Lecture 2
Atmospheric Boundary Layer

H. J. Fernando
Arizona State University

Region of the lower atmosphere where effects of the Earth surface are felt

Surface – fluxes of momentum, buoyancy…..

Neutral, Convective, Stable and Transitional Boundary Layers

Atmospheric Boundary Layer (flat terrain)

$$\frac{\partial \overline{U}_i}{\partial t} + \overline{U}_j \frac{\partial \overline{U}_i}{\partial x_j} + \underset{\sim}{f} \times \underset{\sim}{U} = -\frac{1}{\rho}\frac{\partial P}{\partial x_i} + \frac{\partial}{\partial x_j}\underbrace{\left(\nu \frac{\partial \overline{U}_i}{\partial x_j} - \overline{u_i u_j} \right)}_{\tau ij}$$

Horizontal homogeneity
Steady (Boun Layer)

$$\underset{\sim}{f} \times \underset{\sim}{U} = -\frac{1}{\rho}\frac{\partial P}{\partial x_i} + \frac{\partial \tau_{i3}}{\partial x_3}$$

Geostrophy

$$\underset{\sim}{f} \times \underset{\sim}{U}_g = -\frac{1}{\rho}\frac{\partial P}{\partial x_i}$$

$$\underset{\sim}{f} \times \underset{\sim}{U}_g = -\frac{1}{\rho}\nabla P$$

$$\underset{\sim}{U}_g . \nabla P = 0$$

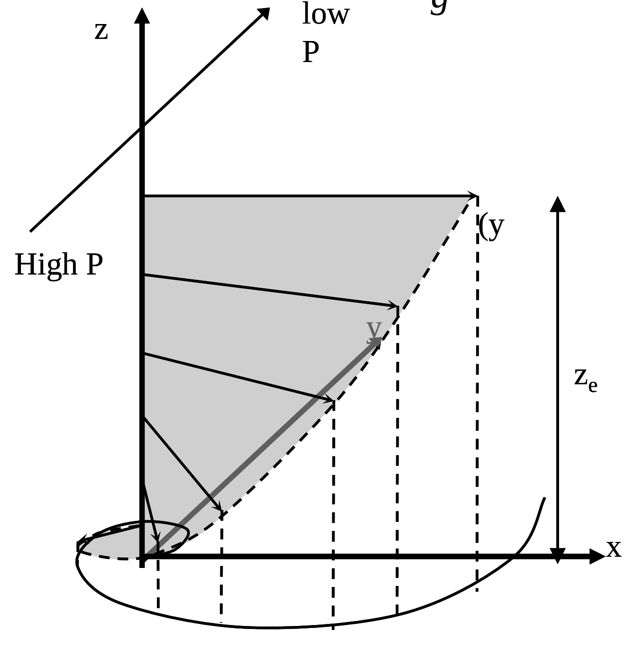
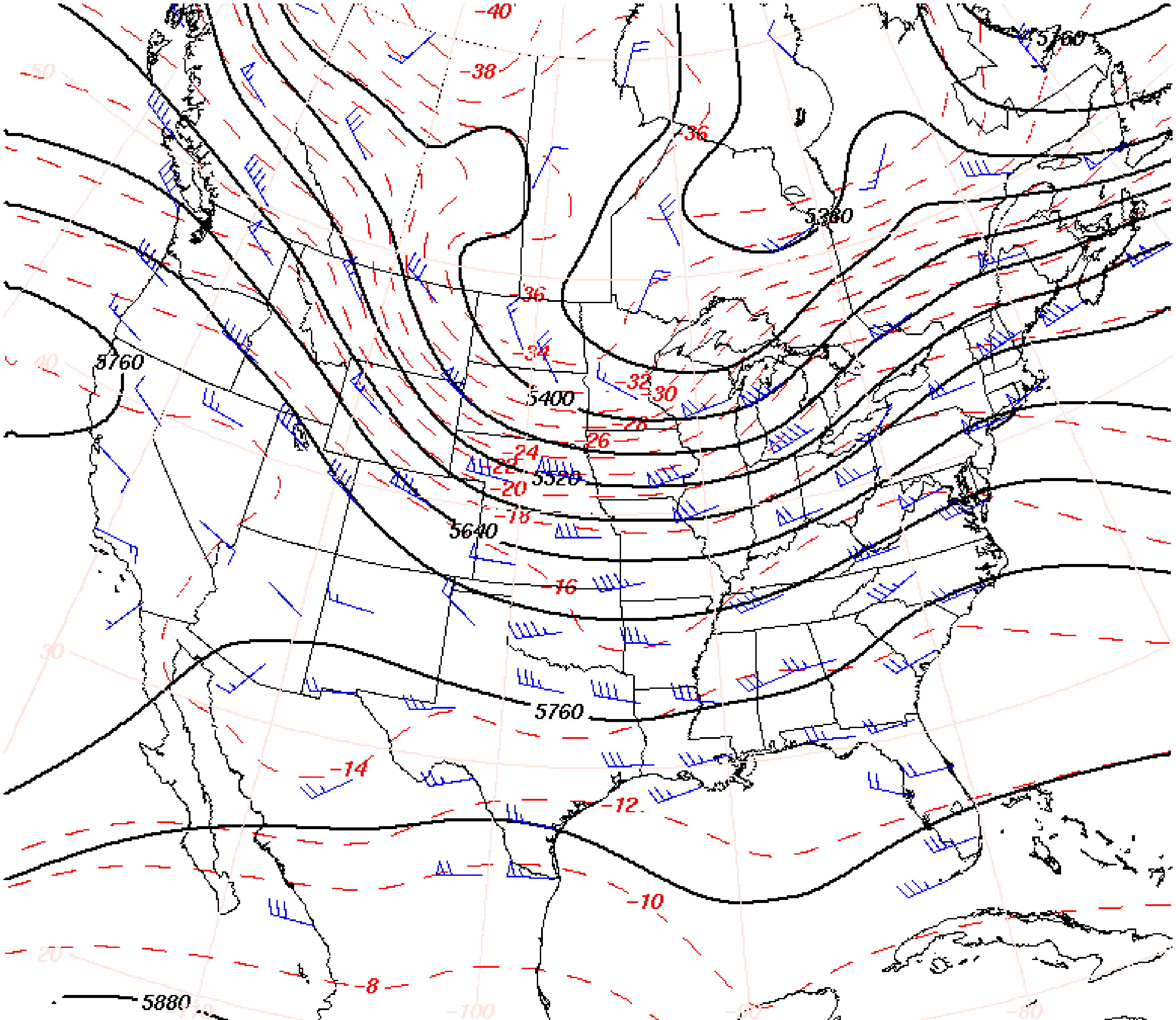


Fig. 4.7.1 Ekman Spiral in the Northern Atmosphere



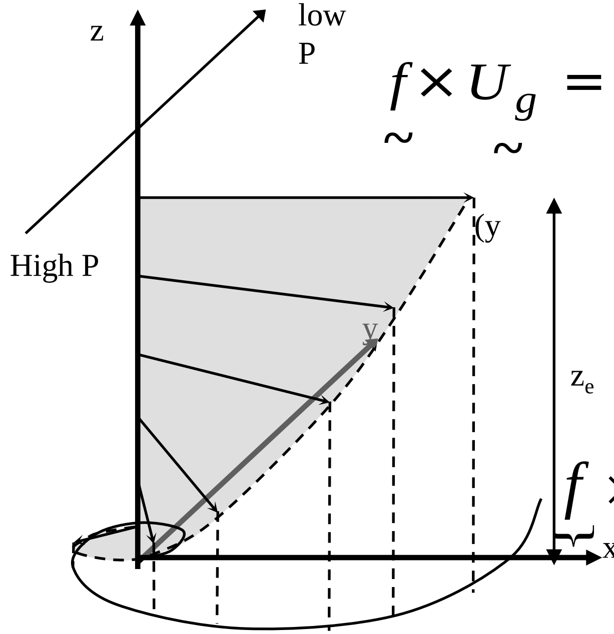
500 mb Wind Barbs (kt) 020403/0000
500 mb Height (m) Analysis 020403/0000
500 mb Temperature (C) Analysis 020403/0000

Boundary Layer fow

$$\underset{\sim}{f} \times \underset{\sim}{U} = f \times U_g + \frac{\partial \tau_{i3}}{\partial x_3}$$

$$- \underset{\sim}{f} \times (\underset{\sim}{U_g} - \underset{\sim}{U}) = \frac{\partial \tau_{i3}}{\partial x_3}$$

$$\underset{\sim}{f} \times \underset{\sim}{U_g} = \frac{1}{\rho} \frac{\partial P}{\partial x_i}$$



$$\underbrace{f} \times \underbrace{U} = -\frac{1}{\rho} \frac{\partial P}{\partial x_i} + \frac{\partial \tau_{i3}}{\partial x_3}$$

Ekman Spiral in the Northern Atmosphere

$$-f\left(\bar{v} - v_g\right) = \frac{\partial \tau_{xz}}{\partial z}$$

$$+f\left(\bar{U} - U_g\right) = \frac{\partial \tau_{yz}}{\partial z}$$

$$\tau_{xz}(H) - \tau_{xz}(0) = \int_0^H -f\left(\bar{v} - v_g\right) dz$$

u_*^2

$$(1 - a)u_*^2 \approx f v_g H$$

$$H \approx \frac{(1 - a)u_*^2}{f v_g} \sim \frac{0.2 \times 50^2}{10^{4} \times 10 \times 10^2} \approx 50\, m$$

Surface layer – small change of stress

$$-f\left(\bar{v}-v_g\right)=\frac{\partial \tau_{xz}}{\partial z}$$

$$f\left(\bar{U}-U_g\right)=\frac{\partial \tau_{yz}}{\partial z}$$

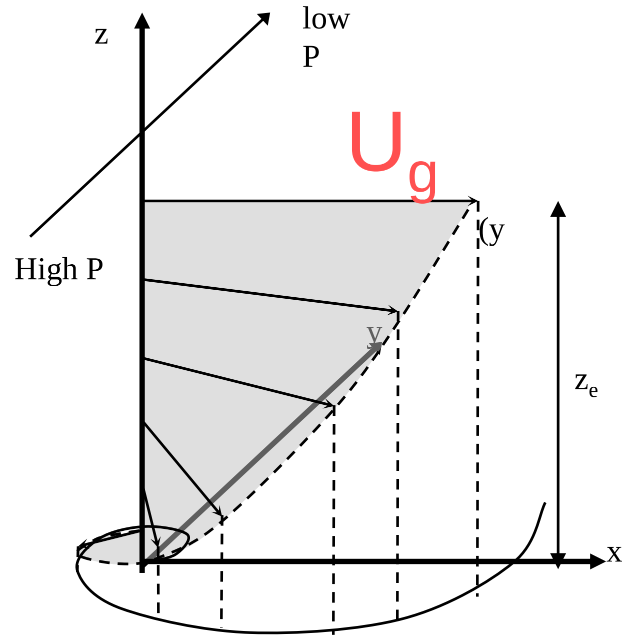


Fig. 4.7.1 Ekman Spiral in the Northern Atmosphere

$\left(\bar{U},\bar{v}\right)=\left(\bar{U}_g,0\right)$ at $z \rightarrow z_e$

$\bar{U}-U_g \rightarrow -U_g \qquad \bar{v}=0$ at $z = 0$

$$\nu\frac{\partial \bar{U}}{\partial z}-\overline{u'w'}=\tau_{xz}=K\frac{\partial U}{\partial z};\quad \tau_{yz}=K\frac{\partial \bar{v}}{\partial z}$$

(K theory)

Ekman Spiral

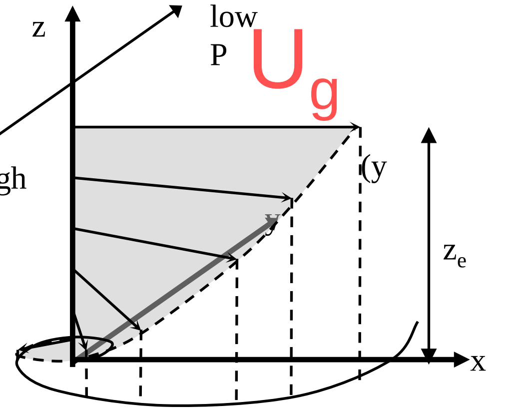


Fig. 4.7.1 Ekman Spiral in the Northern Atmosphere

$$\overline{U} = \overline{U}_g \left[1 - e^{-z/z_e} \cos\left(\frac{z}{z_e} \right) \right]$$

$$\overline{V} = \overline{U}_g \left[e^{-z/z_e} \cos \frac{z}{z_e} \right]$$

$$z_e = \sqrt{2 \frac{K}{f}}$$

Ekman Layer Height

$$h_E = \sqrt{\frac{2K}{f}} \sim 300m$$

Sutton (1953) used this as the ABL height under neutral conditions

$$K = u_* h_E$$

Tennekes (1982)

