SUBGRID-SCALE MOTIONS IN ROUGH WALL BOUNDARY LAYERS

Peter P. Sullivan
National Center for Atmospheric Research

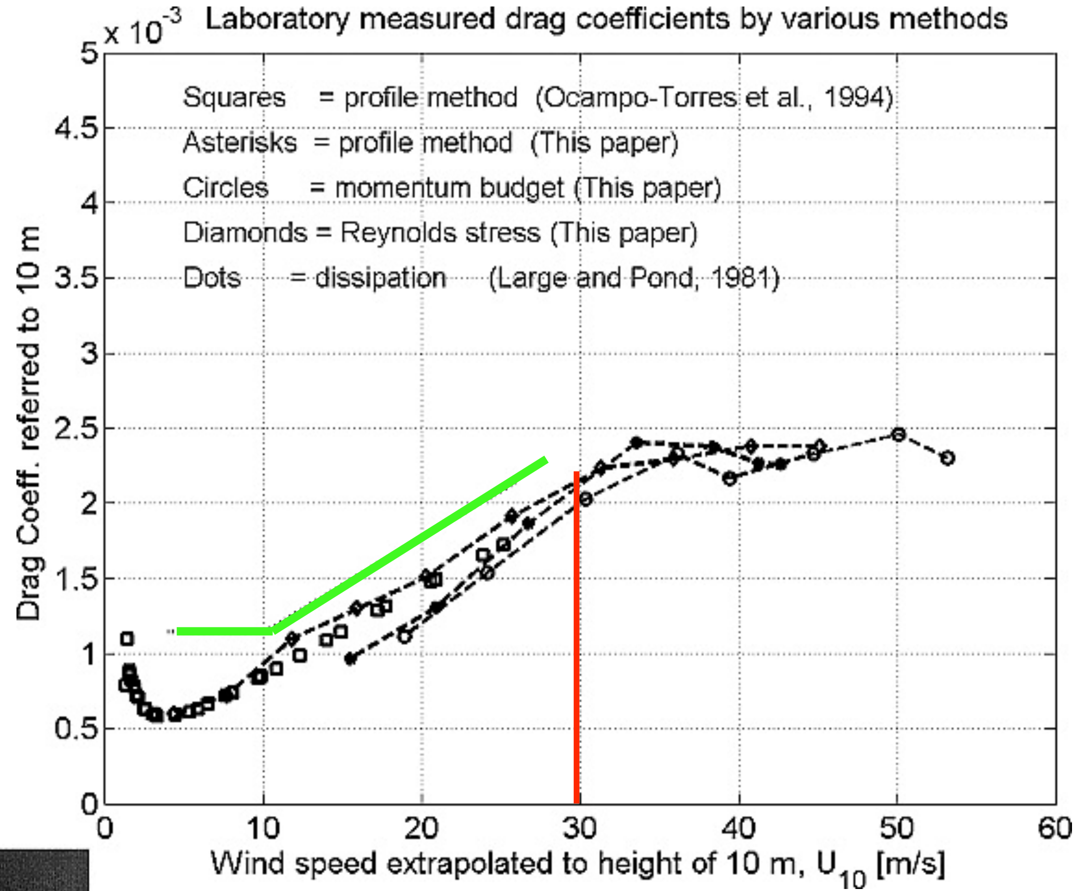
A PBL Turbulence Problem?

