Exploiting Symmetries of MHD Flows (Another Way to Be Cheap)

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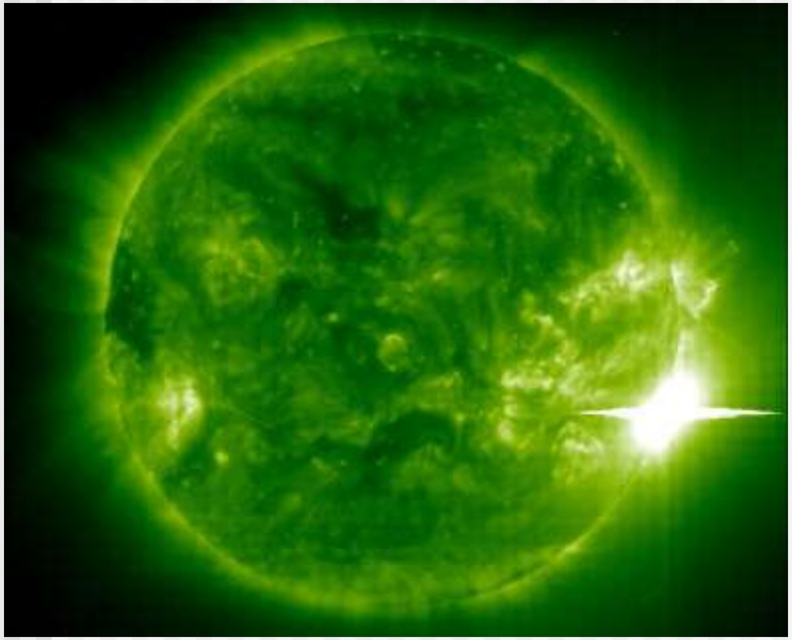
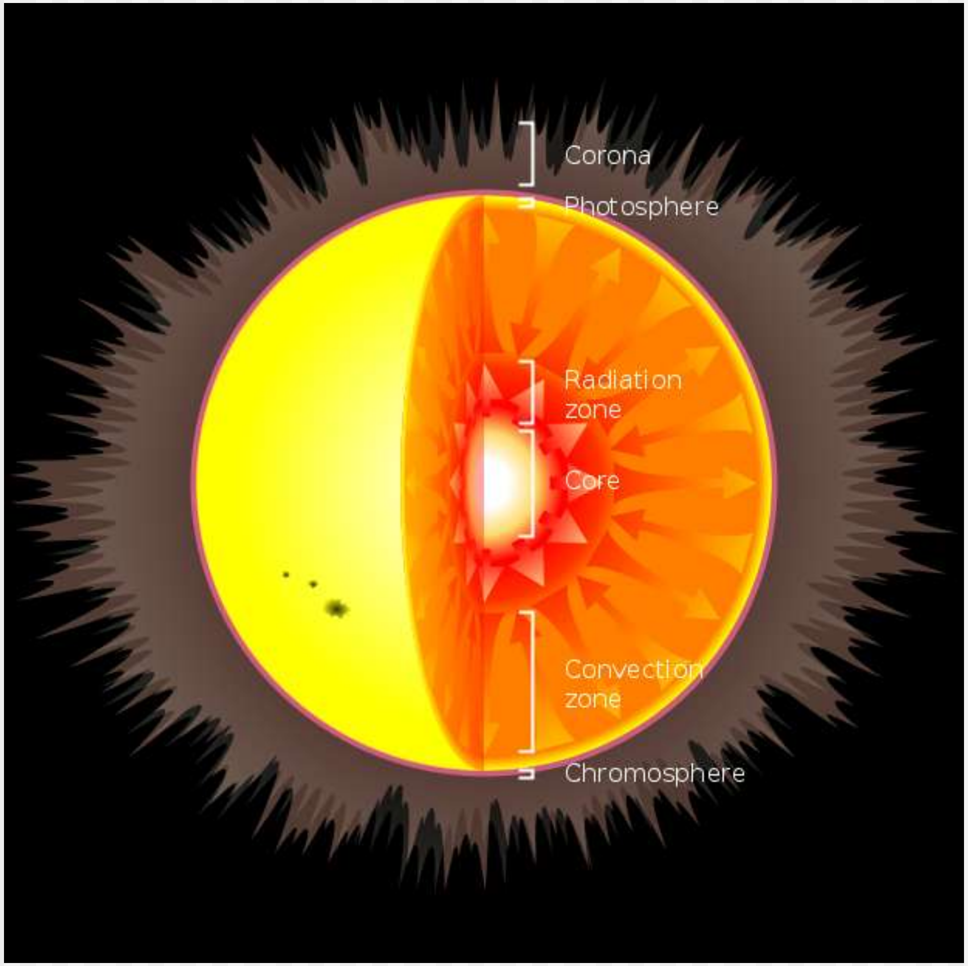
Outline

- I. Plasmas and magnetohydrodynamics (MHD)
  - (A) Plasmas are hot and messy
  - (B) MHD: Let's simplify the universe to two equations
  - (C) Who cares about MHD?
- II. Some "unrealistic" results from a symmetric flow
  - (A) Washing-Machine symmetry
  - (B) Current sheets that just want to be together
  - (C) Turbulence and Alfvén Waves: competition or synergy?