$$h_{ABL} \approx \frac{0.25u_*}{f} \sim 1km$$

With Stratification? -
- Stable or Unstable

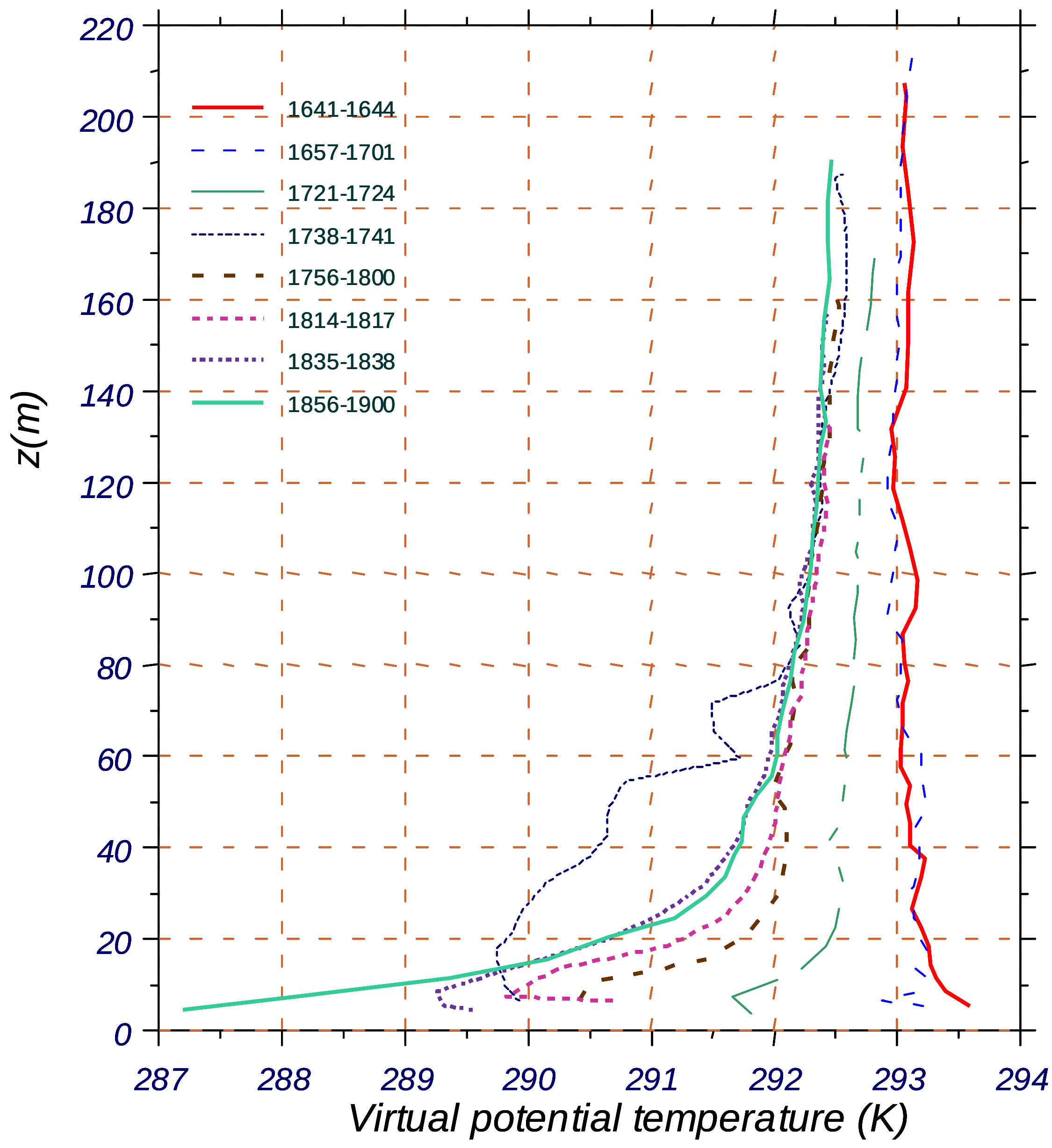
Lecture 2a

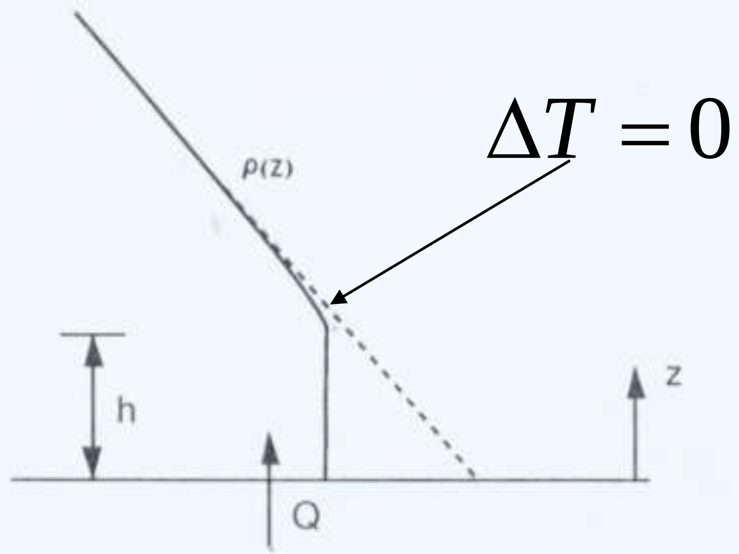
Convective Boundary Layers

And

Convective Flows

ABL EVOLUTION





$$q_0 = \frac{\alpha g Q}{\rho_0 C_p}$$

buoyancy flux

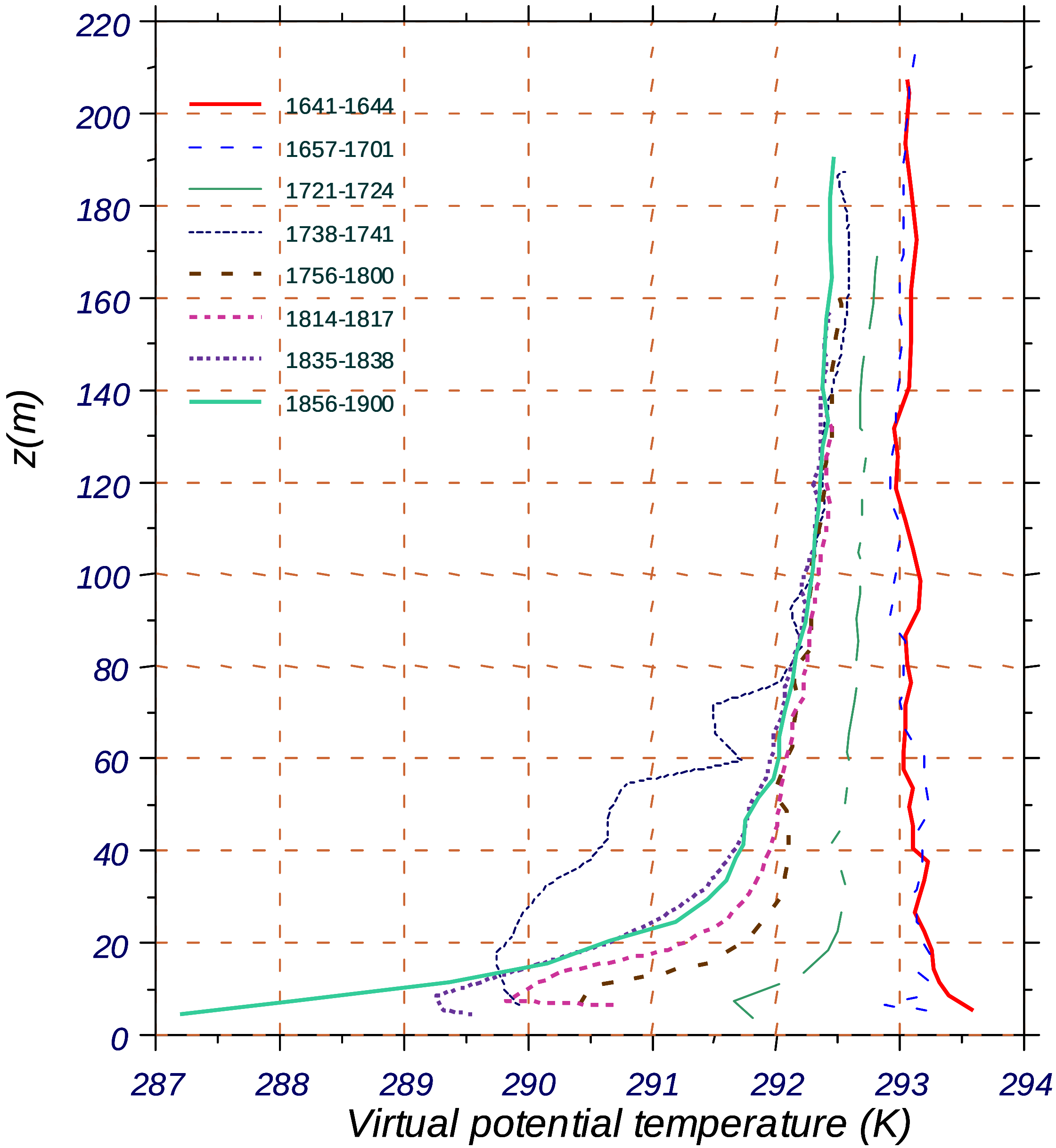
FIG. 1. An idealized experimental configuration: heating of a linearly (temperature) stratified fluid from below with a constant heat flux. Here $\rho(z)$ represents the density distribution resulting from temperature.

Non-Penetrative and Penetrative Convection

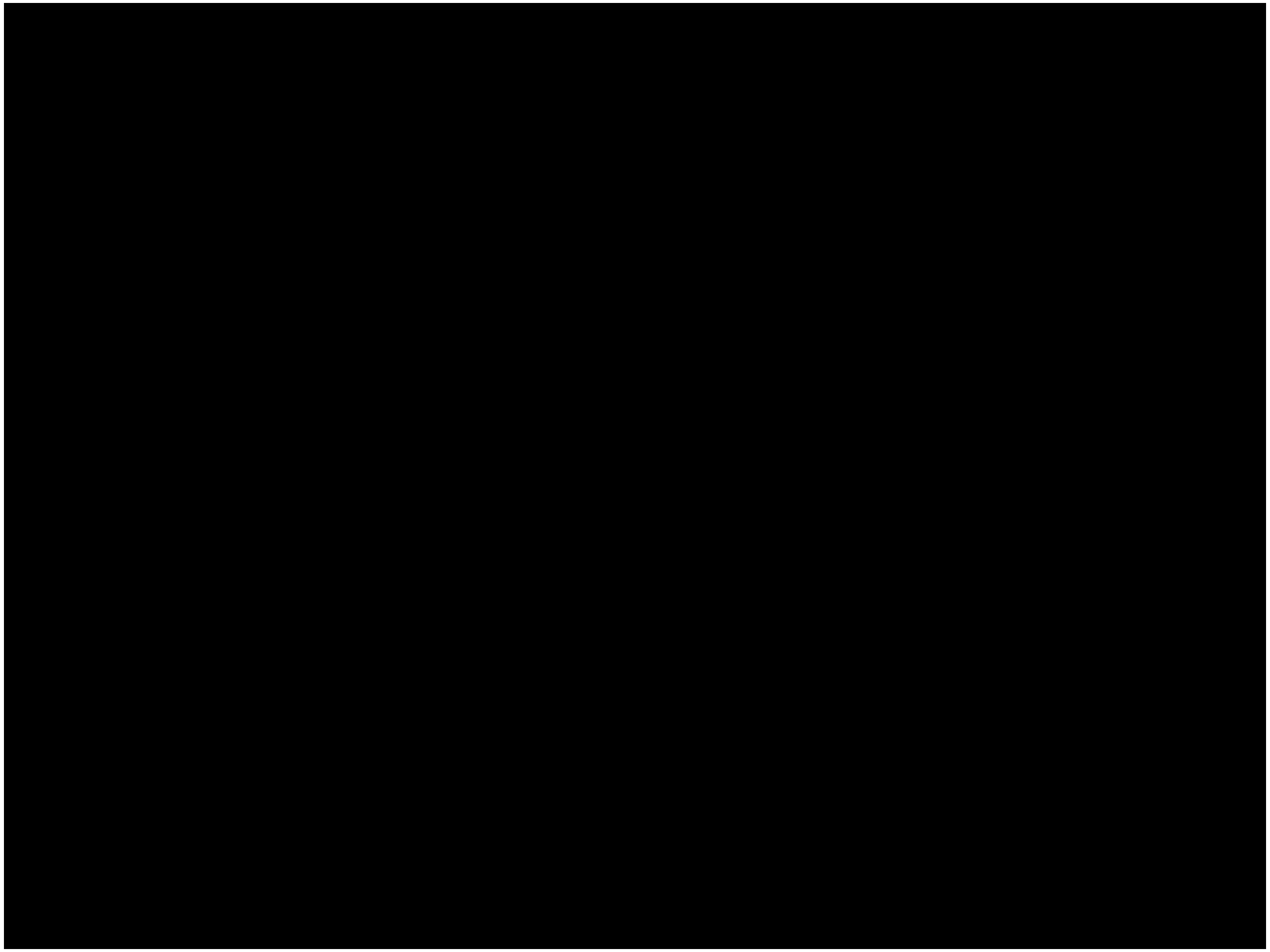
$$\frac{1}{2} N^2 h^2 \approx q_0 t$$

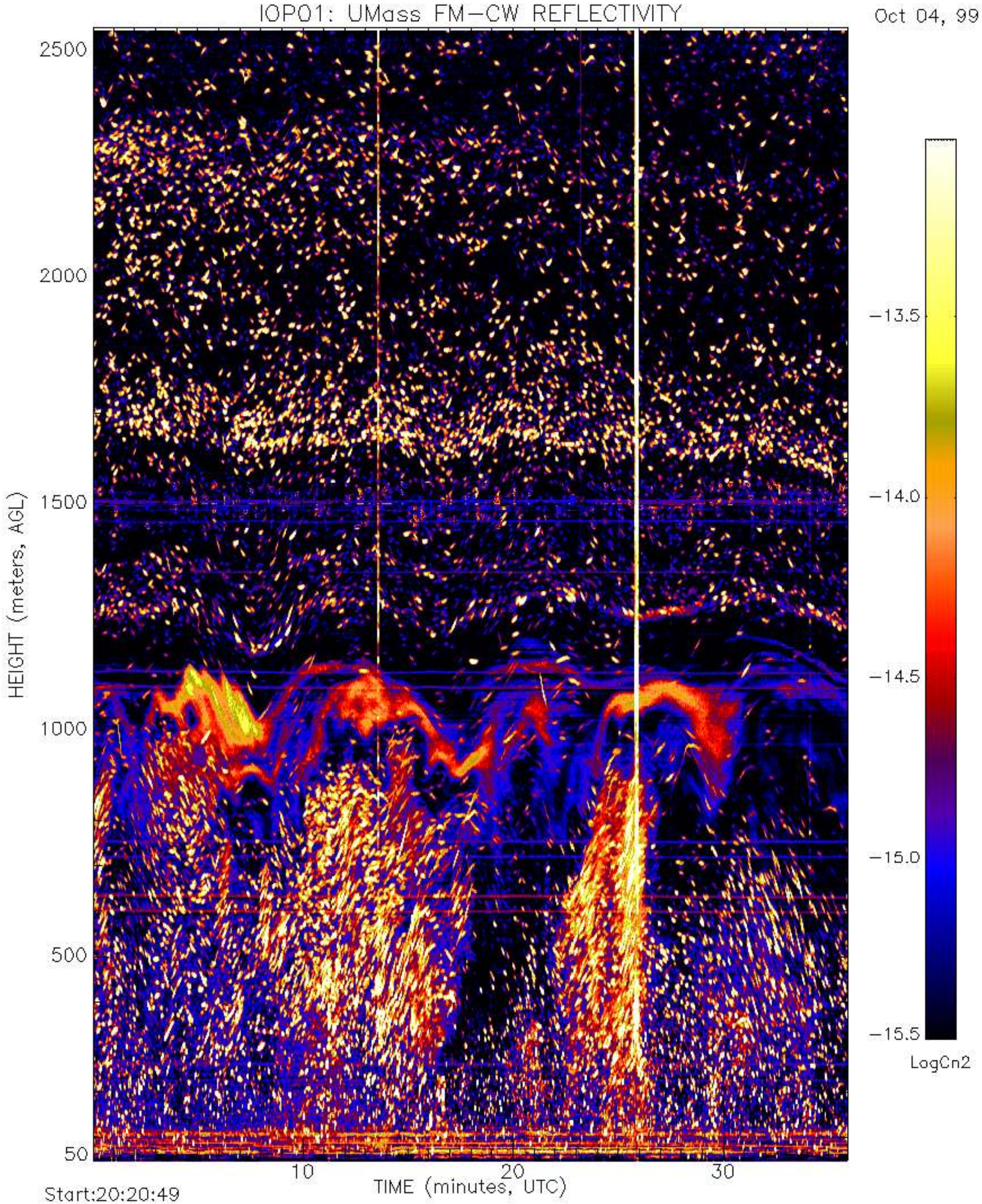
$$h = \sqrt{2}\left(\frac{q_0}{N^2}\right)^{1/2} t^{1/2}$$

ABL EVOLUTION

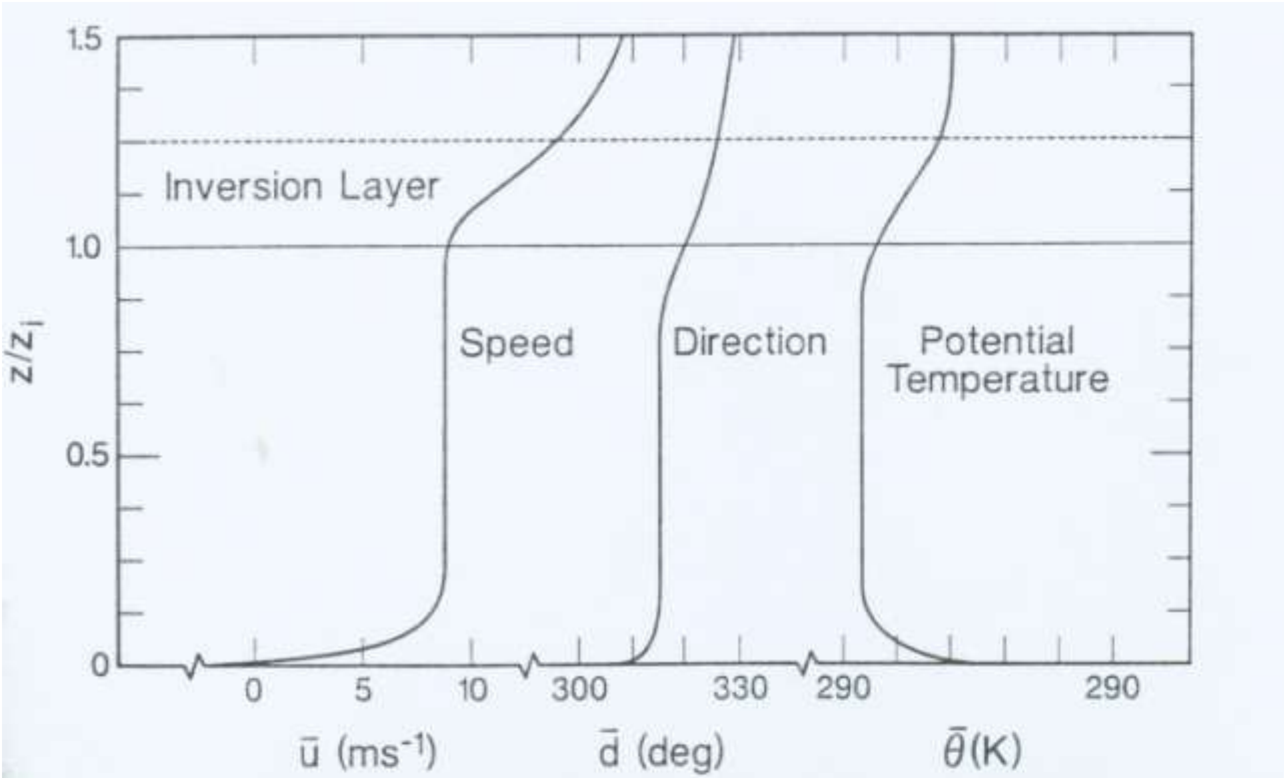


Formation and Breakdown of an Inversion Layer in El Paso

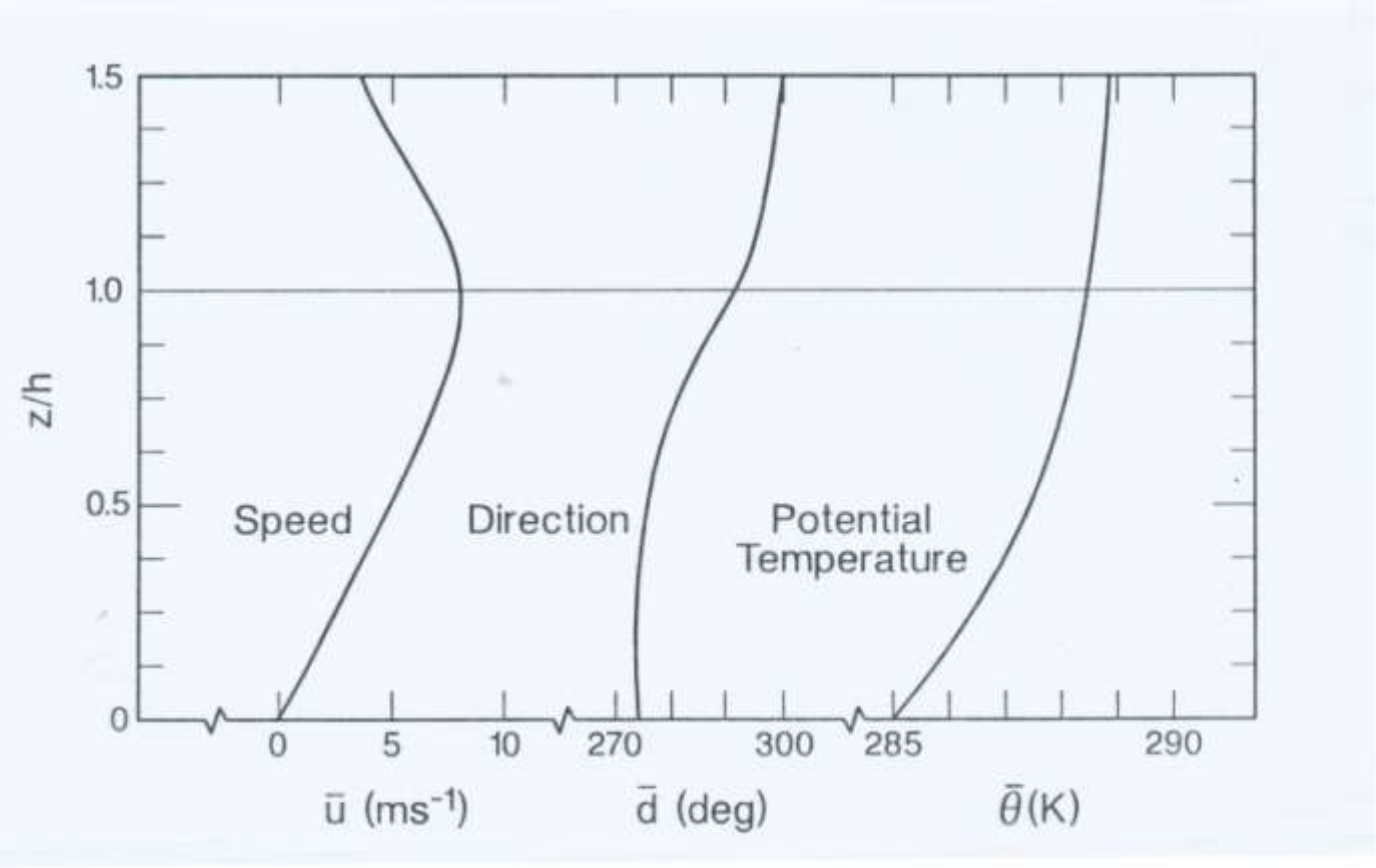



IOP01: UMass FM-CW REFLECTIVITY
Oct 04, 99
2500
2000
1500
1000
500
50
HEIGHT (meters, AGL)
10
20
30
TIME (minutes, UTC)
Start:20:20:49
-13.5
-14.0
-14.5
-15.0
-15.5
LogCn2