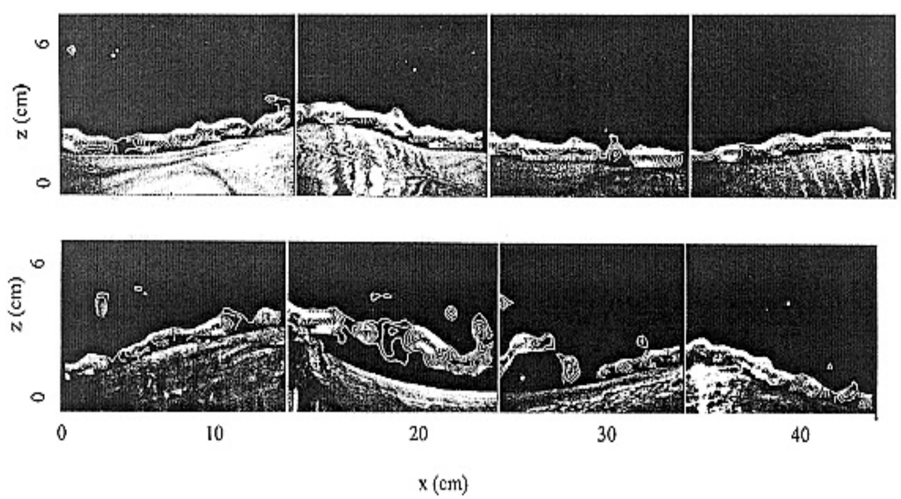
Courtesy NOAA

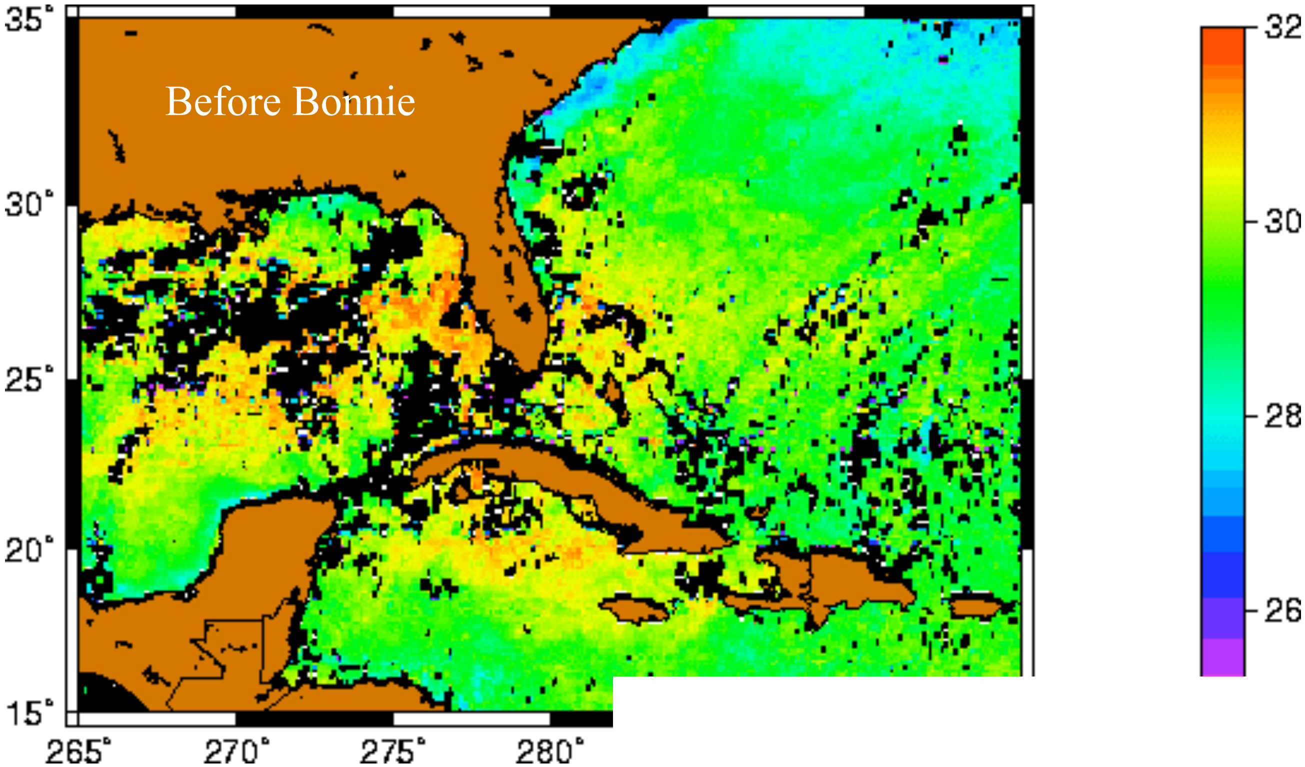
C_D Donelan etal, 2004

$$v_{max}^2 = \frac{T_s - T_o}{T_o}\frac{C_k}{C_D}(k_s^* - k)$$

Maximum azimuthal wind speed (Emanuel, 2004)



