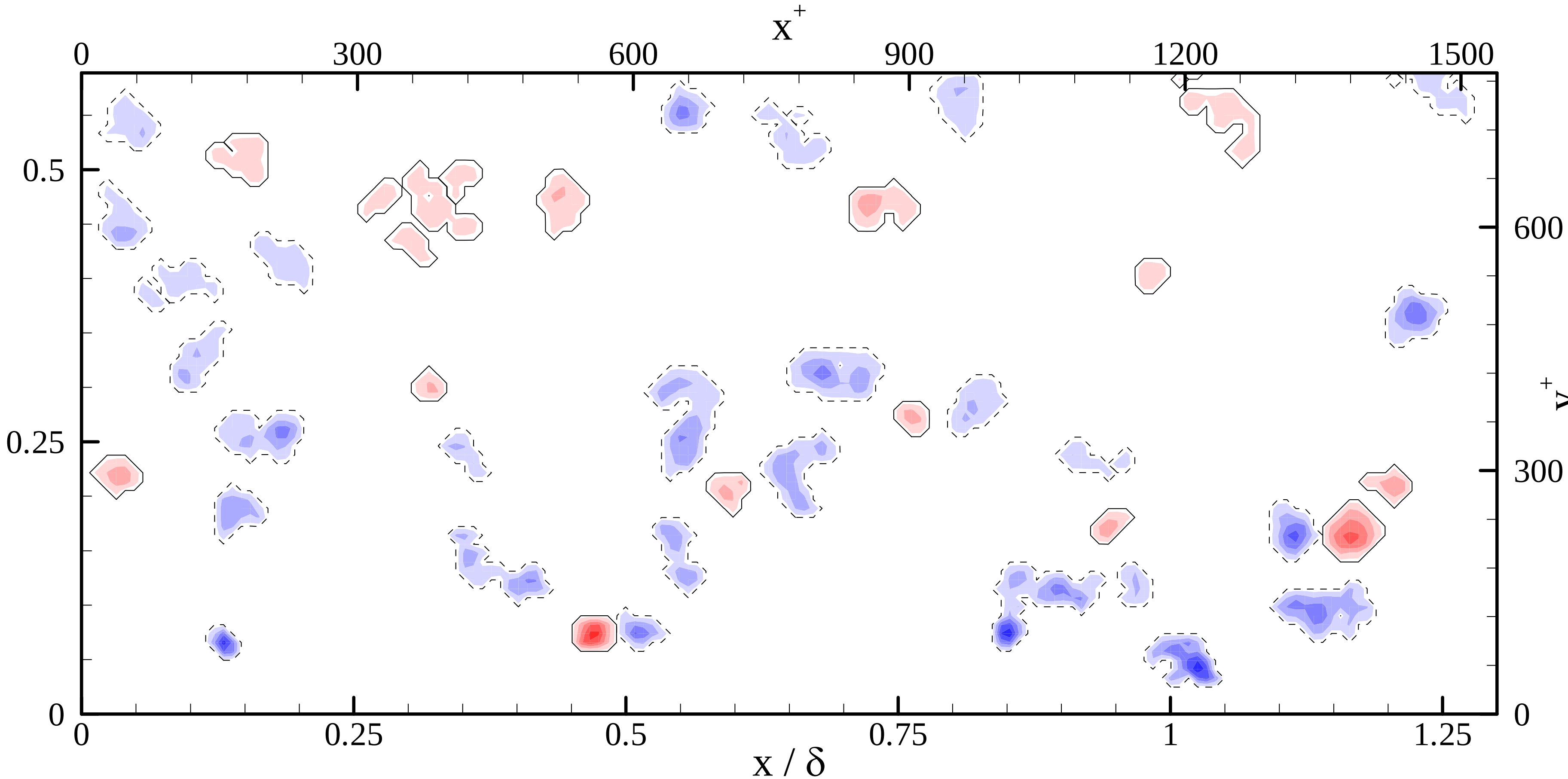
$\Rightarrow \lambda_{ci} \geq 0$

Galilean decomposition of representative instantaneous PIV velocity field



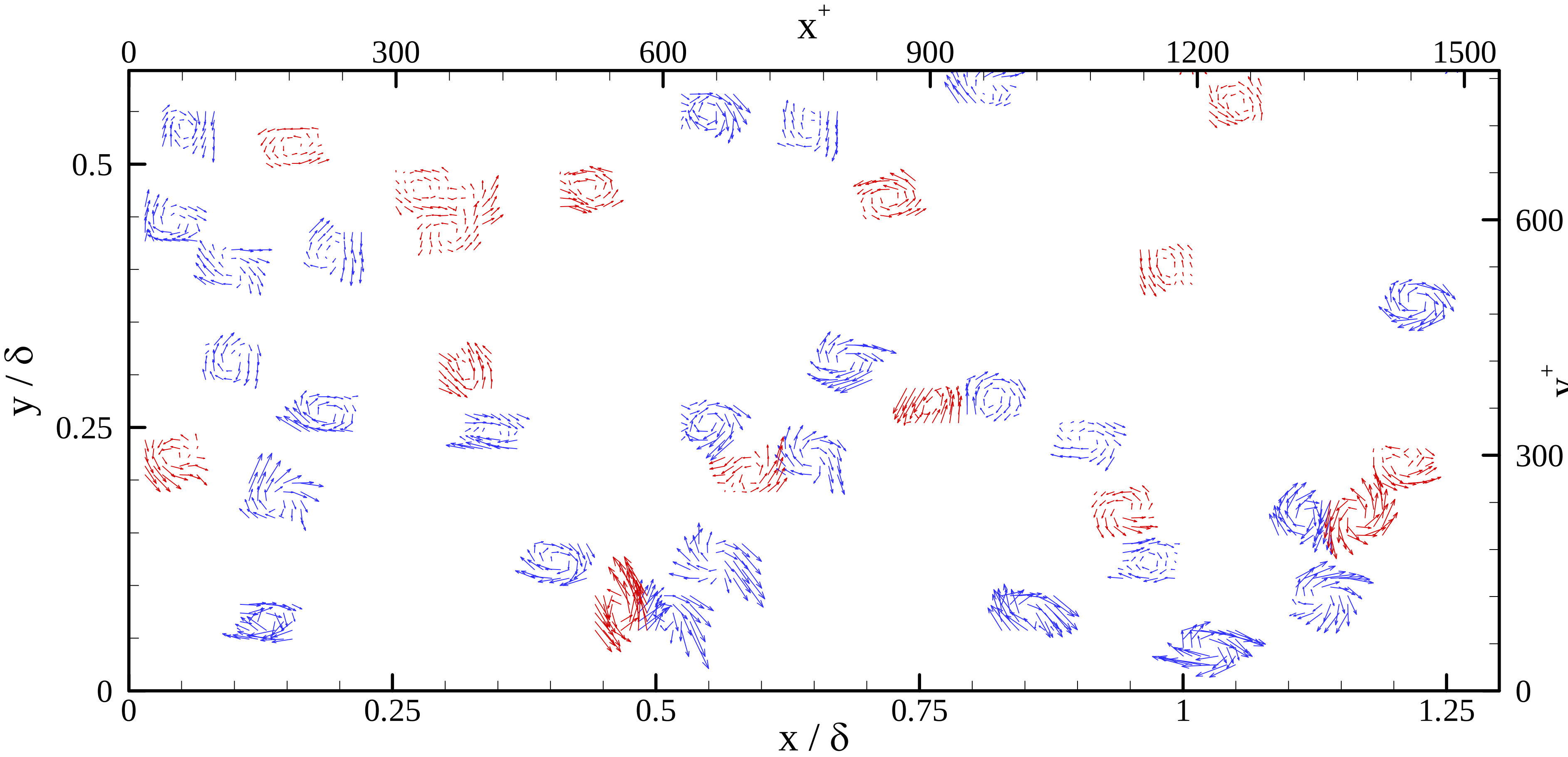
Associated Λ_{ci} field

$$\Lambda_{ci} \equiv \lambda_{ci} \frac{\omega_z}{|\omega_z|}$$

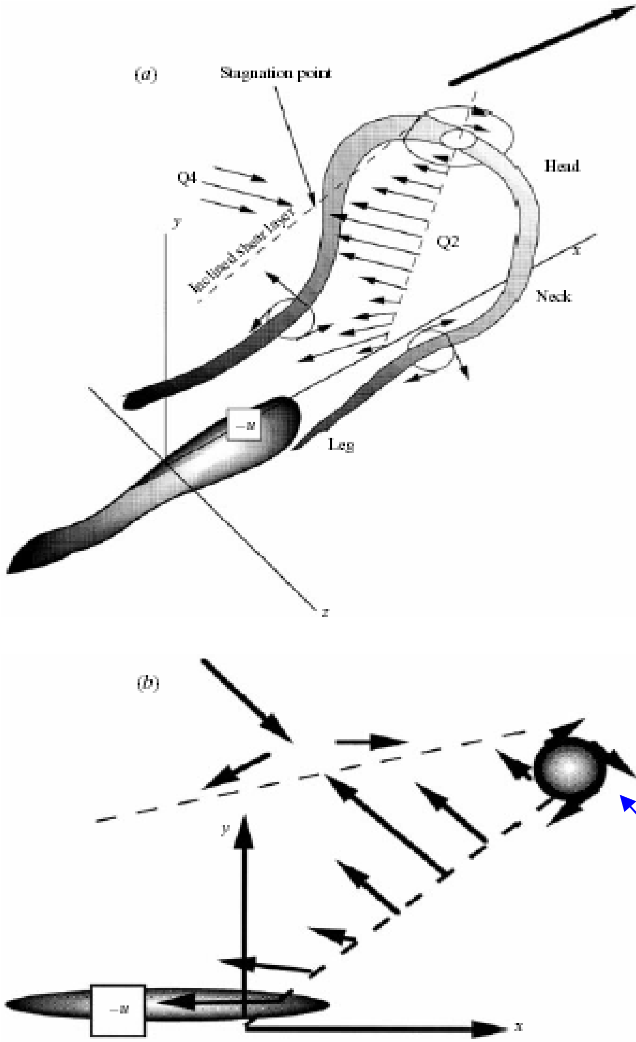


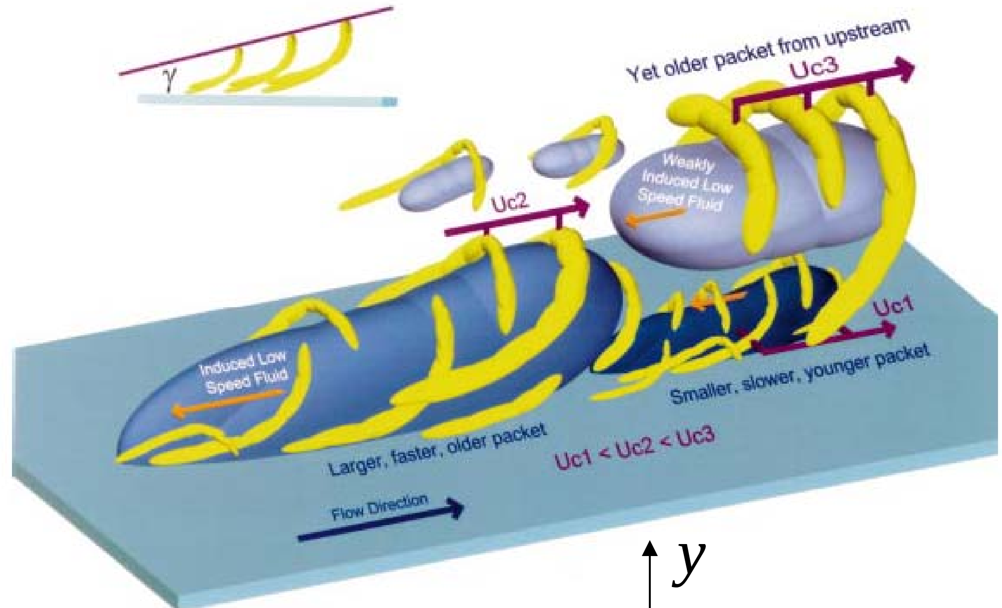
Clockwise Counterclockwise

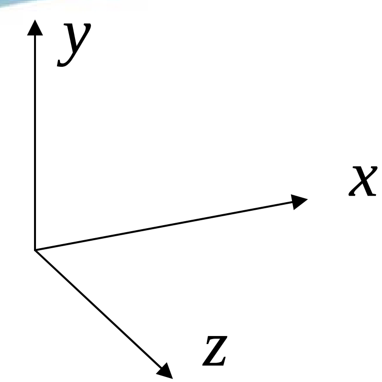
Local Galilean decomposition around Λ_{ci} events



Outer-layer structure of wall turbulence





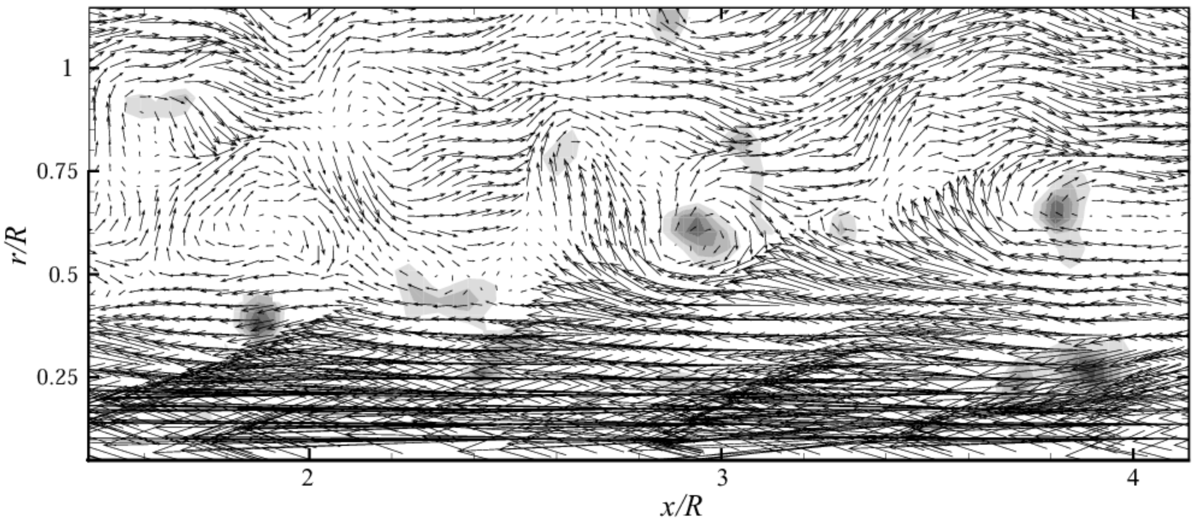


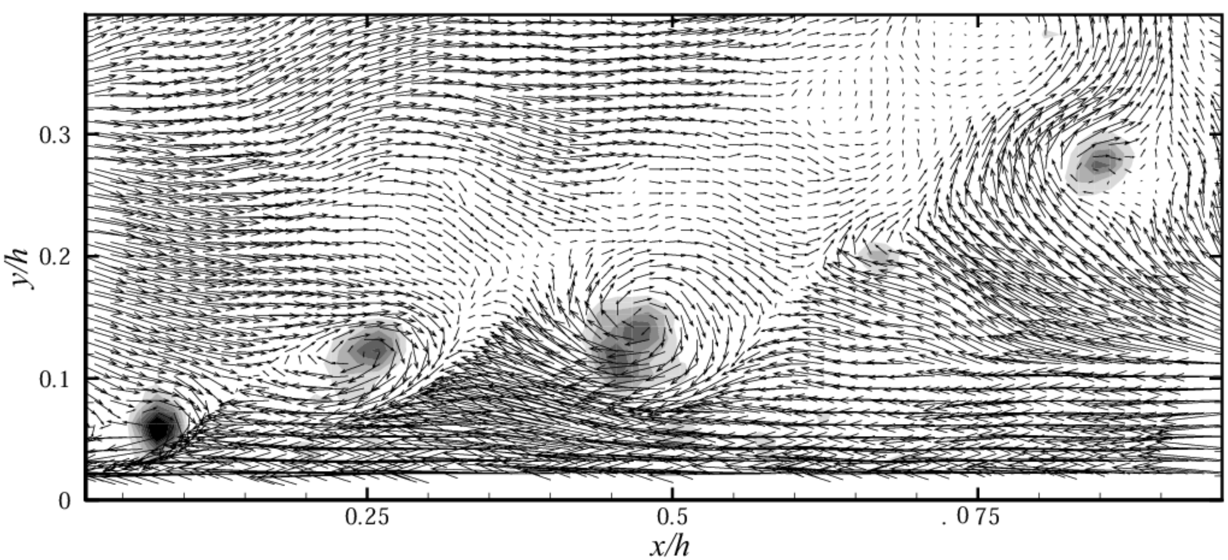
Head of hairpin vortex
(“prograde” spanwise vortex)

