ON NUMERICAL REALIZABILITY OF THERMAL CONVECTION

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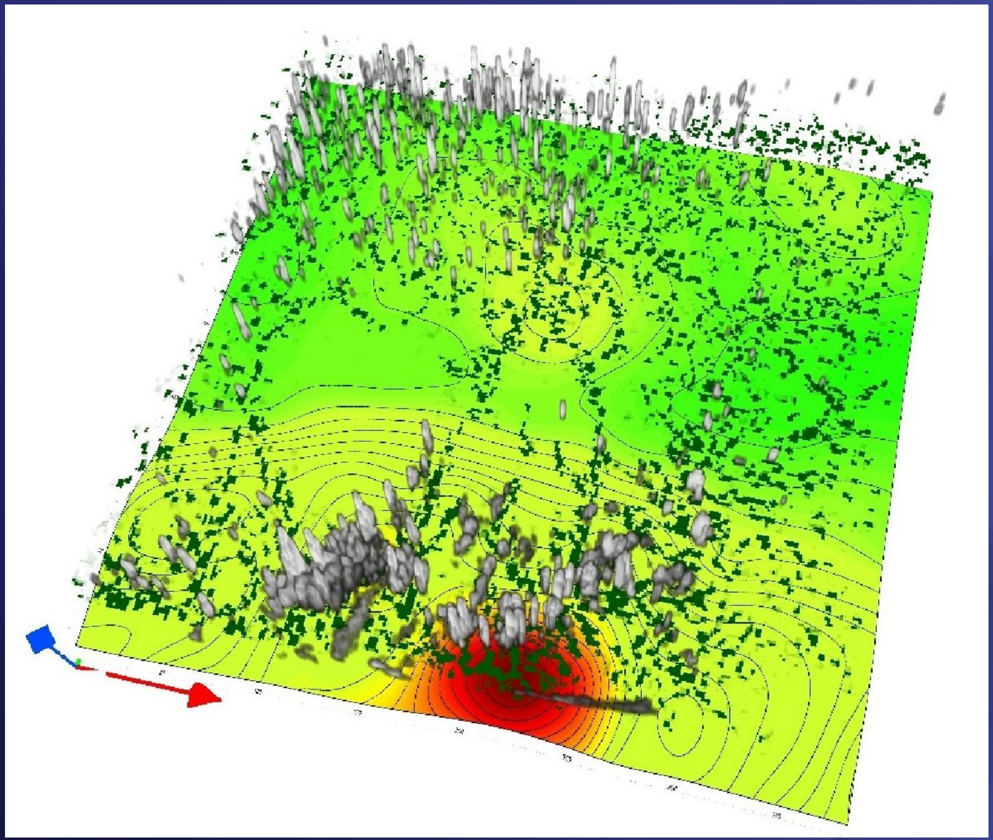
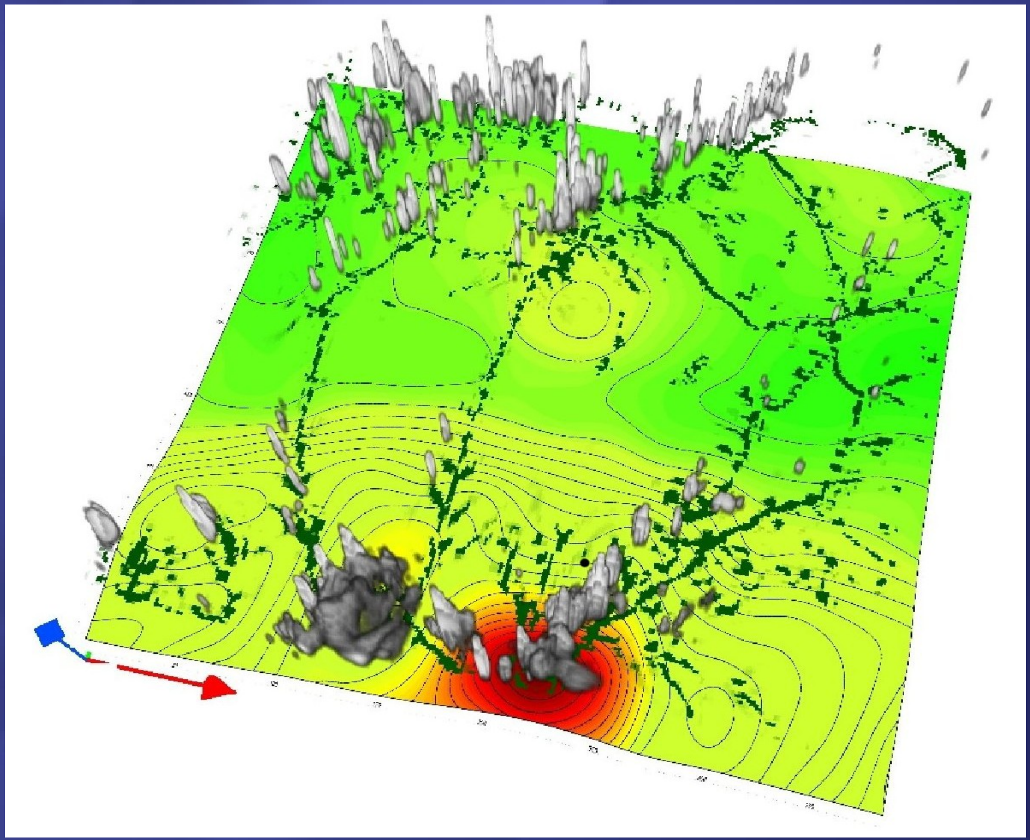
Motivation