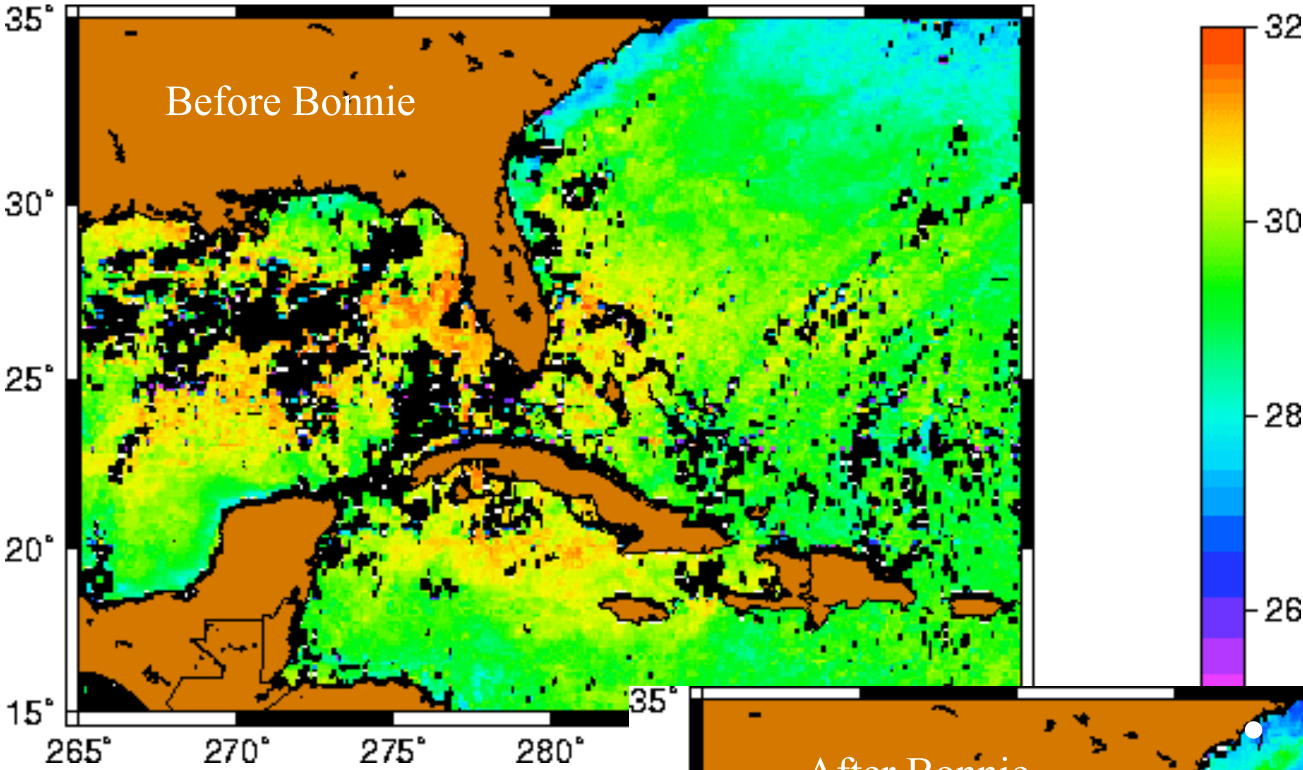


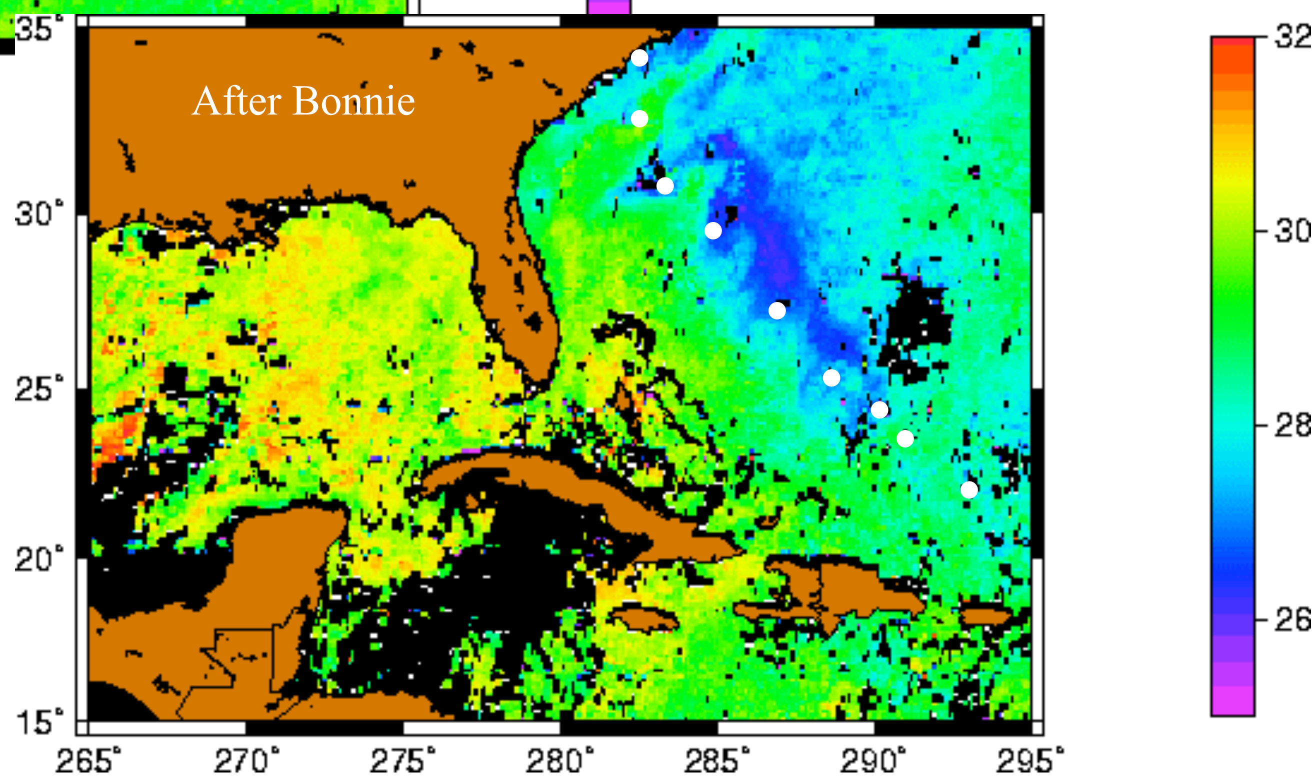


SST Variations, Ocean Mixing and Biology

Shuyi Chen,
RSMAS

SST Variations, Ocean Mixing and Biology