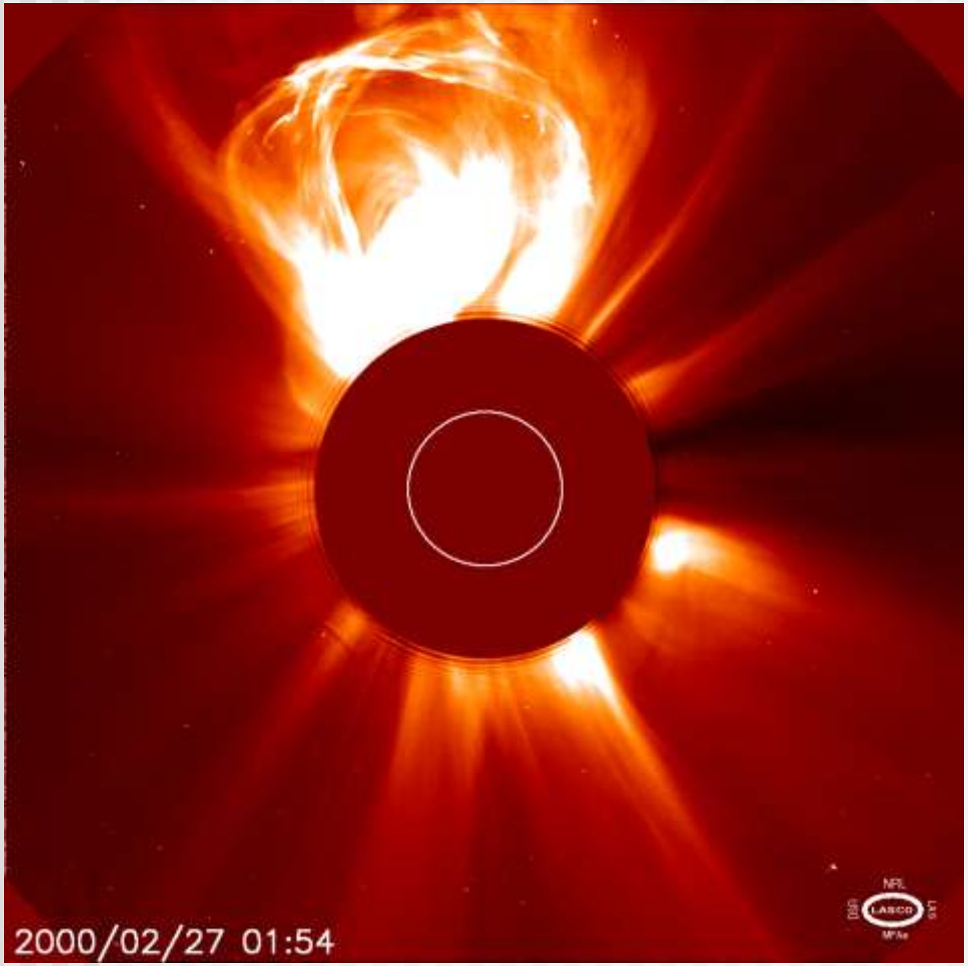
Here Comes the Sun…

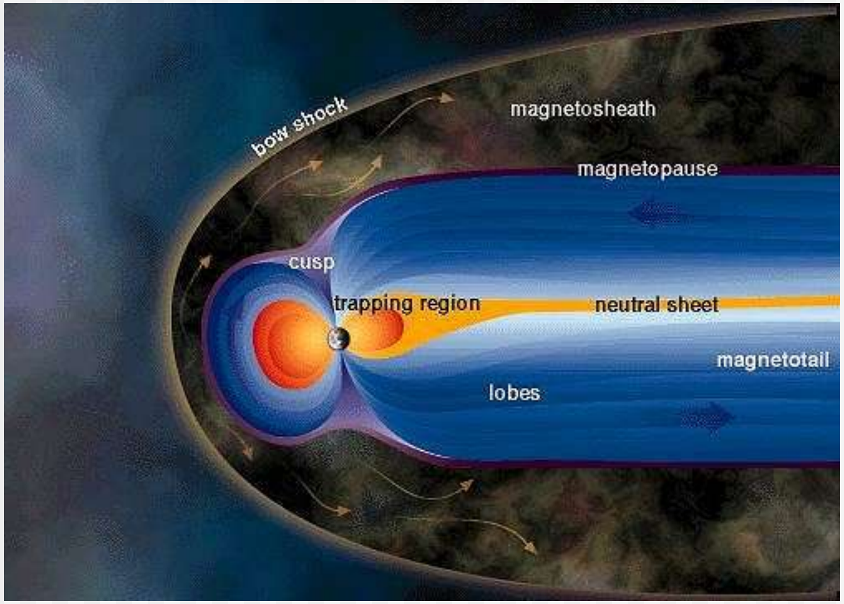


Active Region 10486 (4 Nov 2003), *Photo courtesy of NASA*

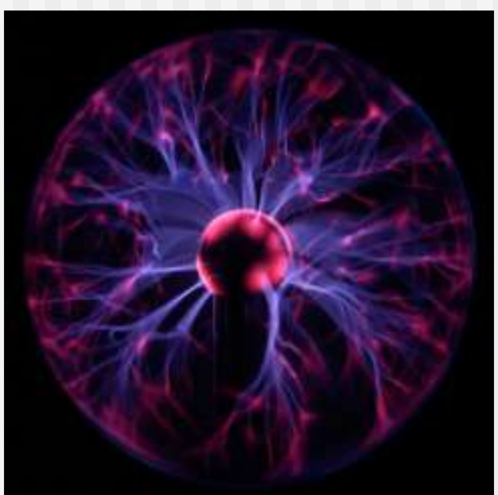


…throwing a fit!

