\[ \int_V \frac{\partial \eta}{\partial t} \, dV + \int_A \eta \mathbf{u}_n \cdot \mathbf{n} \, dA = 0 \] 

\[ \Rightarrow \frac{d}{dt} \int_V \eta \, dV = 0 \text{ in a periodic domain} \]

**Homework**  
Make the analogy between conservation of energy/enstrophy in 2D non-rotating flow and conservation of energy/potential enstrophy in QG flow to argue for an inverse cascade of energy in QG dynamics.
We showed that $\Omega^6$ comes from

$$\frac{\partial a_k}{\partial t} = \sum_{k+p+q} C_{kpq} (a_p)^* (a_q)^*$$

$$k + p + q = 0$$

$$C_{kpq} = \frac{iN(p \times q \cdot z)}{\sigma_k \sigma_p \sigma_q q} (\sigma_q q^2 - \sigma_p p^2)$$

with $$\sigma_k k = (N^2 k_n^2 + F^2 k_z^2)^{1/2}$$

Show that $$k + p + q = 0 \implies$$

$$C_{kpq} + C_{pqk} + C_{qkp} = 0$$

$$\sigma_k^2 k^2 C_{kpq} + \sigma_p^2 p^2 C_{pqk} + \sigma_q^2 q^2 C_{qkp} = 0$$

and therefore trid interactions have 2 quadratic invariants
1. the total energy (kinetic + potential)
\[ |a_k|^2 + |a_p|^2 + |a_q|^2 \]

2. the quadratic part of the potential enstrophy
\[ \sigma_k^2 |a_k|^2 + \sigma_p^2 |a_p|^2 + \sigma_q^2 |a_q|^2 \]