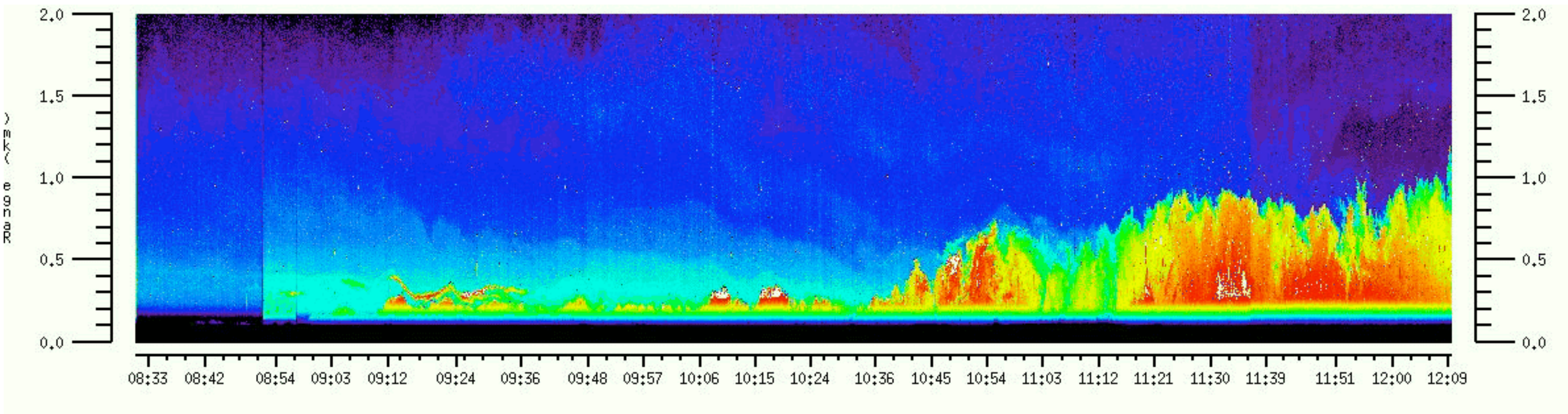


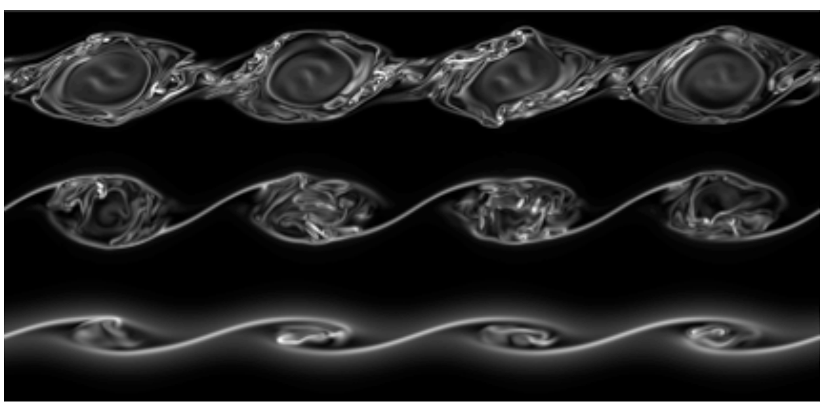


Shuyi Chen, RSMAS

“ … We know what the exact equations for the Planetary Boundary Layer are … let’s just use DNS?”