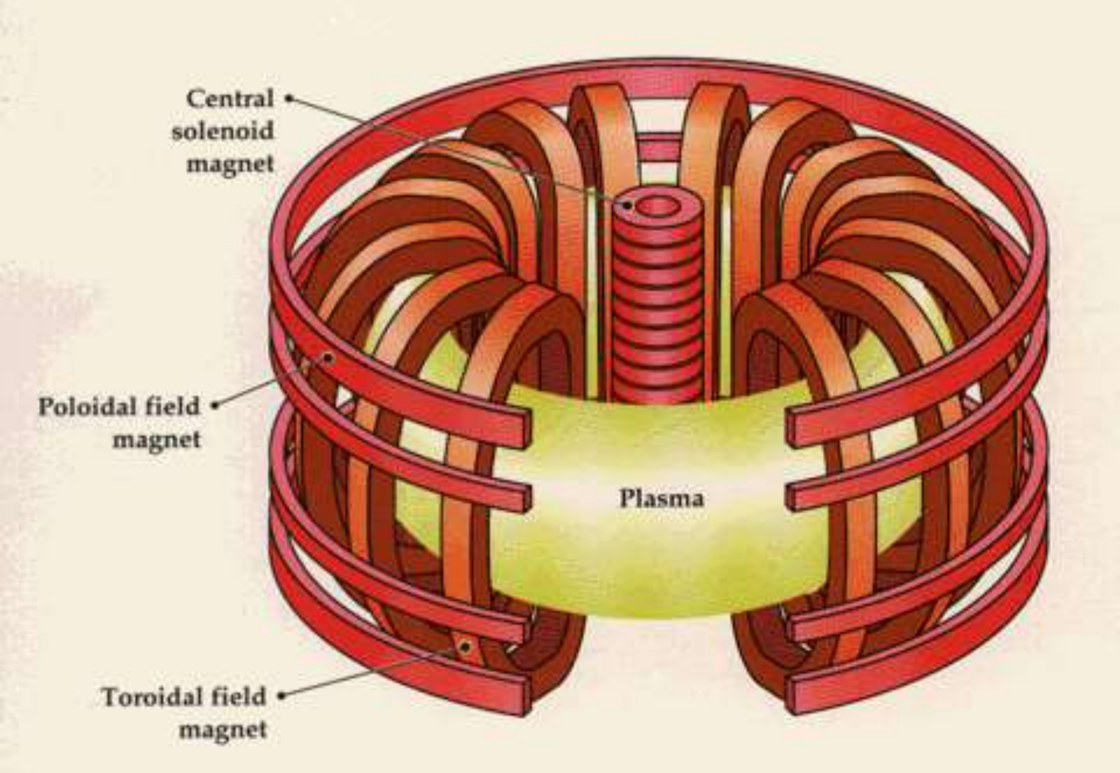




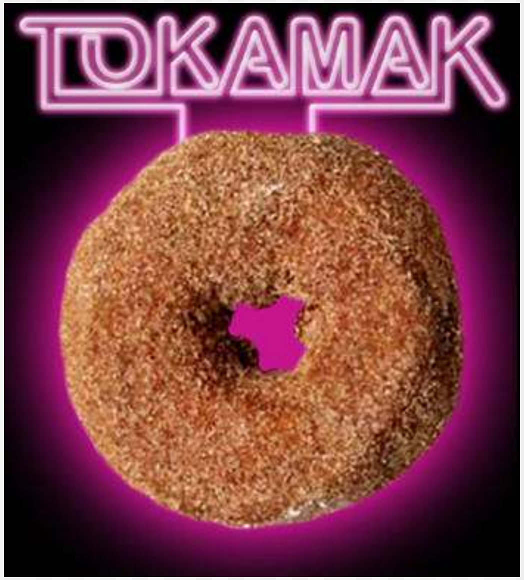
Too close to home



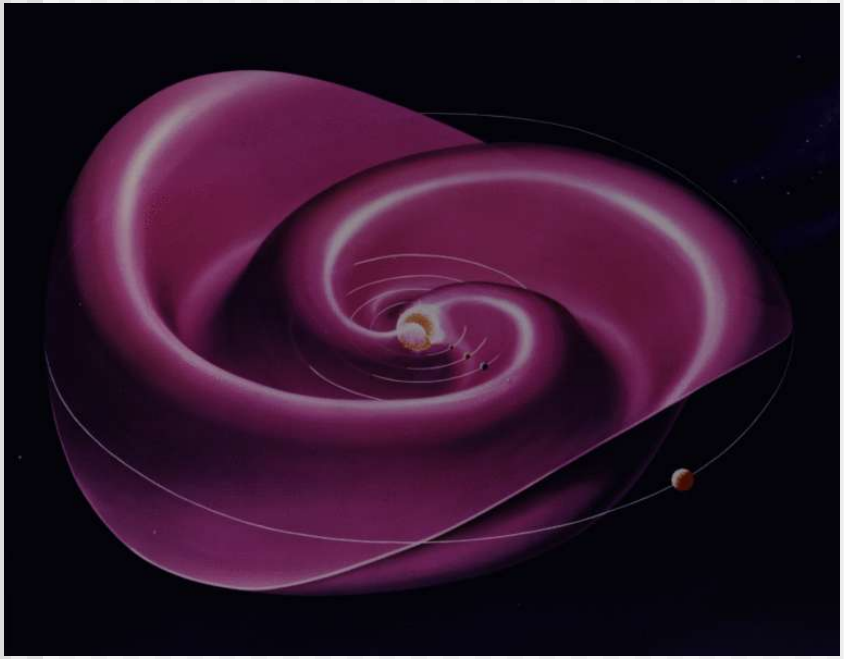
Doughnut-shaped confinement devices that will eventually save the world

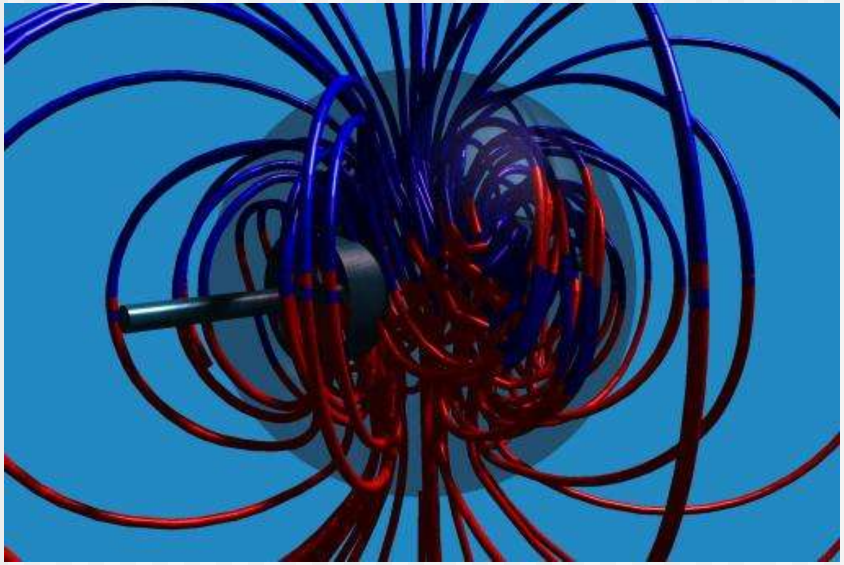


Fusion Tokamak schematic, *Courtesy of Princeton Plasma Physics Laboratory*



Other plasma phenomena





Plasma (hi-)def

- **Do I qualify?**
  1. Strength in numbers ($N >>> 1$):
     High enough density of particles that charges are felt in a *neighborhood* (λ_d), not just next door
  2. Size matters! ($L >> \lambda_d$)
     The size of the plasma is much larger than the neighborhood of influence.
  3. Bullheadedness (ω_p dominant)
     Plasma frequency is much larger than electron-neutral collision frequency, so the plasma acts more like a plasma, not like a gas

→ Quasineutrality

→ Collective effects are possible

→ Bulk internal interactions are more important than boundary effects

→ Large-scale oscillations are effectively shielded out and small-scale oscillations are damped.

Hierarchy of models

- Kinetic Theory
  - Collisionless Boltzmann + Maxwell Eqns => Vlasov Eqn
- Multi-fluid Descriptions
  - Separate momentum equations for electrons, ions, neutrals
- Single-fluid Description
  - If collisions are important, we can describe the plasma as a FLUID

Simplify further!!!

→ Incompressible MHD = Navier-Stokes + Induction

MHD Equations

$$\partial_t \mathbf{v} + \boxed{\mathbf{v} \cdot \nabla \mathbf{v}} + \nabla p = \boxed{\mathbf{j} \times \mathbf{b}} + \boxed{\nu \Delta \mathbf{v}}$$

$$\partial_t \mathbf{b} = \boxed{\nabla \times (\mathbf{v} \times \mathbf{b})} + \boxed{\eta \Delta b}$$

$$\nabla \cdot \mathbf{v} = 0 = \nabla \cdot \mathbf{b}$$

Nondimensional Alfvén units ($v \propto b$):

v = velocity (momentum)

ω = vorticity = curl(**v**)

b = magnetic field

j = current density = curl(**b**)

"IDEAL" $\Leftrightarrow$ $\nu = 0 = \eta$

Who cares about MHD?

- Development of Accretion Discs
  - Angular momentum transport by instability (Balbus & Hawley, 1991, 1998; Ben Jamroz!)
- Heating of the Solar corona
  - Alfvén waves (Haevaerts & Priest, 1983; Davila, 1987; Poedts et al., 1989)
  - Resistive current sheets (Haevaerts & Priest, 1984; Galsgaard & Nordlund, 1996)
  - Turbulence (Haevaerts & Priest, 1992; Cranmer & Ballegooijen, 2003)
- Stellar and Planetary Dynamos
  - Solar dynamo (Gilman, 1983; Glatzmaier, 1985)
  - Geodynamo (Glatzmaier & Roberts, 1993-2003; Kuang & Bloxham, 1999)
- Stellar Winds
  - Acceleration of solar wind by Alfven waves (Isenberg & Hollweg, 1982)
- Planet-Moon Interactions
  - Jupiter-Io (Belcher, 2008)
- Laboratory Plasmas (doughnuts and such)
  - Heating and transport through instabilities (Candy et al., 1997)

Multi-scale interactions

- Physics:
  - Current sheets: large scale structures with small-scale importance and origin
    - Important for reconnection (Biskamp, 1986)
  - Turbulence: 'nuff said!
  - Waves and turbulence
    - "Weak" turbulence (Galtier, Nazarenko, Newell, Pouquet, 2000)

Alfvén Waves:

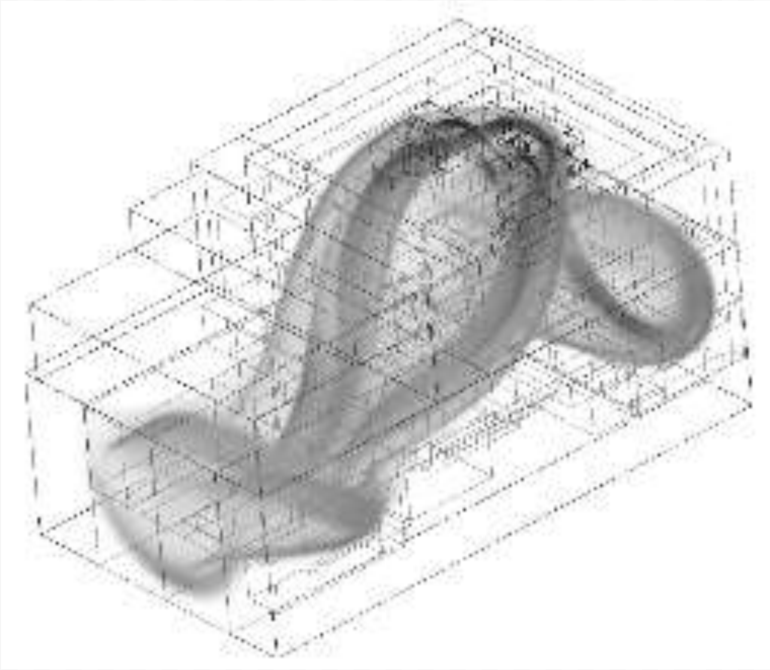
$$\omega^2 \propto \mathbf{k \cdot B_0}$$

