Group1

Majda's Problem #1

B)

$$
\begin{cases}
\theta' = S'_{w,-1} \\
w' = S'_{\theta,-1} \\
\dfrac{D\vec{u}_h'}{Dt} = -\nabla_x p' + \overline{(w'\vec{u}_h')_z} & (1) \\
\mathrm{div}_x \vec{u}_h' + w'_z = 0 & (2)
\end{cases}
$$

Assume $\vec{u}_h' = \begin{pmatrix} u'(x,t) \\ v'(x,t) \end{pmatrix}$,

$$(2) \Rightarrow \partial_x u' + \cancelto{0}{\partial_y v'} + \partial_z w' = 0 \quad \Rightarrow \quad u' = \int -\frac{\partial S'_{\theta,-1}}{\partial z}\, dx \qquad (*1)$$

(Set-up 1:
Since u' is independent of z and y, so $w' = S'_{\theta,-1}$ must be a linear function of z, and independent of y.)

$$(1) \Rightarrow \frac{\partial u'}{\partial t} + (\bar{u}+u')\partial_x u' + (\bar{v}+v')\cancelto{0}{\partial_y u'} + w'\cancelto{0}{\partial_z u'} = -\partial_x p + \overset{\overline{(w'_z)u'}}{\overset{\parallel}{\overline{(w'u')_z}}}$$

$$\Rightarrow p = \int \left(\overline{w'_z u'} - \frac{\partial u'}{\partial t} - (\bar{u}+u')\partial_x u' \right) dx \qquad (*2)$$

$(*2) \Rightarrow p$ is independent of y.

$$(1) \Rightarrow \frac{\partial v'}{\partial t} + (\bar{u}+u')\partial_x v' + (\bar{v}+v')\cancelto{0}{\partial_y v'} + w'\cancelto{0}{\partial_z v'} = -\cancelto{0}{\partial_y p} + \overset{\overline{(w'_z)v'}}{\overset{\parallel}{\overline{(w'v')_z}}}$$

(Set-up 2:
In order to solve this eqn, we need $\overline{(w')_z v'} = w'_z \cdot \bar{v} = 0$, which means $w' = S'_{\theta,-1}$ must be independent of x.)

Then, we can use characteristic line method to the first order ODE for v'. #