LIDAR OBSERVATIONS OF PBL DIURNAL EVOLUTION, COURTESY SHANE MAYOR NCAR

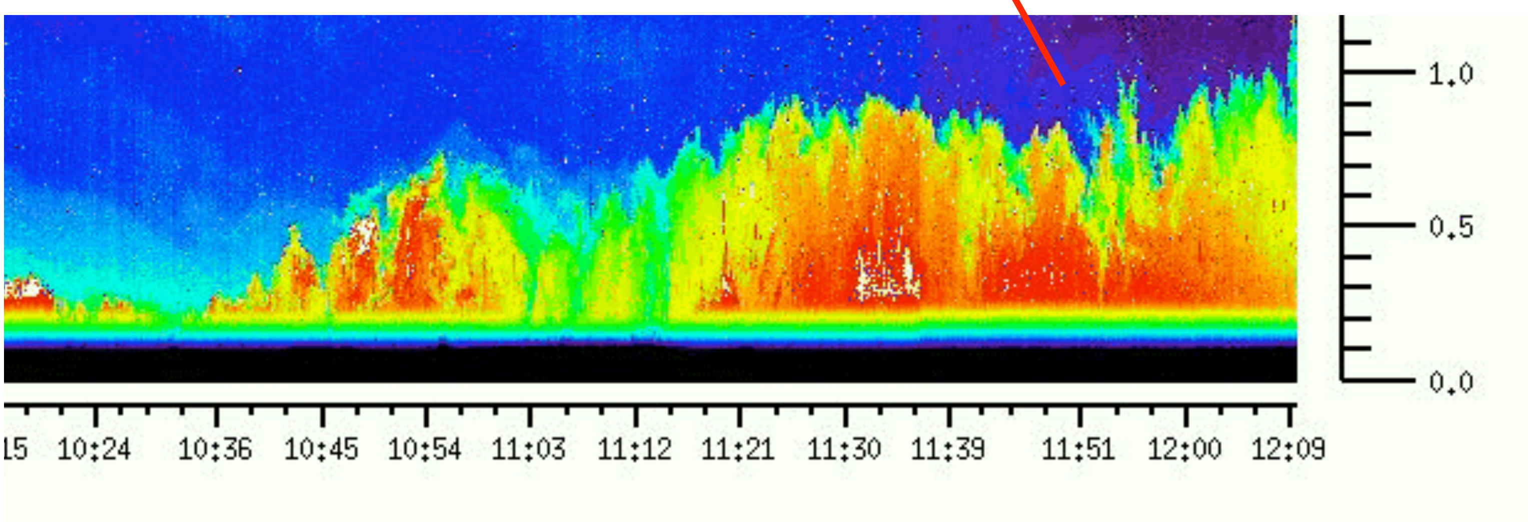




DIRECT NUMERICAL SIMULATIONS OF STRATIFIED KELVIN-HELMHOLTZ FLOW, COURTESY JOSEPH WERNE, CORA

LENGTH AND TIME SCALES OF HIGH-Re CONVECTIVE PBL TURBULENCE

- Energy-containing (large) eddies $\mathcal{L} \sim \mathcal{O}(z_i)$; $\quad z_i \sim 1000$m
- Velocity scale of the (large) eddies $\mathcal{U} \sim (gQ_* z_i/\Theta_o)^{1/3}$; $\quad \mathcal{U} \sim 1$m/s
- Large eddy turnover time $\mathcal{T} = \mathcal{L}/\mathcal{U}$; $\quad \mathcal{T} = 1000$s
- Kolmogorov microscale $\eta = (\nu^3/\epsilon)^{1/4}$; $\quad \eta \sim 1$mm
- Reynolds number $Re_{\mathcal{L}} = \mathcal{U}\mathcal{L}/\nu = 10^6$



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Amount of work (number of mode-steps) is

Pope, p. 348

$$N^3 \cdot N_s \approx Re_{\mathcal{L}}^3$$

Therefore

$$N^3 \cdot N_s \approx 10^{18} \quad \text{for} \quad Re_{\mathcal{L}} = 10^6$$

“ … DNS is mighty useful, however we don’t have enough IBM SP’s to get to a PBL Reynolds number and resolve all scales of a rough wall … We need something else …”

HOW DO WE GET TO OUTDOOR LES

- Spatially filter the full governing equations to eliminate small scale fluctuations
- Subgrid-scale (SGS) challenge
  - Spatial correlations of small scale fluctuations $\neq 0$
  - High Re limit, universality of small scales
  - Building equations for SGS correlations
- Coping with a rough wall boundary

See Tom Lund discussion later for details!

SPATIALLY FILTERED EQUATIONS FOR DRY BOUSSINESQ ROTATING ATMOSPHERIC PBL

$$\frac{\partial \overline{\mathbf{u}}}{\partial t} + (\overline{\mathbf{u}} \cdot \nabla)\overline{\mathbf{u}} \quad = \quad - \mathbf{f} \times \overline{\mathbf{u}} \; - \; \boldsymbol{\nabla}\overline{\pi} \; + \; \hat{\mathbf{z}} g \frac{\overline{\theta'}}{\theta_*} \; + \; \cancelto{0}{\nu \nabla^2 \overline{\mathbf{u}}} \; - \; \boldsymbol{\nabla} \cdot \mathbf{T}$$

$$\frac{\partial \overline{\theta}}{\partial t} \; + \; (\overline{\mathbf{u}} \cdot \nabla)\overline{\theta} \quad = \quad \cancelto{0}{\alpha \nabla^2 \overline{\theta}} \; - \; \boldsymbol{\nabla} \cdot \mathbf{B}$$

$$\boldsymbol{\nabla} \cdot \overline{\mathbf{u}} \; = \; 0 \;\; \Longrightarrow \;\; \nabla^2 \overline{\pi} \; = \; \overline{s}$$

New terms! Subgrid-scale momentum and scalar fluxes

$$\begin{aligned} \mathbf{T} \quad &= \quad \overline{u_i u_j} \; - \; \overline{u_i}\,\overline{u_j} \\ \mathbf{B} \quad &= \quad \overline{u_i \theta} \; - \; \overline{u_i}\,\overline{\theta} \end{aligned}$$

see [*Lilly*(1967), *Deardorff*(1971), *Leonard*(1974), *Moeng*(1984)]

COMMENTS ON THE LES EQUATIONS AND THE SUBGRID SCALE STRESS TENSOR

$$
\begin{aligned}
\frac{\partial \overline{u}_i}{\partial t} + \frac{\partial \overline{u}_i \overline{u}_j}{\partial x_j} &= -\frac{1}{\rho}\frac{\partial \overline{p}}{\partial x_i} - \frac{\partial}{\partial x_j}\left(\mathcal{T}_{ij} - \nu \cancelto{0}{\frac{\partial \overline{u}_i}{\partial x_j}}\right) \\
\mathcal{T}_{ij} &= \overline{u_i u_j} - \overline{u}_i \overline{u}_j
\end{aligned}
$$

