Internal Tide Energetics: Generation, Propagation & Breaking

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Tidal Energy Budget

Tide Dissipation:
M2: 2.5 TW (3.5)
IT: 1.1 TW
Gulf Stream: 1 PW

Bay of Biscay

Garrett & Kunze, 2007; Egbert & Ray, 2001
Internal Tide Energetics

Wide range of scales:

Global Tide  | Internal Tides  | Turbulent Mixing

1000 km day  | 100 – 1 km day – hour | 10 km – 0.01 mm day – ms

Thermo–Haline Circulation
Density anomaly after $3T_0$:
Coastal Ocean Model: Symphonie (NH)

Characteristics:
- Time-splitting
- C-grid
- Generalised σ-coordinates
- Turbulence Closure: Gaspar et al. (1990)

Tools:
- Wavelets / Windowed FT
- (W)eof analysis
- Energy Flux Analysis
- Ensemble modelling

Energy Conserving

\[ \nabla \cdot \vec{v} = 0 \]
\[ \rho = \rho \phi, S, T \]

Applications:
- NW-Mediterranean Sea:
  - forecasting (Estournel et al., 2007)
  - biogeochemistry (Ulses et al., 2008)
  - climate studies (Herrmann et al., 2007)
- Bay of Biscay + Academic + Small
  - internal tide modelling (Pairaud, 2005)

(Marsaleix et al., 2008, doi:10.1016/j.ocemod.2007.07.005)
Symphonie Validation

• Numerically Consistent Energetics (Marsaleix et al., 2008)

• Bay of Biscay IT observations (hydrostatic) (Pairaud, 2005)

• Coriolis-platform (C. Staquet, LEGI, Grenoble)
Laboratory Study

- Oscillating Gaussian Ridge (with A. Paci at SPEA/CNRM-GAME)
  - validation & parameter studies

Channel: 4x0.5x0.4 m

Synthetic Schlieren: Density
(Gostiaux et al., ‘05; Sutherland et al., ‘98)

Gaussian ‘Seamount’
\[ h(x) = h_0 \exp\left(-\frac{x^2}{a^2}\right) \]
\[ h_0 = 11 \text{ cm} \]
\[ a = 5.7 \text{ cm} \]
Evolution of the anomaly of density gradient during the establishment of the internal tide.

Linear Stratification: \( N \sim 1.1 \text{ rad/s} \)

Forcing Frequency: \( 0.1 \leq \omega \leq 1.1 \text{ rad/s} \)

Internal wave slope: \( 5 \leq \theta \leq 83 \text{ (measured)} \)

Note 1. \( \omega^2 = N^2 \sin^2 \theta \)

Note 2. Ridge Oscillates!
IT037 $T_0 = 65s \ T \sim 1.9T_0$

IT042 $T_0 = 15s \ T \sim 2.1T_0$

IT045 $T_0 = 6.5s \ T \sim 2.1T_0$

IT049 $T_0 = 5.7s \ T \sim 2.4T_0$
IT037 $T_0 = 65s \ T \sim 5.8T_0$

IT042 $T_0 = 15s \ T \sim 6.3T_0$

IT045 $T_0 = 6.5s \ T \sim 6.2T_0$

IT049 $T_0 = 5.7s \ T \sim 7.1T_0$
IT037 $T = 65s$, $T \sim 10.1T_0$

IT042 $T_0 = 15s$, $T \sim 10.9T_0$

IT045 $T = 6.5s$, $T \sim 10.8T_0$

IT049 $T_0 = 5.7s$, $T \sim 12.3T_0$
Preliminary Conclusions

- Internal tide establishes more rapidly for small $\omega, \theta$
- Many rays (subharmonics?) generated for $\omega \not\in N$
- $\Delta N_{max}^2$ augments then diminishes with $\theta$:  
  \[
  \begin{align*}
  0.01 \text{ rad}^2/\text{s}^2 & \text{ for } \theta = 5^\circ \\
  \text{MAX for } \theta = 45^\circ & \Rightarrow 0.05 \text{ rad}^2/\text{s}^2 \\
  0.03 \text{ rad}^2/\text{s}^2 & \text{ for } \theta = 83^\circ
  \end{align*}
\]
Energy Transfers: Schematic

- Exterior Energy
- Available Energy
  - Kinetic Energy
    - $\Phi_{KE,A}$
    - $\Phi_{KE->TKE}$
  - Potential Energy
    - $\Phi_{W}$
    - $\Phi_{TKE->PE}$
    - $\Phi_{mix}$
    - $\Phi_{int,F}$
- Background Potential Energy
  - $\Phi_{A+ \Phi_{MA}}$
  - $\Phi_{d}$

Mathematical expressions:

- $\Phi_{KE,A}$
- $\Phi_{A+ \Phi_{MA}}$
- $\Phi_{W}$
- $\Phi_{TKE->PE}$
- $\Phi_{mix}$
- $\Phi_{int,F}$
- $\Phi_{int}$

Equations:

- $\Phi_{KE,A} + \Phi_{A+ \Phi_{MA}} = \Phi_{W}$
- $\Phi_{TKE->PE} = \Phi_{mix}$
- $\Phi_{int,F} + \Phi_{int} = \Phi_{W}$
Energy Equations

\[ \phi = \phi_d + \phi_{ape} = \int_0^1 g \frac{K^z}{D} \frac{\partial \rho}{\partial \sigma} \frac{\partial \mathbf{q}}{\partial \sigma} + Z^* d\sigma \]

\[ \int_0^1 \frac{D}{\rho_0} \int_0^1 \tilde{\rho} g w d\sigma \]

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Primary Energy Transfers in Internal Wave Generation
Summary & Perspectives

- Internal tides estimated 30% of available tidal energy:
  Where does the energy go?
- Symphonie is suitable for IT-modelling at and across different scales
- When $\omega \not\equiv N$ : strong non-linearity & subharmonics form.

- Experimental:
  - Particle Imaging Velocimetry $\bullet \ u, w; E_K$ (August 2008)
  - link model & observations $\bullet$ numerical moving ‘ridge’
    (Gerkema & Zimmerman, 1995)
- Modelling:
  - explore $f \not\equiv 0$, 3D effects
  - quantification of $E_K \bullet \bullet$ APE, diapycnal mixing, $APE_0$ (Shepherd, 1993)
  - 3D NH embedded modelling of tidal ray reflection and soliton generation
    in the Bay of Biscay (with Dauxois, Gostiaux and others)
- Theory:
  - multi-scale analysis ($\lambda_0 >> \lambda_{IGW}, T_0 = T_{IGW}, N, L_B, h_0, H$)
  - explanation $\omega \not\equiv N$ – dynamics
Questions:

• Non-Boussinesq effects: work by compression

• Non-traditional dynamics (horizontal components of Coriolis)

• GGG, PPG (f non-zero)

• What about PPPP?

• Qualitative explanation for \( \omega \bullet N \) observations by Taylor & Sarkar, jfm, 2007?