Unstable Boundary Layer (flat terrain)

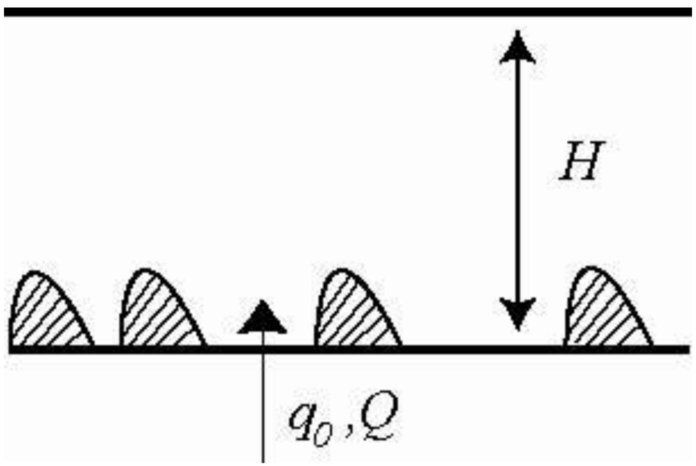


Stable Boundary Layer (flat terrain)



Convection between horizontal surfaces

[Rayleigh-Benard ; Chandrasekhar 1961]



$$q_o, H, k_T, \nu, \left(f = 2\Omega\right)$$

$$\mathrm{Ra}_f = \frac{q_0 H^4}{k_T^2 \nu} \quad \left\{ \frac{g \alpha \Delta T d^3}{k \nu} \right\}$$

$$W_* = \left(q_0 H\right)^{1/3}$$

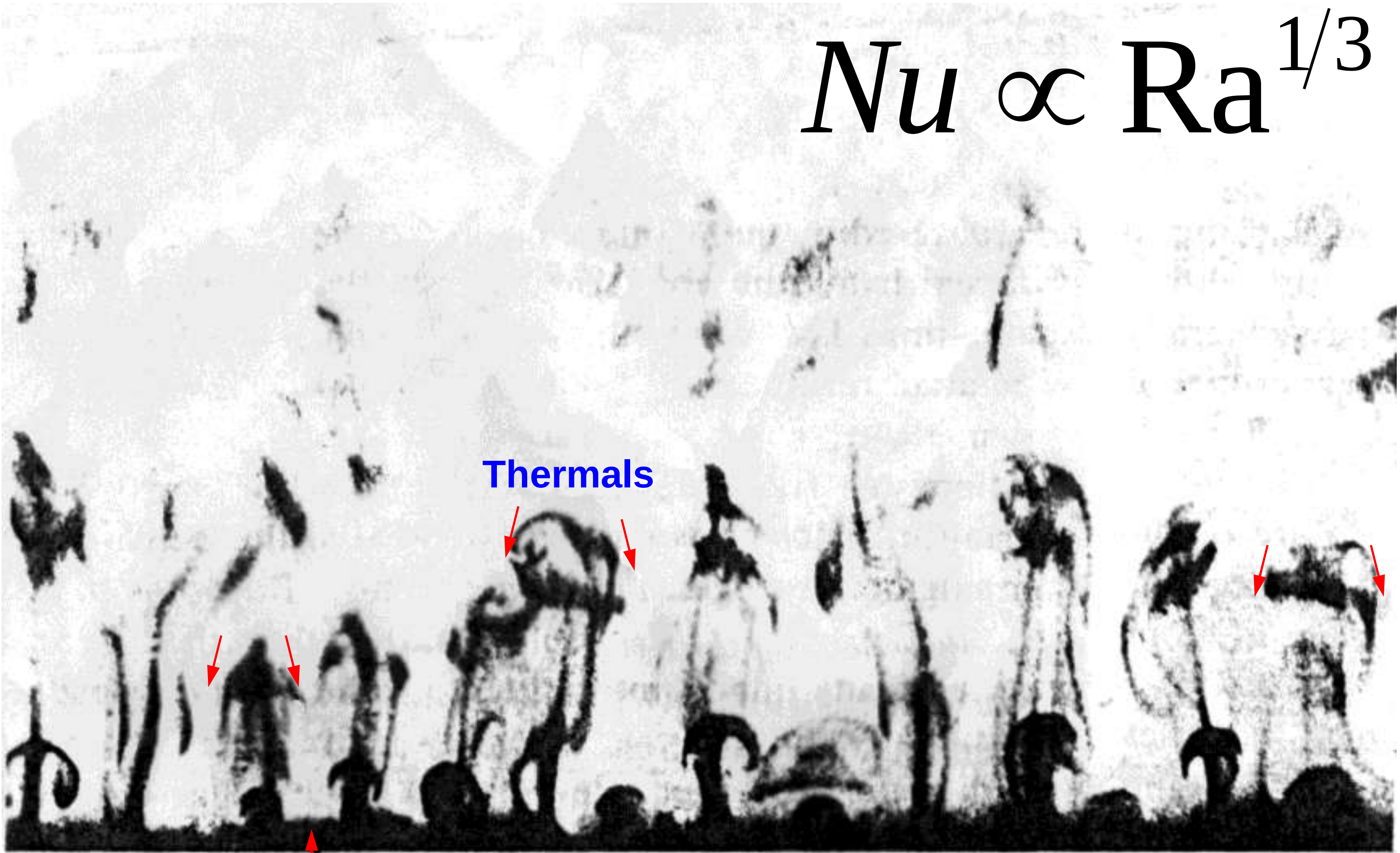
$$T_a = \frac{4\Omega^2 H^4}{\nu^2} \; , \; P_r = \frac{\nu}{k_T}$$

$$L \sim H \qquad R_f \approx 10^8$$

$$B \sim q_0 / W_*$$

Deardorff (1971)

$$Nu \propto \mathrm{Ra}^{1/3}$$



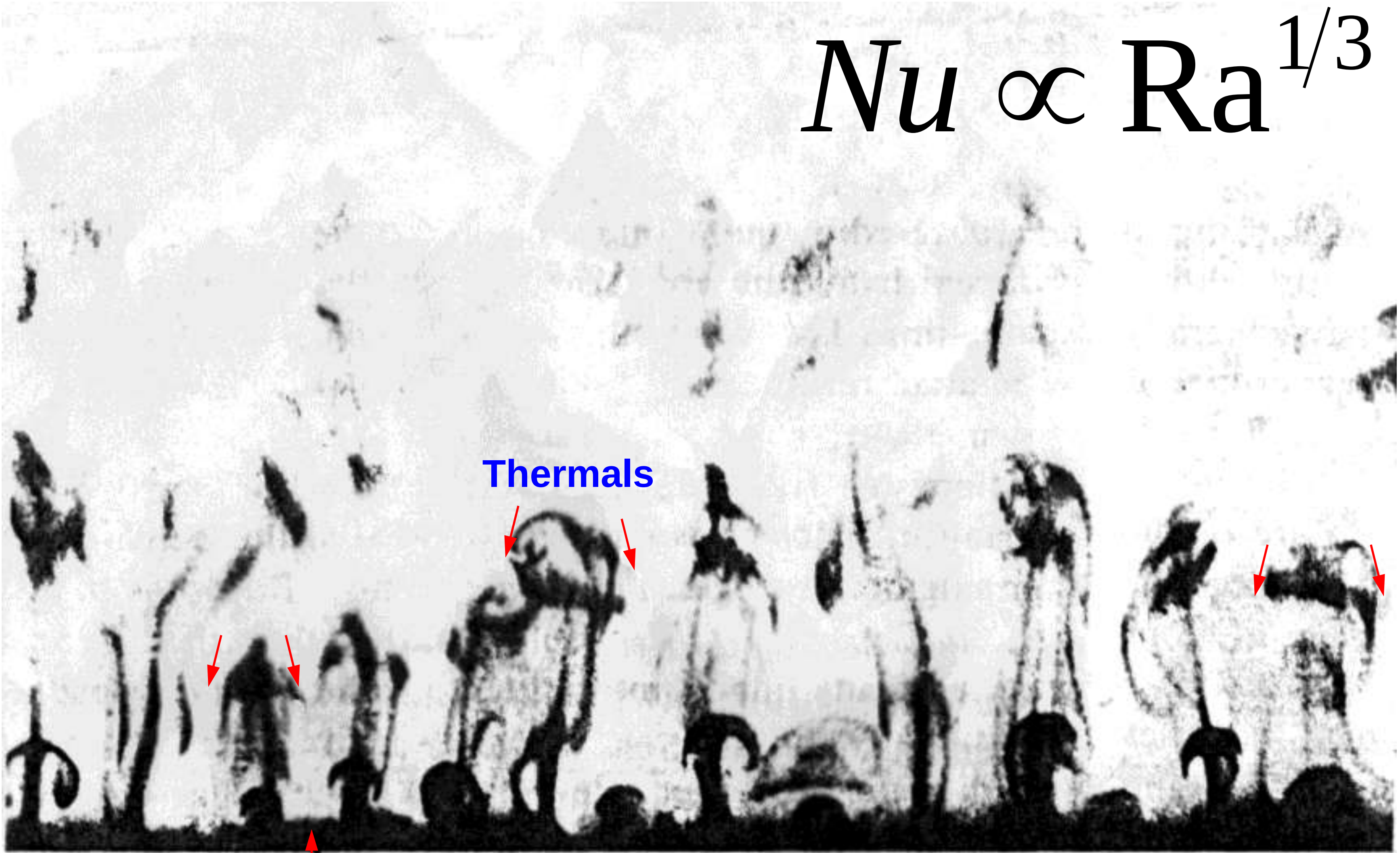
Molecular

Goldstein and Chu (1973)

Sparrow *et. al. (1970)*

Plumes - Convective

$$Nu \propto \mathrm{Ra}^{1/3}$$



Goldstein and Chu (1973)

Sparrow *et. al. (1970)*

Fig. 1. Plan view of an array of Bénard cells developed in a deep rotating layer of water by evaporation from the top surface. Threads of ink from the top surface have moved down the center of each cell. The threads with curlicues at the bottom due to the spiral motion in each cell are seen head-on near the center and obliquely around the periphery. (59–B–1, 040565–4R6). Mean depth = 40 cm, rotation = 1.37 s^{-1}, Taylor number = 3×10^{11}, estimated vertical temperature difference = -0.4°C, estimated Rayleigh number = 4×10^{8}, cell center distance ~4 cm.

Fig. 2. Plan view (59–B–1, 040565–4R14) of circular ring cells in the air-driven Ekman layer at the free surface of a deep rotating layer of water with stable vertical stratification. Conditions same as Fig. 1 except stratification probably stable, ring spacing ~2 cm.

Dave Fultz's experiments

518 *B. M. Boubnov and G. S. Golitsyn*

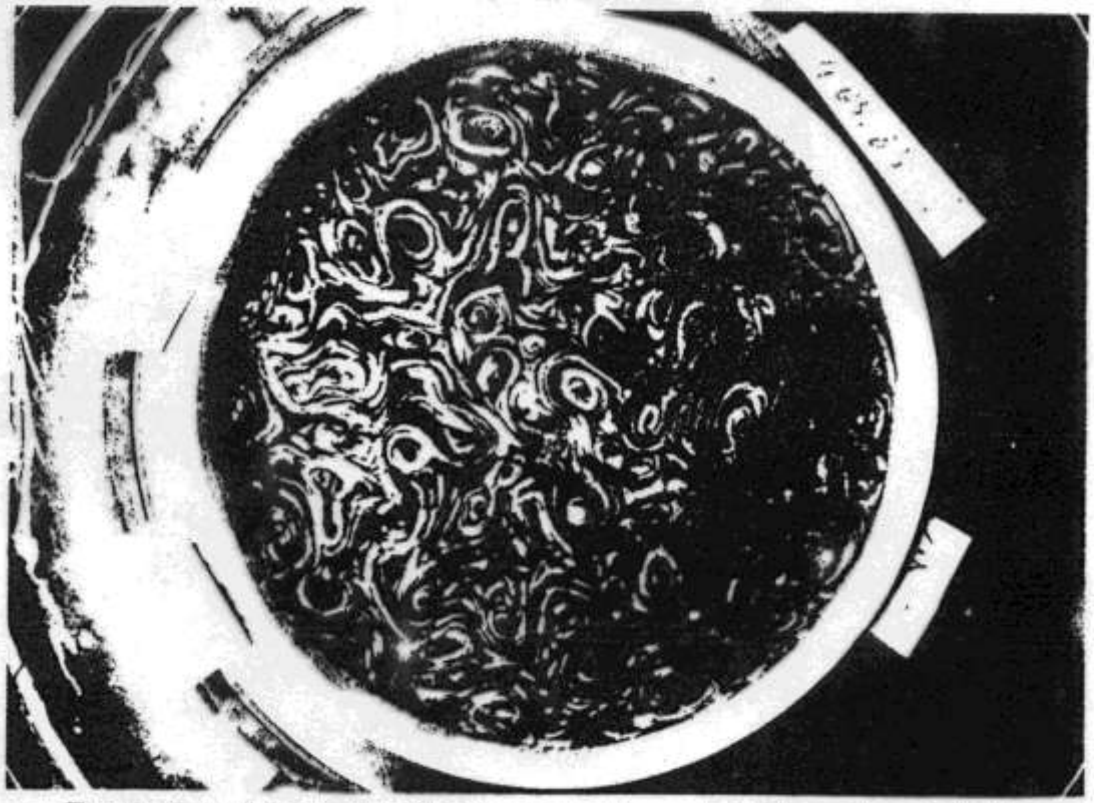


FIGURE 9. An example of an irregular convective vortex pattern; $Ra_f = 2.0 \times 10^8$, $Ta = 7.1 \times 10^7$.

$$u.\nabla u \sim 2\Omega \times u \sim \frac{1}{\rho}\nabla p$$

Irregular vortex patterns

(Higher Ra/smaller T_a)

"Geostrophic Turbulence"

Onset of Rotational Effects

$$\left(\frac{L_H}{u_H T_H}\right)\frac{\partial u_\alpha}{\partial t} + u_\beta \frac{\partial u_\alpha}{\partial x_\beta} + w\frac{\partial u_\alpha}{\partial z} + \left(\frac{1}{Ro}\right)\varepsilon_{\alpha jk}\ell_j u_k = -\left(\frac{p_0}{\rho_0 u_H{}^2}\right)\frac{\partial p}{\partial x_\alpha}$$

$$+\left(\frac{1}{Re}\right)\left[\frac{\partial^2 u_\alpha}{\partial x_\beta \partial x_\beta} + \left(\frac{L_H}{L_V}\right)^2 \frac{\partial^2 u_\alpha}{\partial z^2}\right]$$