Adrian, Meinhart and Tomkins (2000), JFM

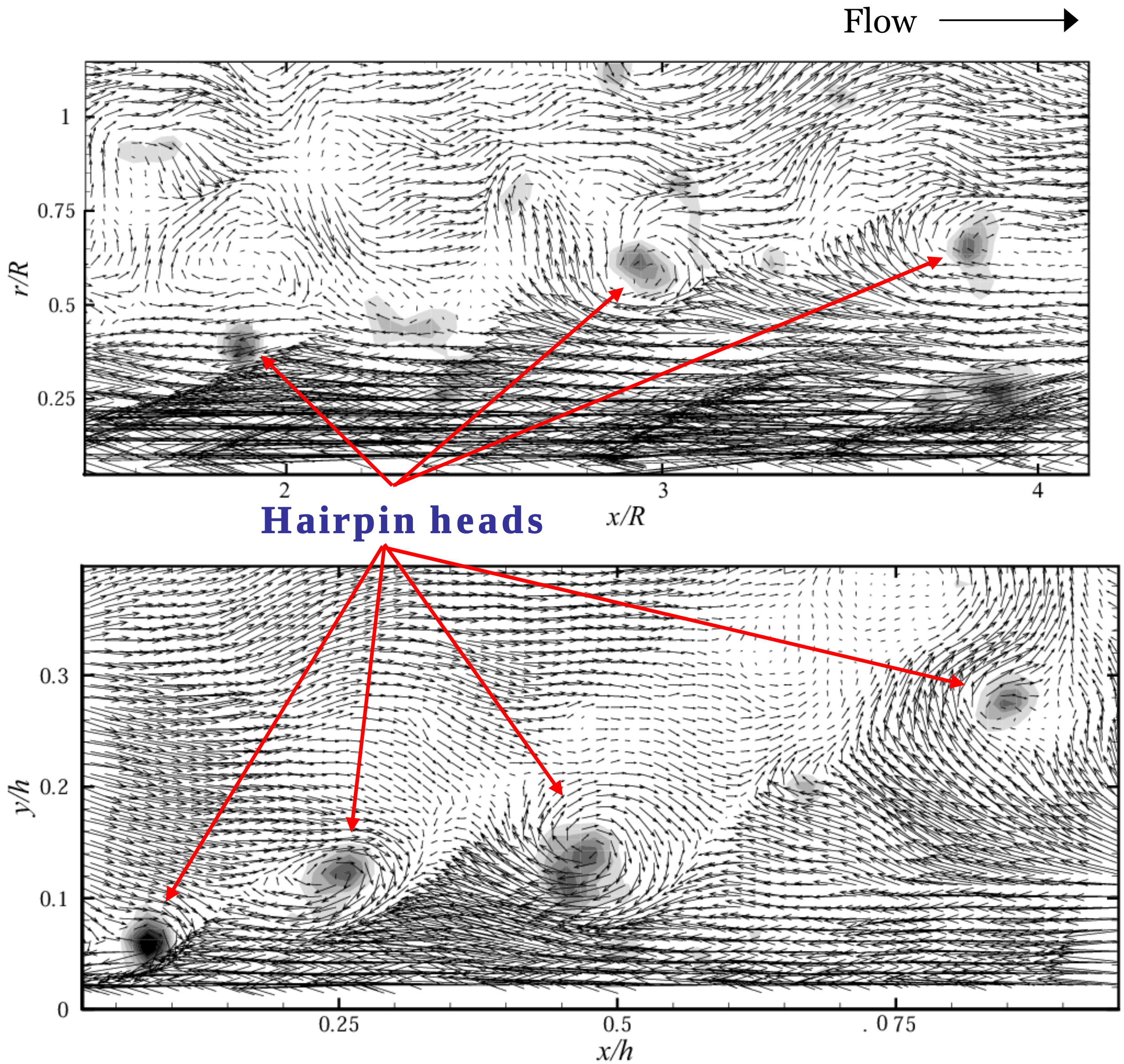
Outer-layer structure of wall turbulence: *x–y plane*

Flow

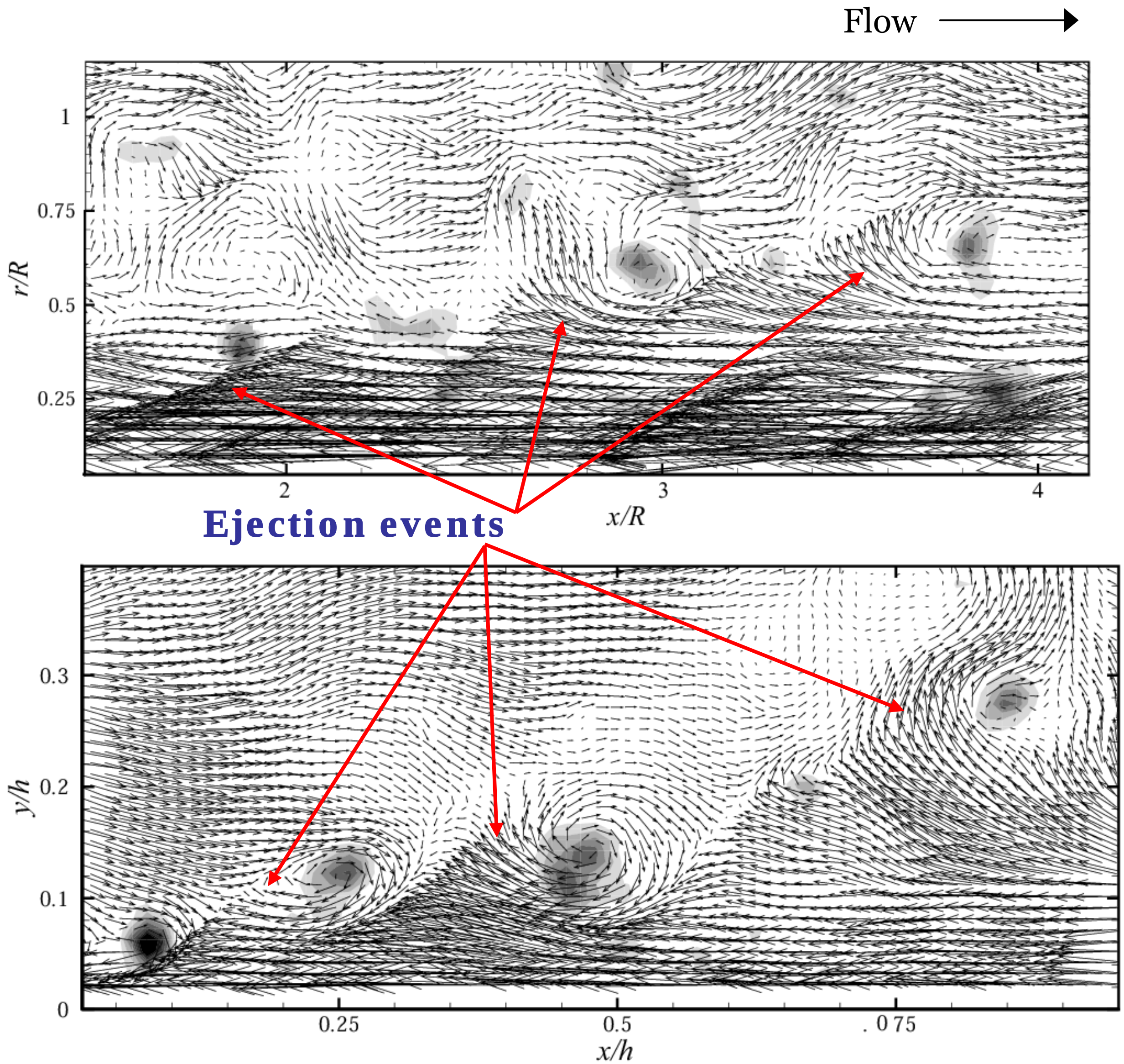




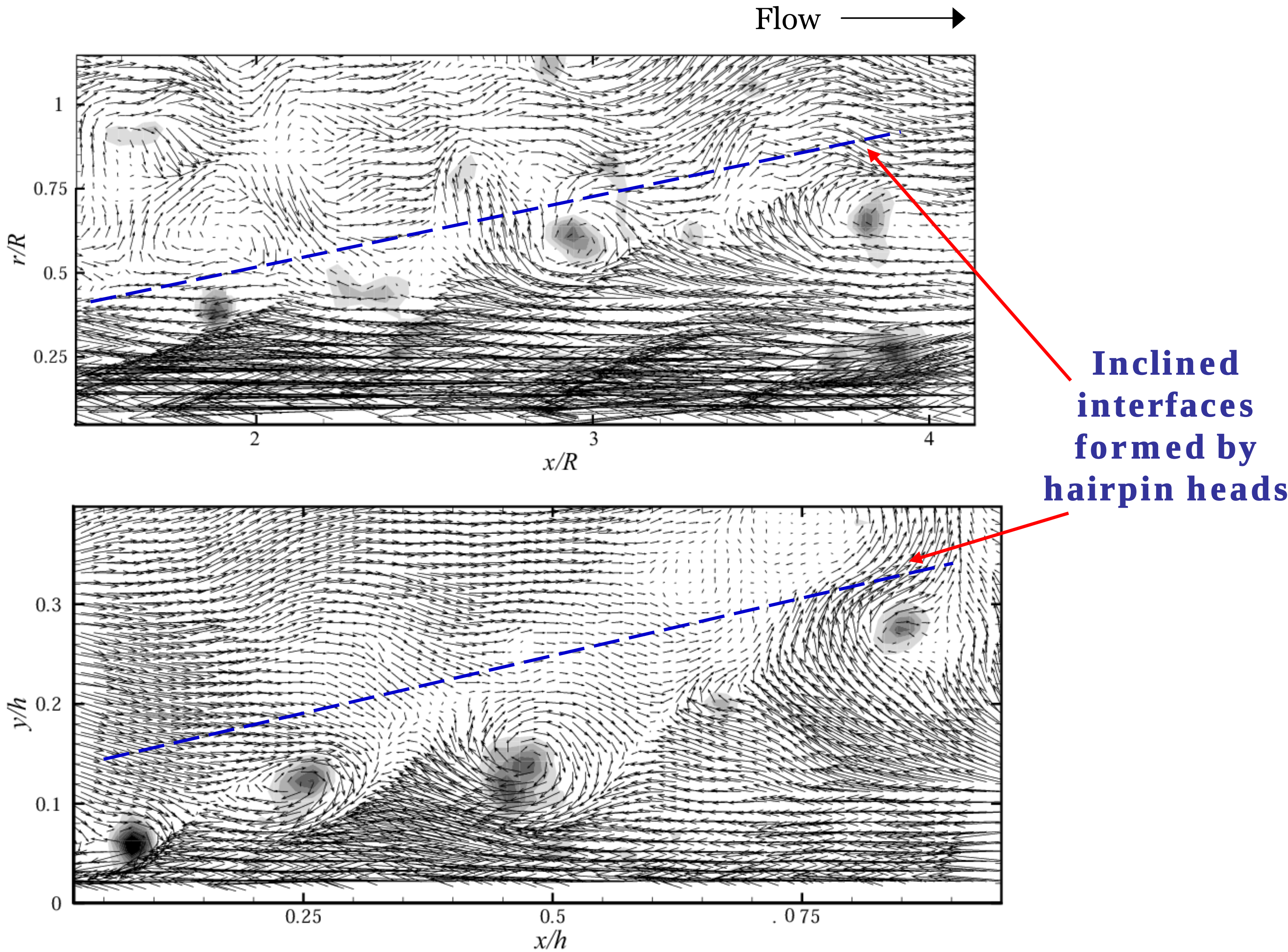
Outer-layer structure of wall turbulence: *x–y plane*



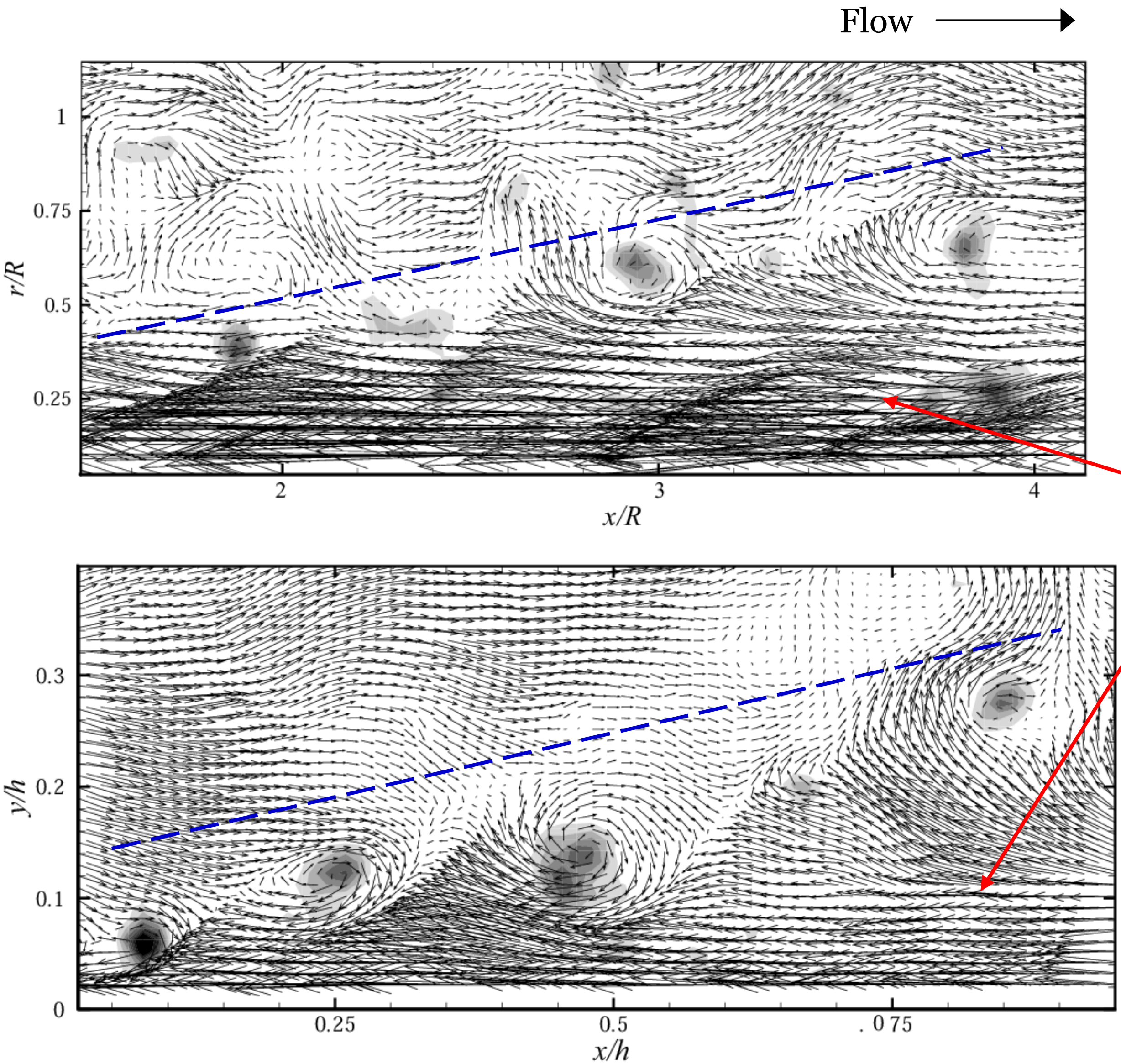
Outer-layer structure of wall turbulence: *x–y plane*



Outer-layer structure of wall turbulence: *x–y plane*



Outer-layer structure of wall turbulence: *x–y plane*

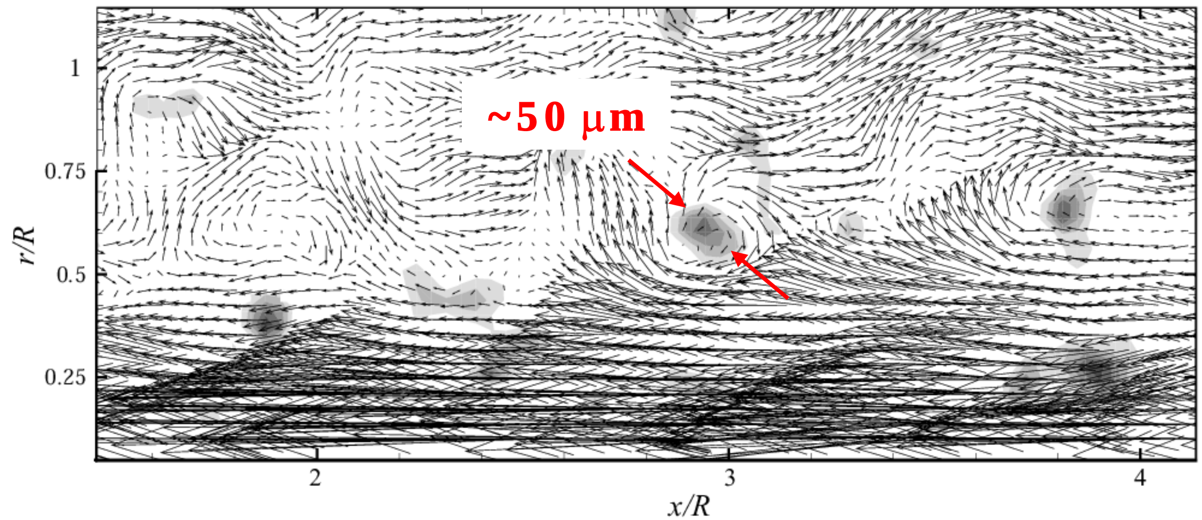


Induced low-momentum regions (LMR's)