$$\omega^2 = v_A^2 \, k_{||}^2$$

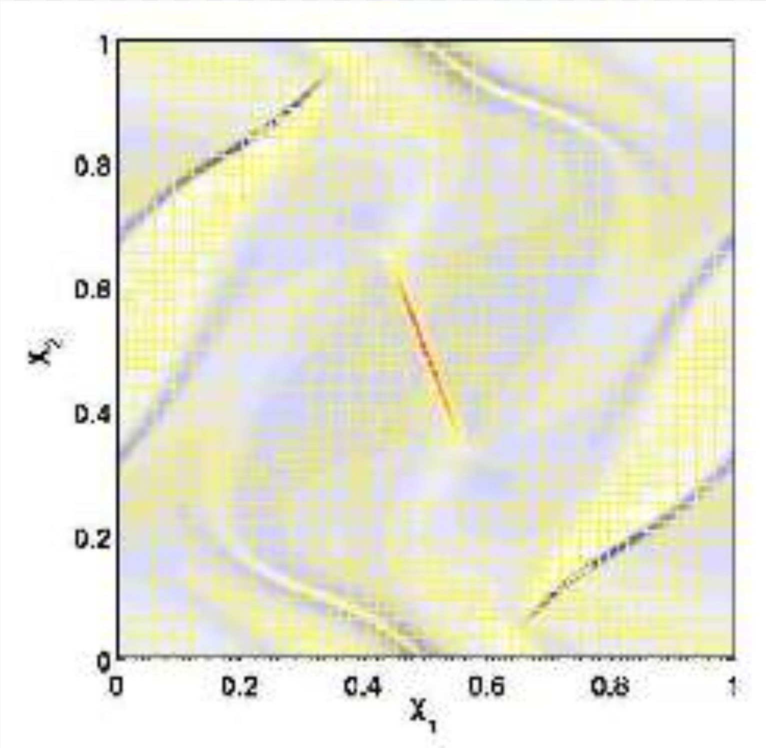
- CFD:
  - Direct Numerical Simulation
  - Adaptive Mesh Refinement
  - Large-Eddy Simulation

Multi-scale interactions

- Adaptive mesh refinement (AMR)



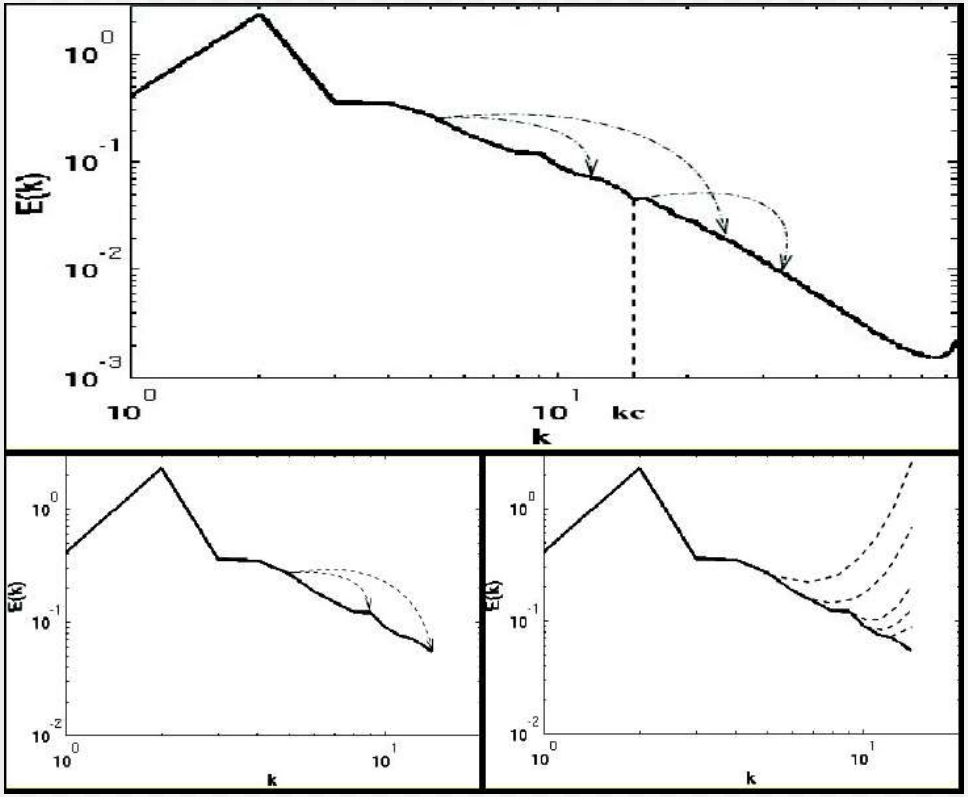
Source: Grauer et al. (1998) Phys. Rev. Lett.

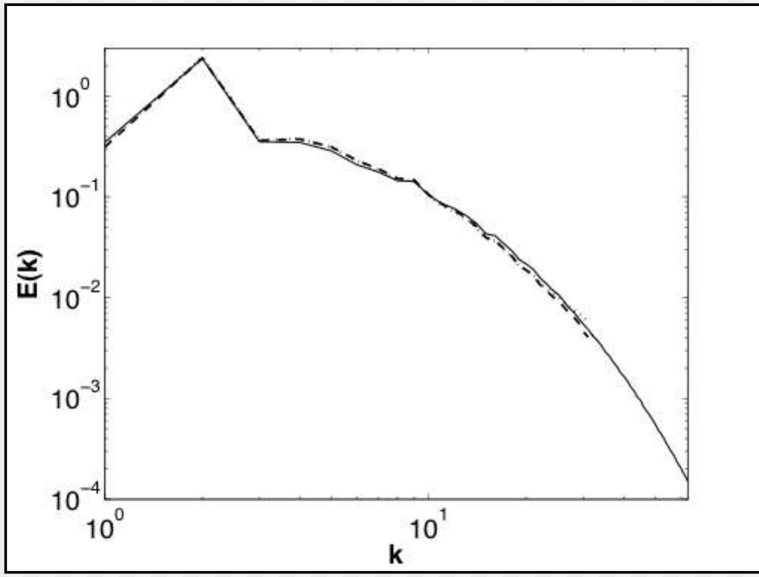


Source: Rosenberg et al. (2007) New J. Phys.

