

Homework Lecture 2 Smith

NL-6

Finally one can rewrite using reality conditions \Rightarrow

$$\frac{\partial}{\partial t} b_{mk} =$$

$$\sum_{\underline{k}+\underline{p}+\underline{q}=0} \sum_{m_p} \sum_{m_q} C_{k p q}^{m_k m_p m_q} b_{mp}^* b_{mq}^* e^{i[\sigma_{mk} + \sigma_{mp} + \sigma_{mq}]t}$$

reality : $\hat{u}(\underline{k}) = \hat{u}^*(-\underline{k})$

Homework Show that there are no 3-wave exact resonances with $\underline{k}+\underline{p}+\underline{q}=0$ and

$$\sigma^{\pm}(\underline{k}) + \sigma^{\pm}(\underline{p}) + \sigma^{\pm}(\underline{q}) = 0$$

in the range $\frac{1}{2} < \frac{N}{F} < 2$.

Homework

Find $C_{kpg}^{s_k s_p s_g}$ explicitly for pure rotation with

$$\underline{\Phi}^s(\underline{k}) = \frac{\underline{k} \times (\underline{k} \times \hat{z})}{k |\underline{k} \times \hat{z}|} + i s \frac{\underline{k} \times \hat{z}}{|\underline{k} \times \hat{z}|}$$

$\underline{k} \neq (0, 0, k_z)$

$$\underline{\Phi}^s(0, 0, k_z) = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 + i \operatorname{sgn}(k_z) & 1 - i \operatorname{sgn}(k_z) & 0 \end{bmatrix}^T$$

$$s = +1, -1 \quad (\text{no PV mode})$$

$$\sigma_s(\underline{k}) = s \frac{k_z}{k}$$

$$\left(\underline{\Phi}^m(\underline{k}) \right)^\dagger \underline{\Phi}^s(\underline{k}) = \begin{cases} 2 & m=s \\ 0 & m \neq s \end{cases}$$

$$\underline{k} \cdot \underline{\Phi}^m(\underline{k}) = 0$$