Measurement of Atmospheric Turbulence by Means of Light, Sound, and Radio Waves

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Sampling Strategies, Technologies, and Applications
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Overview

- Introduction
- Wave propagation through turbulence: the basics
- Anisotropy in optical surface-layer turbulence
- Vertical-velocity biases observed with radars and sodars
- Outlook
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Light, sound, and radio waves in the optically clear atmosphere

- Do not propagate along straight lines
- Change their amplitudes, phases, and angles-of-arrival deterministically and randomly in space and time
- Carry information about mean values and fluctuations of wind, temperature, density, pressure, humidity, and refractive index.
Differential downward ray-bending can make the sun look squeezed ...
Here comes the sun Here comes the sun ……

Number of dipoles: 18,432

Peak power: 1.5 MW

Beam width: 0.8 deg
... or otherwise deformed.

From Pernter and Exner, *Meteorologische Optik* (1910, p. 194)
Upward ray-bending can make you see water puddles on a dry road …
Road mirage
From Pernter and Exner, *Meteorologische Optik* (1910, p. 127)
From Pernter and Exner, *Meteorologische Optik* (1910, p. 128)
Random ray-bending can create changing patterns at the bottom of a swimming pool ...
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Valeryan Tatarskii

Tatarskii, V. I., 1961: 
*Wave Propagation in a Turbulent Medium.* 

Tatarskii, V. I., 1971: 
*The Effects of the Turbulent Atmosphere on Wave Propagation.* 
Israel Program for Scientific Translation, Jerusalem, 472 pp.

Rytov, S. M., Y. A. Kravtsov, and V. I. Tatarskii, 1986-89: 
*Principles of Statistical Radiophysics, Vols. 1-4.* 
The inhomogeneous Helmholtz equation

- Maxwell equations
- Time-harmonic fields
- Simplified constitutive parameters ($\mu=\mu_0$, $\sigma=0$, $\varepsilon$ varies with location)
- Wave equation
- (Inhomogeneous) Helmholtz equation
Methods to solve the inhomogeneous Helmholtz equation

- Eikonal equation
- Born approximation
- Rytov approximation
- Parabolic equation
Consider scalar Helmholtz equation:

\[ \nabla^2 E + k^2 \varepsilon_r E = 0, \]  

where \( E \) is one of the three Cartesian components of the electrical field vector, \( k \) is the wave number of the unperturbed EM wave,

\[ \varepsilon_r = n^2 \]  

is the relative permittivity, and \( n \) is the refractive index.

Let \( \varepsilon_r (r) \) be a deterministic or random field that is smoothly inhomogeneous, such that \( E \) is locally a plane wave,

\[ E (r) = A (r) \exp [iS (r)] \]
\[ = A (r) \exp [ik \varphi (r)], \]  

where \( A (r) \) is the amplitude, \( S (r) \) is the phase,

\[ \varphi (r) = \frac{S (r)}{k} \]  

is the eikonal (or “phase path”).
Per definition of smooth inhomogeneity, the change of $A(\mathbf{r})$ and $\nabla S(\mathbf{r})$ over one EM wavelength is negligible ("locally plane waves").

Expand $E(\mathbf{r})$ in series of powers in $(ik)^{-1}$:

$$E(\mathbf{r}) = \left( A_0(\mathbf{r}) + \frac{A_1(\mathbf{r})}{ik} + \frac{A_2(\mathbf{r})}{(ik)^2} + \ldots \right) \exp[ik\phi(\mathbf{r})]. \quad (5)$$

Insert into Helmholtz equation, (1).