Multi-scale interactions

- Large-eddy simulation



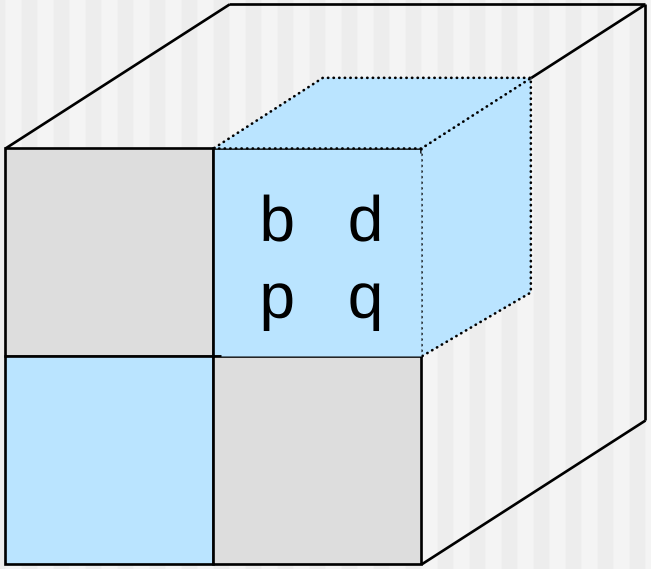


Source: Baerenzung et al. (2008) Phys. Rev. E (submitted)

Part II:

Exploiting symmetries

Taylor-Green vortex

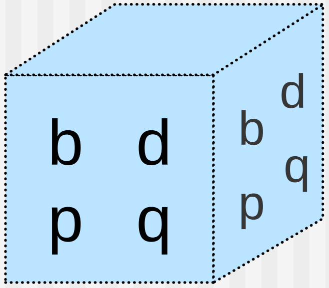


$$v_x = v_0 \sin(x) \cos(y) \cos(z)$$

$$v_y = -v_0 \cos(x) \sin(y) \cos(z)$$

$$v_z = 0$$

- TG vortex:
  - Brachet et al., 1983; Brachet 1991
- Analyticity strip:
  - Brachet et al., 1992;
  - Cichowlas et al., 2005;
- Dynamo:
  - Nore et al., 1997



Magnetic Taylor-Green

- MHD equations

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"IDEAL" $\Leftrightarrow$ $\nu = 0 = \eta$

- Initial velocity field

$$v_x = v_0 \sin(x) \cos(y) \cos(z)$$
$$v_y = -v_0 \cos(x) \sin(y) \cos(z)$$
$$v_z = 0$$

- Initial magnetic field

$$b_x = b_0 \cos(x) \sin(y) \sin(z)$$
$$b_y = b_0 \sin(x) \cos(y) \sin(z)$$
$$b_z = -2b_0 \sin(x) \sin(y) \cos(z)$$

Lee, Brachet, Pouquet, Mininni, Rosenberg (2008) arXiv:0802:1550

Search for singularity

- Euler singularity
  - Beale-Kato-Majda (1984):

    $\limsup_{(t\uparrow T^*)} \|\omega(t)\|_\infty = \infty$ OR $\int \|\omega(t)\|_\infty \, dt < \infty$

- MHD
  - Caflisch-Klapper-Steele (1997):

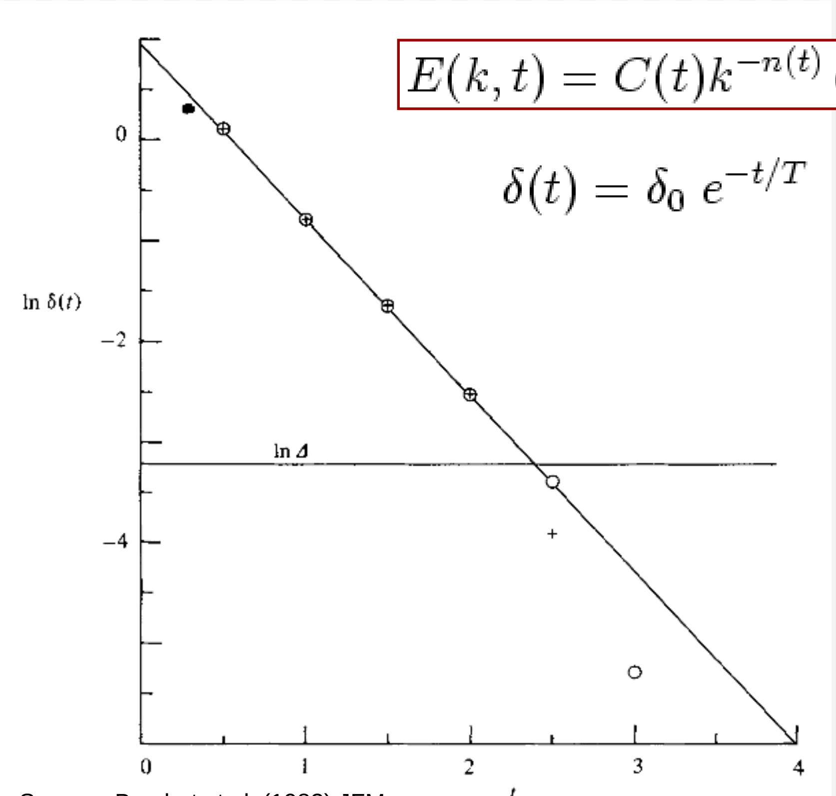
    $\limsup_{(t\uparrow T^*)} \left(\|\omega(t)\|_\infty + \|j(t)\|_\infty \right) = \infty$ if singularity exists

$\omega^i (x^i, t)$ = vorticity

$j^i(x^i, t)$ = current density

Analyticity strip

- Sulem, Sulem, Frisch (1983):
  - Also Frisch, Pouquet, Sulem, Meneguzzi (1983) - 2D MHD
  - Also Brachet et al. (1983) - 3D Euler