Structure of thermal convection over heated terrain.

Vertical velocities after 6h of simulated time are shown within the PBL depth. Grey iso-surfaces represent clouds, and dark green patterns mark updrafts at boundary layer top. Isolines and other colors show the topography. The only difference between the two simulations is the effective viscosity of numerical advection.

Rayleigh number :

$$Ra = \frac{g\Delta\overline{\theta}h^3}{\overline{\theta}\nu\nu_\theta}$$

g – gravity acceleration

h – fluid layer thickness

ν – kinematic viscosity

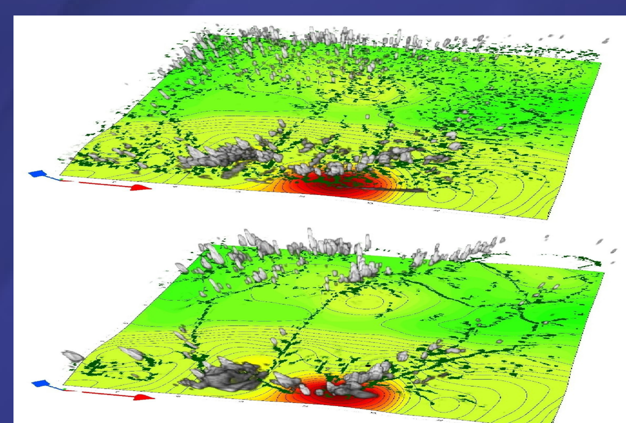
ν_θ – thermal diffusivity

$\Delta\theta/\theta$ – pot. temperature, relative change over h

Ra measures relative magnitude of buoyancy and viscous forces

rigid/stress-free lower/upper boundary

Ra_c=1100.657



>> critical

$\gtrsim$ critical

In the dry atmosphere:

$h = 1000$ m
$\nu = 1.7 \times 10^{-5}$
$\nu_\theta = 1.9 \times 10^{-5}$
$\Delta\theta / \theta = 0.1 \times 10^{-2}$

$\longrightarrow$ **$Ra \approx 10^{16}$!!!**

So, how to explain cellular convection ?

Modified definition (Jeffreys, 1928)

$$Ra = \frac{g \Delta \bar{\theta} h^3}{\bar{\theta} K_m^2}$$

Km - effective „eddy diffusivity"

Note, K_m can be different in the horizontal and in the vertical.