First term, proportional to $(ik)^2$, gives the eikonal equation,

$$\nabla^2 \phi = n^2. \quad (6)$$

Second term, proportional to $(ik)^1$, gives the transport equation,

$$2\nabla A_0 \cdot \nabla \phi + A_0 \nabla^2 \phi = 0. \quad (7)$$

The eikonal equation and the transport equation are the equations for geometrical optics.
# Refraction and diffraction

<table>
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<tr>
<th>Eikonal equation</th>
<th>Born approximation</th>
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<td>(geometrical optics, “ray tracing”)</td>
<td>Describes both refraction and diffraction</td>
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<tr>
<td>Describes refraction but not diffraction</td>
<td>Very good approximation for radio-wave backscatter from clear-air refractive-index perturbations</td>
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<tr>
<td>Approximates variances and frequency spectra of optical angle-of-arrival (AOA) fluctuations very well if aperture diameter is larger than twice the Fresnel length</td>
<td>Fraunhofer approximation valid if turbulence is Bragg-isotropic</td>
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<tr>
<td></td>
<td>Fresnel approximation (or higher-order approximation) necessary if turbulence not Bragg-isotropic of in case of scatter from interfaces</td>
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Turbulence in the inertial range (in a nutshell)

Structure function of a random refractive-index field:

$$D_n (r_1, r_2) = \langle [n (r_1) - n (r_2)]^2 \rangle.$$  \hspace{1cm} (1)

Locally homogeneous refractive-index field:

$$D_n = D_n (r),$$  \hspace{1cm} (2)

where $r - r_1 - r_2$ is the spatial lag vector.

Locally homogeneous AND isotropic refractive-index field:

$$D_n = D_n (r),$$  \hspace{1cm} (3)

where $r = |r_1 - r_2|$.

Inertial-range turbulence (Kolmogorov, Obukhov, 1941):

$$D_n (r) = C_n^2 r^{2/3},$$  \hspace{1cm} (4)

where $C_n^2$ is the structure parameter.

Three-dimensional wave-number spectrum for refractive-index fluctuations in the inertial range:

$$\Phi_n (\kappa) = \frac{5}{18 \pi \Gamma (1/3)} C_n^2 \kappa^{-11/3} = 0.033 C_n^2 \kappa^{-11/3},$$  \hspace{1cm} (5)

where $\kappa = \sqrt{\kappa_1^2 + \kappa_2^2 + \kappa_3^2}$. 
Optical angle-of-arrival fluctuations (from theory of eikonal fluctuations)

Consider vertical angle-of-arrival (AOA) fluctuations (local and instantaneous wave-front tilts) at point receiver located at distance $L$ from source:

$$\alpha(y,z) = \frac{\partial}{\partial z} \left( \int_{x=0}^{L} n'(x,y,z) \, dx \right).$$

(1)

Variance of vertical AOA fluctuations of a spherical wave in locally homogeneous and isotropic turbulence, measured with two-point interferometer with baseline $b$:

$$\langle \alpha^2(b) \rangle = \frac{D_S(b)}{k^2b^2},$$

(2)

where

$$D_S(\rho) = 8\pi^2 L k^2 \int_0^\infty d\kappa \Phi_n(\kappa) \kappa \int_0^1 du \left[ 1 - J_0(\kappa u) \right]$$

(3)

is the phase structure function of a spherical wave in the receiving plane. (Here, $u = x/L$.)

Variance of aperture-averaged AOA fluctuations (circular aperture with radius $a$)

$$\langle \bar{\alpha}^2 \rangle = \frac{3}{8} \frac{8\pi^2 L}{\Phi_n(\kappa)} \kappa^2 \left[ \frac{J_1(\kappa a)}{\kappa a} \right]^2.$$  

(4)

Evaluation gives

$$\langle \bar{\alpha}^2 \rangle = 1.06 C_n^2 L D^{-1/3},$$

(5)

where $D = 2a$ is the aperture diameter.
“Seeing” the ground cooling after sunset

Averaging time = 5 sec

Time (sec)

Angle of arrival (μrad)

Horizontal

Vertical
Sequence of snapshots of single light

30 frames / s

Random ray-bending (lateral and vertical)

Frequency spectrum of AOA fluctuations

$$S_\alpha(f) = \frac{2^{4/3}}{9 \pi^{7/6} \Gamma(5/6)} C^2_b v_b^{5/3} L b^{-2} f^{-8/3} \left[ 1 - \frac{\sin(2\pi b f/v_b)}{2\pi b f/v_b} \right],$$

(14)

where the numerical coefficient has the value 0.06524. Equation (14) is identical to Clifford's main result, his Eq. (28). [His slightly different coefficient, 0.066, is probably due to rounding errors, and his factor $b^2$ instead of $b^{-2}$ is due to an erroneous conversion from $W_{\delta S}(f)$ to $S_\alpha(f)$.] It is quite remarkable that Eq. (14) can be easily obtained via the geometrical-optics approximation, Eq. (3), without the cumbersome integrations that Clifford had to carry out because he did not approximate the cosine term as a constant.

