Tutorial 2
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1. (a) Define atmospheric surface layer
   (b) Monin-Obukhov similarity theory assumes that the surface layer is determined by a few governing parameters: the friction velocity u_*, surface temperature flux $\left(H/\rho C_p\right) = \overline{w'\theta'}$ and the height z. The relevant temperature scale is $T_* = \overline{w'\theta'}/u_*$. The non-dimensional functions pertinent to salient variables are (typically for $z/|L| < 1$):

$\phi_m = \left(kz/u_*\right)\left(\partial \bar{u}/\partial z\right)$	wind shear,
$\left(kz/T_*\right)\left(\partial \bar{\theta}/\partial z\right)$	thermal stratification,
$\phi_w = \sigma_w/u_*$	variability in w,
$\phi_\theta = \sigma_\theta/\lvert T_*\rvert$	variability in θ,
$\phi_\in = kz\varepsilon/u_*^3$	dissipation of turbulent kinetic energy,
$\phi_x = kz\chi/u_*^2 T_*^2$	dissipation of temperature fluctuations,

where the functions are dependent on z/L, L being the Monin-Obukhov lengthscale.

   (i) Show that, under neutral stratification conditions with TKE production $\approx$ dissipation, $\phi_\in \approx \phi_m$ and $\phi_x \approx \phi_h$.

   (ii) Show that the eddy diffusivity ratio, the flux Richardson number Ri_f and the Richardson number Ri can be given as

$$\frac{K_h}{K_m} = \frac{\phi_m}{\phi_h}; \quad Ri = \left(\frac{z}{L}\right)\left(\frac{\phi_h}{\phi_m^2}\right); \quad Ri_f = \left(\frac{z}{L}\right)\phi_m$$

and

$$\frac{Ri_f}{Ri_g} = \frac{K_h}{K_m}.$$

2. (a) Consider one dimensional spectra $F_u(k_1)$ in the k_1 wave-number direction and the frequency spectra $S_u(f)$. The spatial scales can be converted to frequency scales using the Taylor hypothesis, $k_1 = 2\pi f/\overline{U}$. Show that:

$$k_1 F_u(k_1) = f S_u(f)$$

(b) If $F_u(k_1) = \alpha_1 \varepsilon^{2/3} k_1^{-5/3}$, where $\alpha \approx 0.55$, show that:

$$\frac{fS_u(f)}{u_*^2 \phi_\in^{2/3}} = 0.3n^{-2/3}$$

where n is the normalized frequency $n = fz/\overline{U}$.

3. (a) Consider a turbulence stably stratified fluid. If the mean flow and temperature varies over scale L_0, then scales $l < L_0$ is expected to be independent on mean variables. Because of the anisotropy introduced by stable buoyancy, the flow is expected to be axially symmetric relative to the vertical, and uniquely determined by the parameters ε, N, $g\beta$, v and *x*, where *N* is the buoyancy frequency and *x* the buoyancy dissipation rate. These parameters give rise to a lengthscale (known as the Obukhov-Bolgiano scale) L_*. Derive the form of L_* and interpret its physical meaning.

For lengthscales $l >> L_*$, argue that the stratification effects are pronounced and thus, the energy spectrum should take the form:

$$E(k) = C_1 N^{2/5} (g\beta)^{5/4} k^{-11/5}$$
$$E_{\theta\theta}(k) = C_\theta N^{4/5} (g\beta)^{-2/5} k^{-7/5}$$

where C_1 and C_θ are constants.

4. (a) Consider an atmosphere in hydrostatic balance. Assuming that it follows ideal gas equations, show that the pressure distribution is given by an expression of the form:

$$p = p_0 \exp[\Psi(z)],$$

(b) and determine $\Psi(z)$ in terms of the gravitational acceleration *g*, gas constant *R*, local temperature *T(z)* and height from the ground *z*.

Show that for an isothermal atmosphere with temperature T_0

$$p = p_0 e^{-z/H},$$

(c) where the scaling height $H = RT_0/g$ and the symbols have usual meanings.

Very briefly explain how you modify the expression in (b) for a moist unsaturated atmosphere.

5. (a) Show that the equilibrium radiative temperature T_E of the Earth is related to the radiative temperature T_S of the solar photosphere (radius R_S) by

$$T_E = \frac{(1-a)^{1/4}}{\sqrt{2}} \left(\frac{R_S}{R_{ES}} \right)^{1/2} T_S \,,$$

where a is the Earth's planetary albedo and R_{ES} is the distance from the Earth to the Sun. (photosphere – sun's optical radius from where the radiation is emanated).

(b) The distance from Venus to Sun is about $0.72 \ R_{ES}$. The thick cloud blanket of Venus elevates its albedo to about 0.7. The Earth's albedo is 0.3 and its black body radiative equilibrium temperature is 255 *K*. Find the equilibrium temperature of Venus [use the answer in 2(a)].

(c) Consider the planet Earth with a thin layer of atmosphere and its equilibrium radiative temperature is 288 *K*. At the top of the atmosphere, the incoming and outgoing radiation is the same, for equilibrium to exist. If the atmospheric CO_2 is doubled, then the radiation balance is upset because of the reduction of outgoing radiation, say by ΔI. To maintain the radiative equilibrium the Earth's temperature is expected to rise by ΔT_S, thus increasing the outgoing radiation (which is quantified by the "radiative forcing) *G*). Calculate how much Earth's temperature should increase per unit change of radiation, *i.e.*, $G = \Delta T_S / \Delta I$ (take the Stefan-Boltzman constant as $\sigma = 5.67 \times 10^{-8} \ Wm^{-2} K^{-4}$). **(5 marks)**

6. Assume an ideal atmosphere in hydrostatic balance. The pressure at heights z_1 and z_2 are p_1 and p_2, respectively. Show that the <u>averaged</u> virtual potential temperature $\overline{T_V}$ (or just the averaged temperature in the case of dry air) between the two heights can be written as:

$$z_2 - z_1 = \ln p_2 - \ln p_1 \frac{R\overline{T_V}}{g}$$

where *R* is the gas constant of dry air.

(b) The speed of sound in dry air at 27° and water at 0° *C*, respectively, are 347 *m/s* and 1500 *m/s*. Is it possible to apply the Bossinesq approximation for:

1. Deep ocean circulation with horizontal and vertical scales of 1000 *km* and 2 *km*, respectively?

2. Atmospheric thunderstorms with horizontal and vertical scales of 250 *km* and 10 *km*, respectively?

3. Atmospheric nocturnal boundary layer turbulence with horizontal scale 100 *m* and vertical scale 10 *m?*