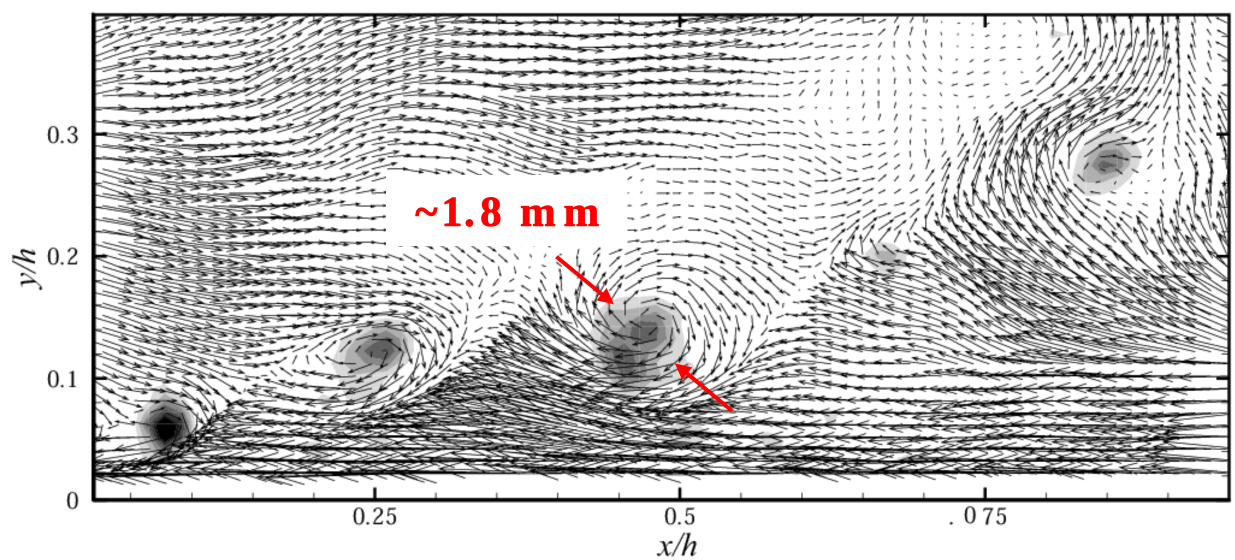
Outer-layer structure of wall turbulence: *x–y plane*

Turbulent capillary flow
R = 0.268 mm
Re_τ = 167
Micro-PIV result

Natrajan, Yamaguchi and Christensen (2007), Microfluidics and Nanofluidics ***3****(1)*



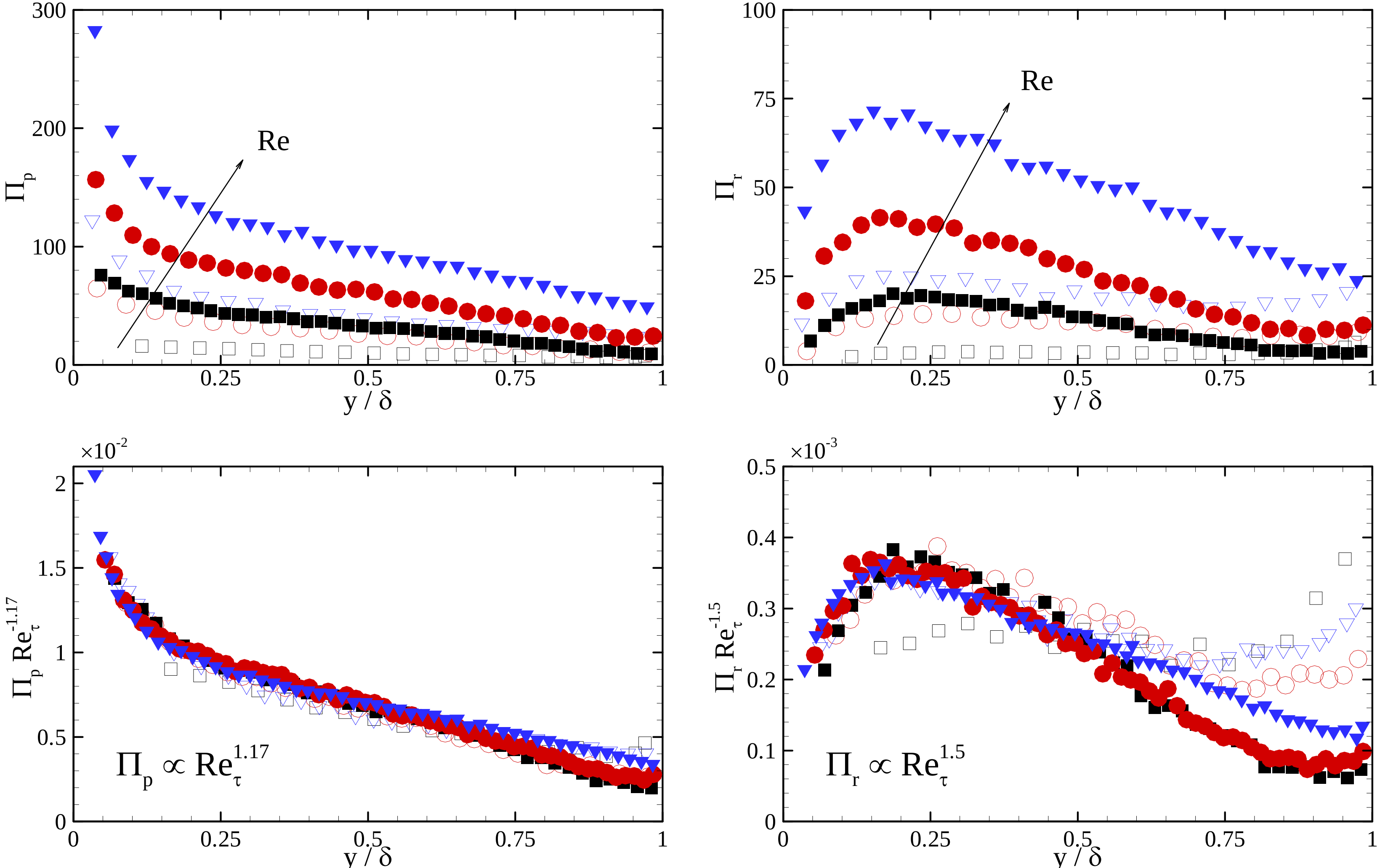
Macroscale turbulent channel flow
h = 25.0 mm
Re_τ = 550
Lightsheet PIV result



Vortex population statistics

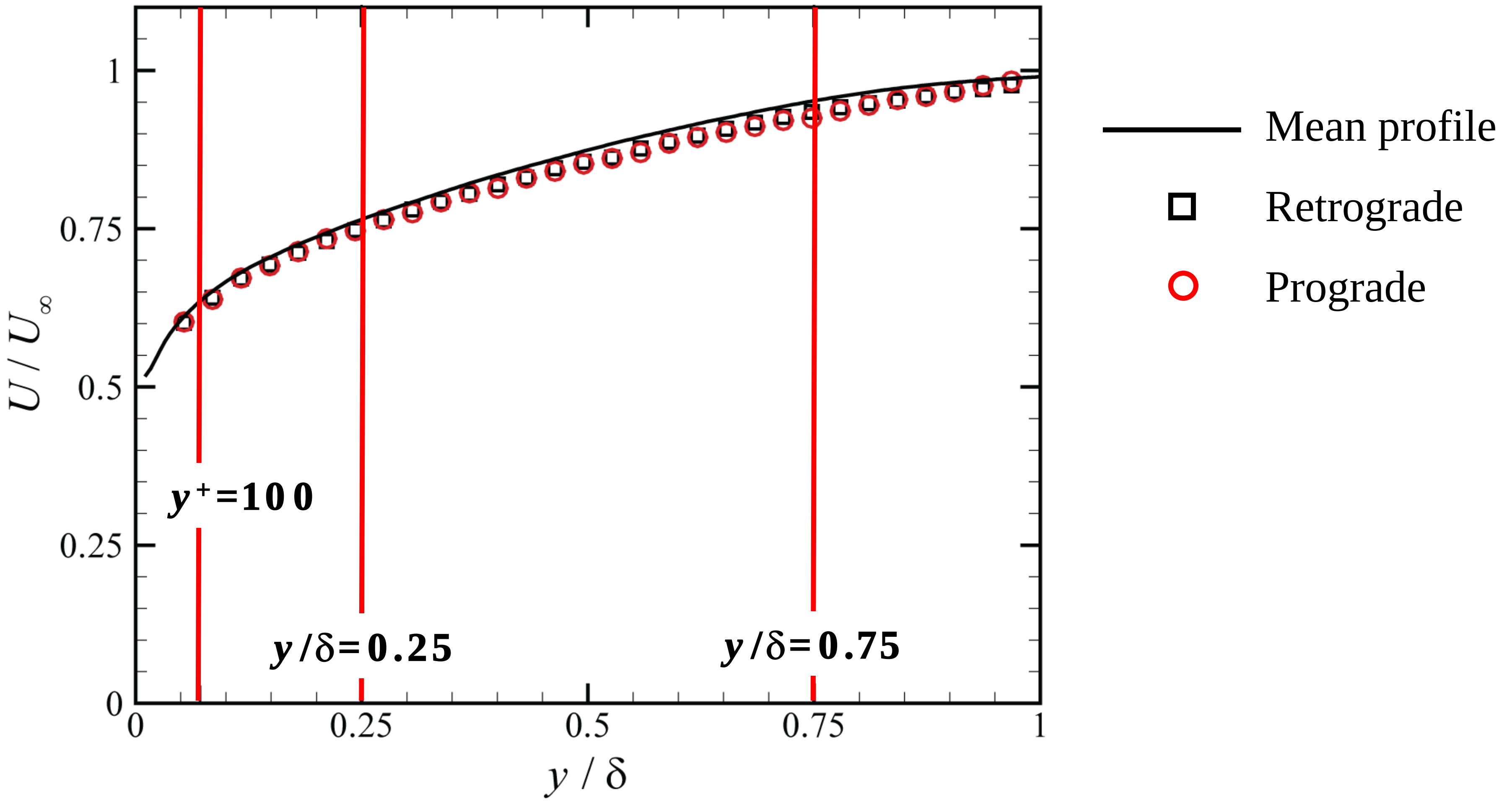
$$\Pi_{p(r)} \equiv \frac{N_{p(r)}}{\frac{3\Delta y}{\delta}\,\frac{L_x}{\delta}}$$