$$\boxed{E(k,t) = C(t)k^{-n(t)} \exp[-2\delta(t)k]}$$

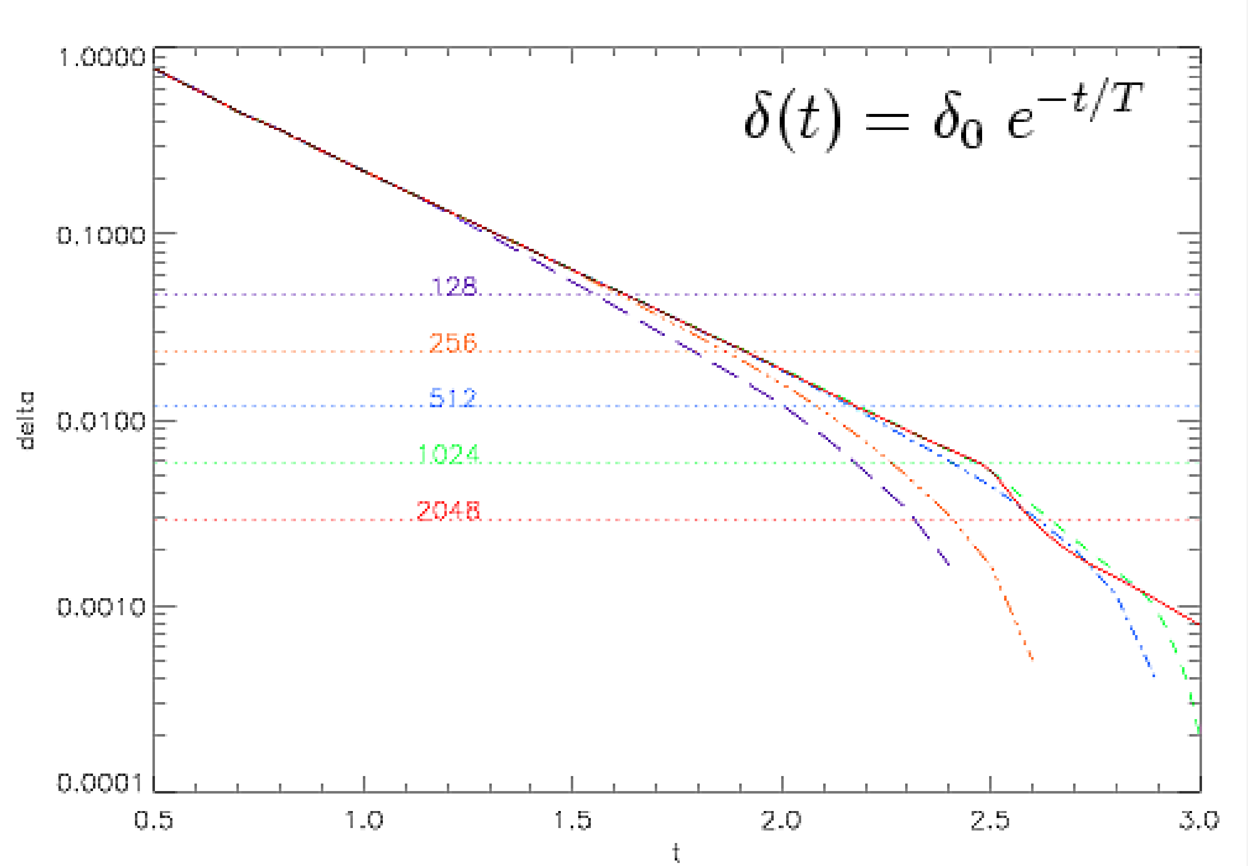
$$\delta(t) = \delta_0 \, e^{-t/T}$$

Source: Brachet et al. (1983) JFM

Ideal MTG

- Evolution of delta

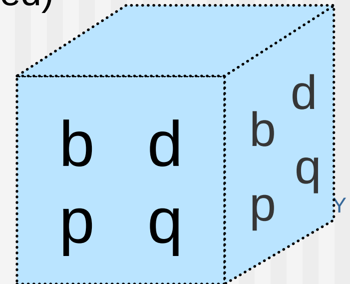
$$E(k,t) = C(t)k^{-n(t)} \exp[-2\delta(t)k]$$



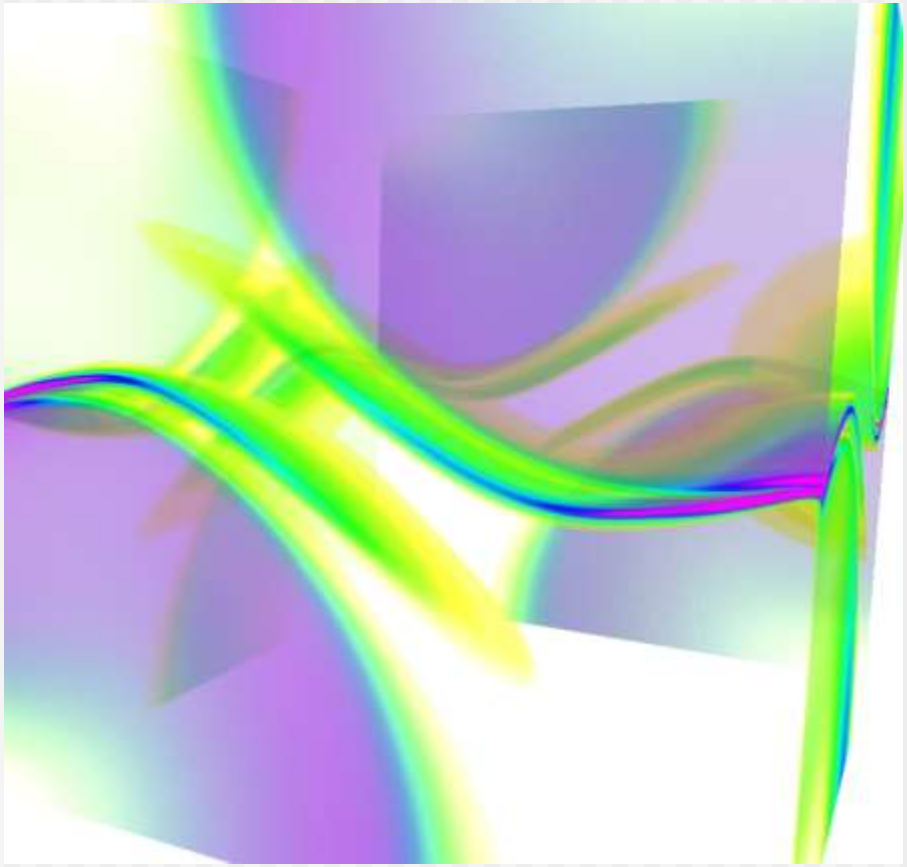
Lee, Brachet, Pouquet, Mininni, Rosenberg (2008) arXiv:0802:1550

High-res simulation results

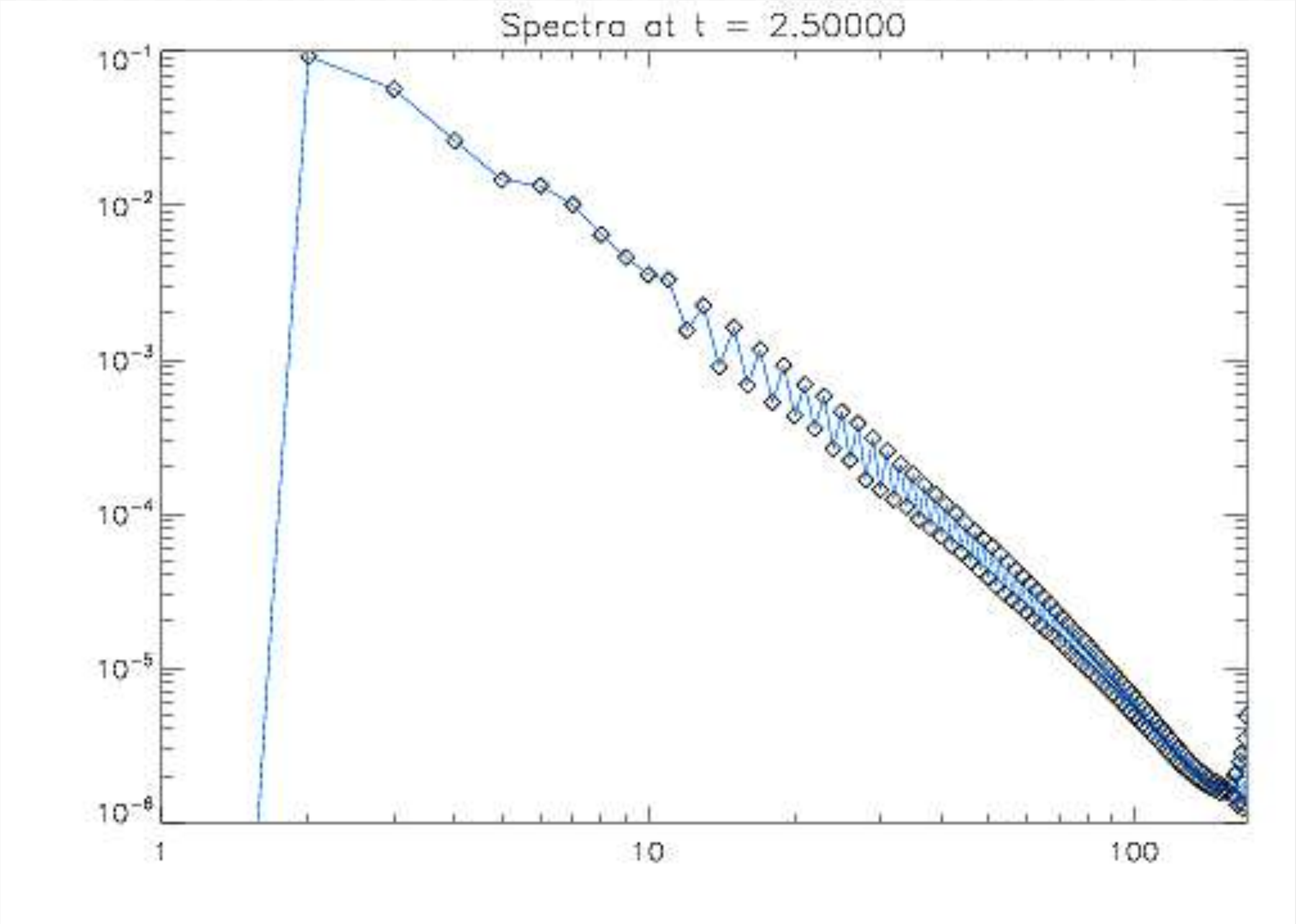
- IDEAL CASE ($\nu, \eta = 0$)
  - 2084^3 resolution
- Integration:
  - NCAR IBM BlueGene/L ("Frost")
  - 80K CPU hrs (to t=3)
  - Pseudospectral, periodic BC, w/ symmetries implemented in code
    - Also code without imposed symmetries
  - 2nd-order RK timestepping
    - Also 4th-order
- Visualization:
  - VAPOR (Clyne et al., 2007; Mininni et al., 2008 submitted)