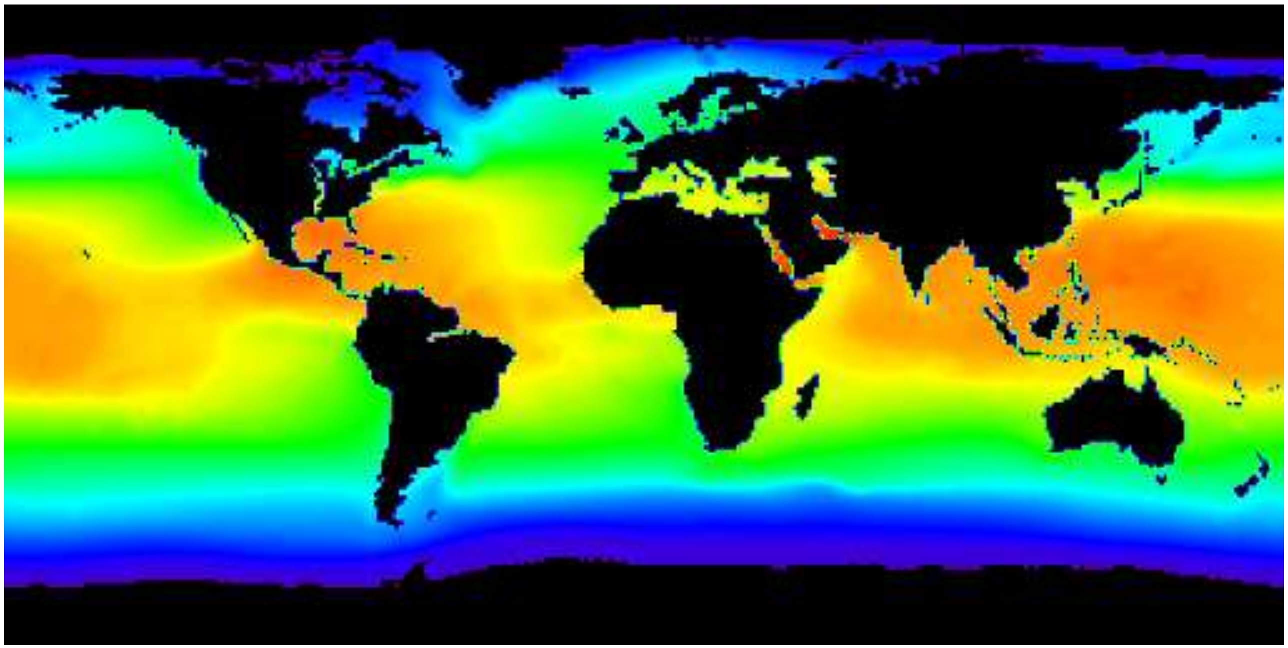
$$Ro = \frac{u_H}{f L_H} \sim 1$$

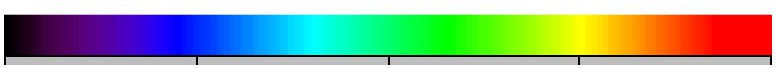
$$U_H = (q_0 L_H)^{1/3}$$

$$U \sim \left(q_o L_H\right)^{1/3} \approx 2(q_0 / f)^{1/2}$$

$$L_R \approx 10\,(q_0 / f^3)^{1/2} \sim 100\ km$$

Sea Surface Temperature, July

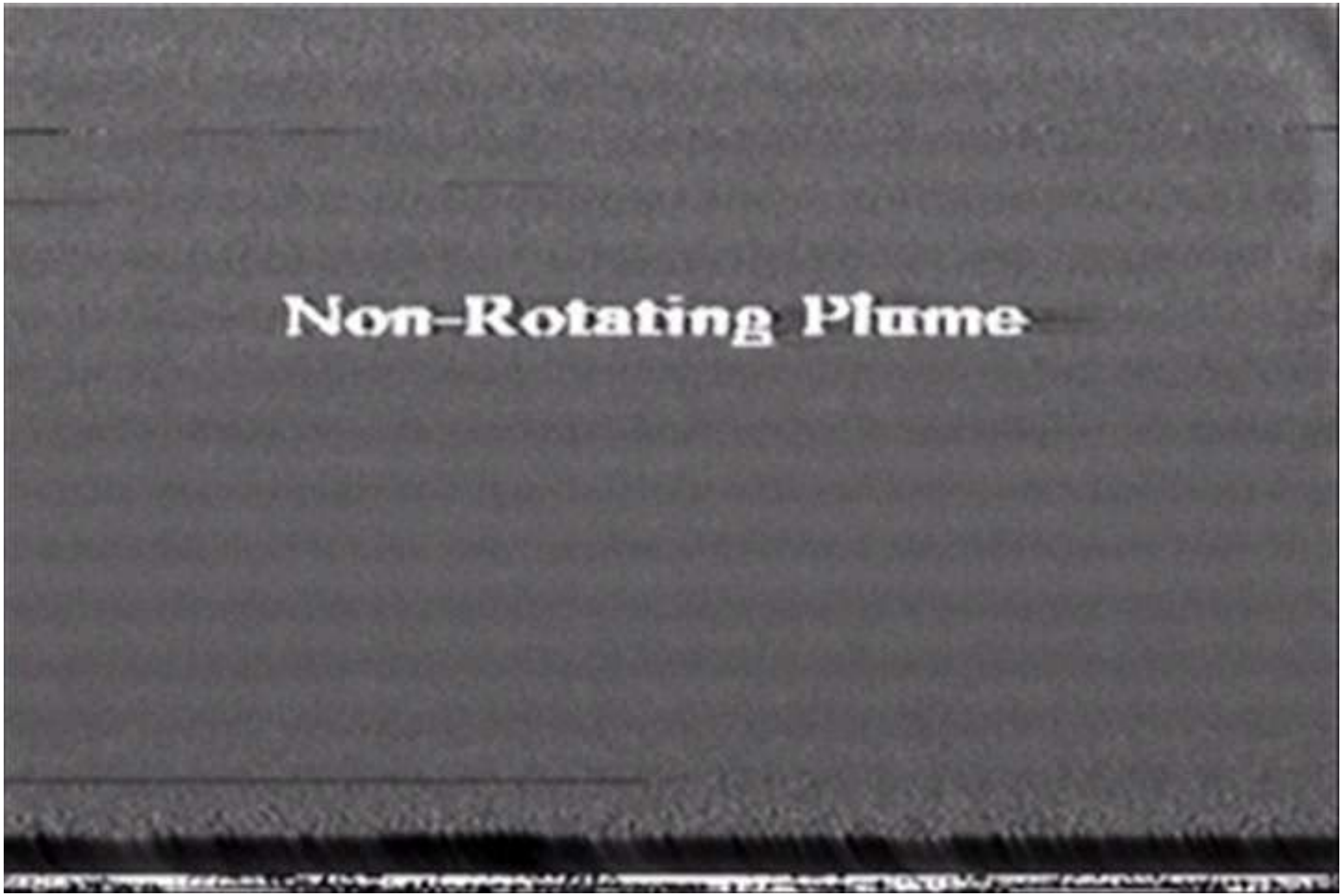




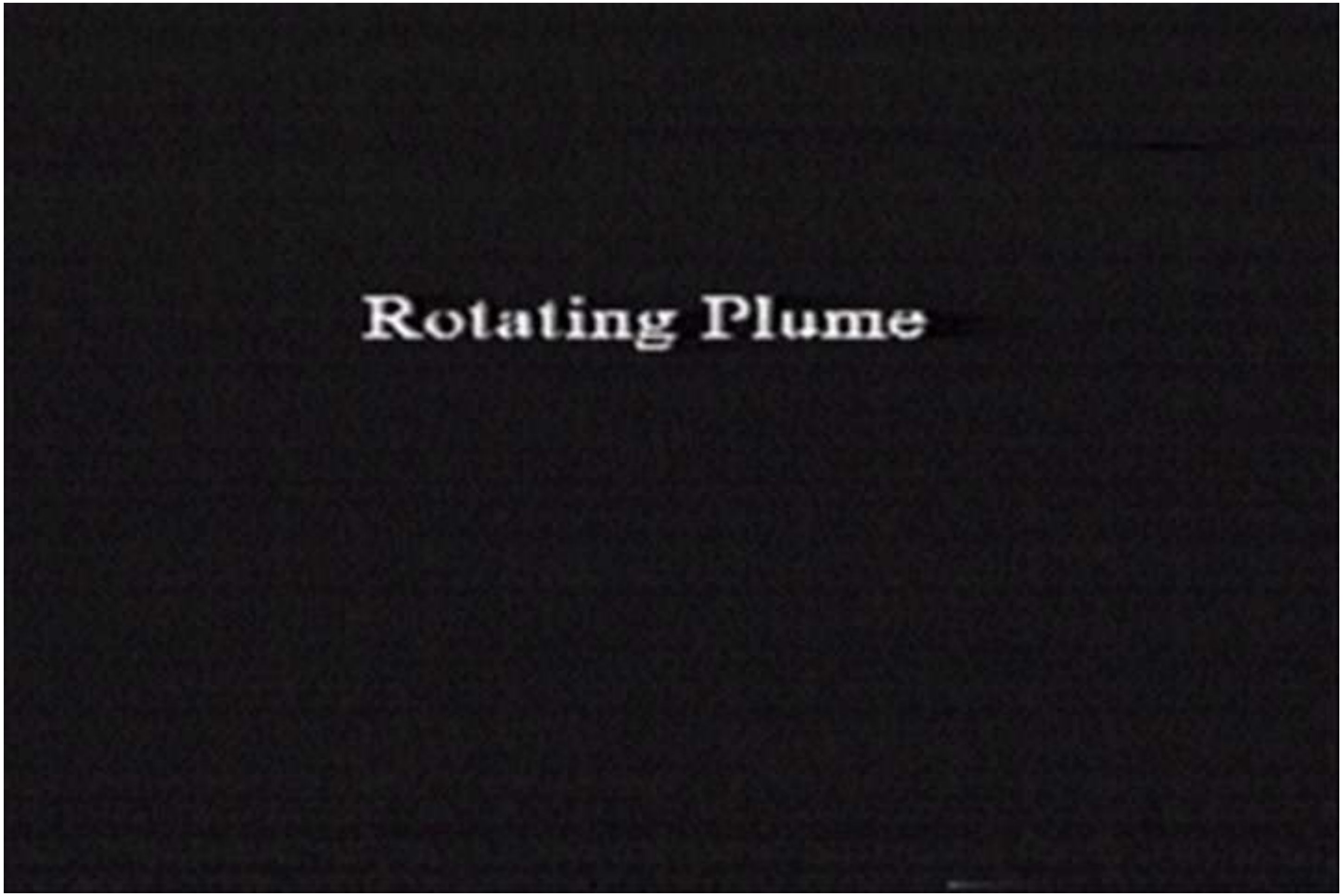
-10.0 33.2 (^{o}C)


Hurricane Floyd
NOAA-14 AVHRR HRPT
Multi-spectral False Color Image
September 15, 1999 @ 2018 UTC

Non-Rotating Plume



Rotating Plume



Fernando, Dyn. Atmos. Oceans, 2000

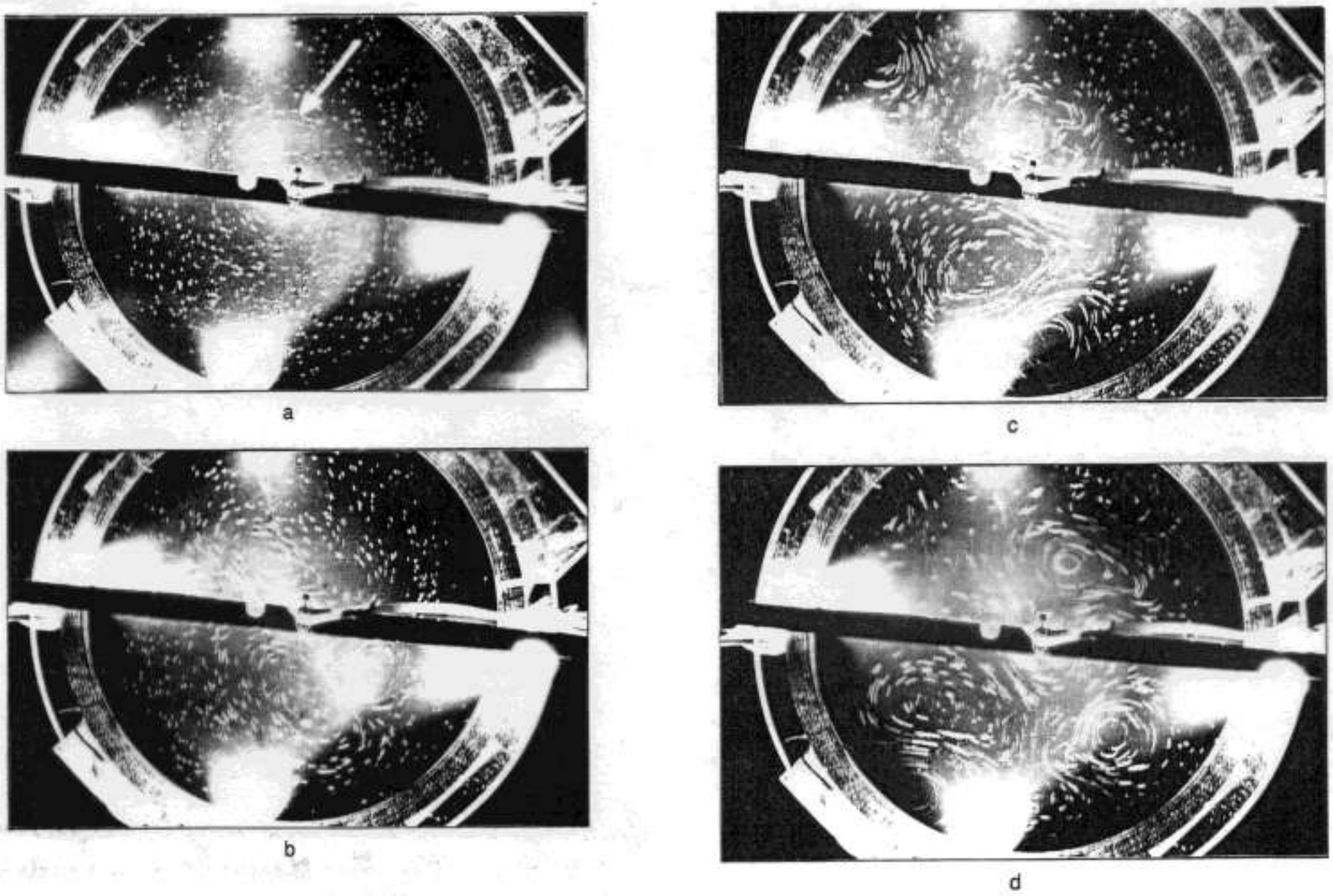


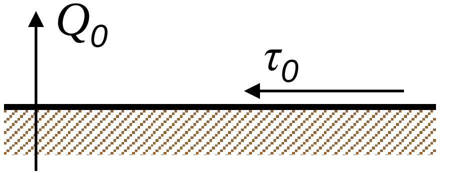
Figure 6. A sequence of streakline photographs of particles placed at the top of a homogeneous rotating fluid when a dense plume is released from a source of d_0 = 1.27 cm. The particle streaks show the velocity field at times (a) 15 (b) 60 (c) 90 and (d)120 s after the start of the plume. The experimental parameters are Ω= 0.5 rads^{-1}, B_0= 12 cm^{4}s^{-3}.

Atmospheric Surface layer

Monin-Obukhov (1954) Similarity Theory
-- For flat terrain surface layer

Parameters

Heat Flux Q_0, Stress $\tau_0 = u_*^2$



buoyancy flux $q_0 = \dfrac{g\alpha Q_0}{\rho_0 C_p}$

temperature flux $H = (\overline{\theta' w'})_0 = \dfrac{Q_0}{\rho C_p}$

Define the scaling variables:

velocity scale $u_* = \left[\left(-\overline{u'w'}\right)_0\right]^{1/2}$ $\qquad \overline{\theta' w'} > 0$ Convection $(T_* < 0)$

temperature scale $T_* = \left(\dfrac{-\overline{w'\theta'}}{u_*}\right)_0$ $\qquad \theta' w' < 0$ Stratification $(\text{stable}: T_* > 0)$

Monin-Obukhov scale

$$L_* = \frac{1}{\kappa} \frac{u_*^3}{g \frac{\overline{\theta' w'}}{\Theta}} = \frac{1}{\kappa} \frac{u_*^3}{q_0}$$

Non dimensional relations

$$Any = F(u_*, z, q_0)$$
$$= G(u_*, z, L_*)$$

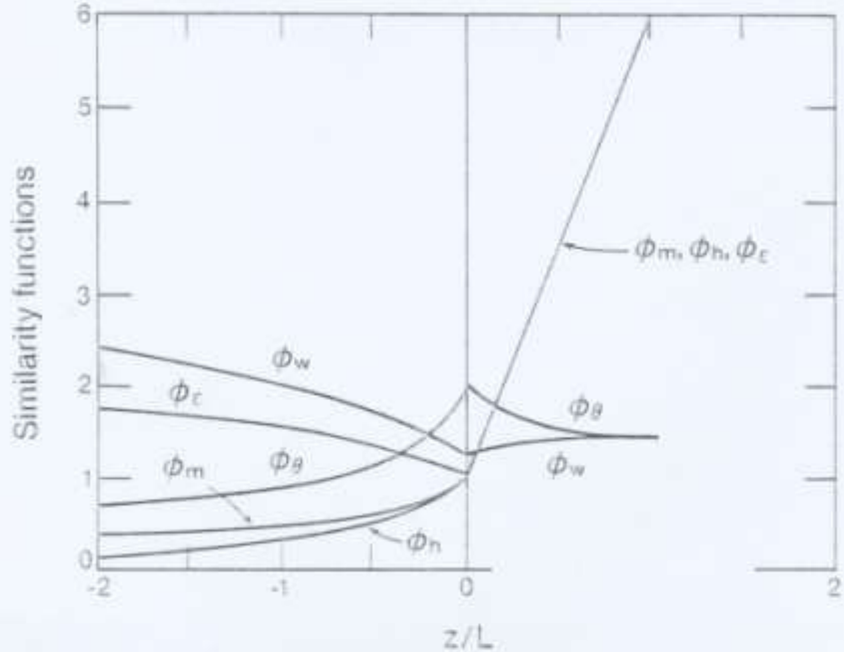
$$\frac{\partial \overline{U}}{\partial Z} = \frac{u_*}{\kappa z} \varnothing_m (z/L_*)$$ wind shear)

$$\frac{\partial \overline{\theta}}{\partial Z} = \frac{T_*}{\kappa z} \varnothing_h (z/L_*)$$ (thermal stratification)

$$\varnothing_w = \frac{\sigma_w}{u_*}$$ (variability of *w*)

$$\varnothing_\theta = \frac{\sigma_w}{|T_*|}$$ (variability in *θ*)

$$\varepsilon = \frac{u_*^3}{\kappa z} \varnothing_\varepsilon (z/L_*)$$ (dissipation)



Similarity functions in the surface layer.