Prograde | Retrograde



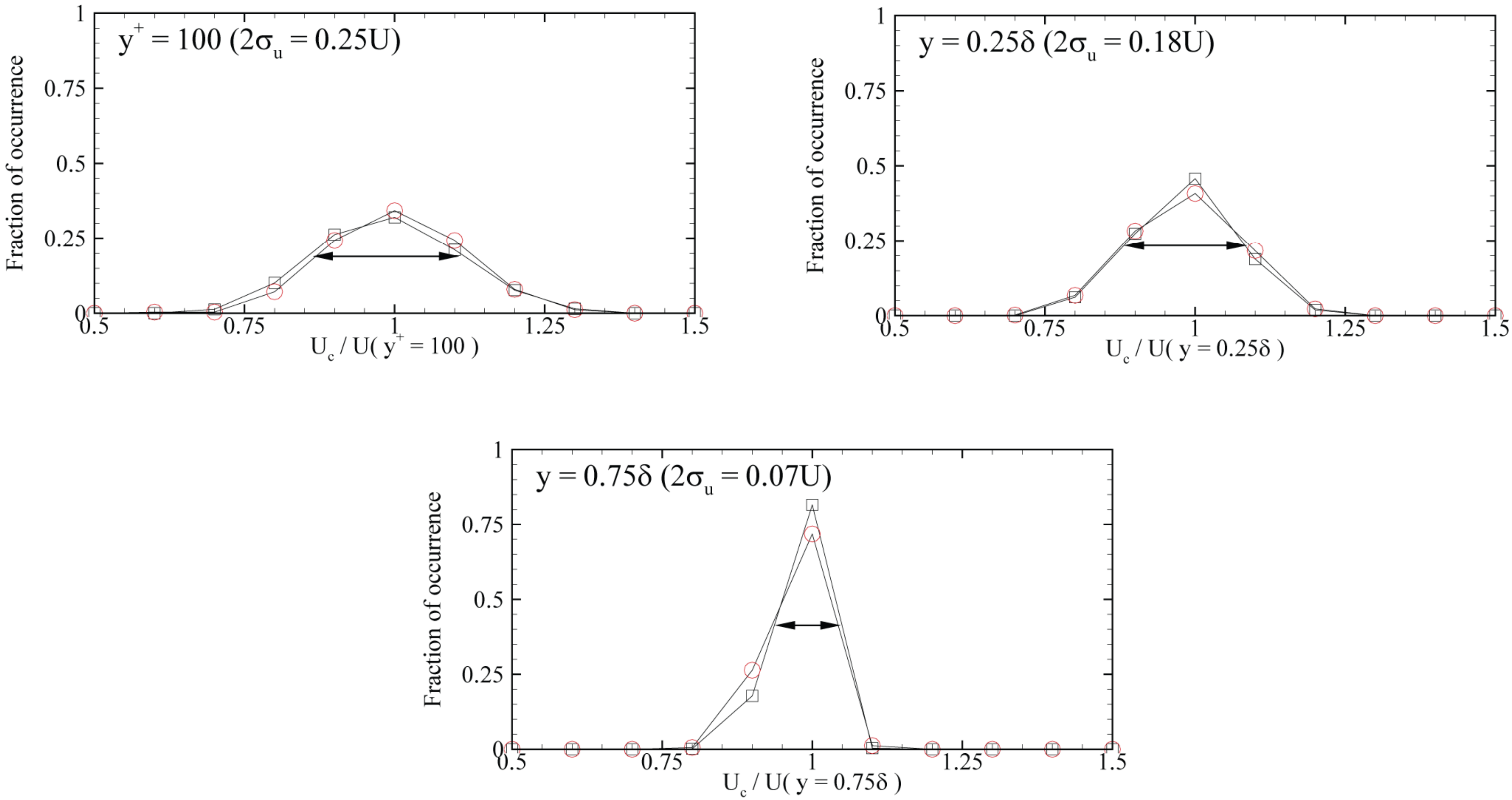
Wu and Christensen (2006), JFM

Vortex advection velocities: TBL



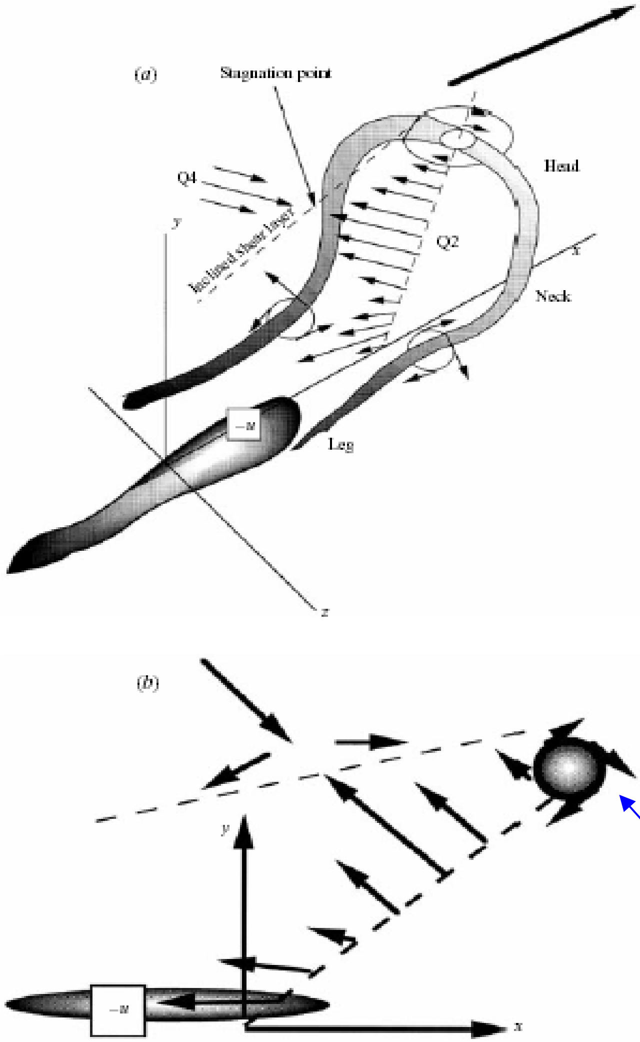
Wu and Christensen (2006), JFM

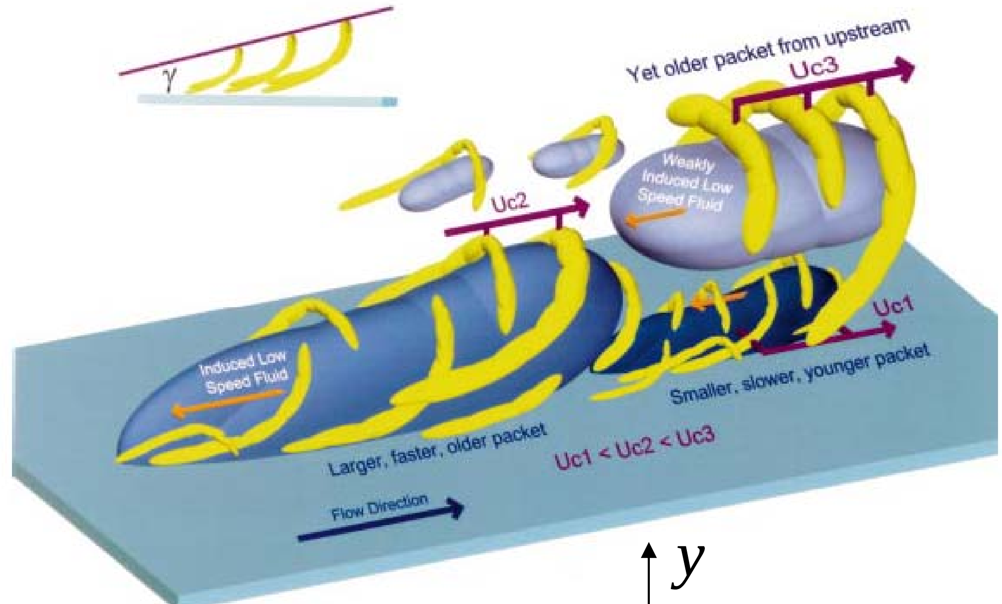
Histograms of advection velocities

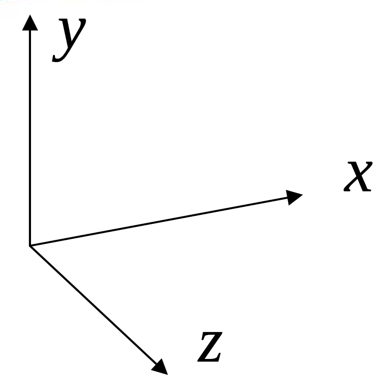


Wu and Christensen (2006), JFM

Outer-layer structure of wall turbulence







Head of hairpin vortex
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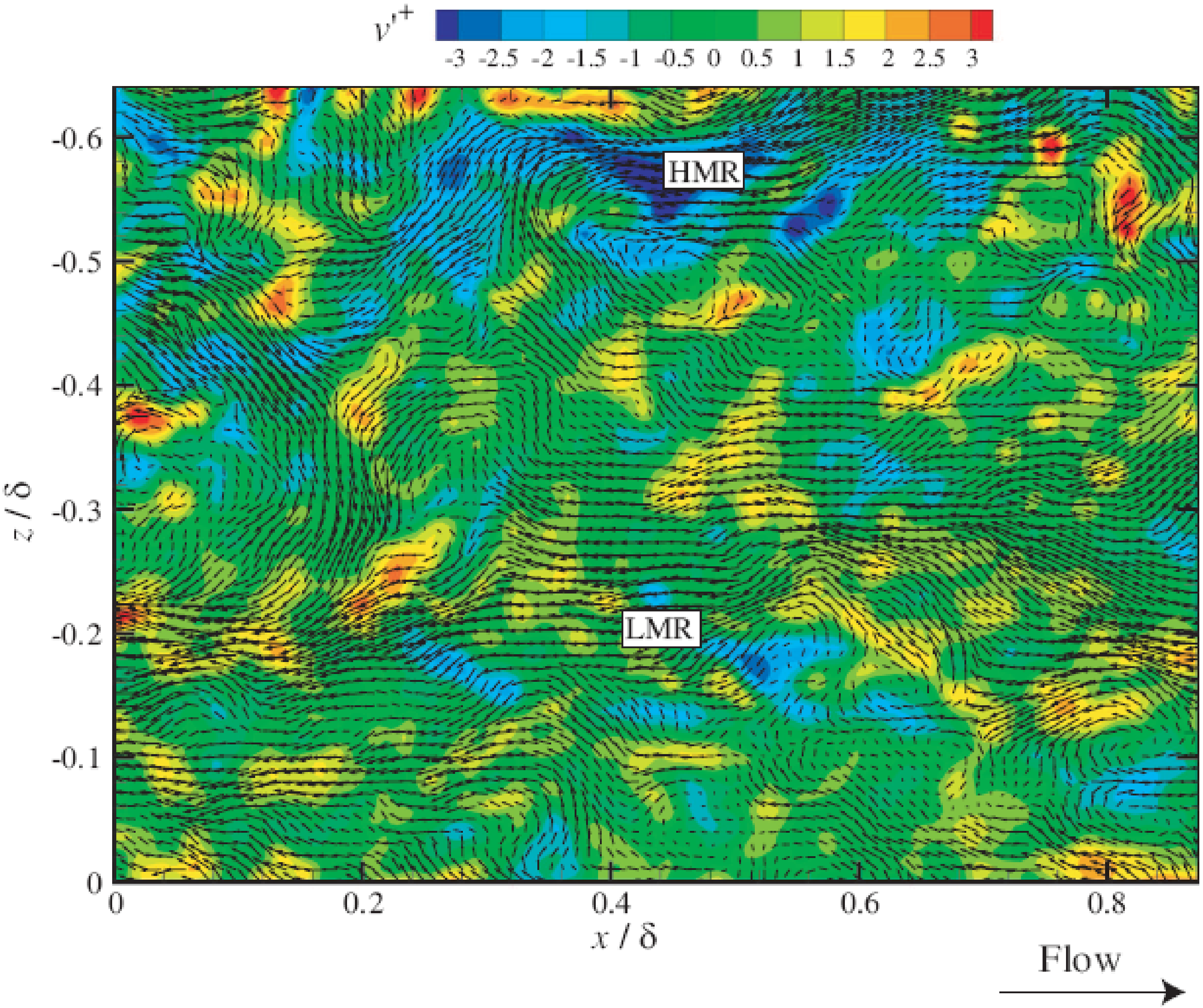
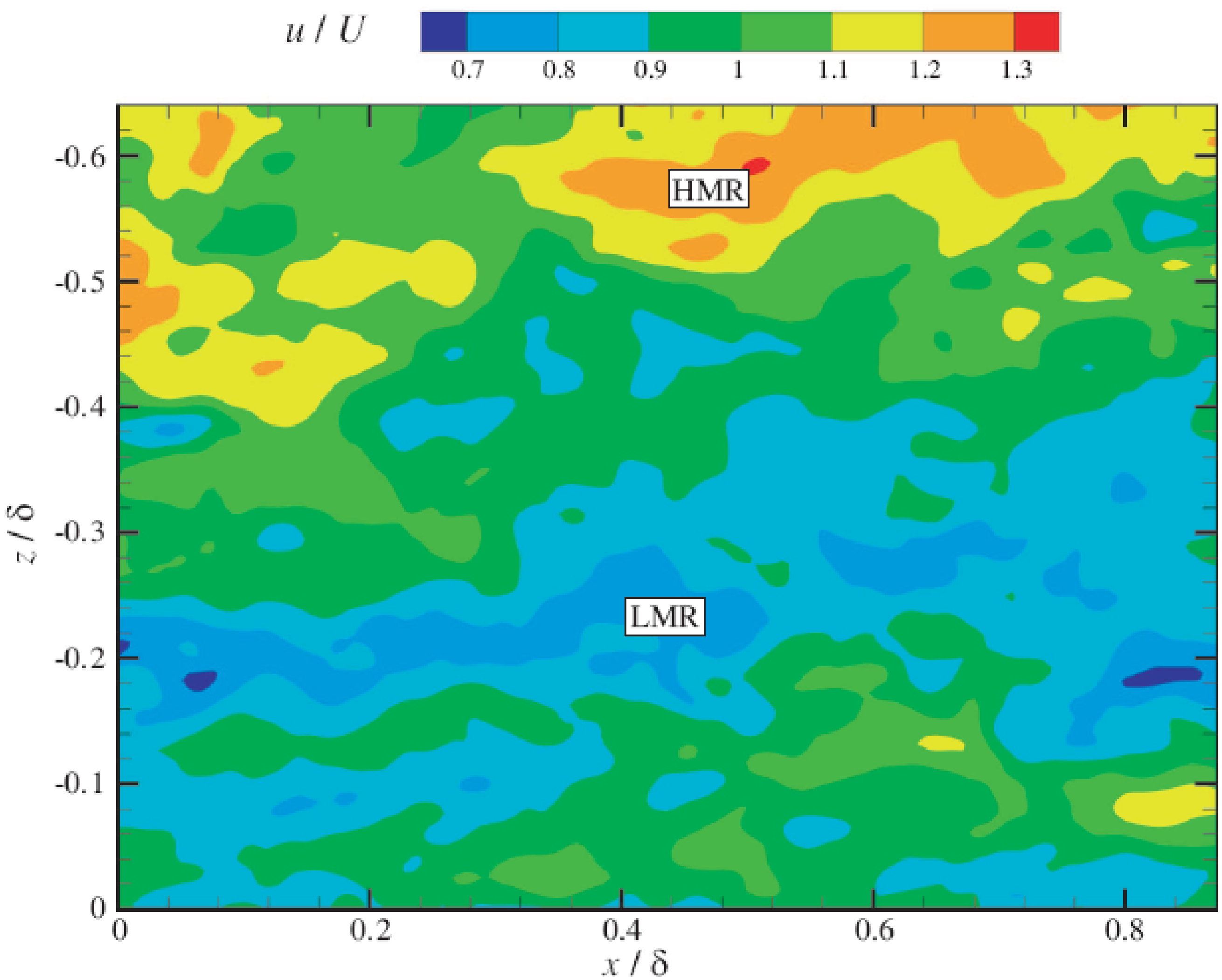
Adrian, Meinhart and Tomkins (2000), JFM

Outer-layer structure of wall turbulence: *x–z plane*



Outer-layer structure of wall turbulence: *x–z plane*



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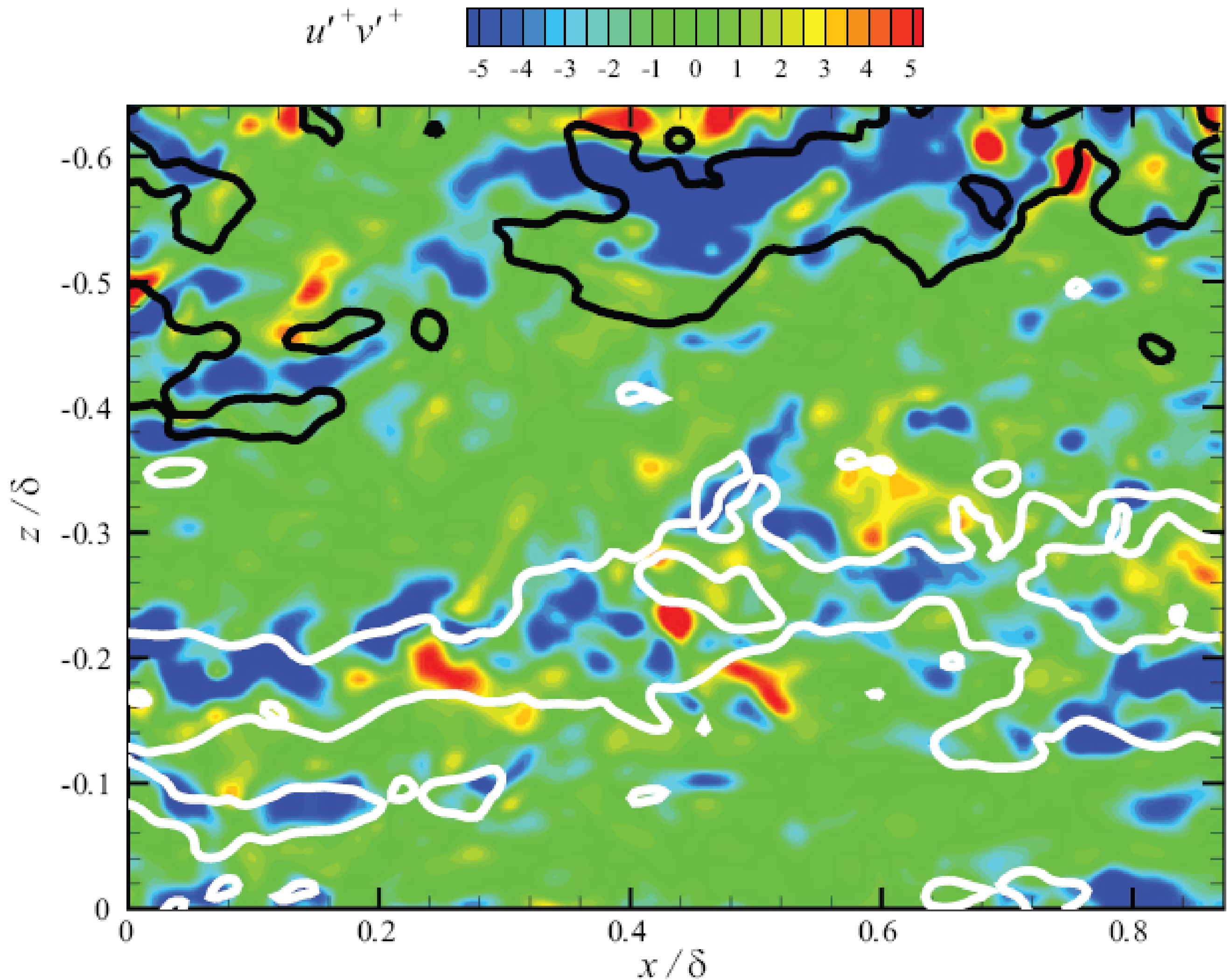
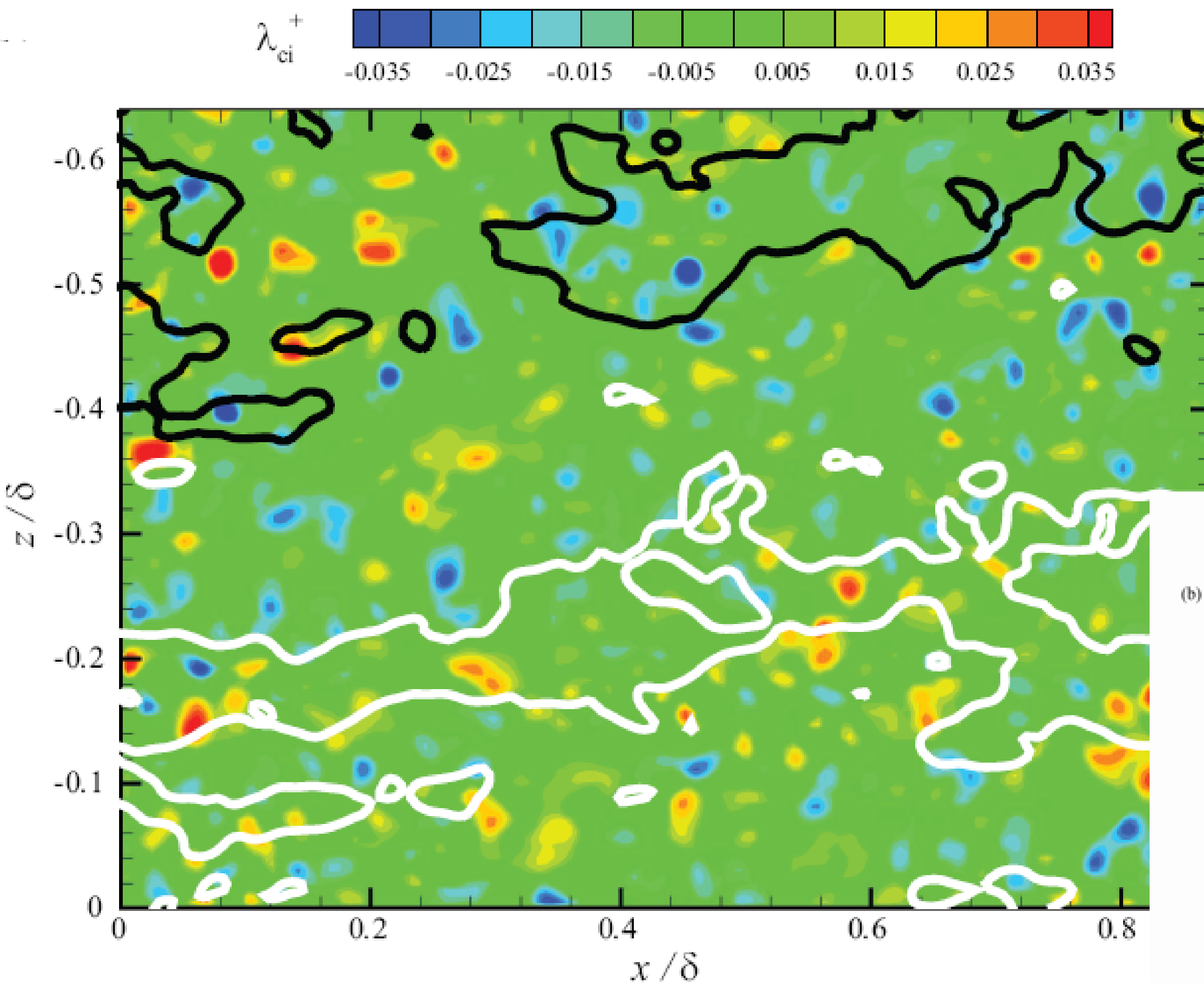
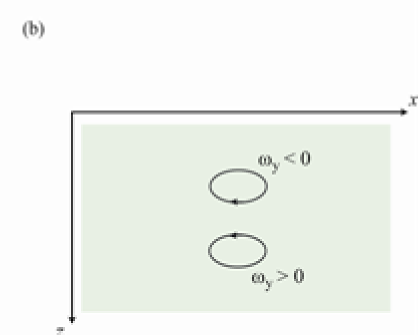


Outer-layer structure of wall turbulence: *x–z plane*





Contributions of LMR's to single-point statistics

- Define low-momentum threshold and identify gridpoints satisfying the threshold:

$$I(x_j, z_j; U_{th}) = \begin{cases} 1, & \text{when } u(x_j, z_j) \le U_{th} \\ 0, & \text{otherwise,} \end{cases}$$