- Current sheets

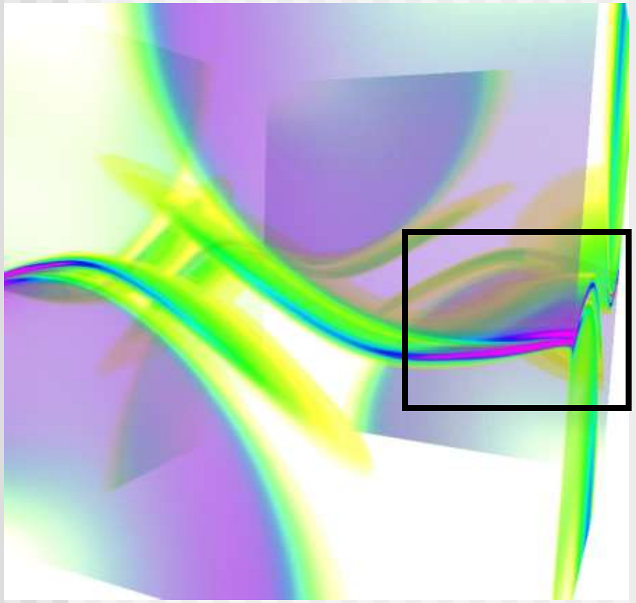


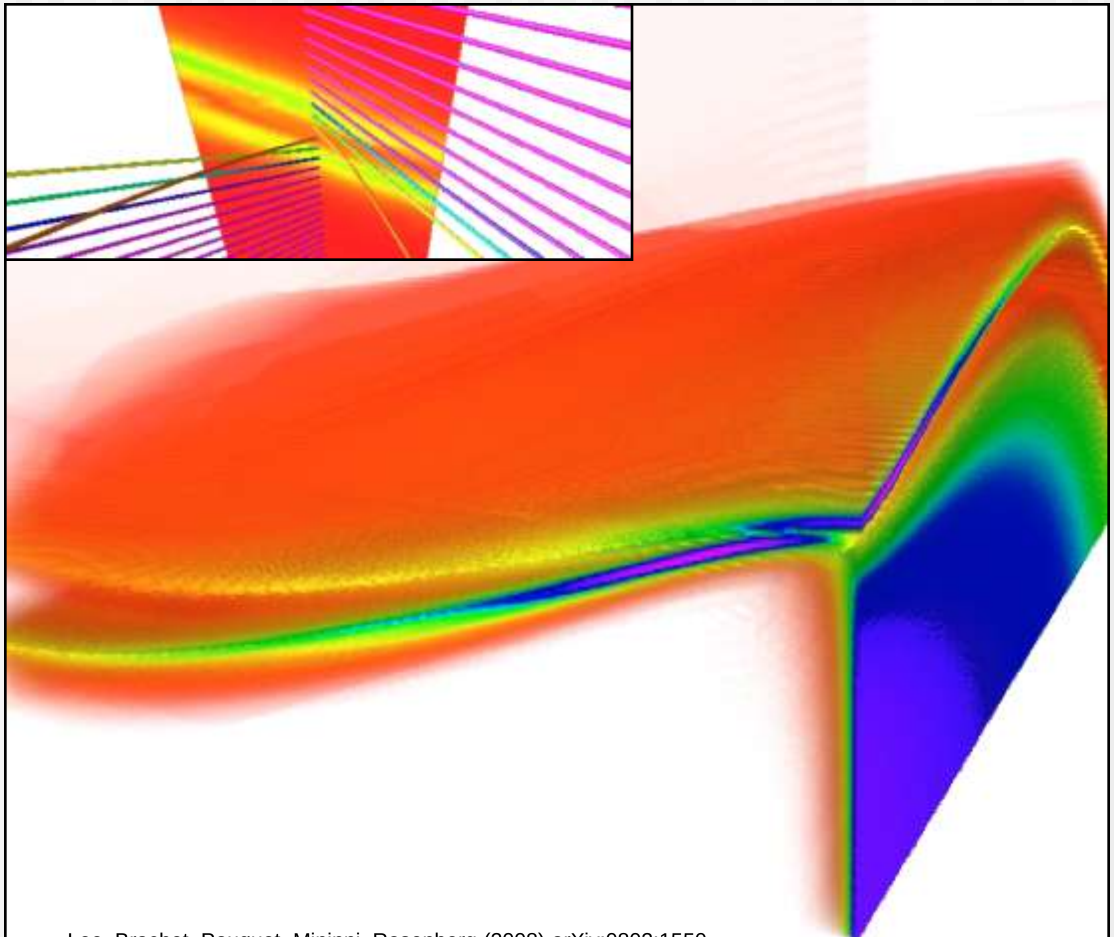
Ideal 2048: comparison



High-res simulation results

- Current sheets
  - Thinning, merging
  - Accompanied by rotational “discontinuity” of **B** (cf. Whang et al., 2004)

