\[
E(k) = A k^m \quad k_0 < k < k_L \\
E(k) = \alpha \varepsilon^{2/3} k^{-5/3} \quad k_L < k < k_v
\]

Since at \(k\), the energy \(E(k)\) should be the same, we have

\[
A k^m = \alpha \varepsilon^{2/3} k^{-5/3} \Rightarrow k_L^{3m+5} = \alpha \varepsilon^{2/3} \Rightarrow k_L = \alpha \varepsilon^{2/3} = C_1 \varepsilon^{2/3}
\]

so

\[
\frac{q^2}{2T} = \int_{k_v}^{k_L} E(k) \, dk = \int_{k_v}^{k_L} E(k) \, dk + \int_{k_L}^{k_v} E(k) \, dk
\]

\[
= \int_{k_v}^{k_L} \frac{Ak^m}{m+1} \, dk + \int_{k_L}^{k_v} \alpha \varepsilon^{2/3} k^{-5/3} \, dk
\]

\[
= \frac{m+1}{m+1} \left[ k_v^{m+1} \right]_{k_0}^{k_L} - \frac{3}{2} \varepsilon^{2/3} k^{-4/3} \bigg|_{k_v}^{k_L}
\]

\[
= \frac{m+1}{m+1} + \frac{3}{2} \varepsilon^{2/3} k_L
\]

\[
= C_1 \varepsilon^{2/3} + C_2 \varepsilon^{2/3} \varepsilon^{-4/3(3m+5)}
\]

\[
= c_3 \varepsilon^{2/3}
\]

here we assumed the energy in both very large scale \(k_0\) and very small scale \(k_v\) are small. Also, from the derivation, we can see \(c_3\) is a constant with respect to \(\varepsilon\).

so

\[
\frac{q^2}{q_0} = \frac{c_3 \varepsilon^{2/3}}{c_0 \varepsilon^{2/3}} = \left( \frac{c_0}{c_0} \right)^{2m+2} = \left( \frac{1}{c_0} \right)^{2m+2}
\]