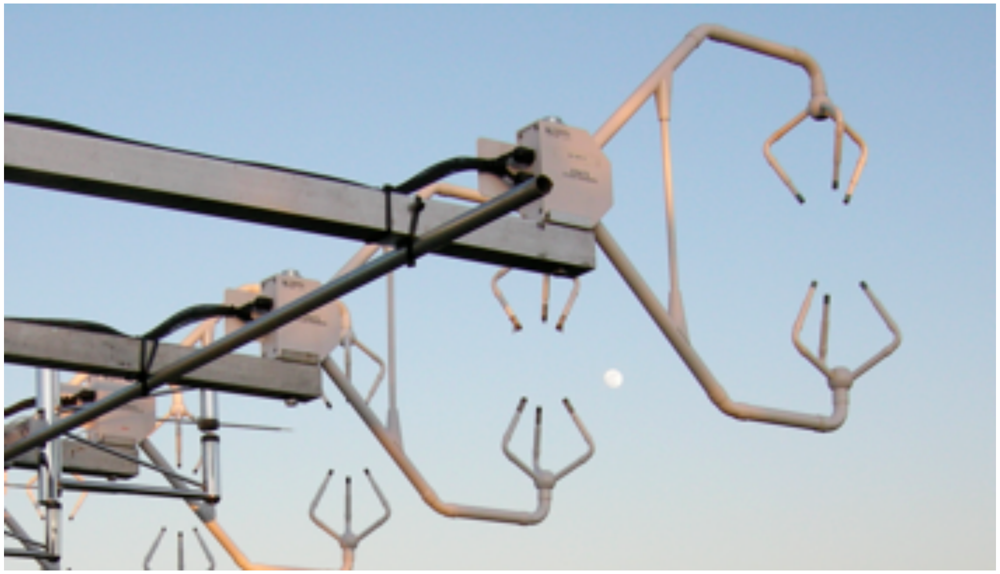
- The LES equations contain two parameters, Re and the filter properties (loosely the shape of the filter and its cutoff k_c)

- Solutions of the LES equations are *stochastic*, $i.e.$, $\overline{u}_i$ is a random variable in $(\boldsymbol{x}, t)$

- $\mathcal{T}_{ij}$ is unknown! It needs to be expressed in terms of known resolved fields $\overline{u}_i$

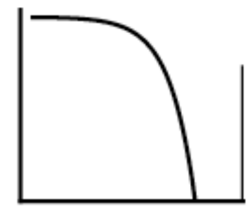
- Subgrid scale $\mathcal{T}_{ij}$ is stochastic and depends on the filter. This makes $\overline{u}_i$ also filter dependent.

SIMPLE (CHEAP) FILTERING EXAMPLE ...