Although research on eddy viscosity effects on atmospheric cellular convection has continued for nearly a century ---

[1] H. Jeffreys, Some cases of instability in fluid motion, Proceedings of the Royal Society of London. Series A, Containing Papers of a Mathematical and Physical Character (1905-1934) 118 (779) (1928) 195–208.

[2] C. Priestley, Width-height ratio of large convection cells, Tellus 14 (1962) 123–124.

[3] D. Ray, Cellular convection with nonisotropic eddys., Tellus 17 (1965) 434–439.

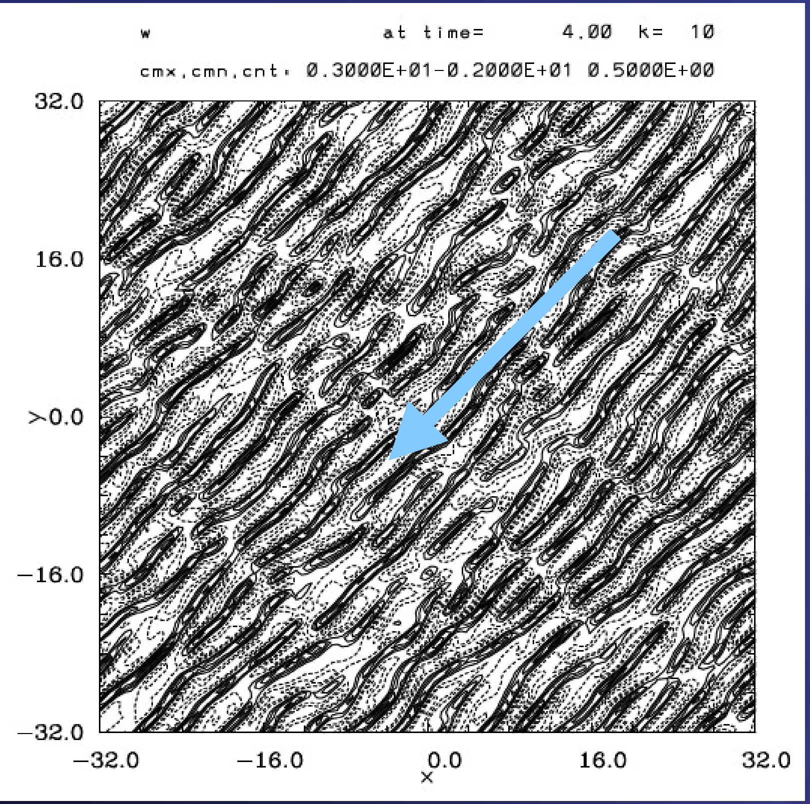
[4] P. Sheu, E. Agee, J. Tribbia, A numerical study of physical processes affecting convective cellular geometry, J. Meteor. Soc. Japan, 58 (1980) 489–499.

[5] B. Atkinson, J. Zhang, Mesoscale shallow convection in the atmosphere, Rev. Geophys. 34 (1996) 403–431.

--- the problem lacks conclusion, and calls for attention with the advent of $O(1)$ km resolution NWP.