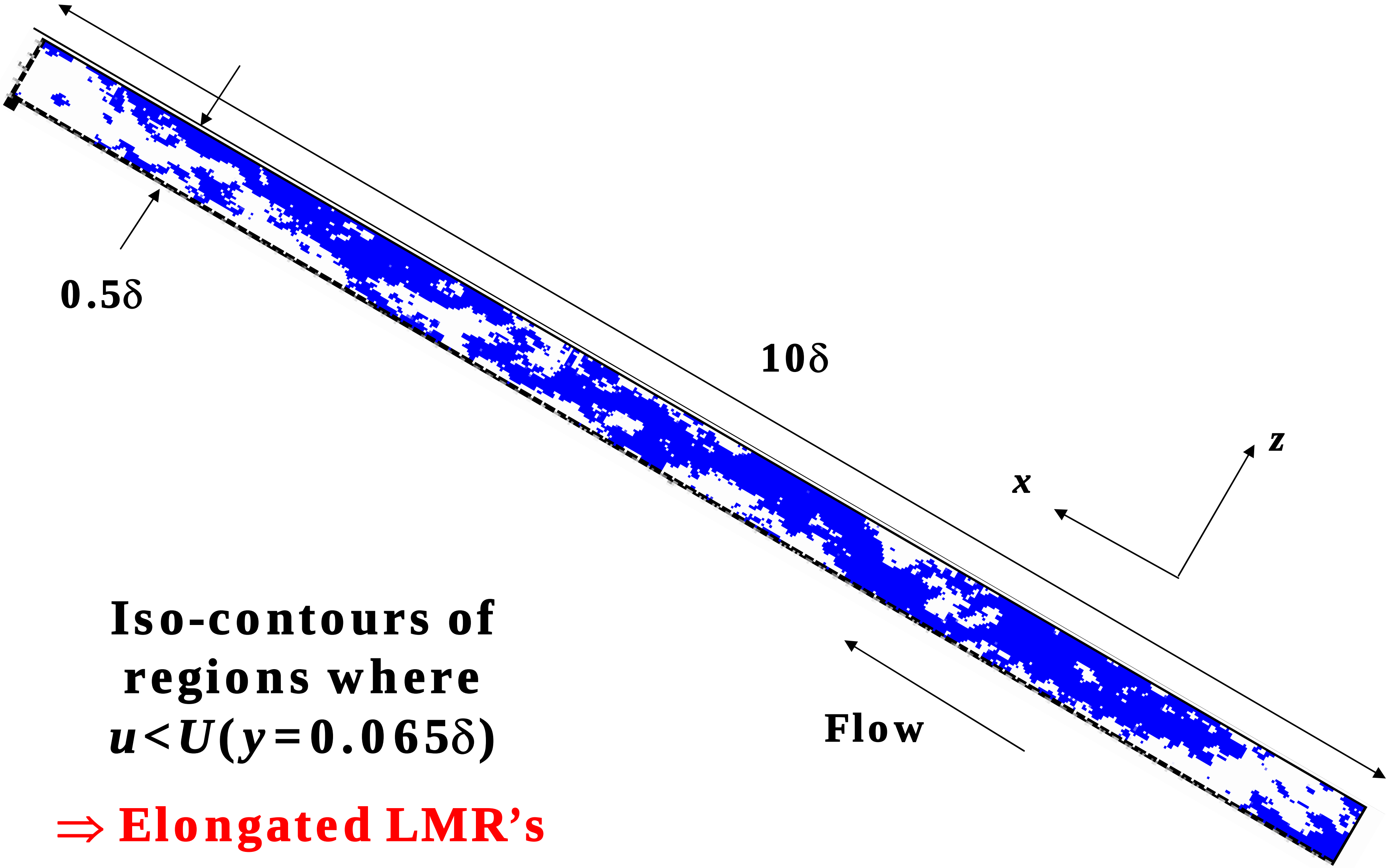
- Average quantity of interest, *S*, satisfying threshold:

$$\langle S \rangle (U_{th}) = \frac{1}{M \times P} \sum_{\text{all}x} \sum_{\text{all}z} \sum_{j=1}^{M} S(x_j, z_j) I(x_j, z_j; U_{th}),$$

$y = 0.065\delta$ ($y^+=200$)

Threshold	$-\langle u'v' \rangle$	$\langle u'^2 \rangle$	$\langle v'^2 \rangle$	$\langle w'^2 \rangle$	$\langle q^2 \rangle$	Space occupied
$0.9U$	46%	43%	14%	29%	34%	21%
$0.8U$	18%	16%	4%	5%	12%	4%
$0.7U$	1.3%	1.2%	0.3%	0.1%	0.8%	0.2%

'Super'-structures at $y = 0.065\delta$



Iso-contours of regions where $u < U(y = 0.065\delta)$

$\Rightarrow$ Elongated LMR's

Statistical imprints of structure

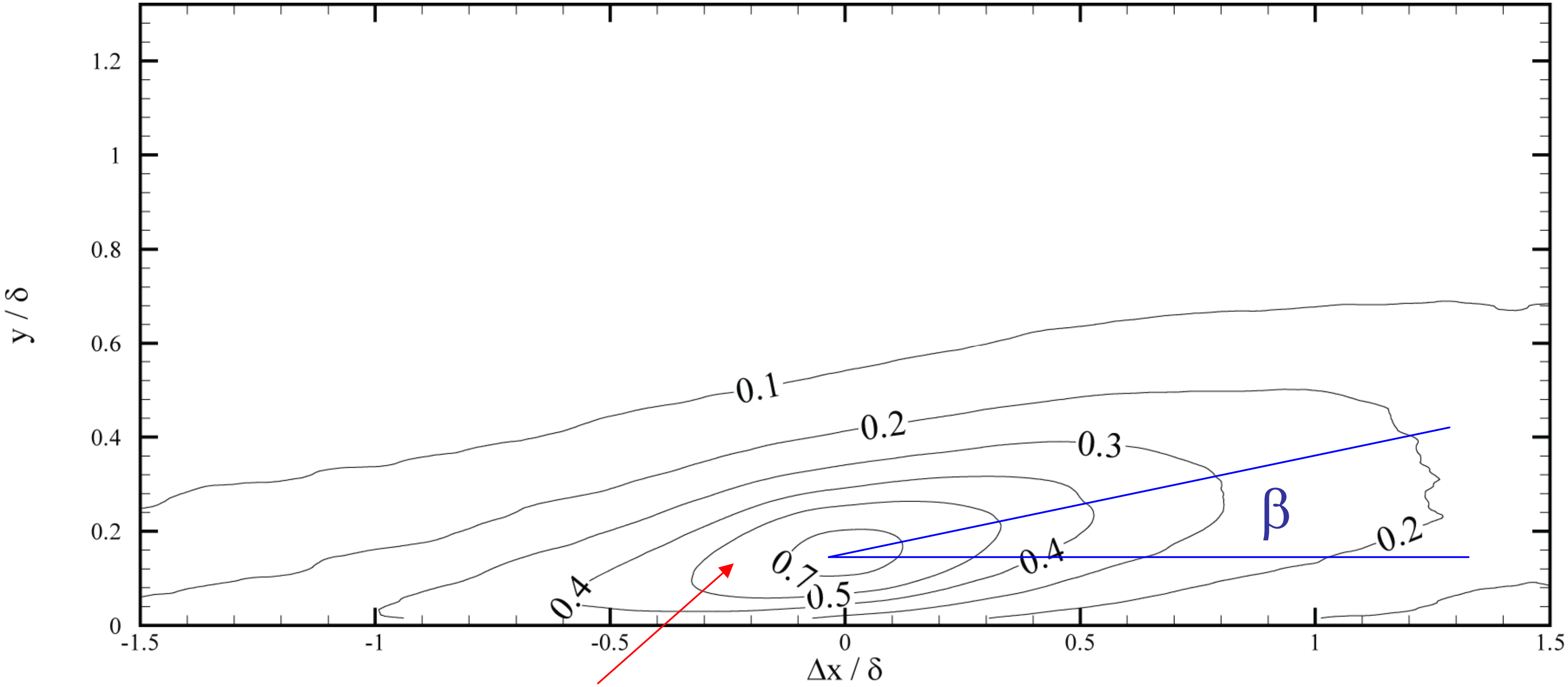
- Do hairpin vortices and their organization into larger-scale vortex packets leave their imprint upon the spatial statistics of wall turbulence?
  - Patterns must occur often.
  - Characteristics must not vary appreciably in order to survive the averaging process.

- Two-point spatial correlations
  - Streamwise velocity (ρ_{uu})
  - Swirling strength ($\rho_{\lambda\lambda}$)

- Conditional averaging based on dominant structural characteristics

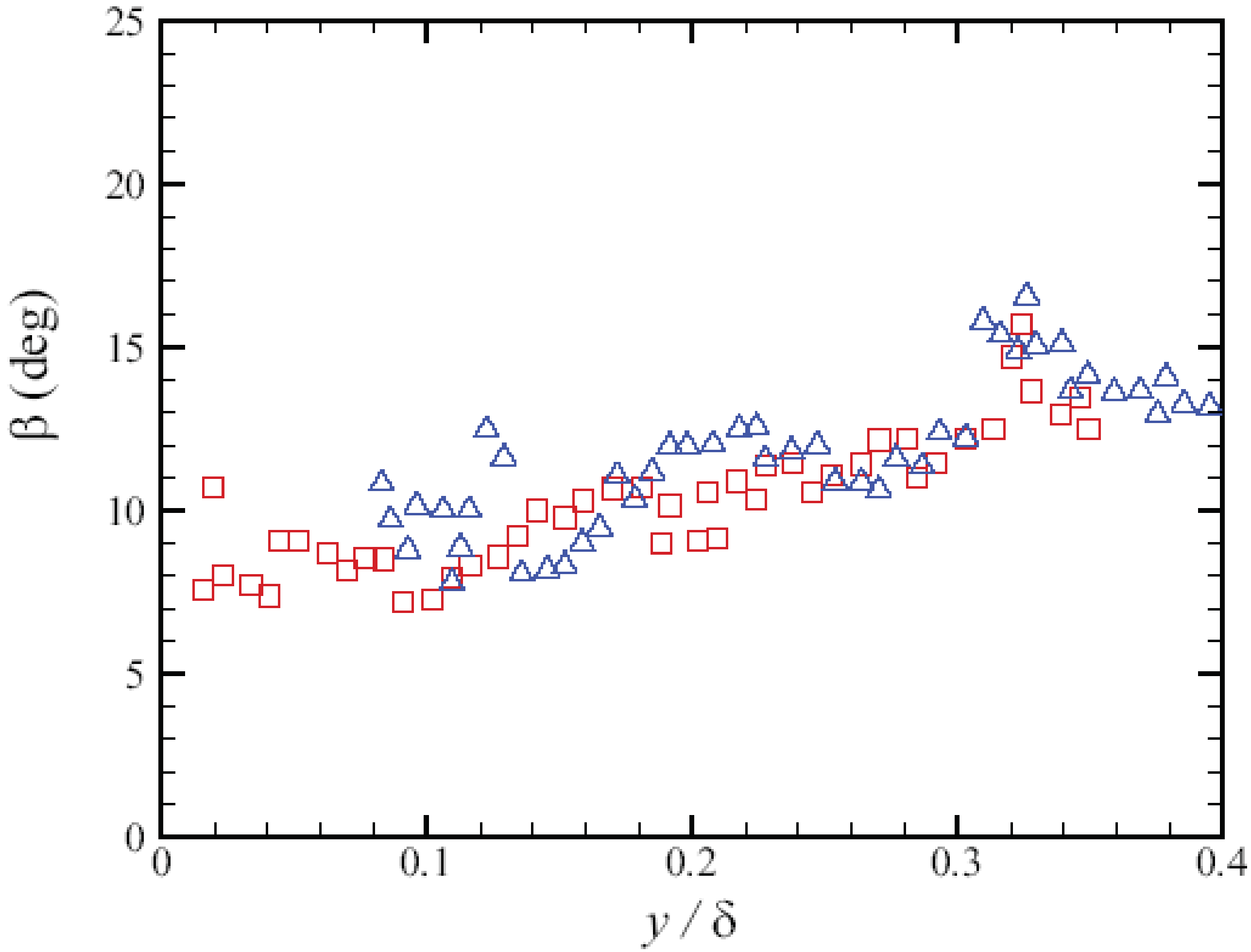
ρ_{uu} in x–y plane at y=0.15δ

$$\rho_{uu}\left(\Delta x, y; y_{ref}\right) = \frac{\left\langle u'\left(x, y_{ref}\right)u'\left(x+\Delta x, y\right)\right\rangle}{\sigma_u\left(y_{ref}\right)\sigma_u\left(y\right)}$$