Kaimal & Finnigan 1994

$$\phi_m = \begin{cases} (1 + 16\,|z/L|)^{-1/4}, & -2 \le z/L \le 0 \\ (1 + 5\,z/L), & 0 \le z/L \le 1 \end{cases}$$

$$\phi_h = \begin{cases} (1 + 16\,|z/L|)^{-1/2}, & -2 \le z/L \le 0 \\ (1 + 5\,z/L), & 0 \le z/L \le 1 \end{cases}$$

$$\phi_w = \begin{cases} 1.25(1 + 3\,|z/L|)^{1/3}, & -2 \le z/L \le 0 \\ 1.25(1 + 0.2\,z/L), & 0 \le z/L \le 1 \end{cases}$$

$$\phi_\theta = \begin{cases} 2(1 + 9.5\,|z/L|)^{-1/3}, & -2 \le z/L \le 0 \\ 2(1 + 0.5\,z/L)^{-1}, & 0 \le z/L \le 1 \end{cases}$$

$$\phi = \{ (1 + 0.5\,|z/L|^{2/3})^{3/2}, \quad -2 \le z/L \le 0$$

$$/L \le 1.$$

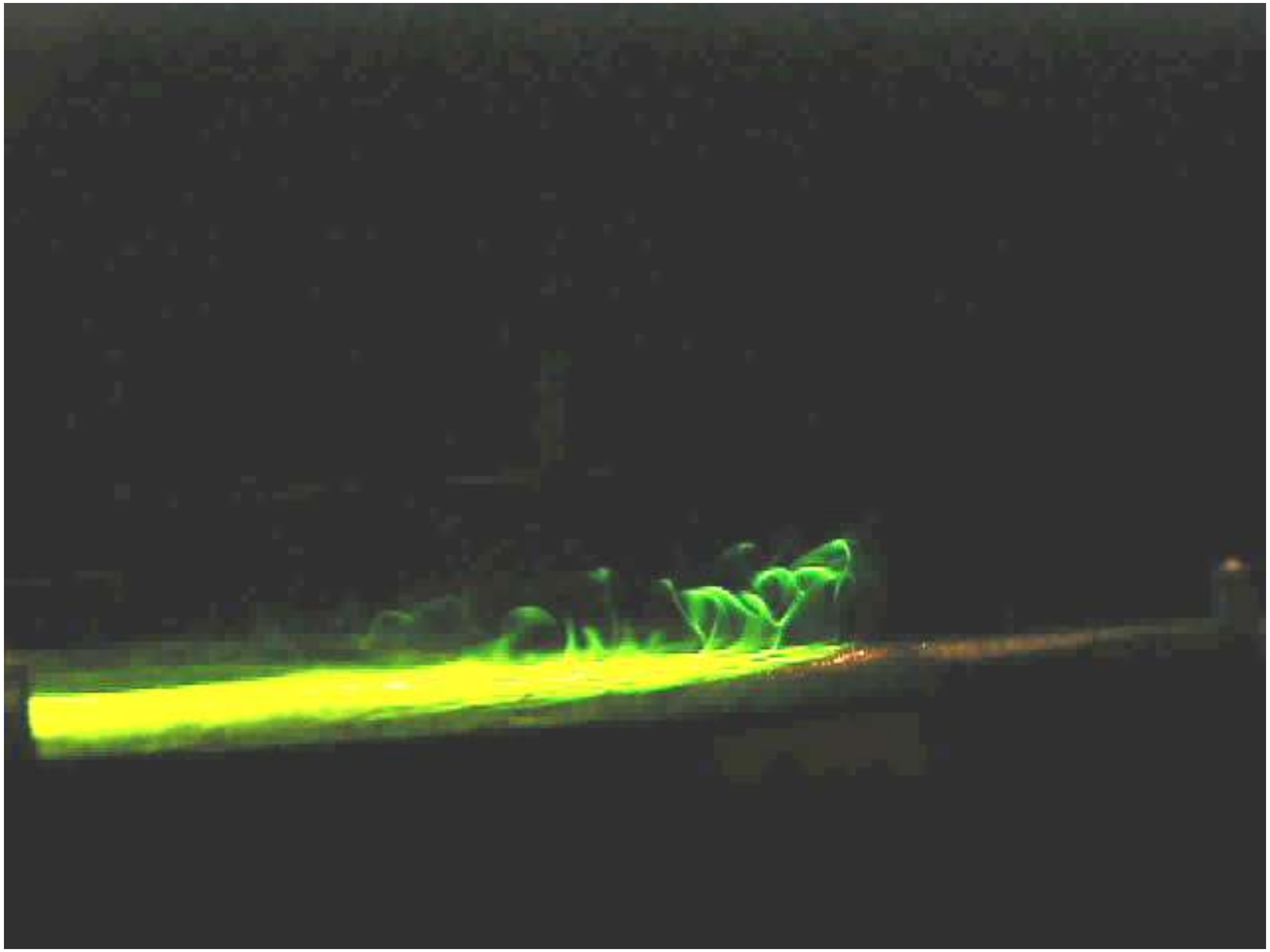
$$\frac{Z}{L_*} = -\frac{g}{\Theta}\frac{\overline{w'\theta'}}{\frac{{u_*}^3}{\kappa Z}} \approx \frac{\overline{-b'w'}}{-\overline{u'w'}\frac{d\overline{U}}{dZ}} = Ri_f$$

given that

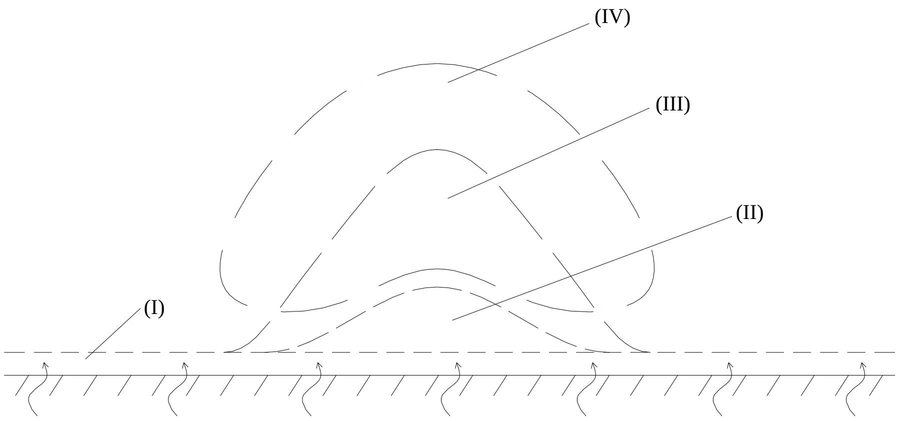
$$\frac{d\overline{U}}{dZ} = \frac{\kappa u_*}{Z} \quad ; \quad -\overline{u'w'} \approx {u_*}^2$$

$z < |L_*| \Rightarrow$ shear dominates $z > |L_*| \Rightarrow$ Buoyancy (outer layer)

With a slope

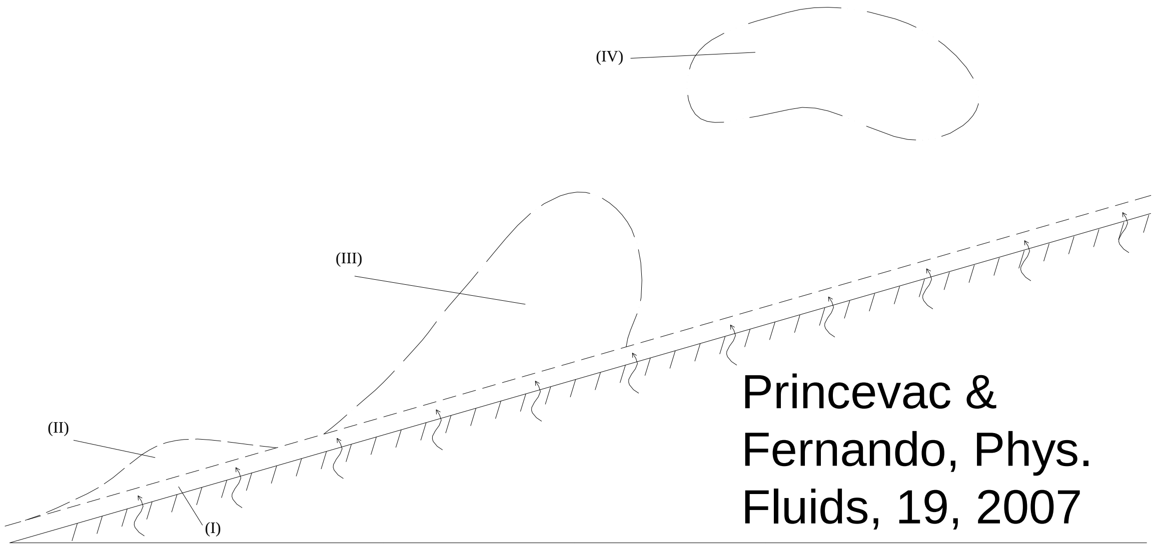


Thermal blob



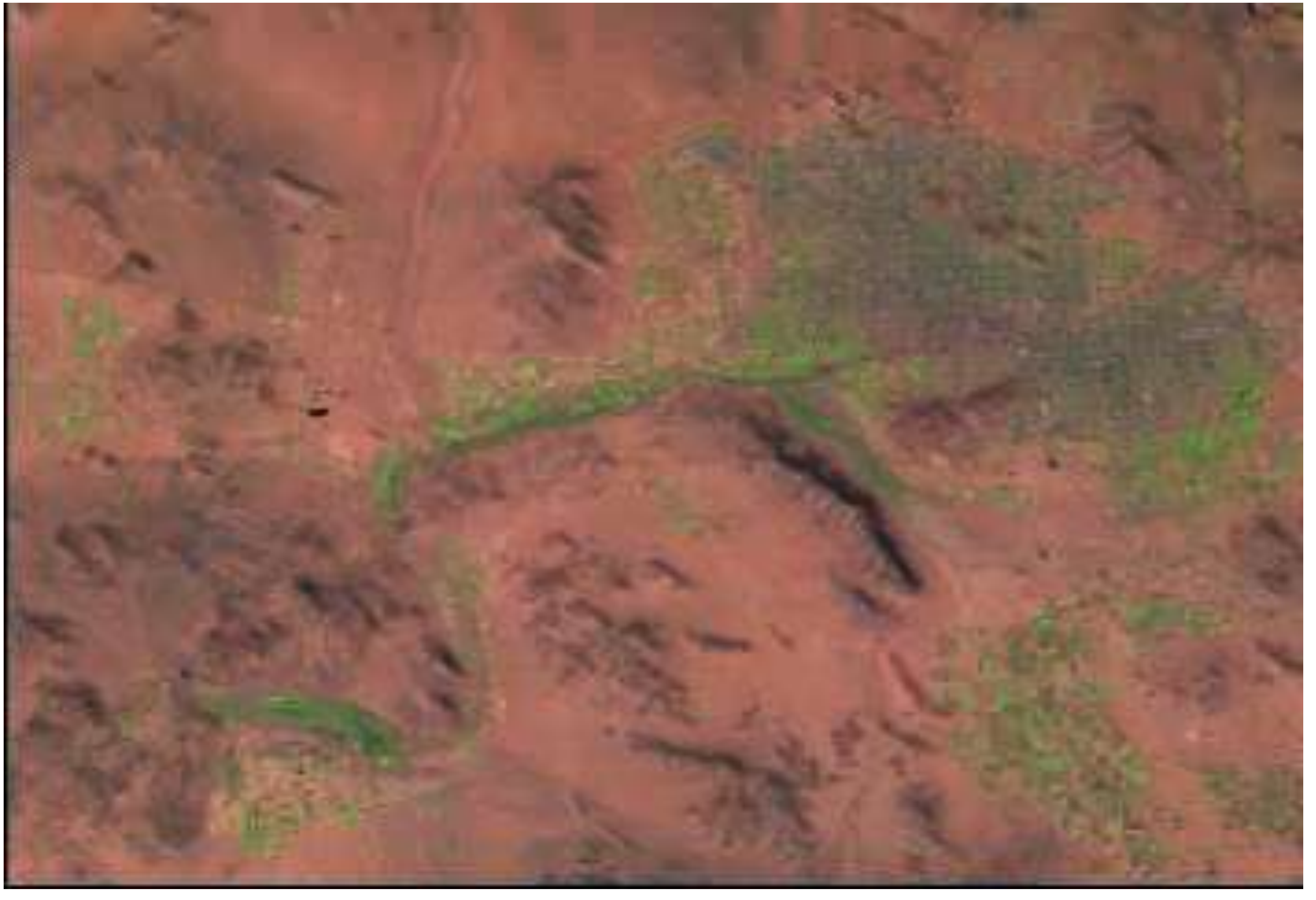
Detachment occurs when

$$Ra = Ra_{\mathbf{c}} = \frac{g\alpha\Delta T\delta_c^3}{\nu\kappa} \approx 10^3$$

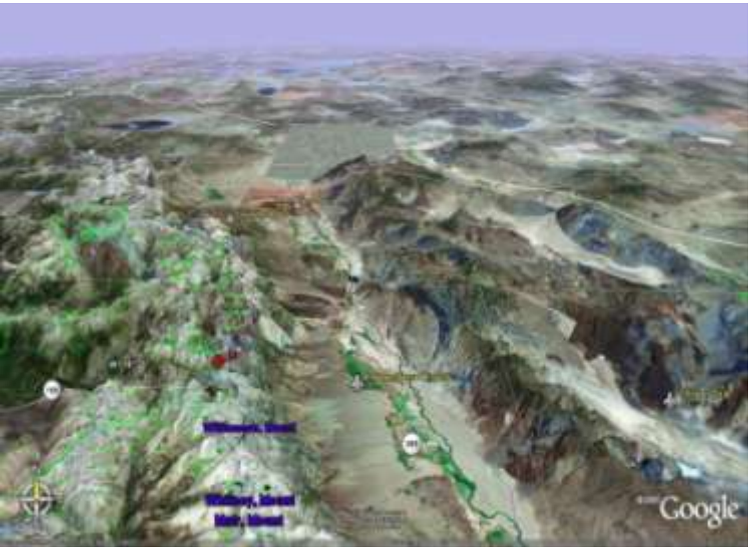


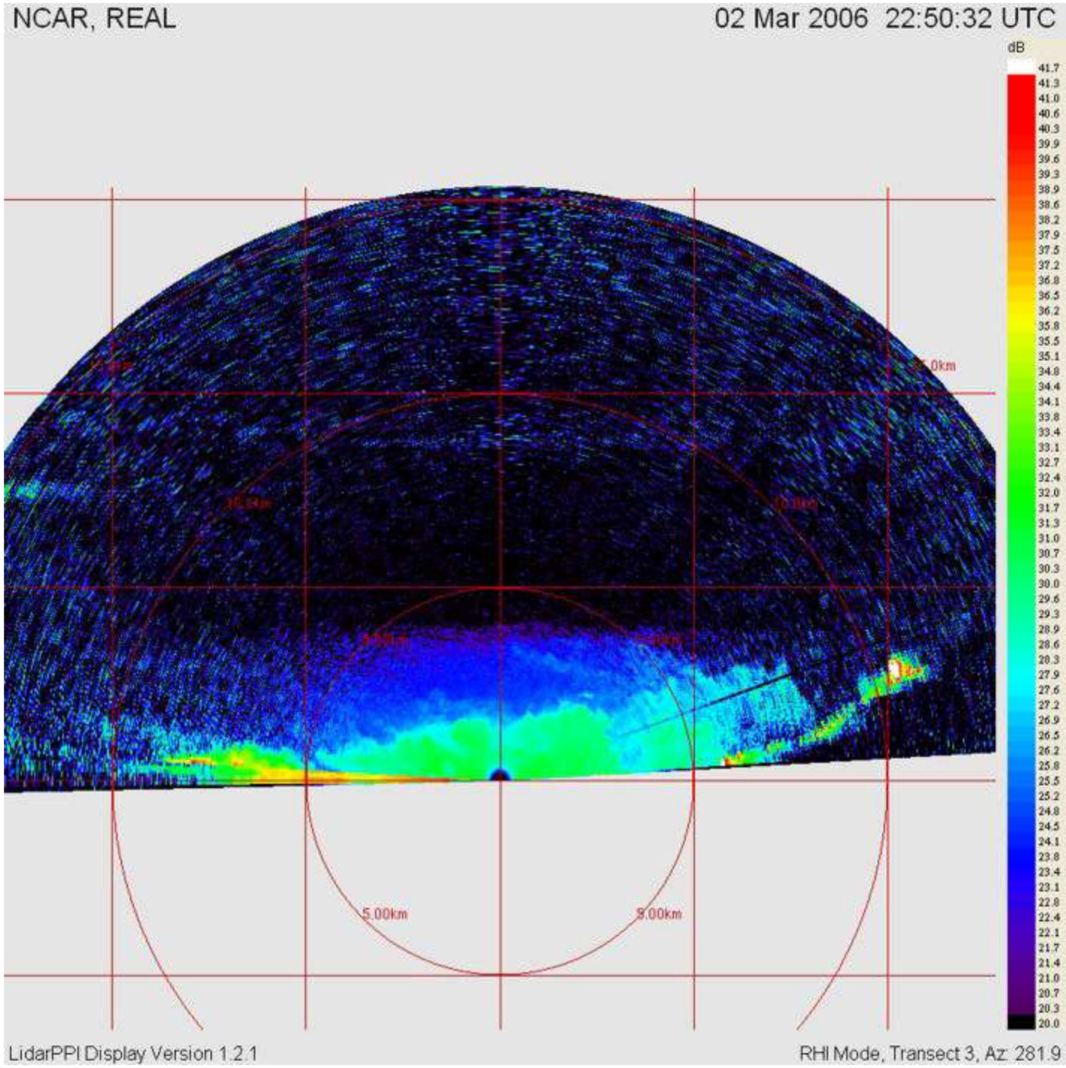
Princevac & Fernando, Phys. Fluids, 19, 2007