Numerical substantiation

Series of LES using the EULAG (MPDATA) model



V = [-10,-10] m/s

dz=50 m

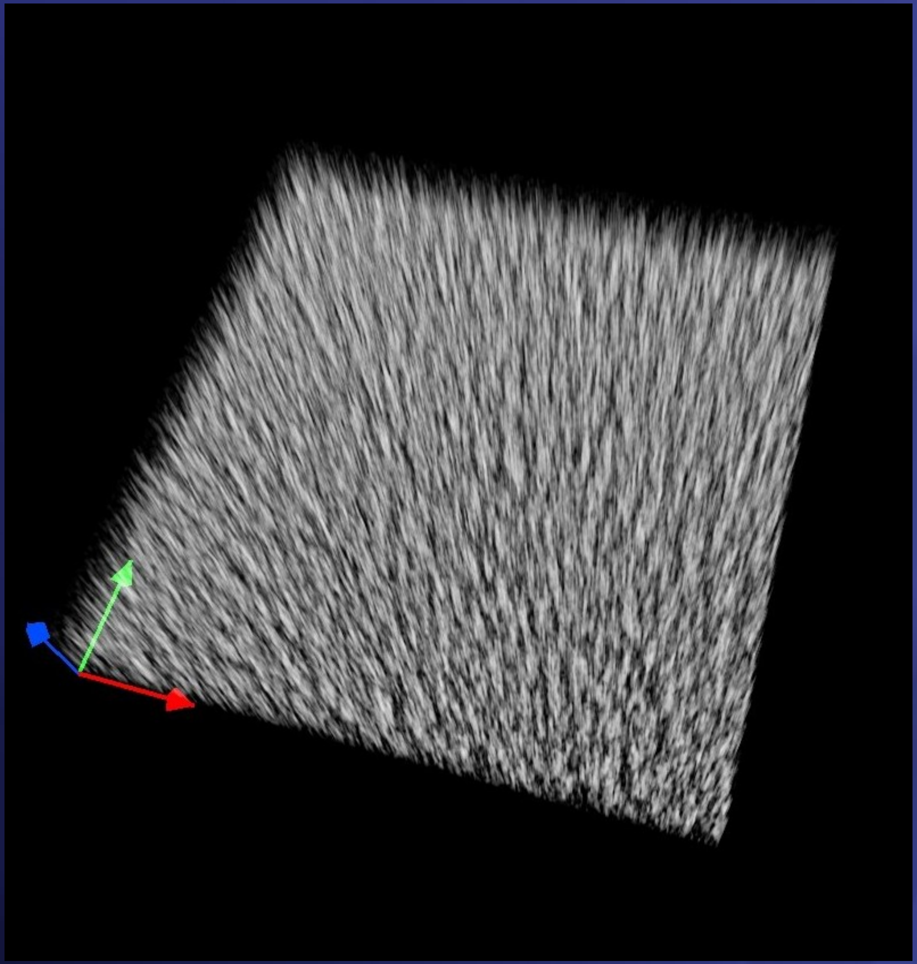
dx=dy≈500 m

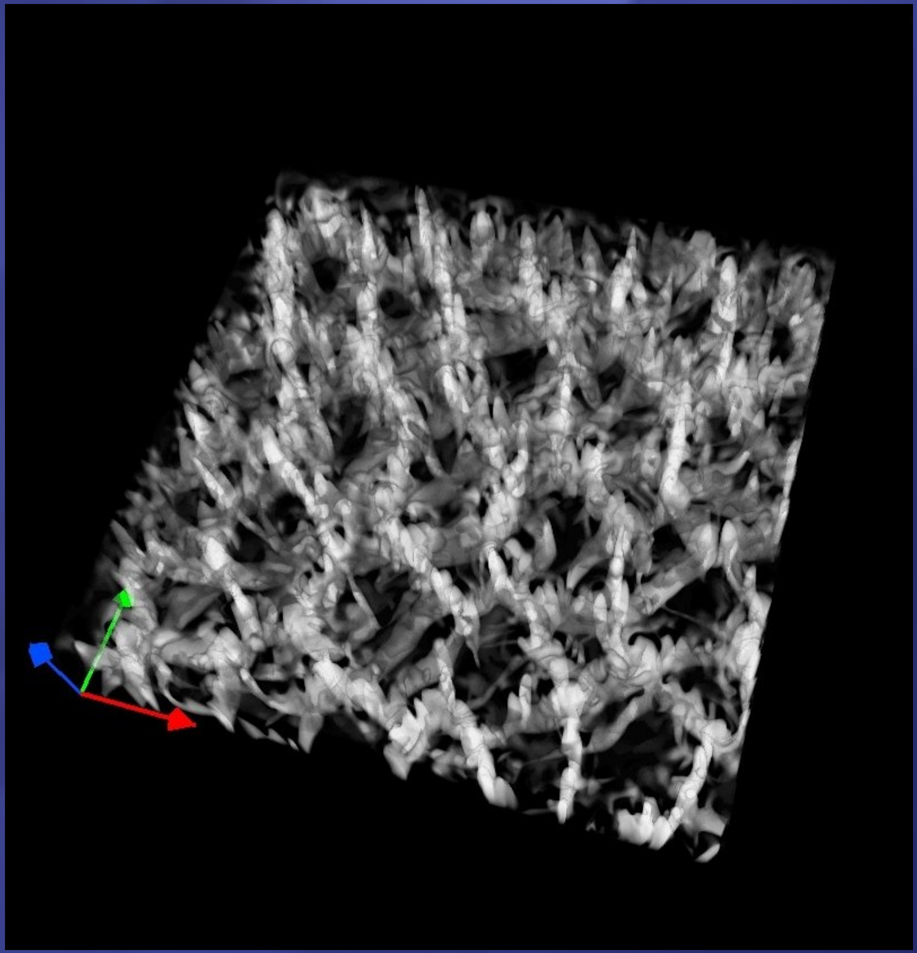
Heat flux
hfx=200 W/m^2