Consistent with inclined LMR's beneath interface formed by hairpin heads

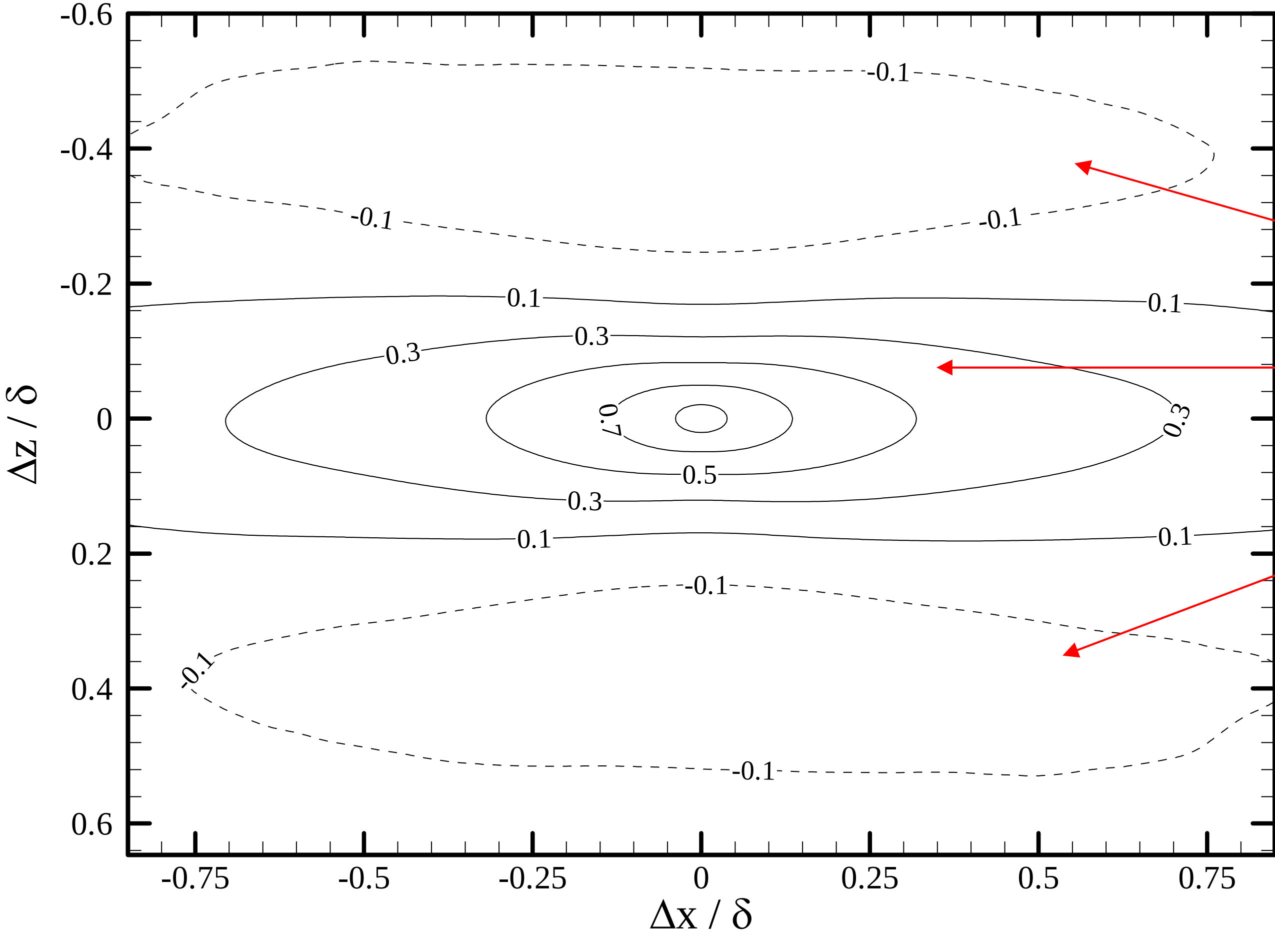
Inclination angle of ρ_{uu}



Smooth
Rough

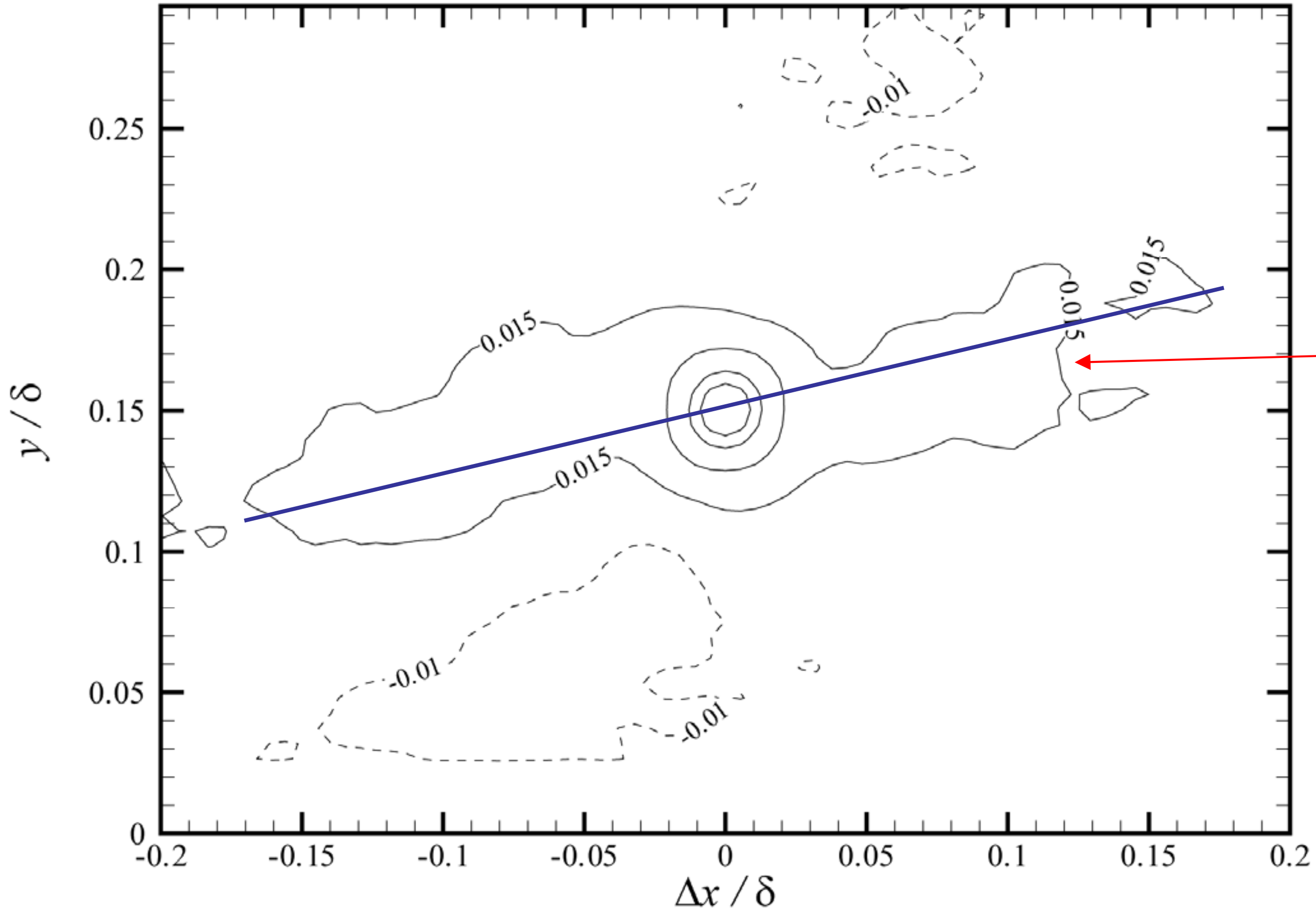
ρ_{uu} in x–z plane at y=0.15δ

$$\boxed{\rho_{uu}\left(\Delta x, \Delta z; y\right) = \frac{\left\langle u'\left(x, z; y\right) u'\left(x+\Delta x, z+\Delta z; y\right)\right\rangle}{\sigma_u^2\left(y\right)}}$$



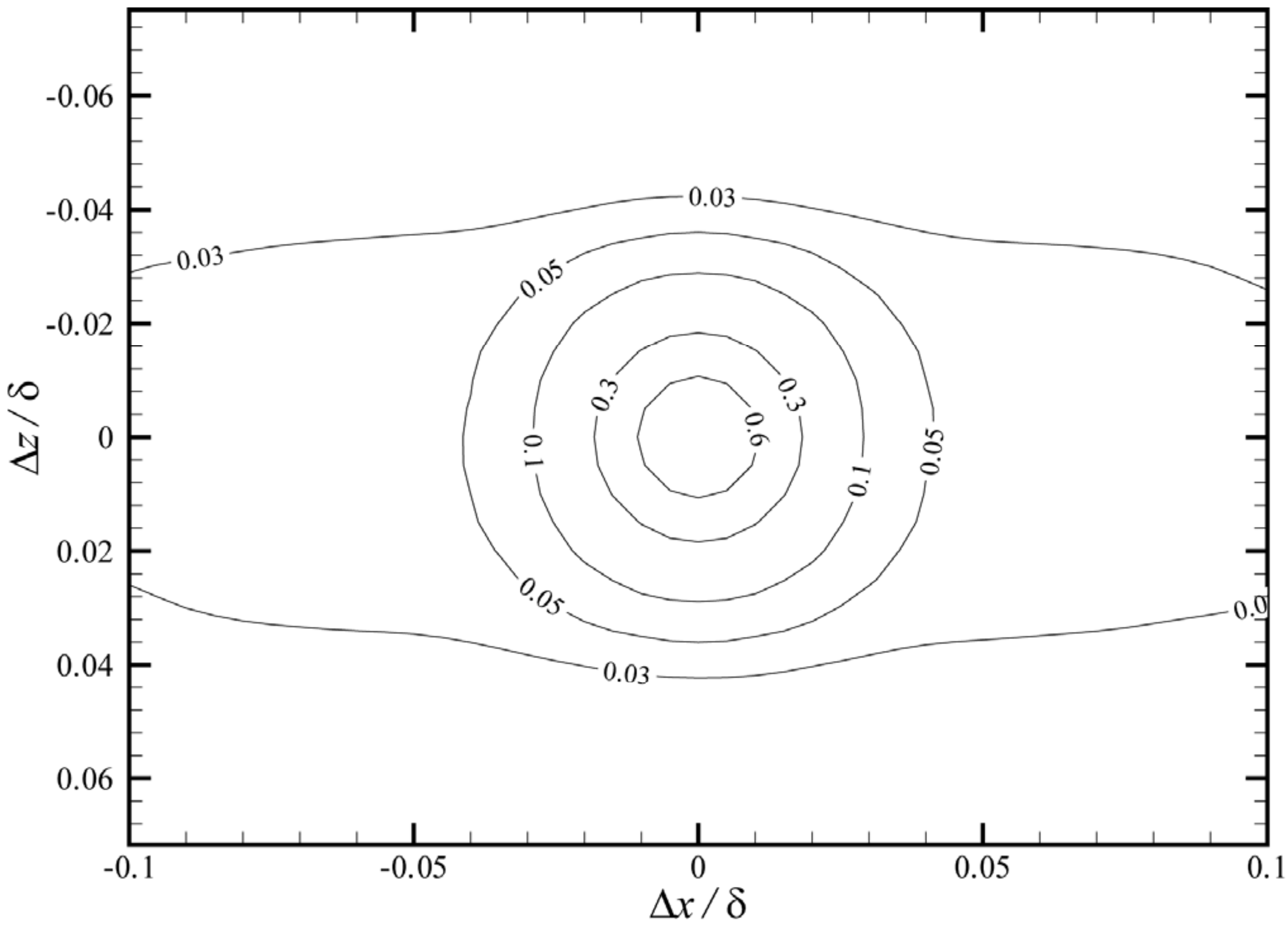
Consistent with spanwise-alternating LMR's and HMR's

$\rho_{\lambda\lambda}$ in x–y plane at y=0.15δ



Consistent with streamwise-aligned hairpin heads inclined slightly away from wall

$\rho_{\lambda\lambda}$ in x–z plane at y=0.15δ



Again consistent with streamwise-aligned hairpin structures

Conditional averaging to reveal characteristics and importance of embedded structure

Example: *What is the most probable velocity field associated with a spanwise vortex core*?

$$\left\langle u_j\left(\mathbf{x}'\right)\middle|\lambda_{ci}\left(\mathbf{x}\right)\right\rangle$$

Linear stochastic estimate of this conditional average:

$$\left\langle u_j\left(\mathbf{x}'\right)\middle|\lambda_{ci}\left(\mathbf{x}\right)\right\rangle \approx L\lambda_{ci}\left(\mathbf{x}\right)$$

Minimization of mean-square error yields

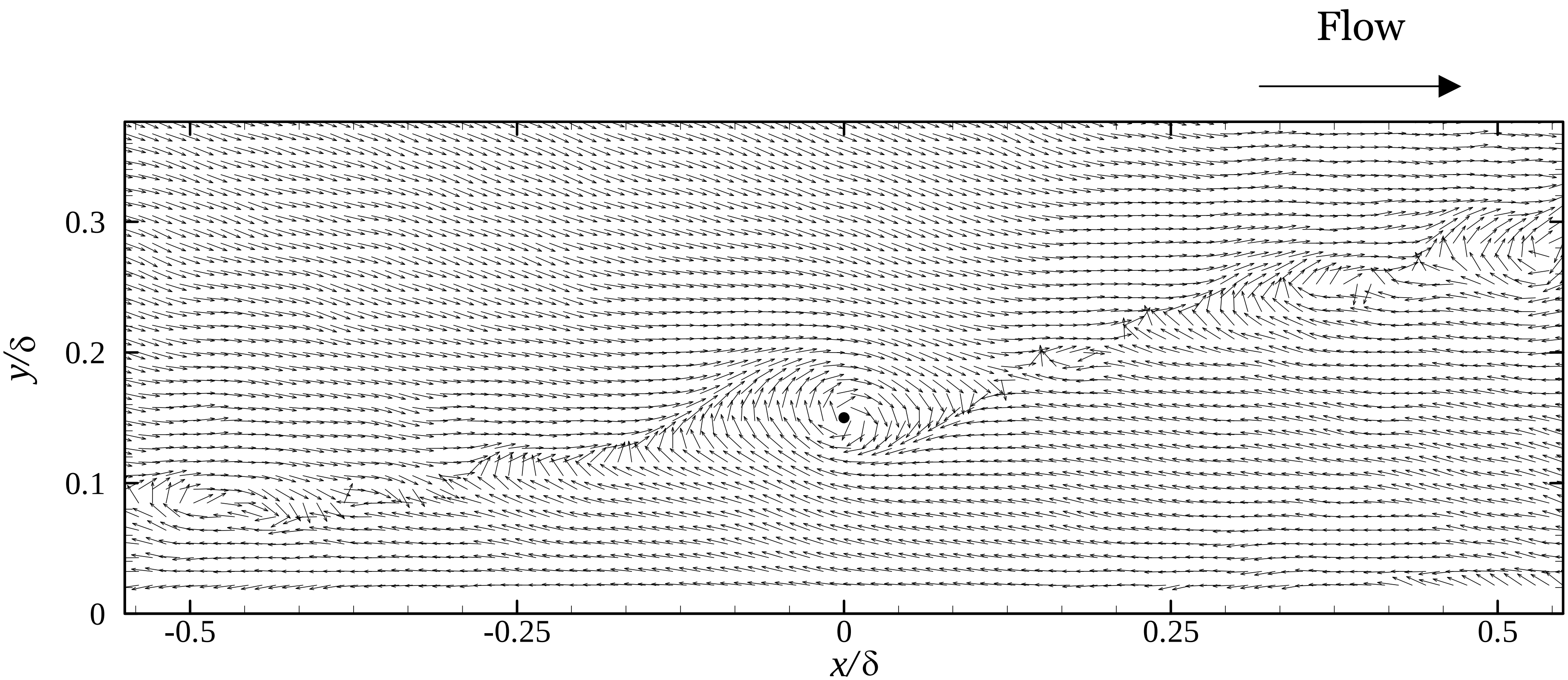
$$\left\langle u_j\left(\mathbf{x}'\right)\middle|\lambda_{ci}\left(\mathbf{x}\right)\right\rangle \approx \frac{\left\langle \lambda_{ci}\left(\mathbf{x}\right)u_j\left(\mathbf{x}'\right)\right\rangle}{\left\langle \lambda_{ci}\left(\mathbf{x}\right)\lambda_{ci}\left(\mathbf{x}\right)\right\rangle}\lambda_{ci}\left(\mathbf{x}\right)$$

Conditional average of the velocity field can therefore be estimated via unconditional two-point spatial correlations:

$$R_{\lambda u}\left(r_x, y\right) = \left\langle \lambda_{ci}\left(x, y_{\mathrm{ref}}\right)u_j\left(x+r_x, y\right)\right\rangle$$

Christensen and Adrian (2001), JFM

$\left\langle u_j(\mathbf{x}') \middle| \lambda_{ci}(\mathbf{x}) \right\rangle$ in turbulent BL at δ^+=3000



Statistical imprint of outer-layer vortex organization

Christensen and Adrian (2001), JFM

Another example: Large-eddy simulation (LES)

- **IDEA:** Only resolve a subset of the dynamically-important spatial scales in order to reduce the overall cost of the computation.
  - Larger scales are solved for directly while the smaller scales are modeled in some fashion.
  - One can run an LES at a much higher Re for the same cost as a lower-Re direct numerical simulation (DNS).

- Implementation
  - Equations of motion are low-pass filtered, yielding a set of "filtered" equations for the resolved scales.

- Difficulties
  - One must define a spatial-scale boundary between the resolved and unresolved scales as well as an appropriate filtering methodology.
  - The influence of the smaller (unresolved) scales on the evolution of the larger (resolved) scales must be modeled.
  - ***What role do hairpin vortex packets play in SGS physics?***

LES governing equations

Filtered continuity and momentum

$$\left\{\begin{aligned} &\frac{\partial \tilde{u}_i}{\partial x_i} = 0 \\ &\frac{\partial \tilde{u}_i}{\partial t} + \frac{\partial \tilde{u}_i \tilde{u}_j}{\partial x_j} = -\frac{1}{\rho}\frac{\partial \tilde{p}}{\partial x_i} + \nu \frac{\partial^2 \tilde{u}_i}{\partial x_j \partial x_j} - \frac{\partial \tau_{ij}}{\partial x_j} \end{aligned}\right.$$

Subgrid-scale (SGS) stresses

$$\tau_{ij} = \widetilde{u_i u_j} - \tilde{u}_i \tilde{u}_j$$

Filtered kinetic energy

$$\frac{\partial \tilde{q}^2/2}{\partial t} + \tilde{u}_j \frac{\partial \tilde{q}^2/2}{\partial x_j} = -\frac{1}{\rho}\frac{\partial \tilde{u}_j \tilde{p}}{\partial x_j} - \frac{\partial \tilde{u}_i \tau_{ij}}{\partial x_j} + \nu \frac{\partial^2 \tilde{q}^2/2}{\partial x_j \partial x_j} - \nu \frac{\partial \tilde{u}_i}{\partial x_j}\frac{\partial \tilde{u}_i}{\partial x_j} - \varepsilon_{sgs}$$

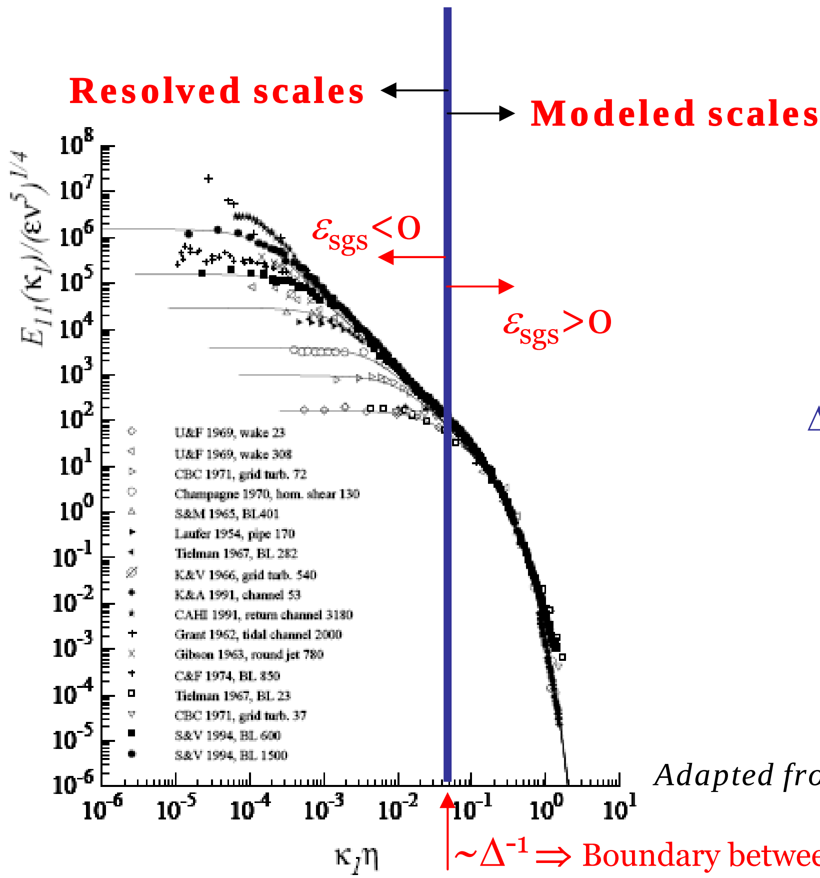
SGS dissipation

$$\varepsilon_{sgs} = -\tau_{ij}\tilde{S}_{ij}$$

ε_{sgs}: Represents the energy transfer across the boundary between the resolved and unresolved scales.