Convection in Complex Terrain

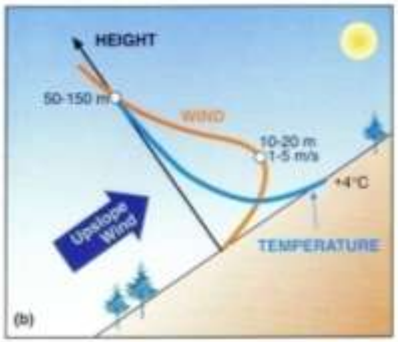


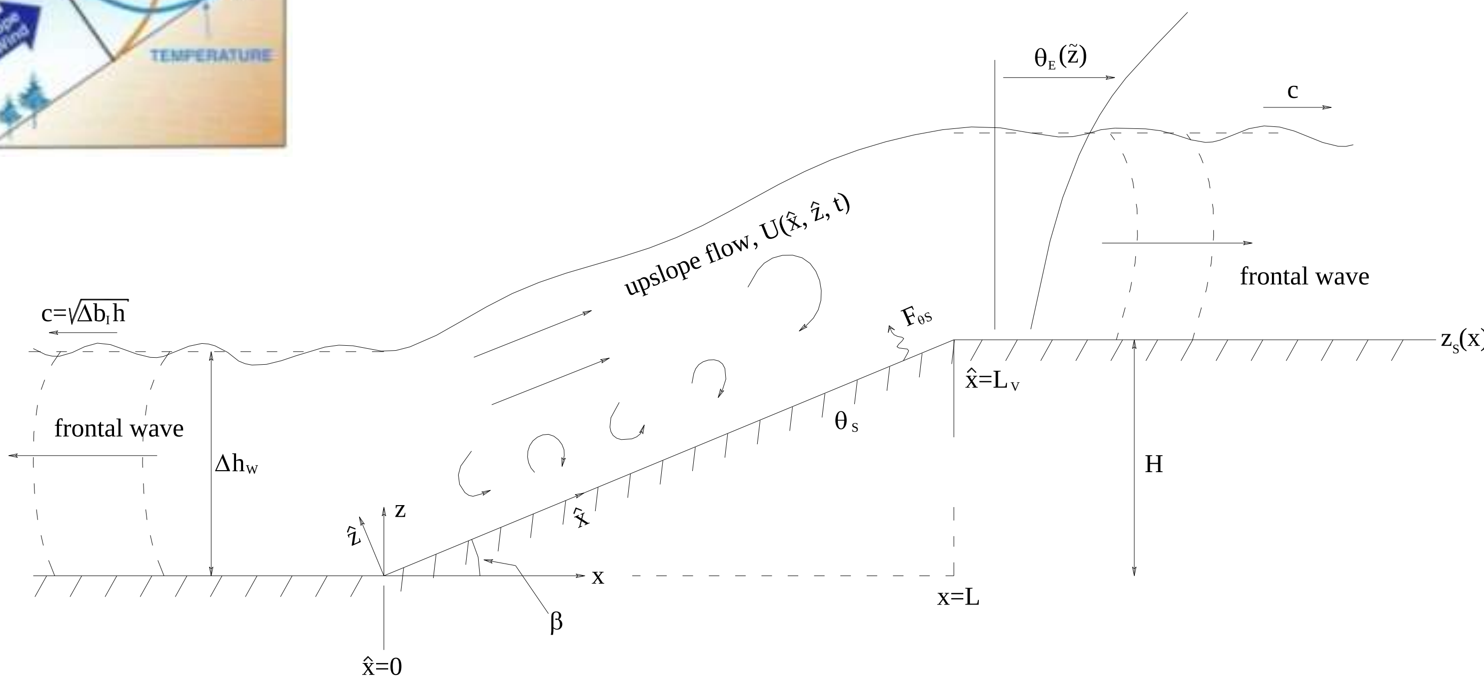
T-Rex Observations (NCAR)





Fully developed upslope flow





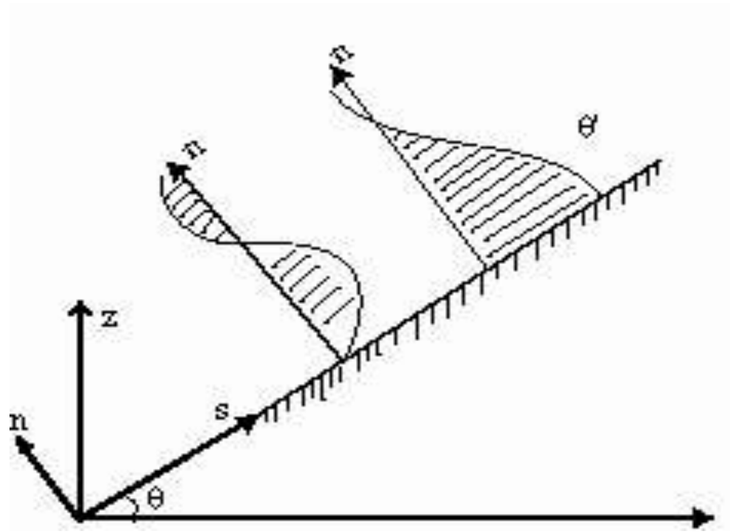
Prandtl's Solutions

Initial temp distribution

$$T = T_0 + \Gamma z$$

$$\bar{b} = -\left(\bar{\rho} - \rho_0\right)_g / \rho_0 = \alpha g\left(\bar{T} - T_0\right)$$

$$\frac{d\bar{b}}{dz} = N^2 = g\alpha\Gamma$$



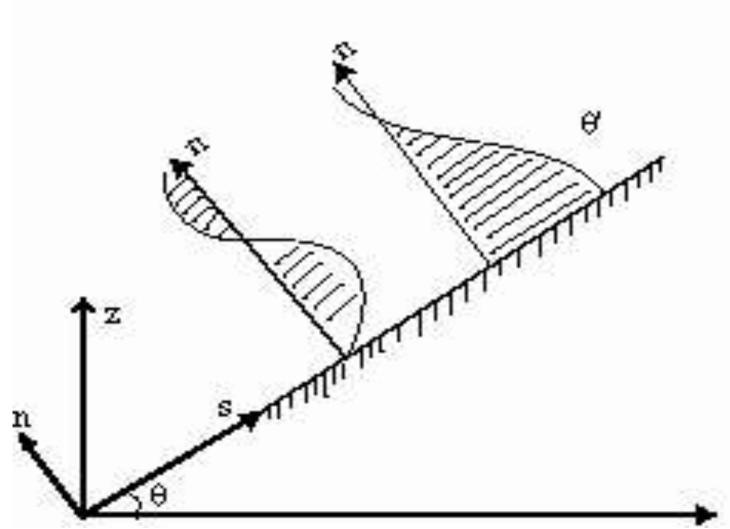
Initial hydrostatic

$$0 = -\frac{1}{\rho_0}\frac{\partial \bar{p}}{\partial s} + \bar{b}\sin\theta$$

$$z = s\sin\theta + n\cos\theta$$

Now give a perturbation, b' and corresponding velocity u

$$0 = b' \sin\theta + v \frac{\partial^2 u}{\partial n^2}$$



$$b' = g\alpha\theta' \text{ and } \partial p'/\partial s = 0$$

$$u \frac{\partial \bar{b}}{\partial s} = \kappa \frac{\partial^2 b'}{\partial s^2}$$

$$\partial \bar{b} / \partial s = \left(\partial \bar{b} / \partial z\right)\left(\partial z / \partial s\right) = N^2 \sin\theta$$

$$\frac{\partial^4 b'}{\partial n^4} + \frac{N^2 \sin^2\theta}{\nu\kappa} b' = 0$$

$$l = \left(\frac{4\nu\kappa}{N^2 \sin^2\theta}\right)^{1/4}$$

$$b' = Ae^{-n/l} \cos nl$$

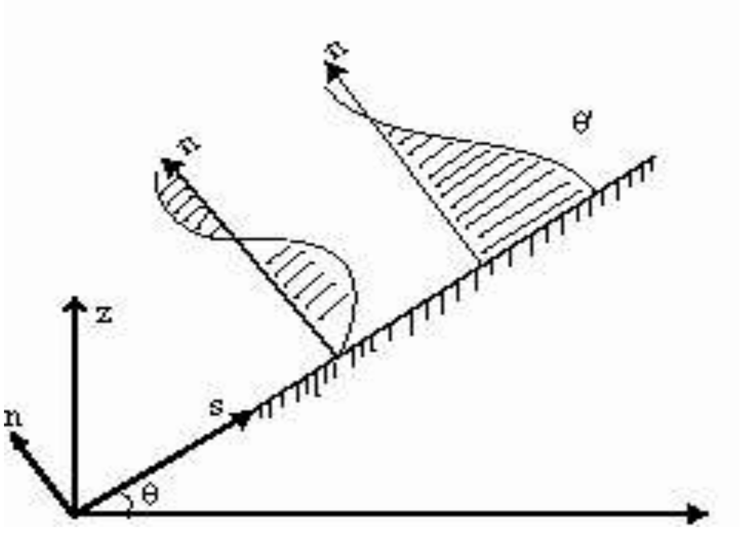
Velocity along the slope, constant (eddy?) coefficients

$$u = A\left(\frac{\kappa}{\nu N^2}\right)^{1/2} e^{-n/l} \sin\left(\frac{n}{l}\right)$$

constant heat flux boundary condition

$$q_0 = -\kappa\left(\partial \bar{b} / \partial n\right)$$

$$A = q_0 l / \kappa$$

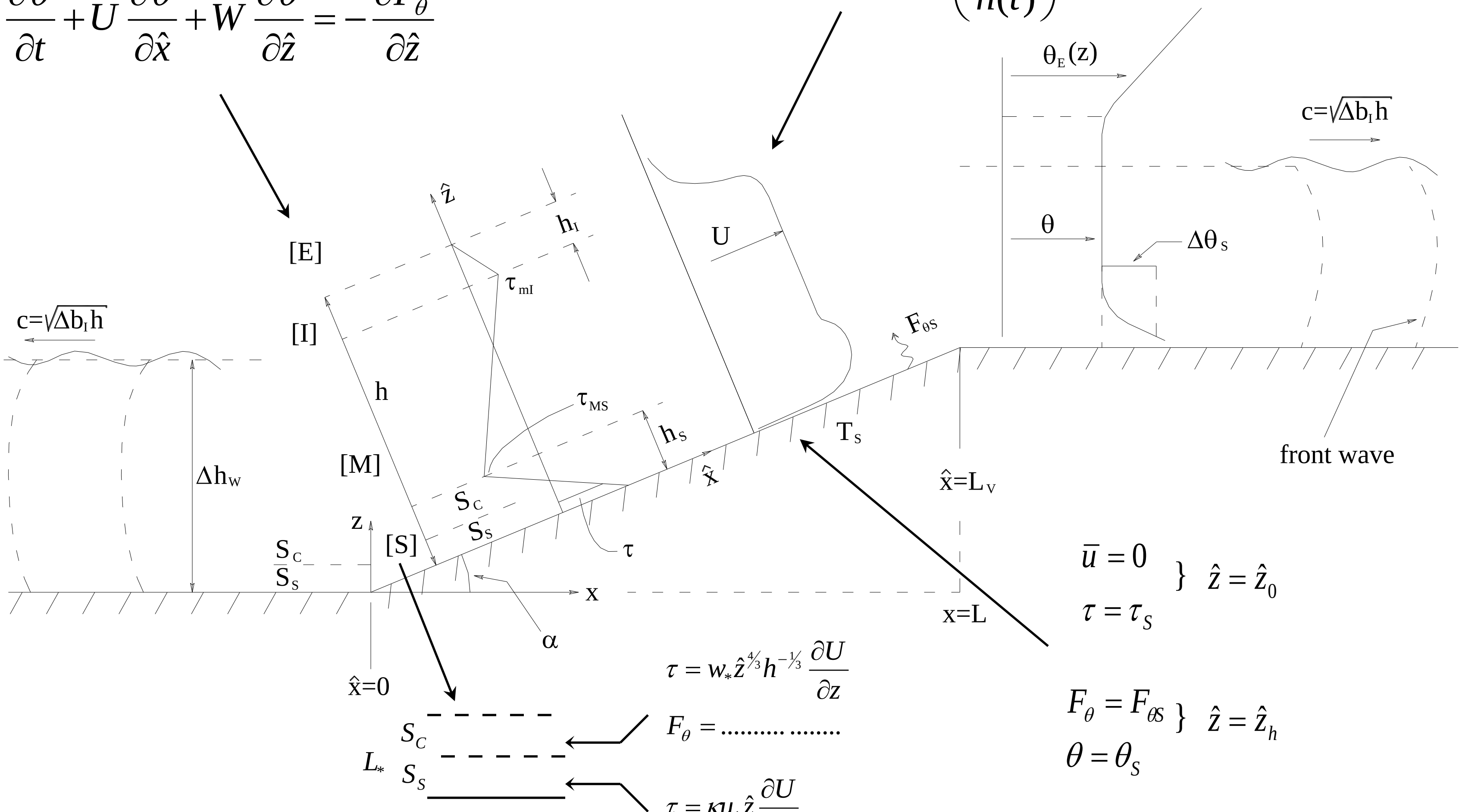


Upslope - Theoretical Model

$$\frac{\partial U}{\partial t} + U\,\frac{\partial U}{\partial \hat{x}} + W\,\frac{\partial U}{\partial \hat{z}} = -\frac{\partial P}{\partial \hat{x}} - \alpha \Delta b + \frac{\partial \tau}{\partial z}$$

$$\frac{\partial \theta}{\partial t} + U\,\frac{\partial \theta}{\partial \hat{x}} + W\,\frac{\partial \theta}{\partial \hat{z}} = -\frac{\partial F_\theta}{\partial \hat{z}}$$

$$U = U_m(t)\, f\left(\frac{\hat{z}}{h(t)}\right)$$



$$\tau = w_* \hat{z}^{4/3} h^{-1/3} \frac{\partial U}{\partial z}$$

$$F_\theta = \ldots\ldots\ldots\ \ldots\ldots\ldots$$

$$\tau = \kappa u_* \hat{z} \frac{\partial U}{\partial z}$$

$$F_\theta = \ldots\ldots\ldots\ \ldots\ldots\ldots$$

$$\left.\begin{array}{l} \bar{u} = 0 \\ \tau = \tau_S \end{array}\right\} \quad \hat{z} = \hat{z}_0$$

$$\left.\begin{array}{l} F_\theta = F_{\theta S} \\ \theta = \theta_S \end{array}\right\} \quad \hat{z} = \hat{z}_h$$

Hunt, Fernando & Princevac

J. Atmos. Sci., 60, 2003

Theory - Up-Slope Velocity

For small α

$$U_M \approx \lambda_u \alpha^{1/3} w_*$$

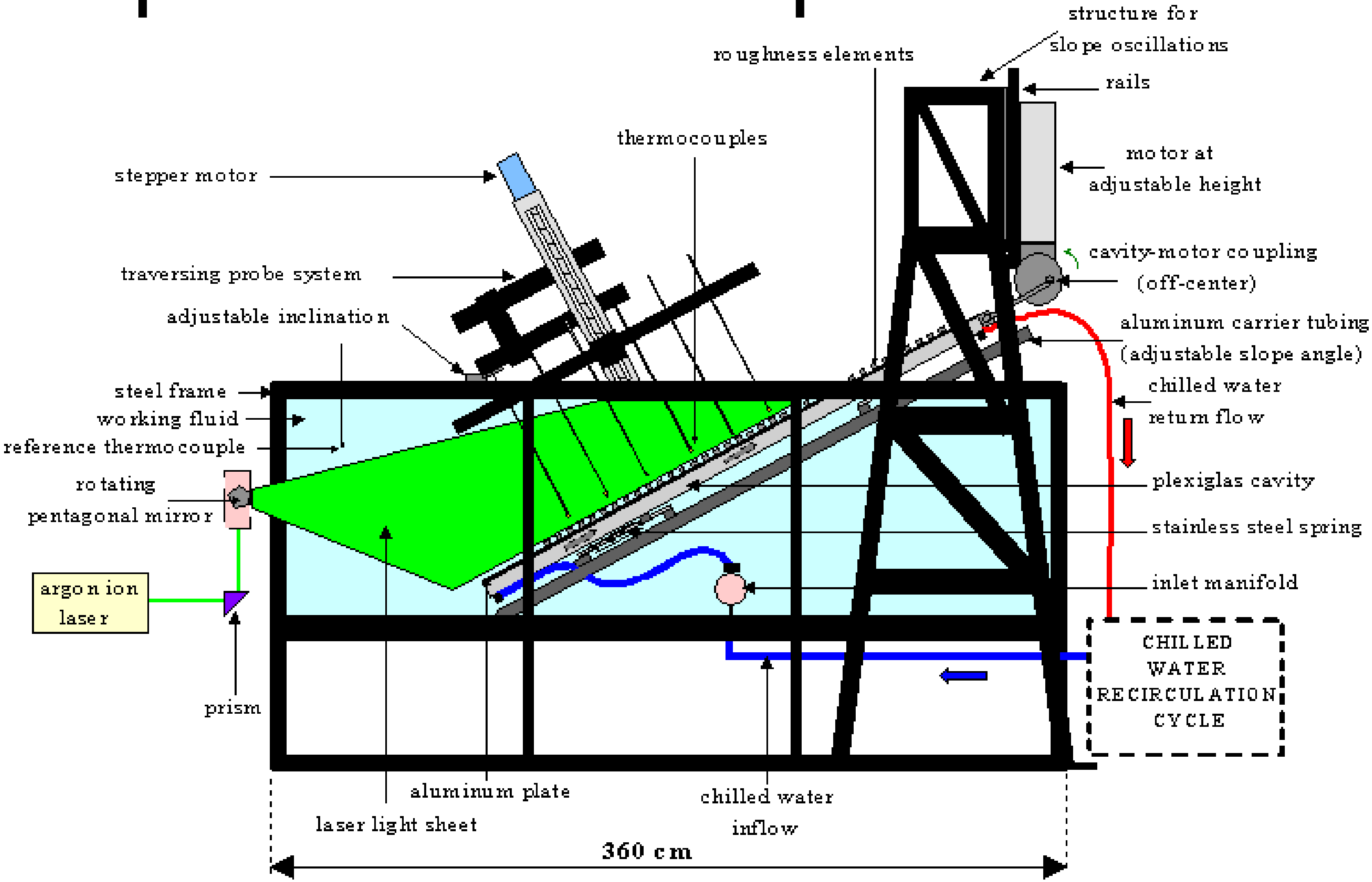
where

$$w_* = \left(F_{\theta S}\beta g\right)^{1/3} h^{1/3} = (q_0 h)^{1/3}$$

$$\lambda_u \approx 4 \quad (?)$$

(Experiments)