Flat lower boundary, doubly periodic horizontal domain, Boussinesq option

Reference setup alludes to contemporary, mesoscale cloud-resolving NWP

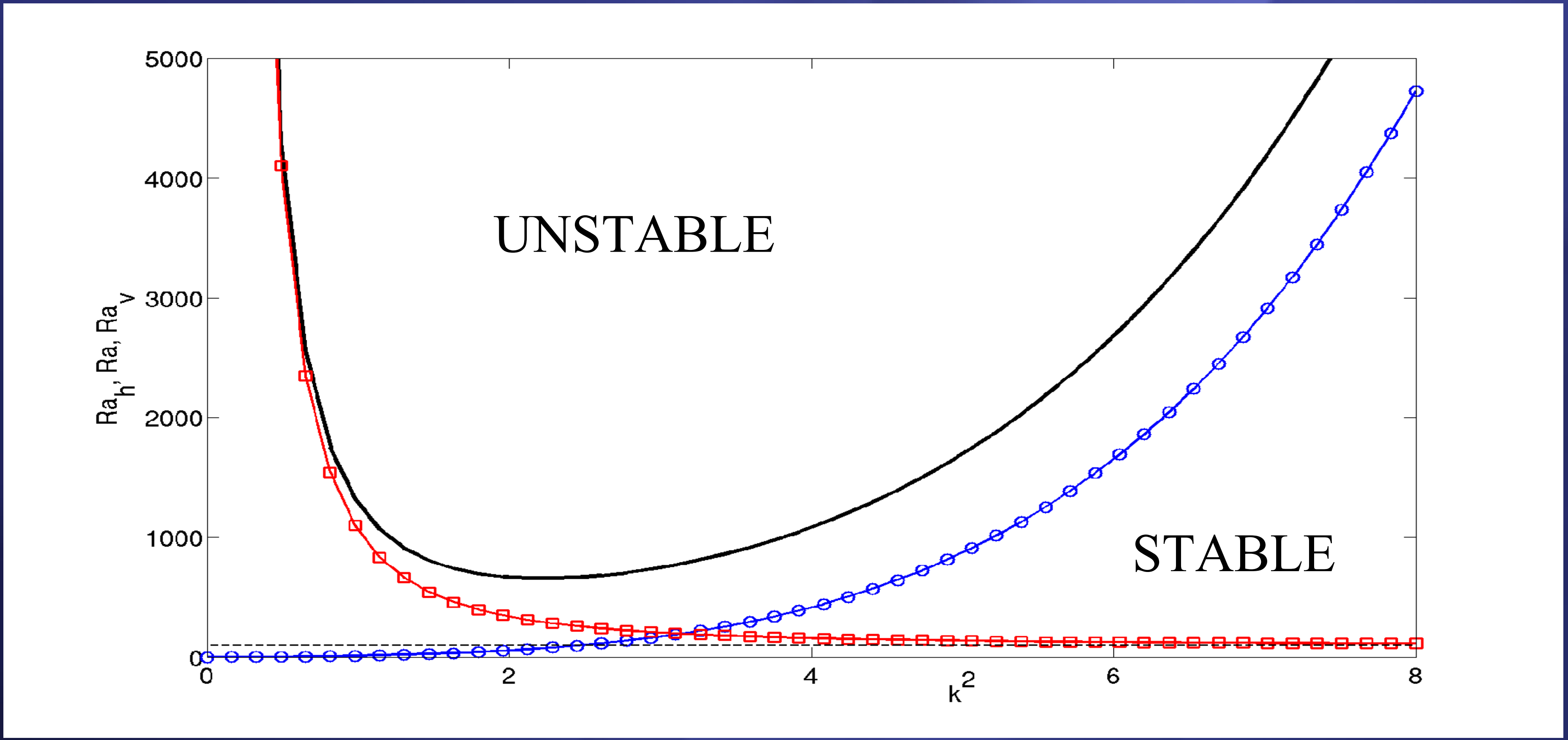
Canonical case: V=[0,0] and constant viscosities