Qualitative description of SGS energy transfer

Representative turbulent energy spectrum



Adapted from Pope (2000)

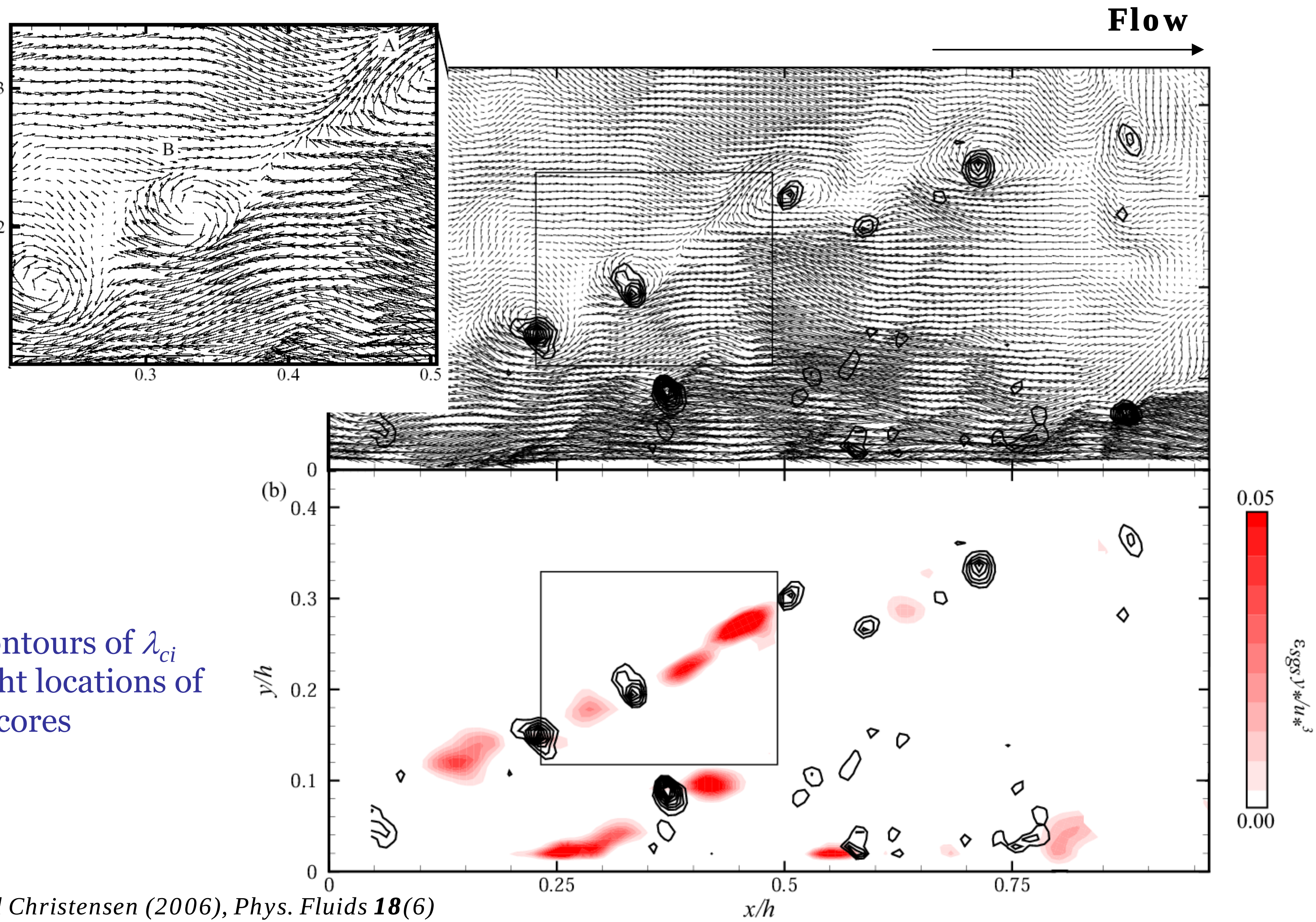
Δ: Filter length scale

$\sim\Delta^{-1} \Rightarrow$ Boundary between resolved and unresolved scales

$\varepsilon_{sgs}>0$: Energy transfer from the resolved to the unresolved scales $\Rightarrow$ Forward scatter

$\varepsilon_{sgs}<0$: Energy transfer from the unresolved to the resolved scales $\Rightarrow$ Backward scatter

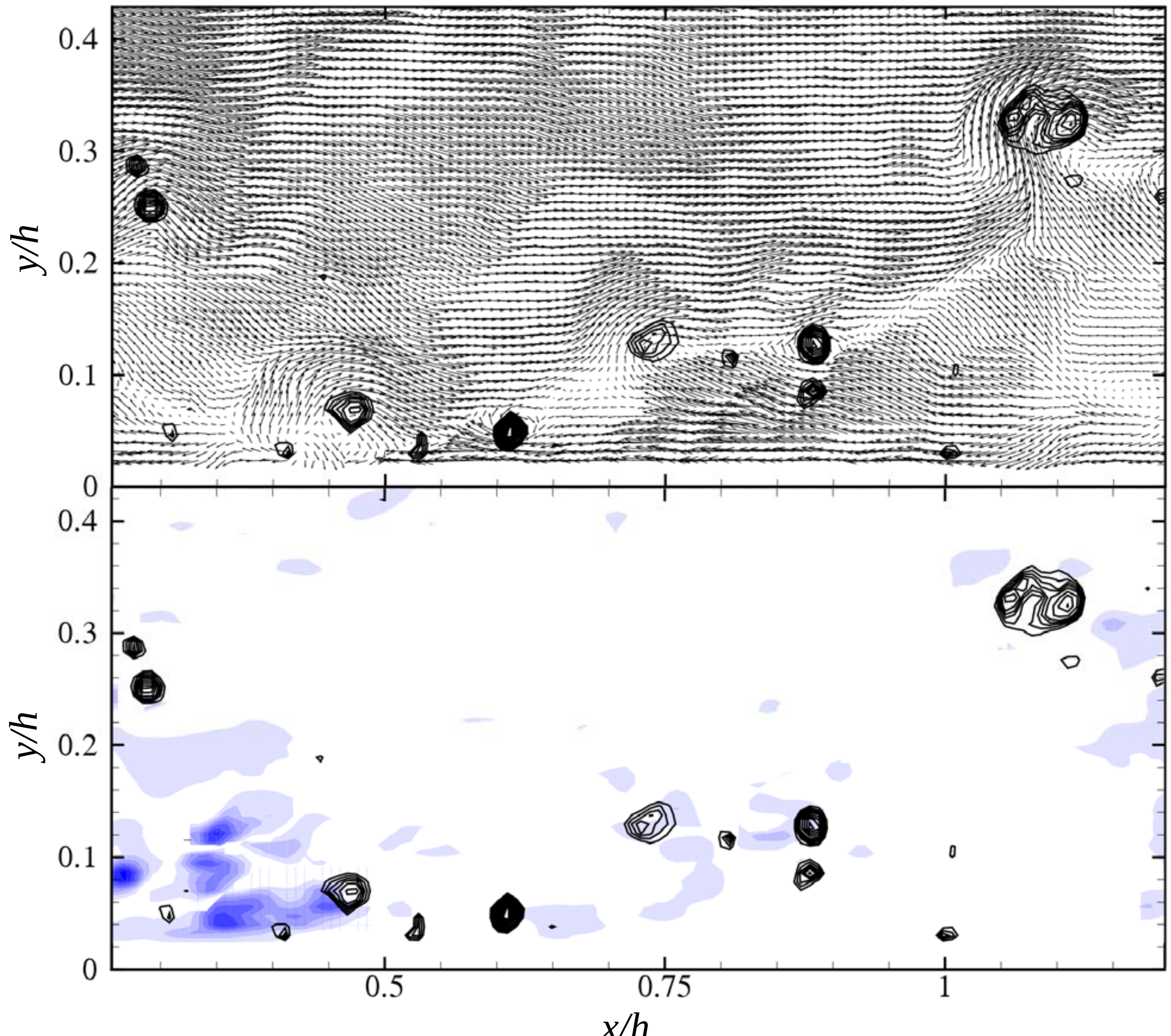
Instantaneous forward scatter



Line contours of λ_{ci} highlight locations of vortex cores

Natrajan and Christensen (2006), Phys. Fluids ***18****(6)*

Instantaneous backscatter



*Natrajan and Christensen (2006), Phys. Fluids **18**(6)*

Statistical analysis

Best estimate of the average forward scatter (or backscatter) fields induced by a hairpin vortex and a vortex packet is $\langle \Phi(\mathbf{x}') | \lambda_{ci}(\mathbf{x}) \rangle$.

Linear stochastic estimate of the conditionally averaged dissipation field given a vortex core:

$$\langle \Phi(\mathbf{x}') | \lambda_{ci}(\mathbf{x}) \rangle \approx L\lambda_{ci}(\mathbf{x})$$

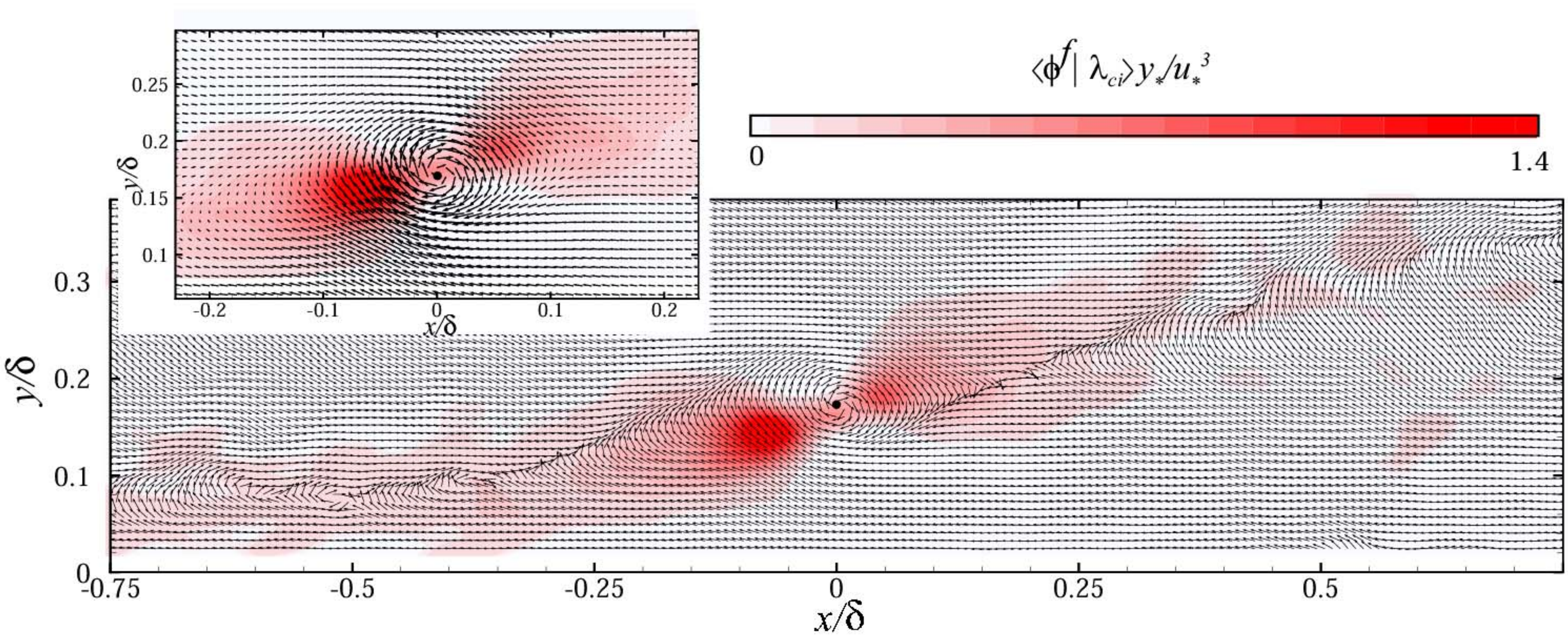
Minimization of mean-square error yields

$$\langle \Phi(\mathbf{x}') | \lambda_{ci}(\mathbf{x}) \rangle \approx \frac{\langle \lambda_{ci}(\mathbf{x}) \Phi(\mathbf{x}') \rangle}{\langle \lambda_{ci}(\mathbf{x}) \lambda_{ci}(\mathbf{x}) \rangle} \lambda_{ci}(\mathbf{x})$$

Conditional average of the forward scatter and backscatter fields can therefore be estimated via unconditional two-point spatial correlations

$$R_{\lambda\Phi}(r_x, y) = \langle \lambda_{ci}(x, y_{\mathrm{ref}}) \Phi(x + r_x, y) \rangle$$

Forward scatter given a hairpin head

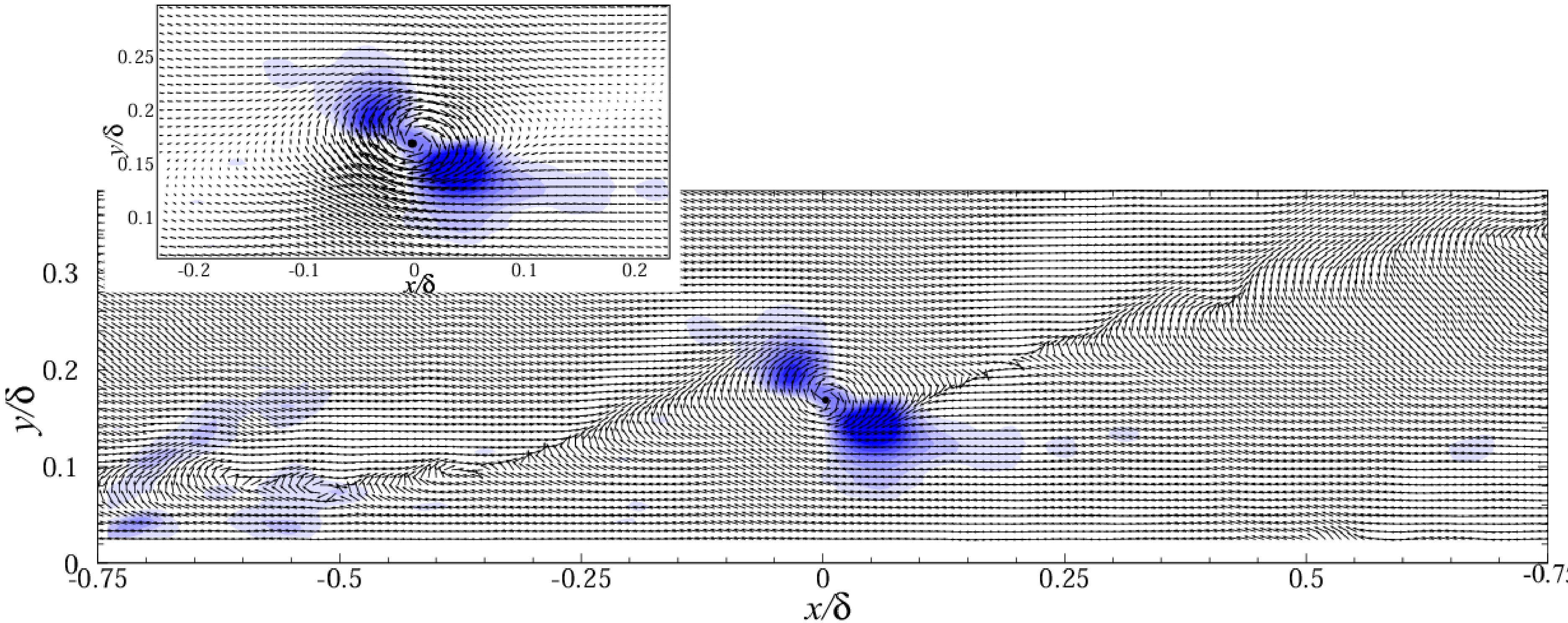


Contours of $\left\langle \phi^f \middle| \lambda_{ci} \right\rangle$ at $y_{\text{ref}} = 0.15\delta$

Overlaid is $\left\langle u'_j \middle| \lambda_{ci} \right\rangle$ (Christensen and Adrian, 2001)

Natrajan and Christensen (2006), Phys. Fluids ***18****(6)*

Backscatter given hairpin head



Contours of $\left\langle \phi^b \middle| \lambda_{ci} \right\rangle$ at $y_{\text{ref}} = 0.15\delta$

Overlaid is $\left\langle u'_j \middle| \lambda_{ci} \right\rangle$ (Christensen and Adrian, 2001)