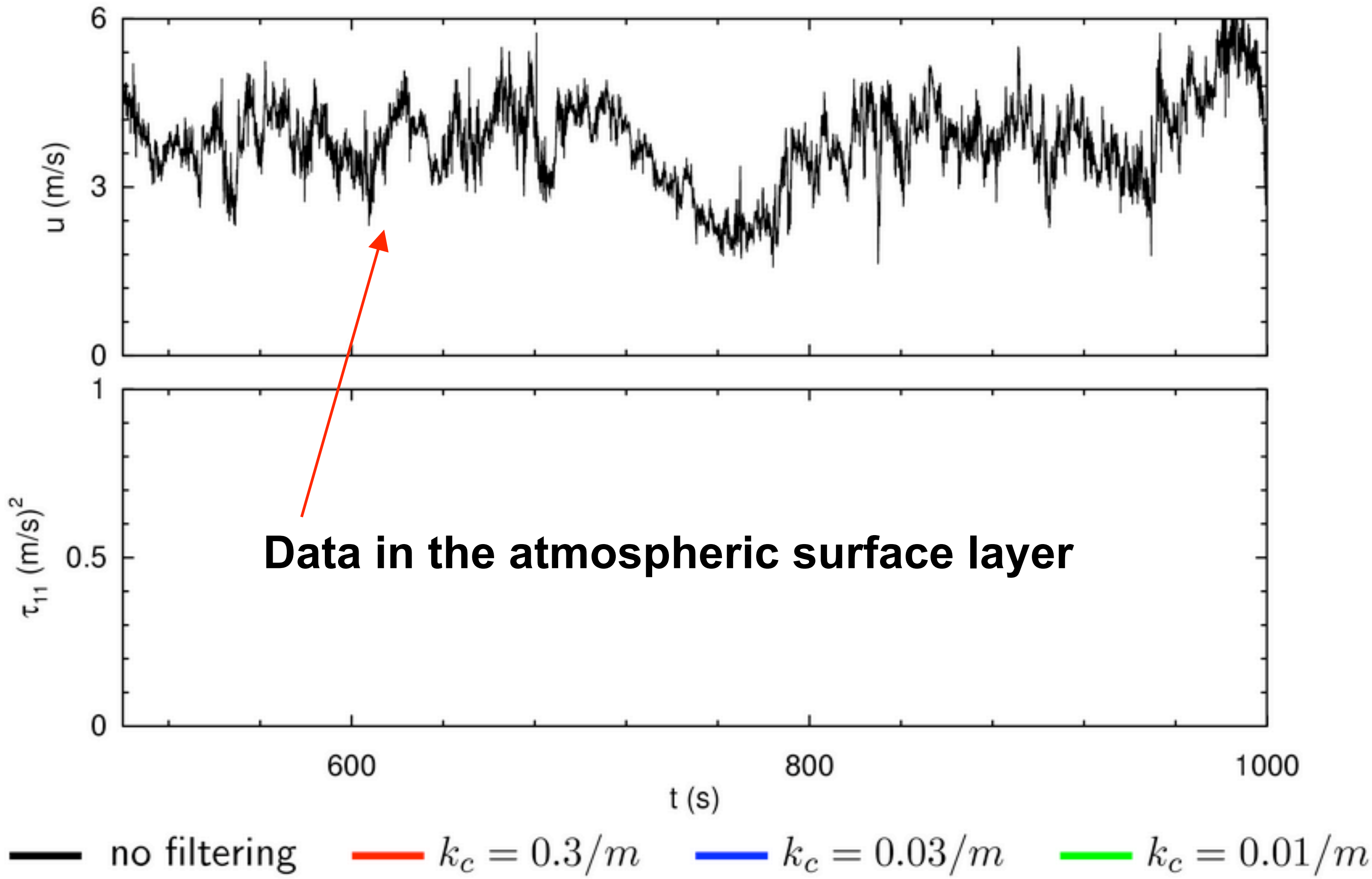


$$\tau_{11} = \overline{u_1 u_1} - \overline{u}_1\,\overline{u}_1$$

MOVING BETWEEN DNS $\Longleftrightarrow$ LES $\Longleftrightarrow$ RANS



What happens to $\overline{u}_i$ and $\mathcal{T}_{ij}$ as we vary the filter cutoff k_c ?



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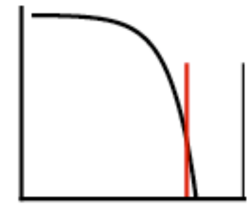

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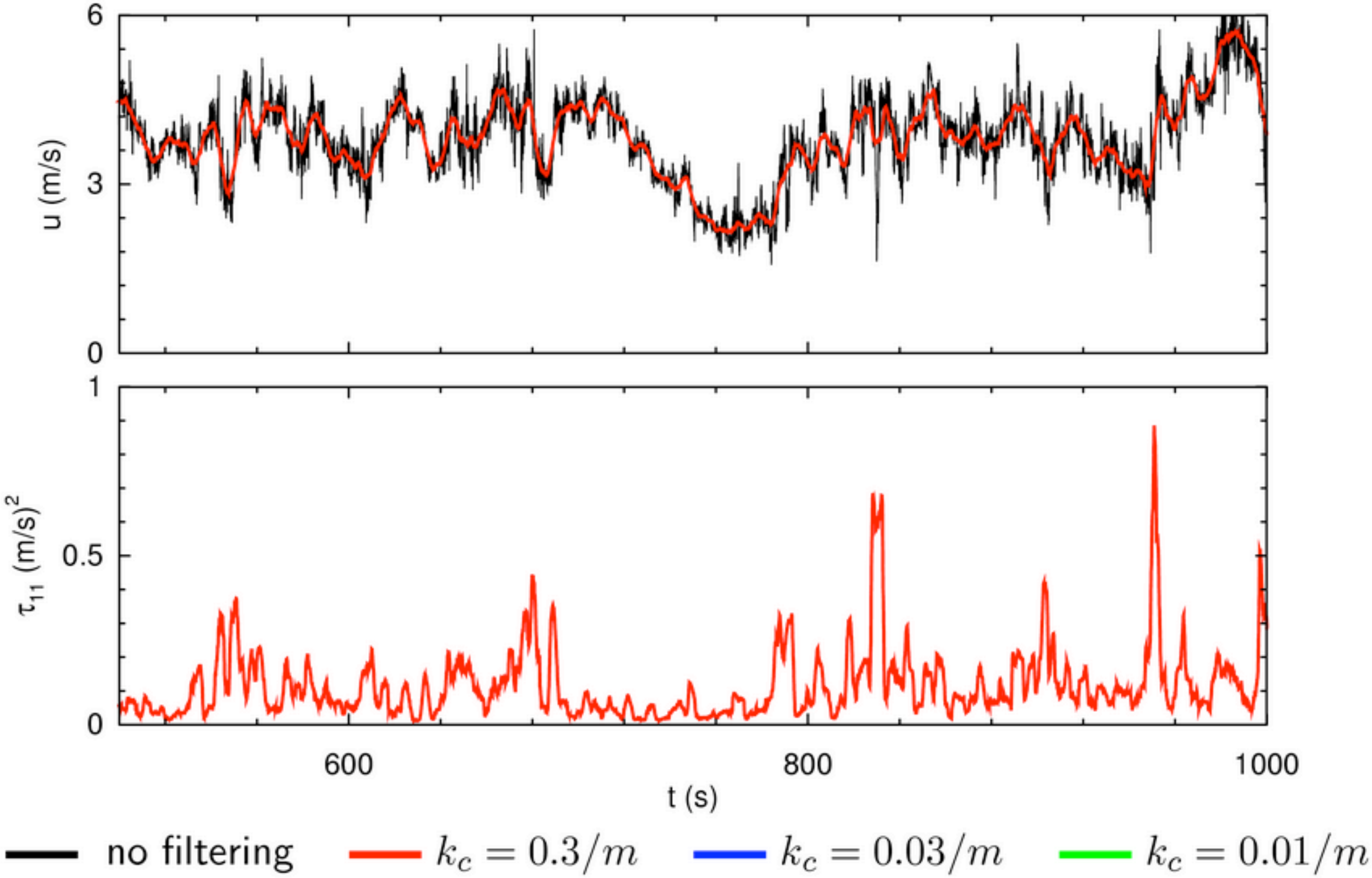