Structure of thermal convection over heated plate. Vertical velocities after 6h of simulated time are shown within the PBL depth. Bright and dark volumes denote updrafts and downdrafts, respectively. The only difference between the two solutions is the value of viscosity in horizontal entries of the stress tensor, $\nu_h = 2.5\ m^2 s^{-1}$ *and* $\nu_h = 70\ m^2 s^{-1}$ *; the vertical entry* $\nu_v = 2.5\ m^2 s^{-1}$ *in both cases.*

Linear theory



Asymptotic marginal stability relations for a finite Prandtl number and $\nu_h = \nu_v$ (black solid), $\nu_v = 0$ (blue circles) and $\nu_h = 0$ (red squares). Respective Rayleigh numbers Ra_h, Ra and Ra_v are shown in function of the squared horizontal wave number. Stability region is below the curves.

Sources of *Ra* anisotropy

- Numerical dissipation ~**V** (flow magnitude), as oppose to ~∂**V**; e.g., first-order upwinding, or composite schemes
- Using numerical schemes with different dissipative properties in the horizontal and in the vertical
- Explicit anisotropic filtering

Domain and resolution required to faithfully represent convective structures:

- $D = O(10)$ km x $O(10)$ km in the horizontal
- $\Delta = O(10)$ m horizontal resolution

Grid spacing required to resolve width of convective rolls appears to be O (10) m