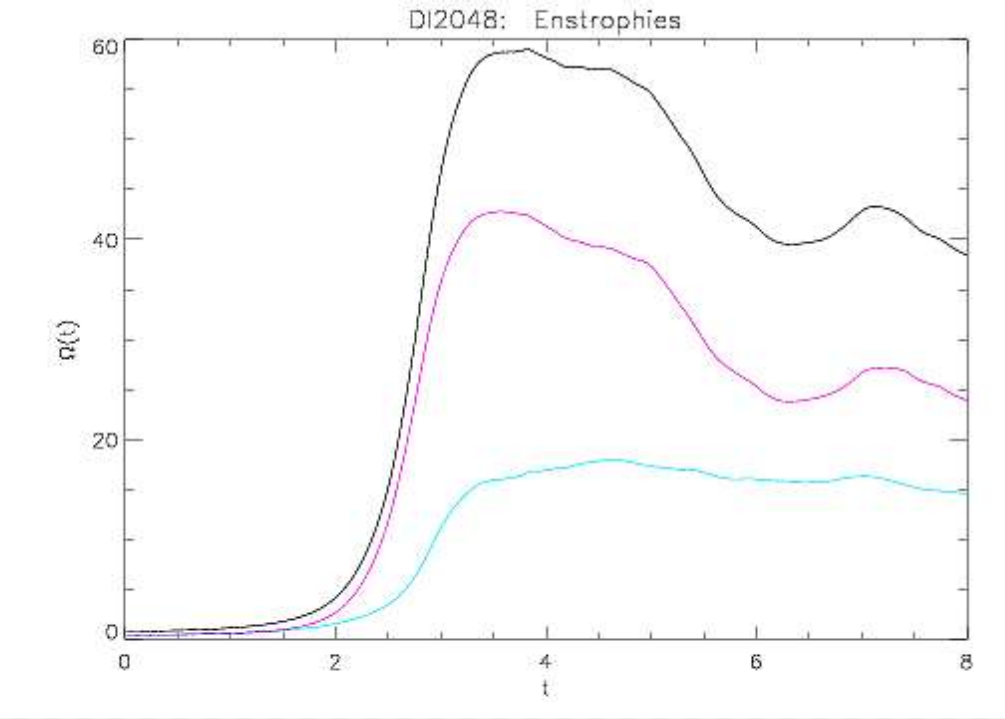




Lee, Brachet, Pouquet, Mininni, Rosenberg (2008) arXiv:0802:1550

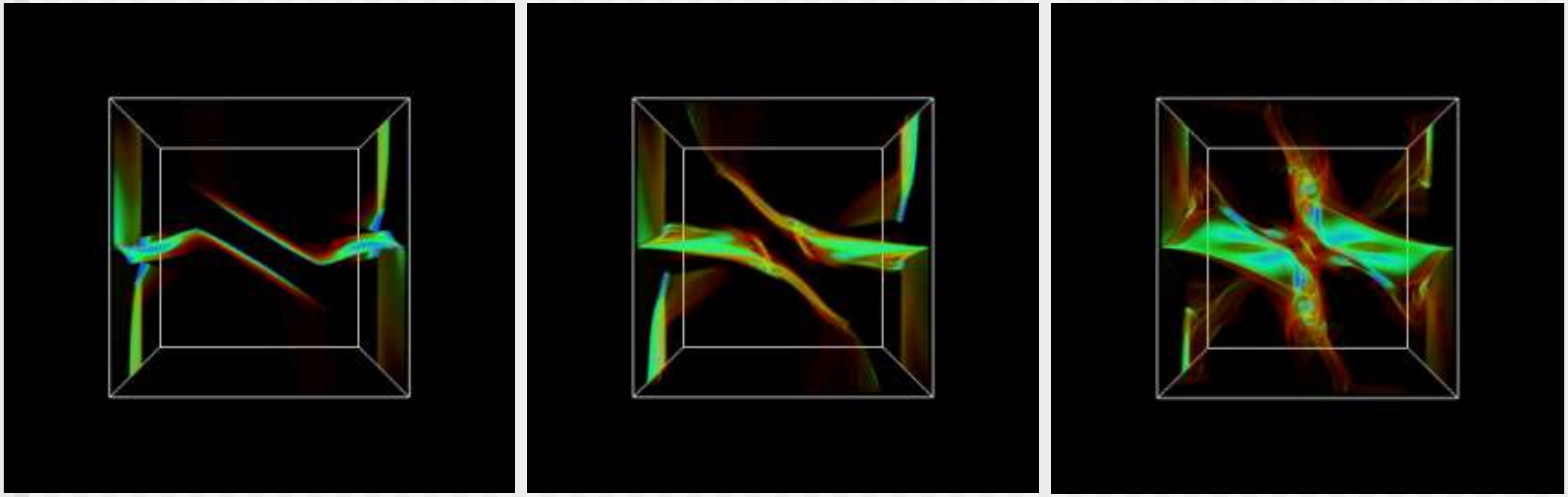
High-res simulation results

- DISSIPATIVE CASE
  - 2048^3 resolution
- Integration:
  - NCAR IBM POWER5+ (“Blueice”)
  - 10K CPU hrs (to t=8)
  - Pseudospectral, periodic BC, w/ symmetries implemented in code
    - Also code without imposed symmetries
  - 2nd-order RK timestepping
- Visualization:
  - VAPOR
- Dissipation



III. Breakthrough prospects with petascale resources:

Dissipative MTG



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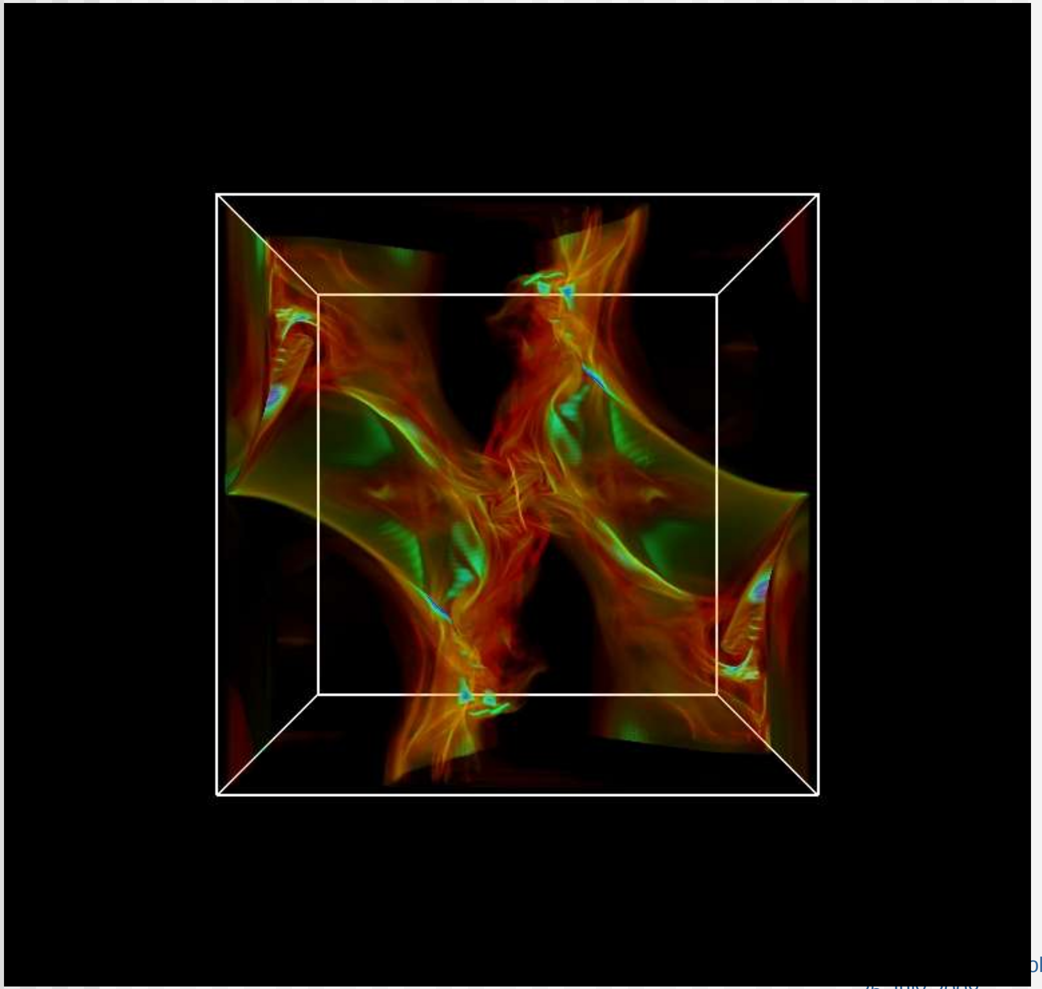

- Structures
  - Current sheets
  - Reconnection
  - Instabilities
- Wave turbulence
  - Spectra
  - Structure functions
  - Time scales

Conclusions

- Taylor-Green symmetries
  - "Fully" resolved
- Ideal MTG
  - Development of current sheets
  - Need for higher resolution to study behavior of smaller scales
  - Evolution of complex-space singularities
- Dissipative MTG
  - Turbulence
  - Waves and turbulence
  - Current sheets and other coherent/dissipative structures
  - Scaling laws
- Application to the "Real World"