Frequency spectrum of AOA fluctuations (measured, lateral and vertical)

Fig. 7. Averaged frequency spectra of AOA fluctuations for the horizontal direction (solid line) and the vertical direction (dashed line) of the bottom left-hand light measured on September 27, 2006. Each spectrum is for 10 s of time duration. The black dots and open circles are the frequency spectra averaged over intervals of equal logarithmic width for horizontal and vertical directions of 2 min of time duration, respectively. The observation time was 21:29:10–21:22:10 LT.

“Seeing” the wind speed transverse to the line of sight

Random ray-bending (lateral and vertical)

“Seeing” quasi-periodic changes in the vertical temperature gradient (gravity waves)

METCRAX: Looking for “atmospheric seiches” in the Atmospheric Meteor Crater
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Downward bias in vertical velocities observed with VHF radars in the free troposphere

Fig. 2. Vertical profile of the mean vertical velocity at Flatland during 22 January–2 April 1990. The number of 3-h means averaged at each height is given; the error bars extend plus and minus one standard error of the mean.

Upward bias in vertical velocities observed with a sodar in the lower CBL

Fig. 2. Comparison of time series of $w$ and $\sigma_w$ by sonic anemometer and SODAR in convective conditions averaged over nine clear days during September 2000 at Falkenberg, Germany. A moving average of 30 min was used.
Geophysical causes of biases in vertical clear-air radar or sodar winds

- Nonzero covariance between local Cn$_2$ and local w (due to gravity waves in the stable free troposphere, Nastrom and VanZandt 1994)

- Horizontal advection of asymmetrically corrugated interfaces ("KHI bias", Muschinski 1996)

- Nonzero covariance between local Cn$_2$ and local w ("intermittency flux" or "reflectivity flux" in the CBL, Muschinski et al., to be published)
“KHI bias”

Fig. 1. Geometry of refractivity surfaces in trains of Kelvin–Helmholtz billows above and below the horizontal wind speed maximum of a jet stream (schematic). Note the opposite orientation of the billows above and below the jet.


“KHI bias”

*Fig. 2. A sinusoidal quasi-specular refractivity surface in a Kelvin–Helmholtz billow, illuminated by a vertically directed VHF radar beam. Note the difference between the radar’s nominal (vertical) and effective beam direction.*
Small-scale intermittency:
Lognormality of local epsilon and local CT2

Small-scale intermittency:
Joint lognormality of local epsilon and local CT2

Intermittency flux and sodar velocity bias

Let \( \left( \overline{C_T^2} \right)_r \) be the local temperature structure parameter, estimated from a volume of radius \( r \), and let \( w_r \) be the vertical wind velocity averaged over the same volume.

Then \( \left( \overline{C_T^2} \right)_r \) and \( w_r \) are random variables, where

\[
C_T^2 = \left\langle \left( \overline{C_T^2} \right)_r \right\rangle.
\]

Then the mean vertical wind velocity measured with a sodar is

\[
\left\langle v_D \right\rangle = \frac{\left\langle \left( \overline{C_T^2} \right)_r w_r \right\rangle}{\left\langle \left( \overline{C_T^2} \right)_r \right\rangle} = \left\langle w \right\rangle + \frac{\left\langle \left( \overline{C_T^2} \prime \right)_r w_r' \right\rangle}{C_T^2}.
\]
Muschinski, A., M. Behn, V. Hohreiter, and Y. Cheon, 2007: Vertical fluxes of the temperature structure variable [...]. Unpublished manuscript.
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Outlook

- Faster data acquisition
- Faster computation
- Better sensors
- Better theoretical understanding
- Better numerical simulations
- Better data assimilation.

- Better observations and predictions.
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