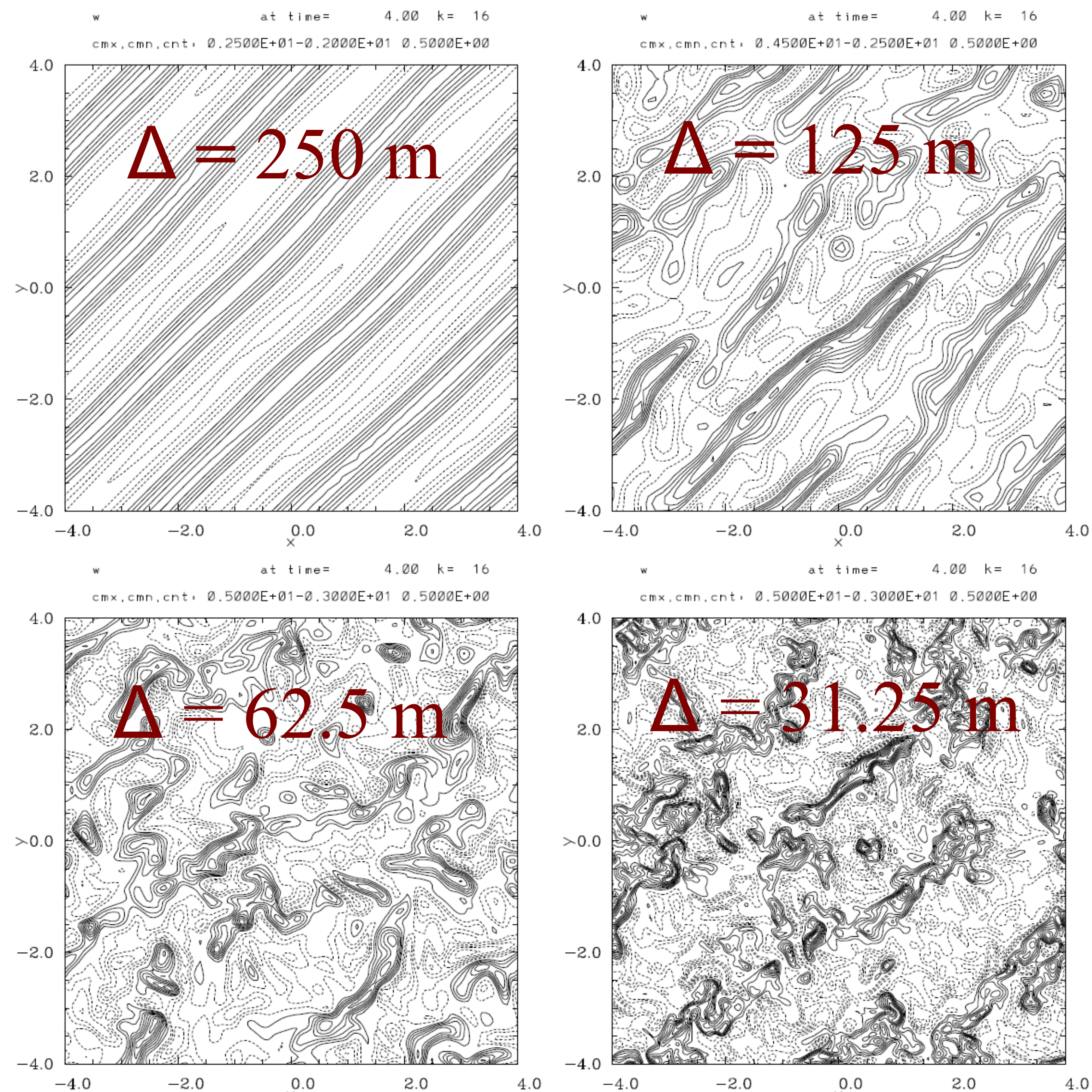


Fig. 7. Structure of vertical velocity at 450 m for fixed domain 8km x 8km x 9km and number of grid points in horizontal 32, 64, 128, 256, respecitvely, while keeping 301 grid points in vertical

