$\boxed{\text{Homework}}$ Make the analogy between conservation of energy/enstrophy in 2D non-rotating flow and conservation of energy/potential enstrophy in QG flow to argue for an inverse cascade of energy in QG dynamics.

$\boxed{\text{Homework}}$

We showed that QG comes from

$$\frac{\partial}{\partial t} a_k^0 = \sum_{\underline{k}+\underline{p}+\underline{q}=0} C_{kpq} (a_p^0)^* (a_q^0)^*$$

$$C_{kpq} = \frac{iN(\underline{p} \times \underline{q} \cdot \hat{z})}{\sigma_k k \, \sigma_p p \, \sigma_q q} (\sigma_q^2 q^2 - \sigma_p^2 p^2)$$

with $\sigma_k k = (N^2 k_h^2 + F^2 k_z^2)^{1/2}$

Show that $\underline{k} + \underline{p} + \underline{q} = 0 \Rightarrow$

$$C_{kpq} + C_{pqk} + C_{qkp} = 0$$

$$\sigma_k^2 k^2 C_{kpq} + \sigma_p^2 p^2 C_{pqk} + \sigma_q^2 q^2 C_{qkp} = 0$$

and therefore triad interactions have 2 quadratic invariants

① the total energy (kinetic + potential)

$$|a_k^0|^2 + |a_p^0|^2 + |a_q^0|^2$$

② the quadratic part of the potential enstrophy

$$\sigma_k^2 k^2 |a_k^0|^2 + \sigma_p^2 p^2 |a_p^0|^2 + \sigma_q^2 q^2 |a_q^0|^2$$