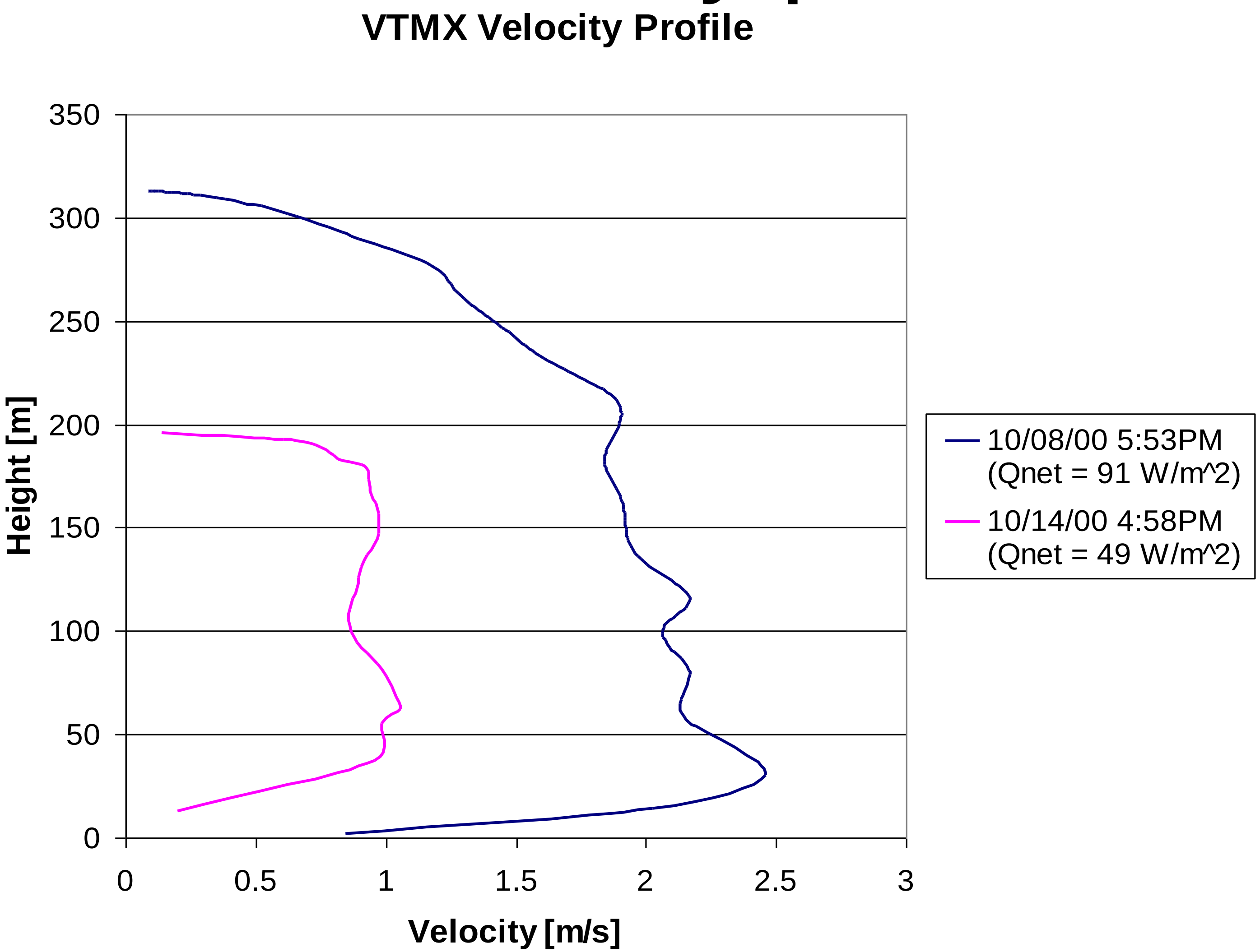
Experimental setup - Schematic



Arizona State University
Environmental Fluid Dynamics
Program

Balloons

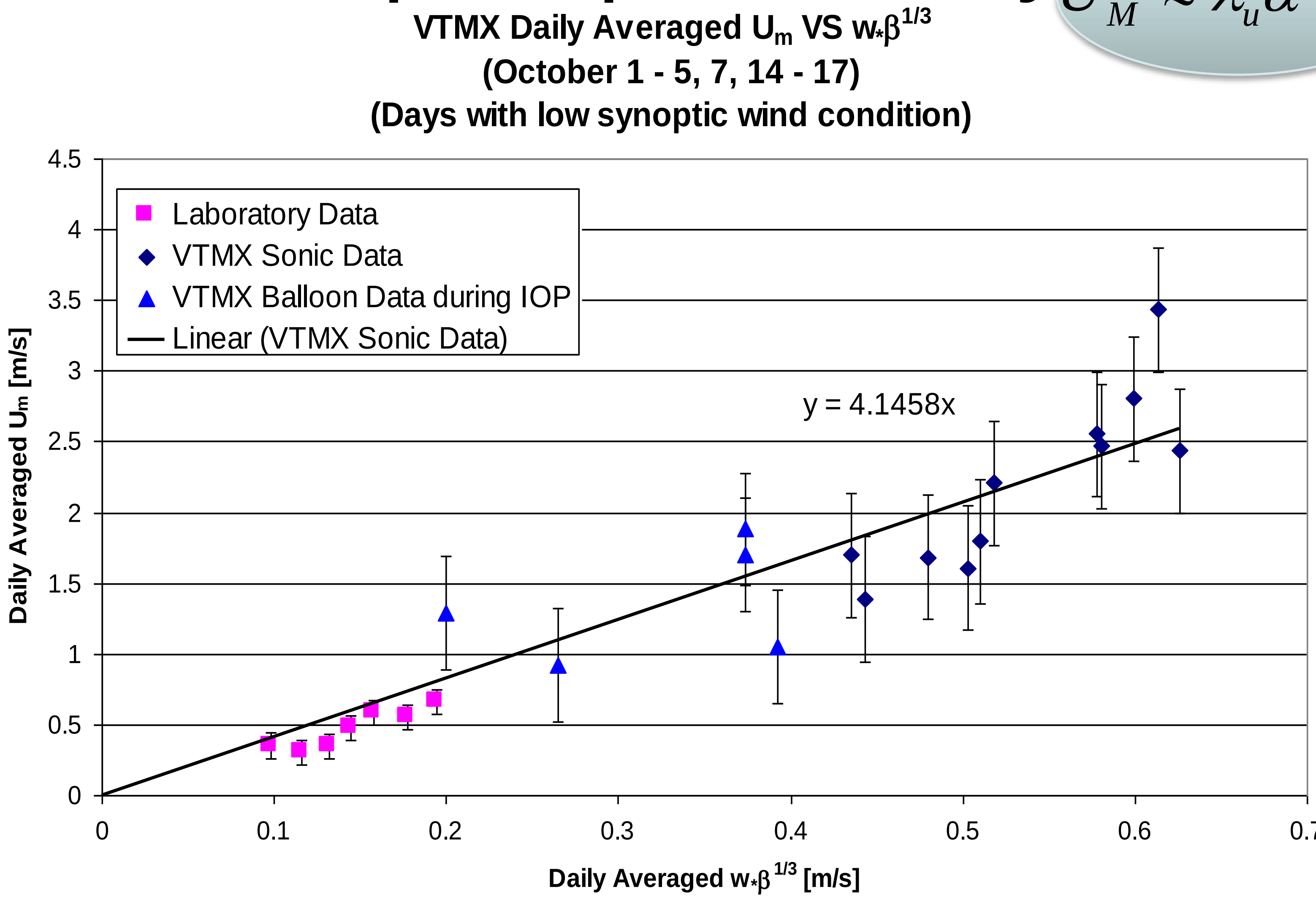
VTMX velocity profile



Arizona State University
Environmental Fluid Dynamics
Program

Up-slope velocity

$$U_M \approx \lambda_u \alpha^{1/3} w_*$$



Arizona State University
Environmental Fluid Dynamics
Program

Geophysical Convection

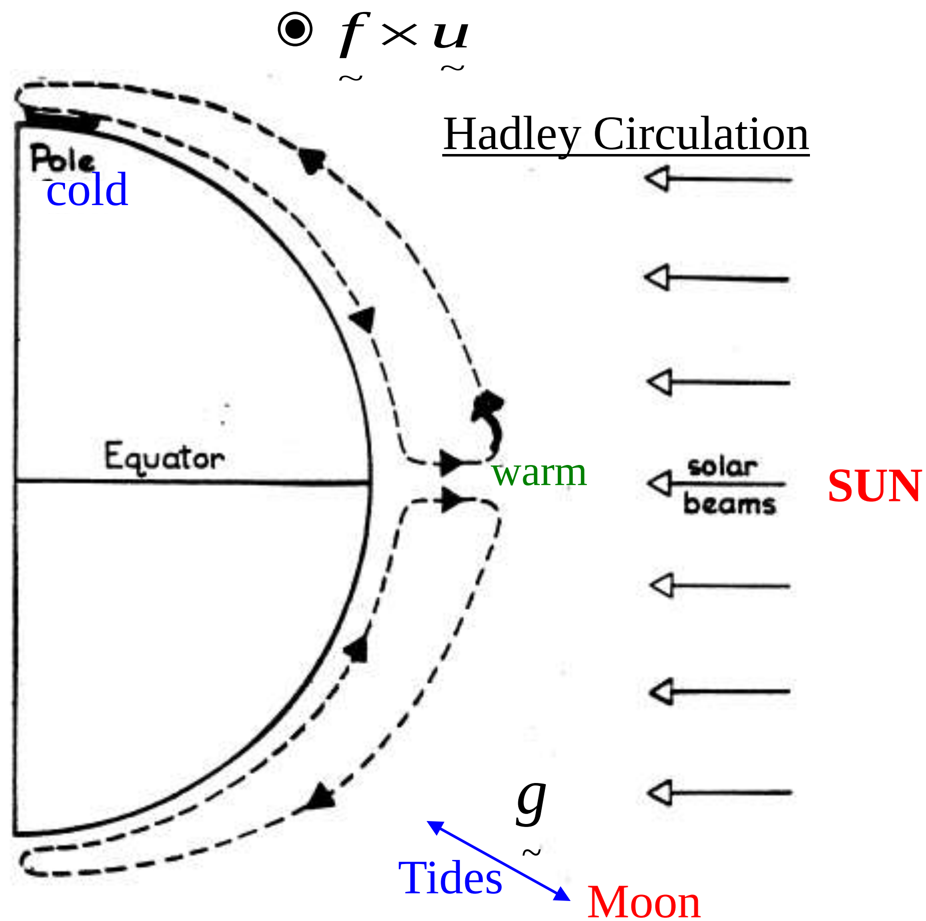
A continuum of scales

- Large scale -- deep convection/Hadley Cells (~ 10000 km)
- Thunderstorms (~250mkm)
- Slope flows (10-100 km)
- Atmospheric Plumes -- Microbursts (2 km)
- CBL (100m to km)

Drivers of Environmental Motions

b

$$-\frac{1}{\rho}\nabla p$$



Av. Distance: 93 million miles

Inclination: 23.5º to the orbital plane

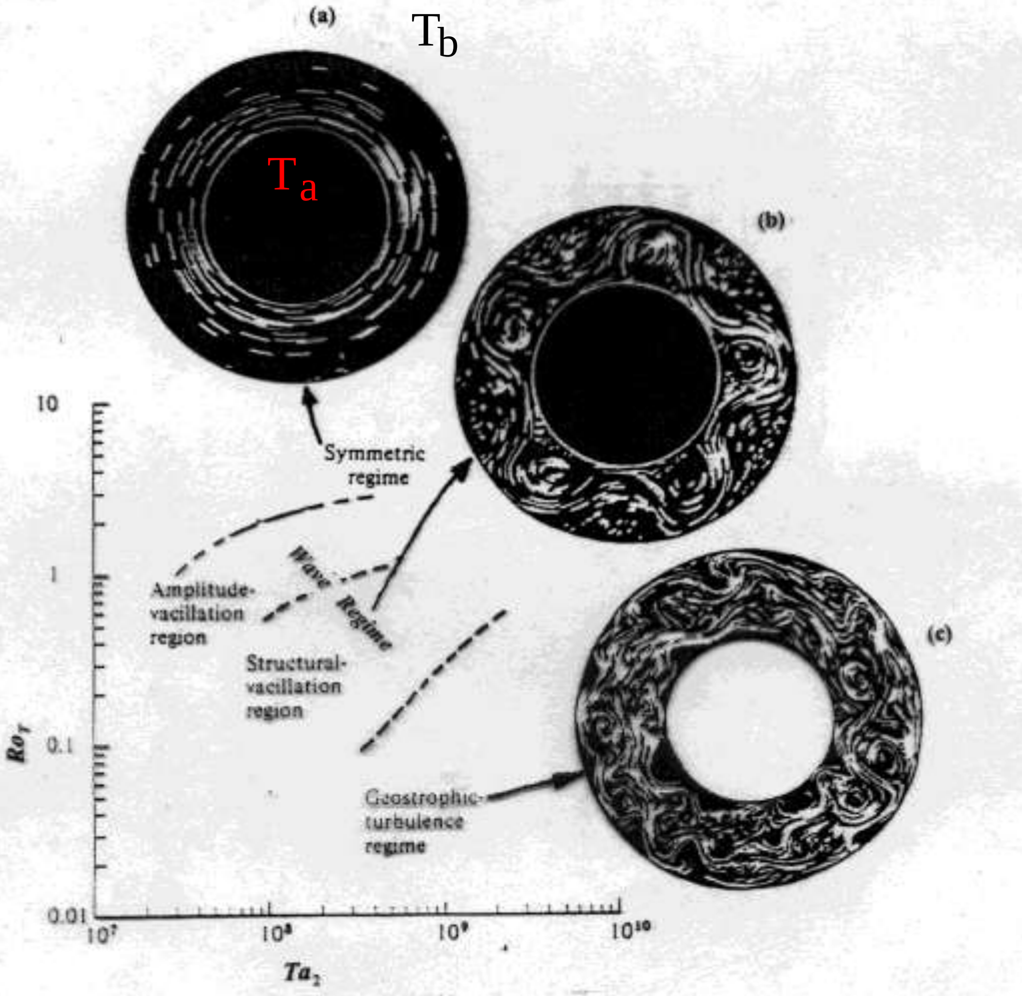
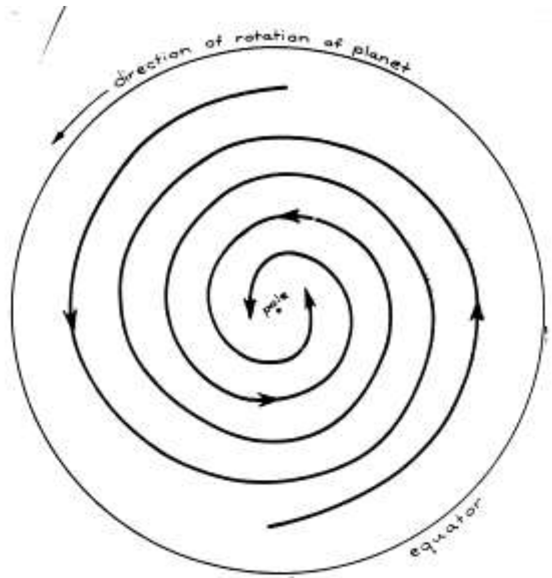


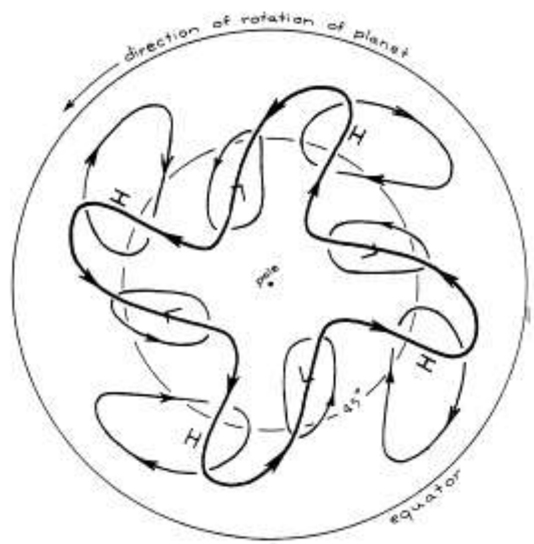
Figure 2: Flow regimes observed in a "dishpan experiment" with $(b-a)/d \approx 2$ and $Pr \approx 21$. These experiments mimic baroclinic waves and slantwise convection observed in the atmosphere. [Compiled from Hide & Mason (1985) and Buzyna, G., Pfeffer, R.L. and Kung, R. (1984, Transition to geostrophic turbulence in a rotating differentially heated annulus of fluid, Journal of Fluid Mechanics, 145, 377-403)]