Horizontal domain size required to capture two convective rolls

> 4 km x 4 km

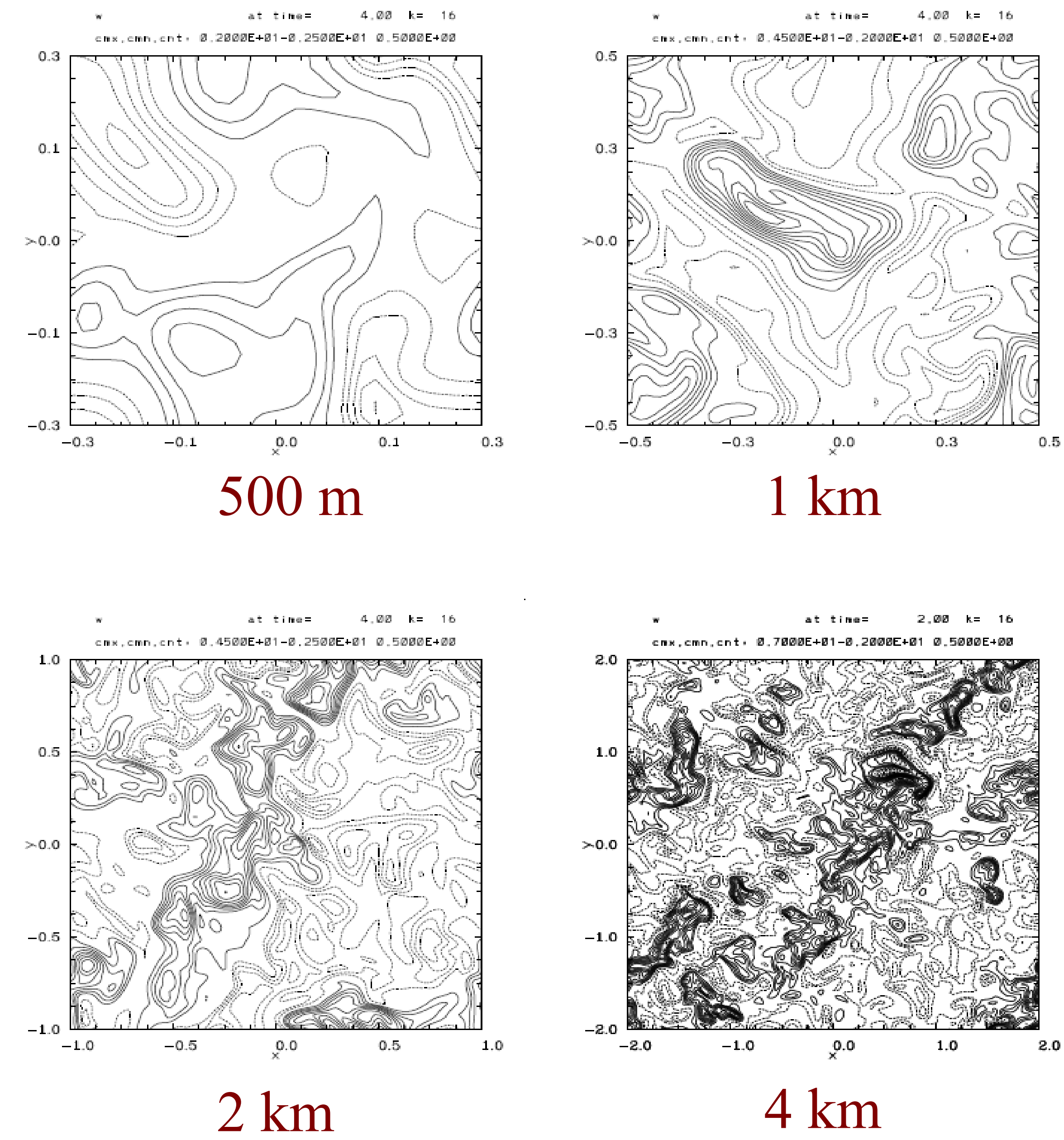


Fig. 9. Structure of vertical velocity at 450 m for fixed resolution and changing domain, while keeping 301 grid points in vertical

“Tenets” of convective-fields simulation

- Control of numerical viscosity: not every dissipative numerics has adequate implicit LES property.
- Awareness of the numerical model design; e.g., avoidance of a first-order dissipative numerics, and ad-hoc filters.
- Verification of the adequacy of subgrid-scale models using convective benchmarks
- Skepticism for the ``eye-pleasing’’ convective structures appearing in large-scale cloud-resolving simulations

Conclusions

- Cellular convection simulated with meso- and large-scale models may be only a spurious result of the effective anisotropic viscosity
- Implicit numerical viscosity and dispersion are well known. There appears to be a need for appreciating ``implicit numerical topology'' while analyzing under-resolved convective structures and cloud coverage
- Non-oscillatory forward-in-time (NFT) methods based on MPDATA advection appear convenient for cloud-resolving simulations, as they:

i) do not depend on explicit subgrid-scale models;

ii) do not require filters for numerical stability; and

iii) are numerically isotropic