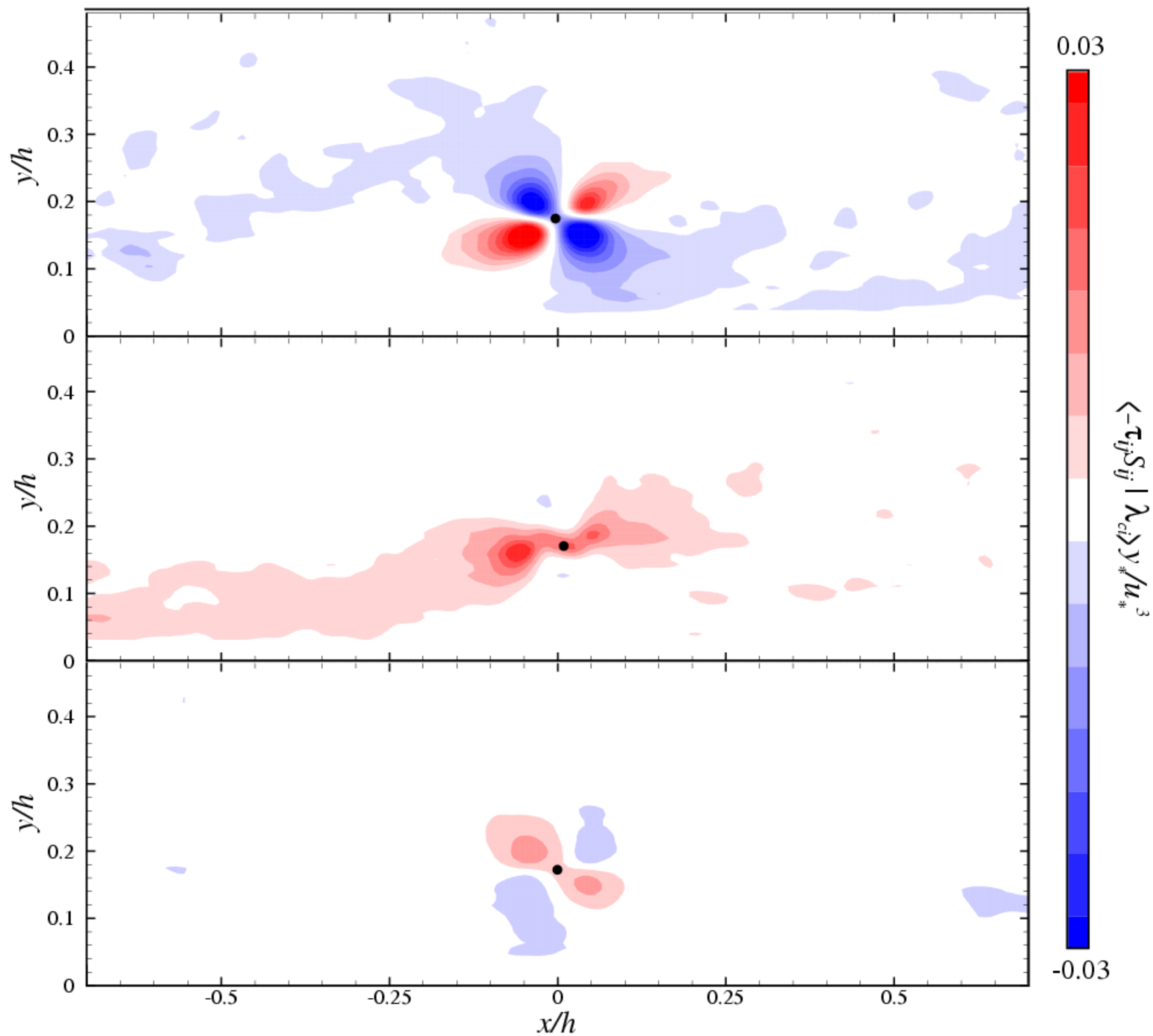
Natrajan and Christensen (2006), Phys. Fluids ***18***(6)

Contributions to forward and backward scatter

$$\left\langle -\tau_{11}\tilde{S}_{11} \,\middle|\, \lambda_{ci} \right\rangle$$

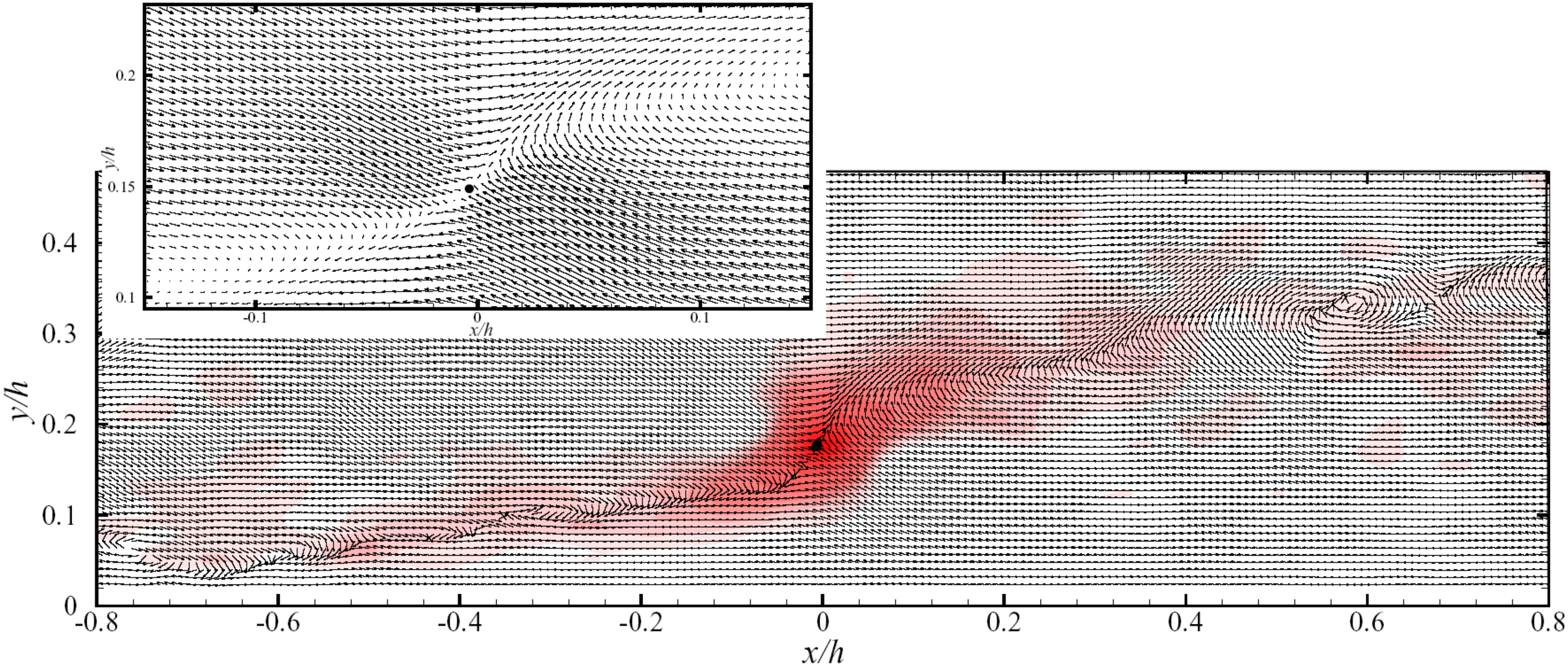
$$\left\langle -\tau_{12}\tilde{S}_{12} \,\middle|\, \lambda_{ci} \right\rangle$$

$$\left\langle -\tau_{22}\tilde{S}_{22} \,\middle|\, \lambda_{ci} \right\rangle$$



Natrajan and Christensen (2006), Phys. Fluids ***18****(6)*

Most probable velocity field given a forward scatter event

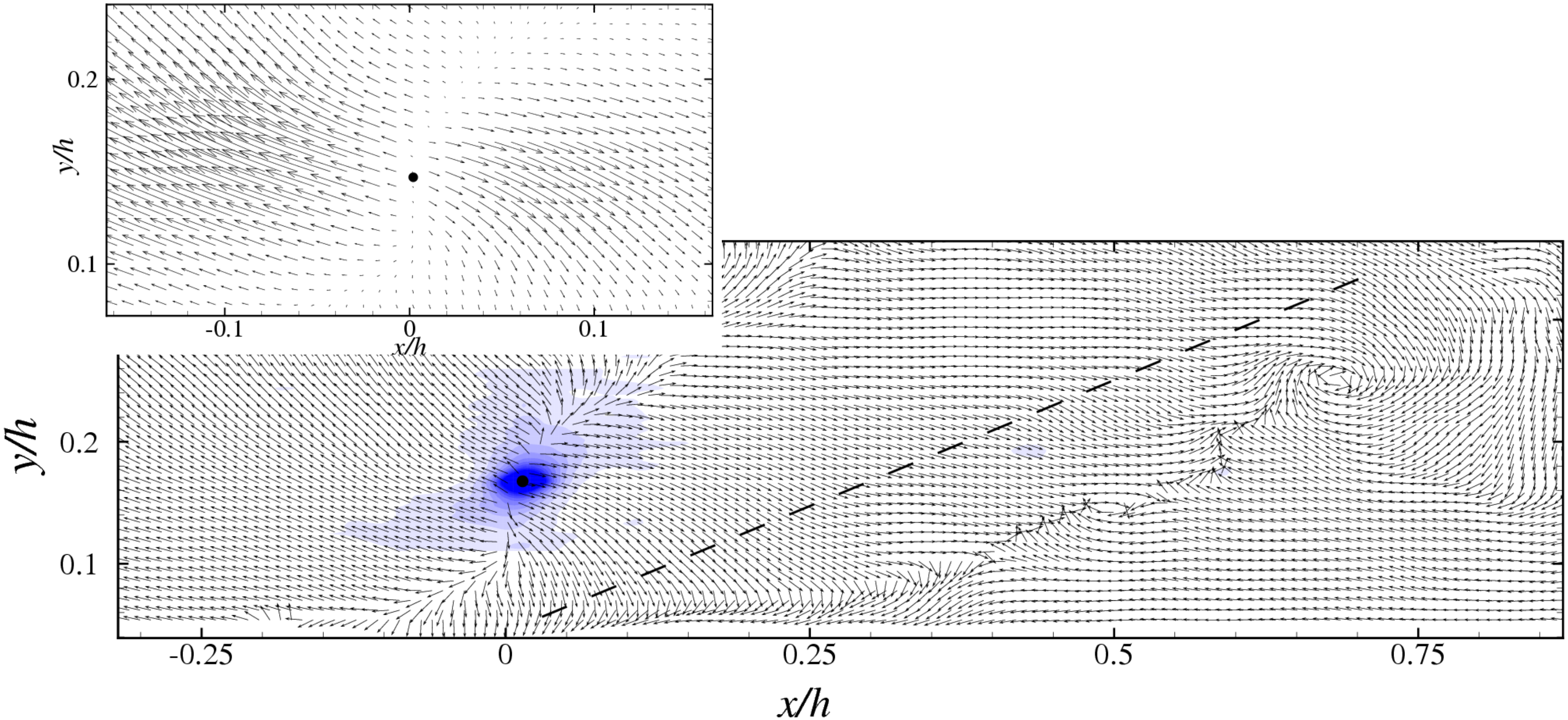


Vector field illustrating $\left\langle u_j' \middle| \phi^f \right\rangle$

Overlaid are contours of $\left\langle \phi^f \middle| \phi^f \right\rangle$

Natrajan and Christensen (2006), Phys. Fluids ***18****(6)*

Most probable velocity field given a backward scatter event

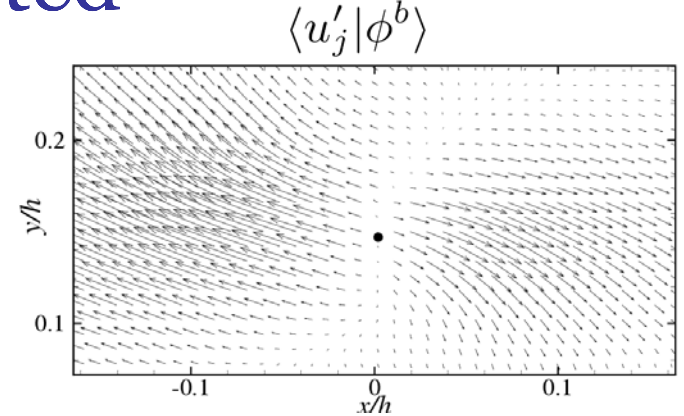


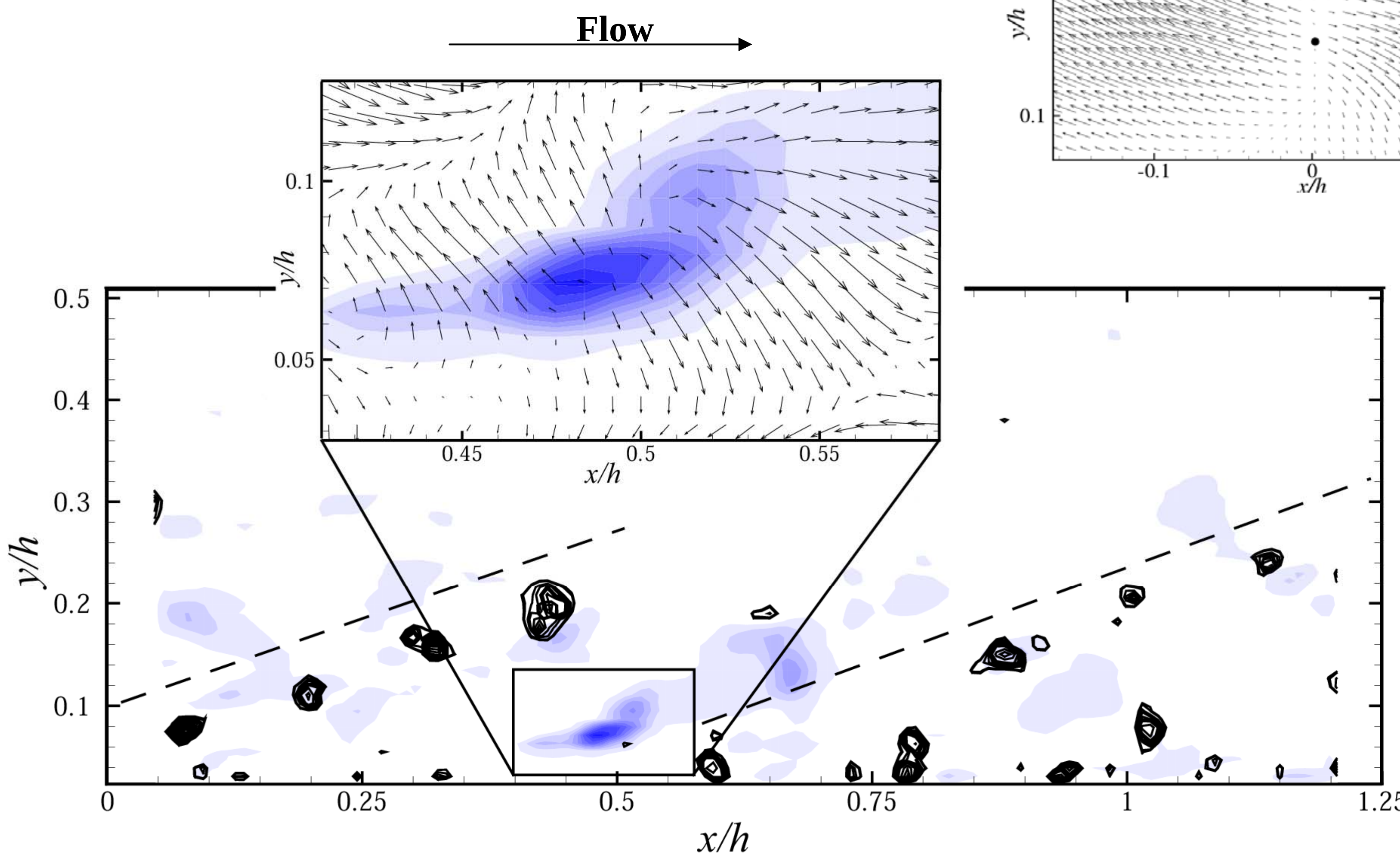
Vector field illustrating $\left\langle u'_j \middle| \phi^b \right\rangle$

Overlaid are contours of $\left\langle \phi^b \middle| \phi^b \right\rangle$

Natrajan and Christensen (2006), Phys. Fluids ***18****(6)*

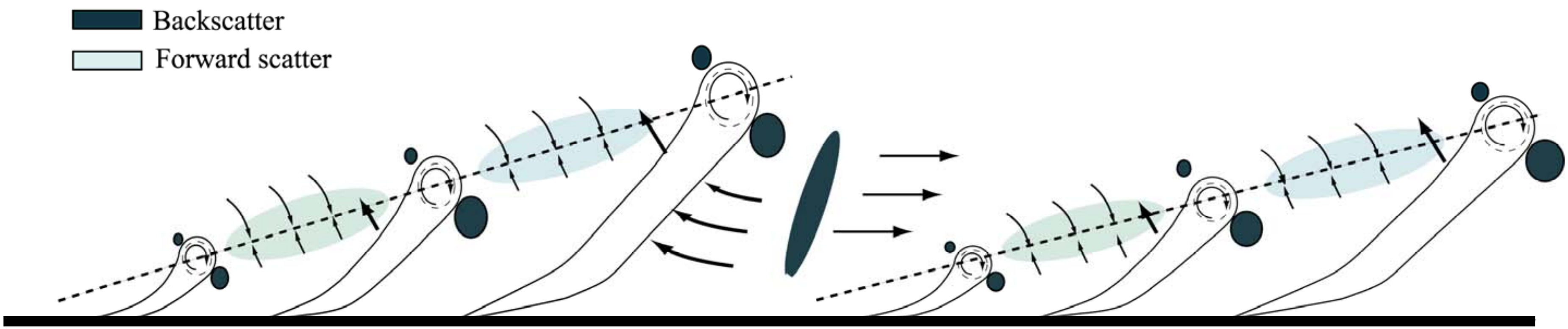
Instantaneous backscatter revisited





Natrajan and Christensen (2006), Phys. Fluids ***18****(6)*

Conceptual model of structural contributions to SGS dissipation



- Forward scatter around a hairpin head is coincident with the ejection induced by the vortex due to $-\tau_{12}\tilde{S}_{12}$. The most intense forward scatter is observed via additional contributions from $-\tau_{11}\tilde{S}_{11}$ when this ejection is countered by a sweep event which collectively generate an inclined shear layer.
- In addition to the localized backscatter observed upstream/above and downstream/below each hairpin head, the most intense backscatter is observed at the trailing end of a hairpin vortex packet, particularly when a second packet is observed upstream. $-\tau_{11}\tilde{S}_{11}$ is the dominant contributor to backscatter events.

Natrajan and Christensen (2006), Phys. Fluids ***18***(6)

Summary

- Whatever the experimental/computational protocol employed, identification of coherent structures in data is pivotal to understanding the evolution of turbulent flows.

- Robust identification methodology must be applied
  - "Quality" of data can impact choice

- Once structures are identified, key challenge lies in extracting their influence and importance.
  - Success tightly coupled to clarity of goals
  - Conditional averaging methods can be extremely helpful in this regard
  - Key is choosing appropriate averaging conditions