$$\mathrm{Ro}_T = \frac{g\alpha\Delta T d}{\Omega^2 (b-a)^2}$$

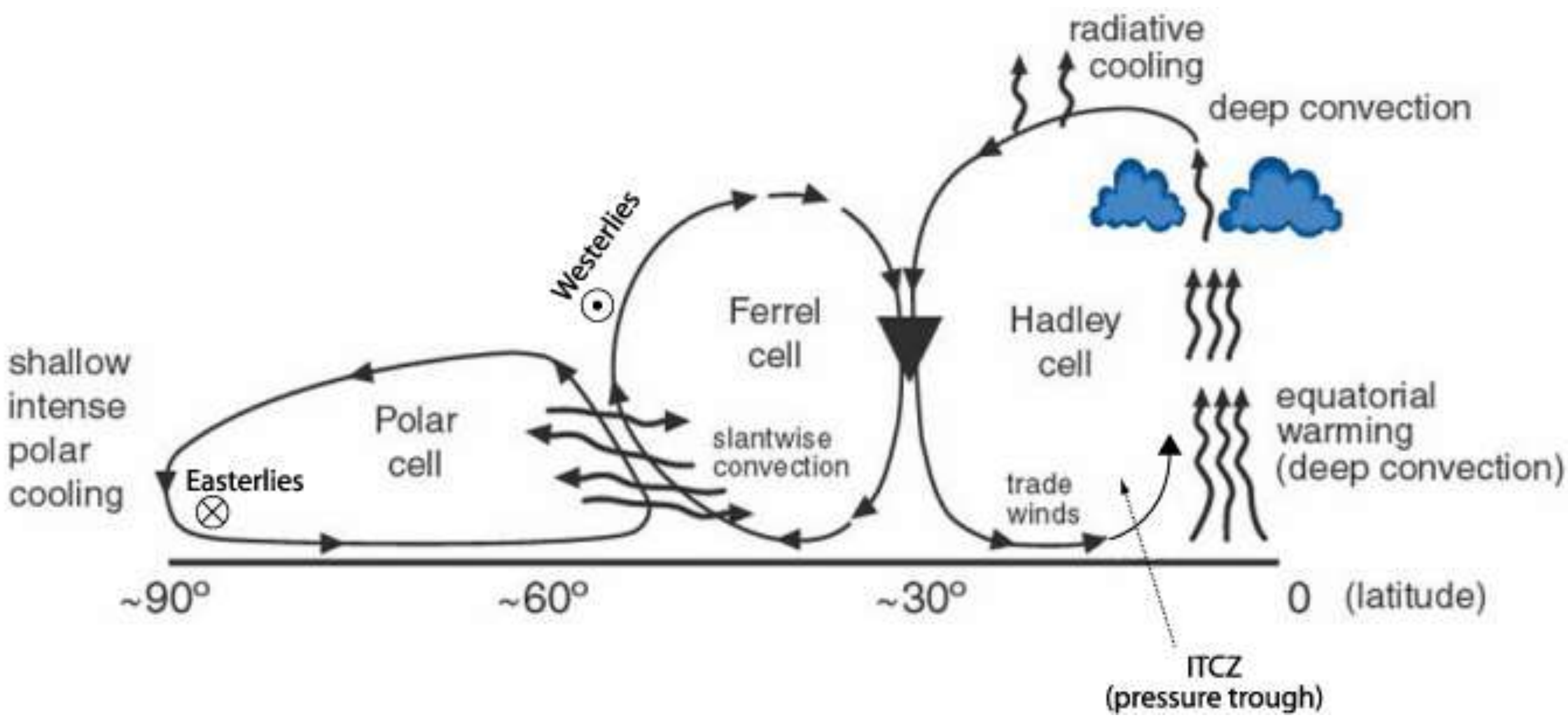
$$T_{a_1} = \frac{4\Omega^2 (b-a)^4 (b-a)}{\nu^2 d}$$

$$T_{a_2} = 4\frac{\Omega^2 d^4}{\nu^2}; \; P_r = \frac{\nu}{k}$$


direction of rotation of planet
pole
equator

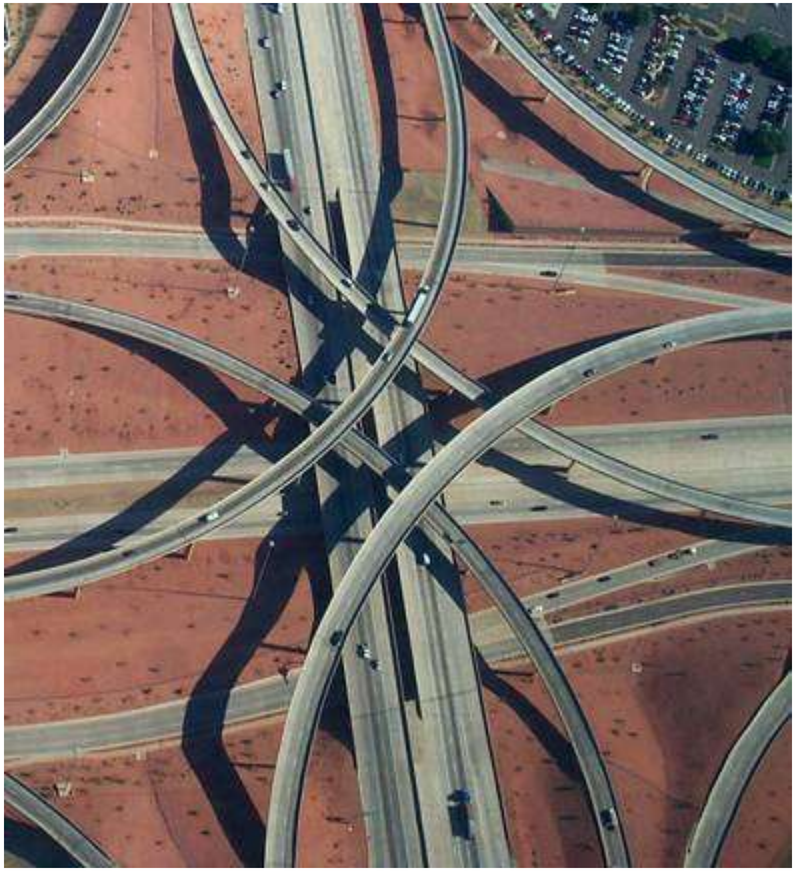

direction of rotation of planet
H
H
H
H
L
L
L
L
pole
45°
equator

Atmospheric Convection

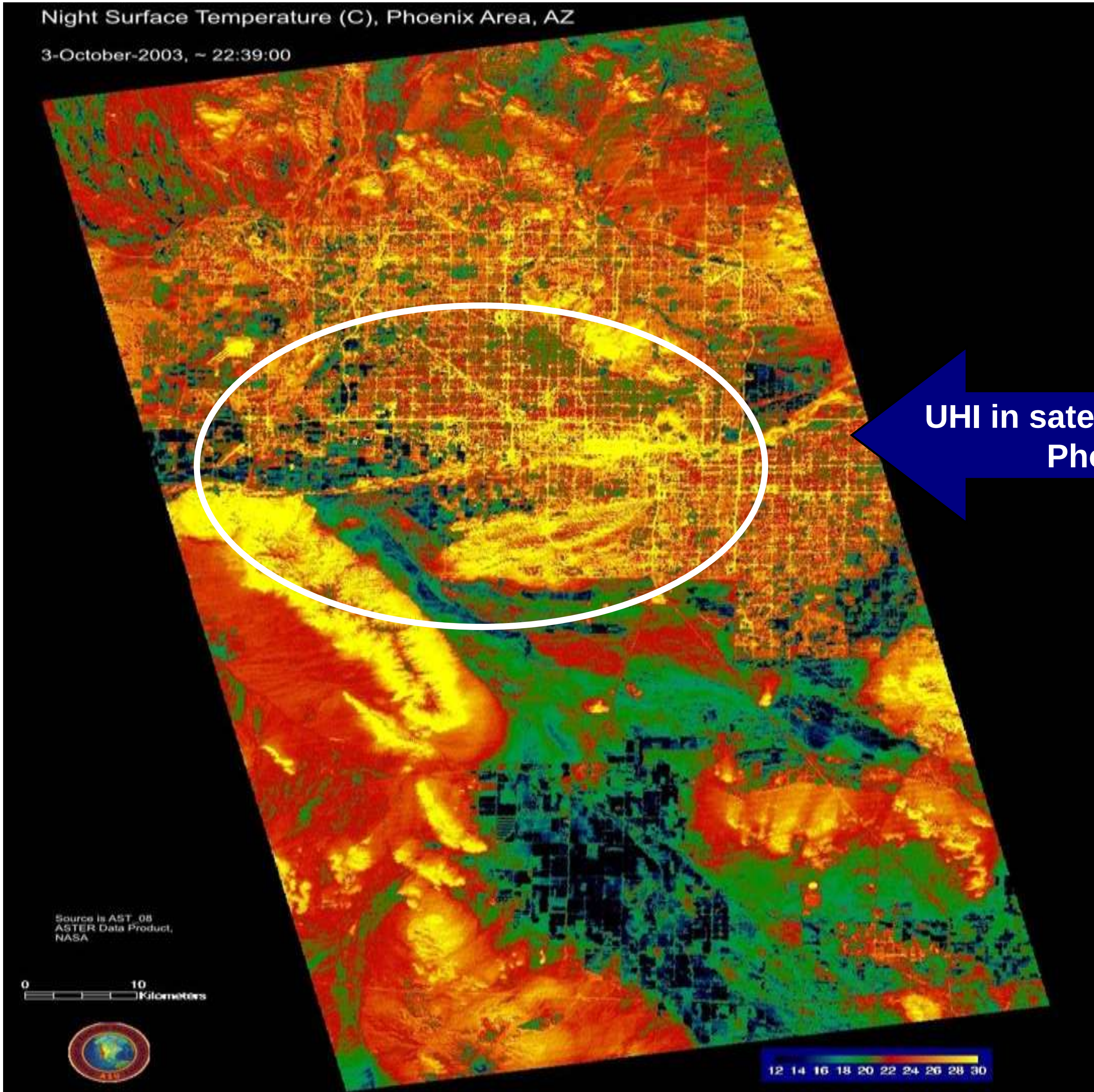


CONVECTION OVER URBAN AREAS

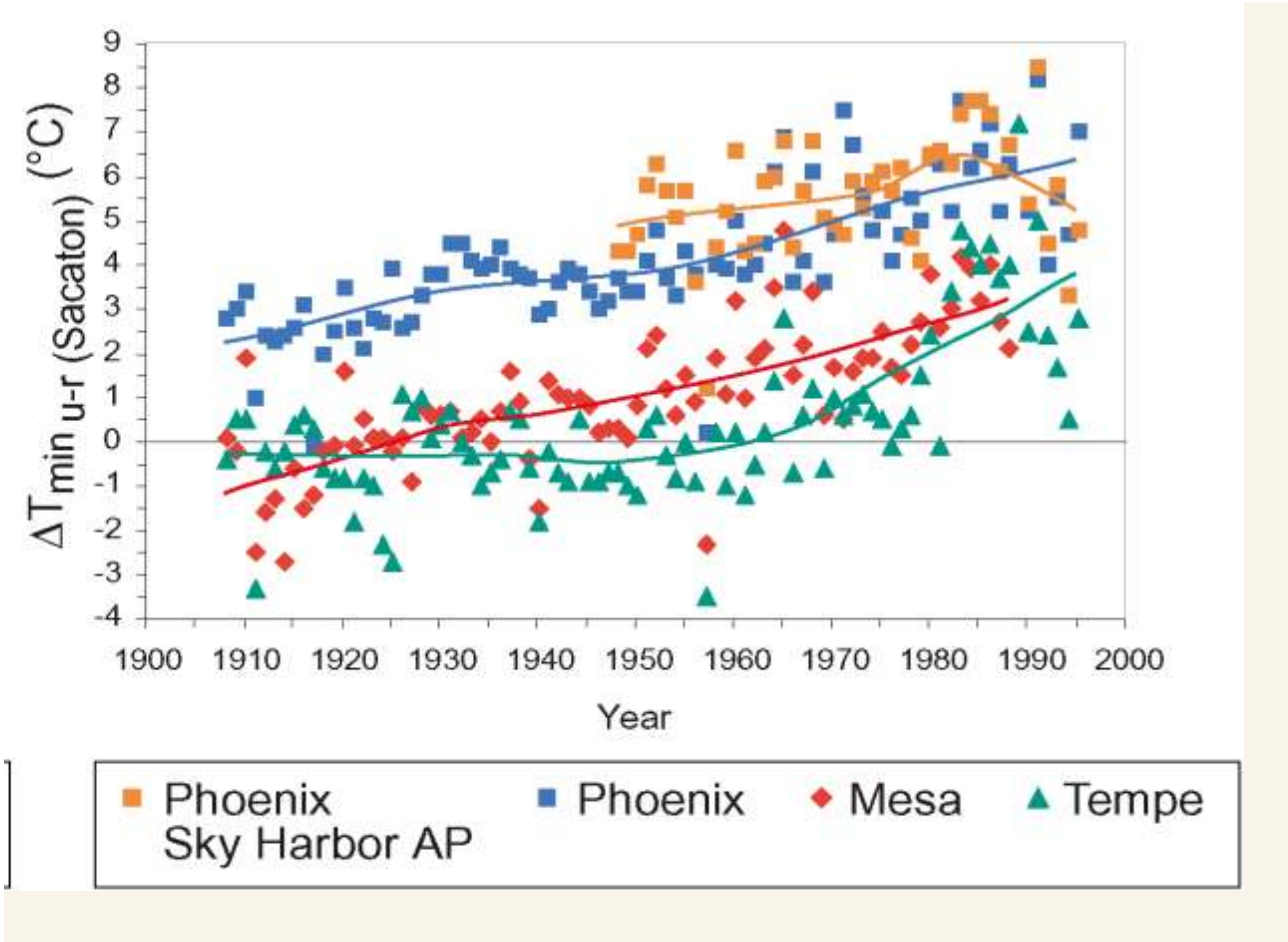
Phoenix Metropolis



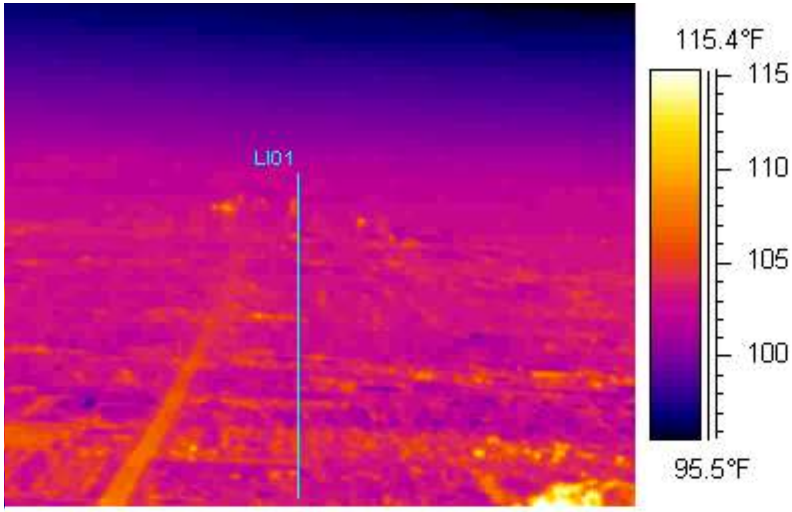
Urban Heat Island -- Urban air can be significantly hotter than the countryside



UHI in satellite image of Phoenix



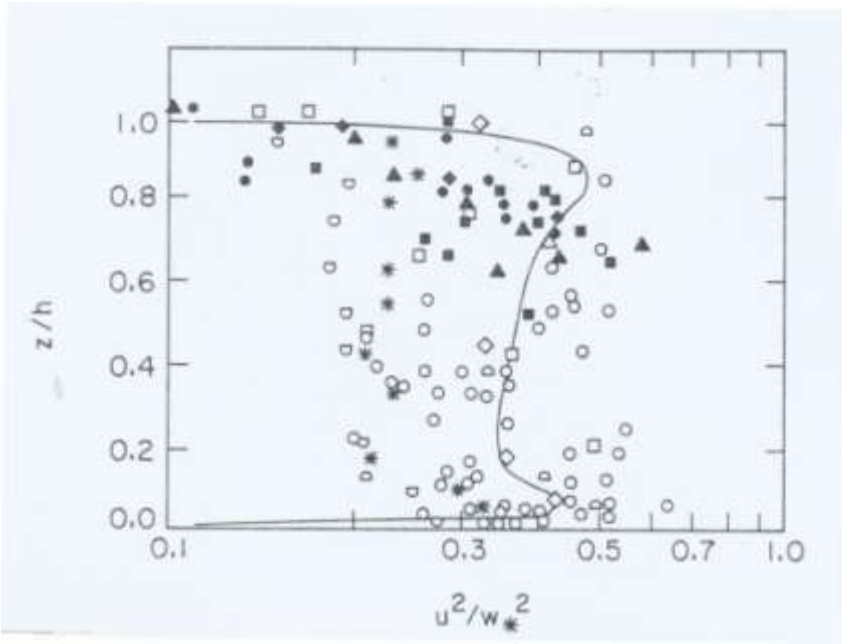
Brazel et al. 2000



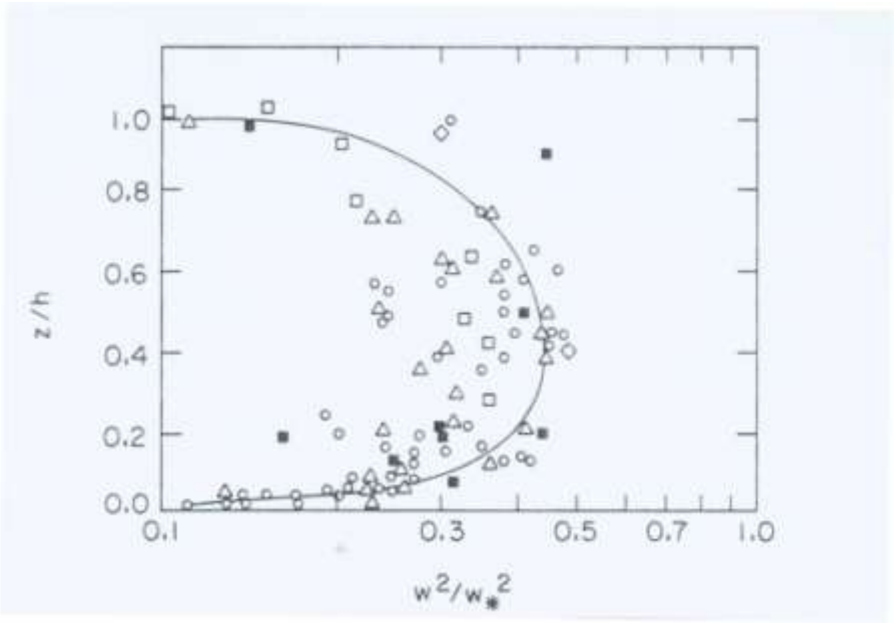
March 19, 2008, 5pm to 10pm
Infrared imaging of Phoenix

UHI Experiment

Convective Scaling Vs. Data



Variation or horizontal variance (normalized). (Solid Curve -- Laboratory)
(Fernando et al., Dyn. Atmos. Oceans, 13,95-121, 1989)



Variation of vertical variance (normalized).

Wind Shear Found at all Altitudes