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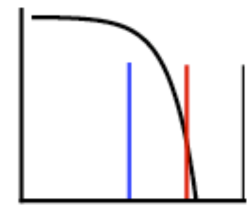

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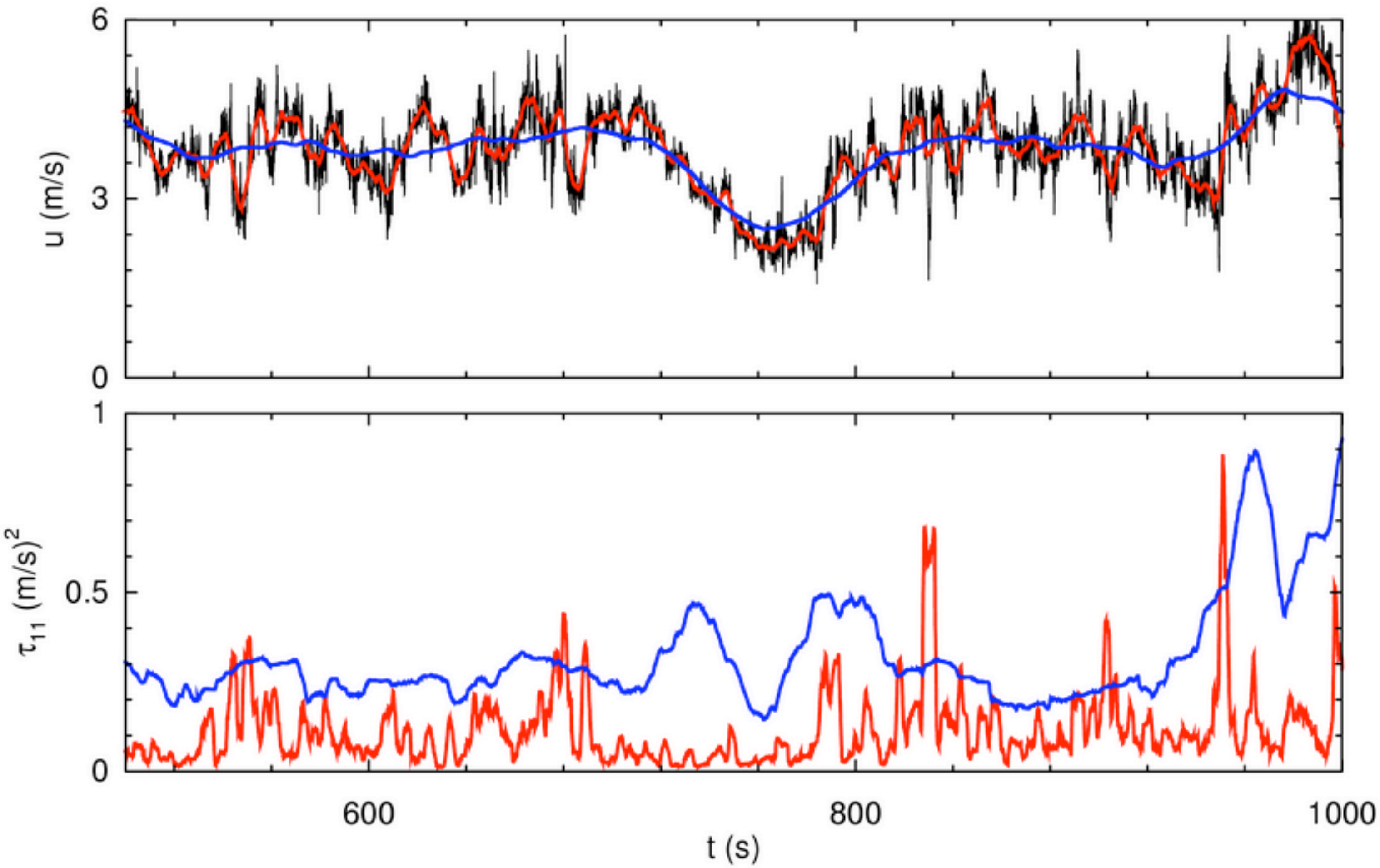
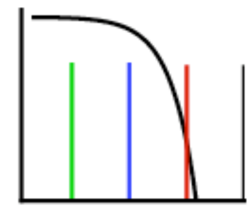


no filtering $k_c = 0.3/m$ $k_c = 0.03/m$ $k_c = 0.01/m$

MOVING BETWEEN DNS $\Longleftrightarrow$ LES $\Longleftrightarrow$ RANS



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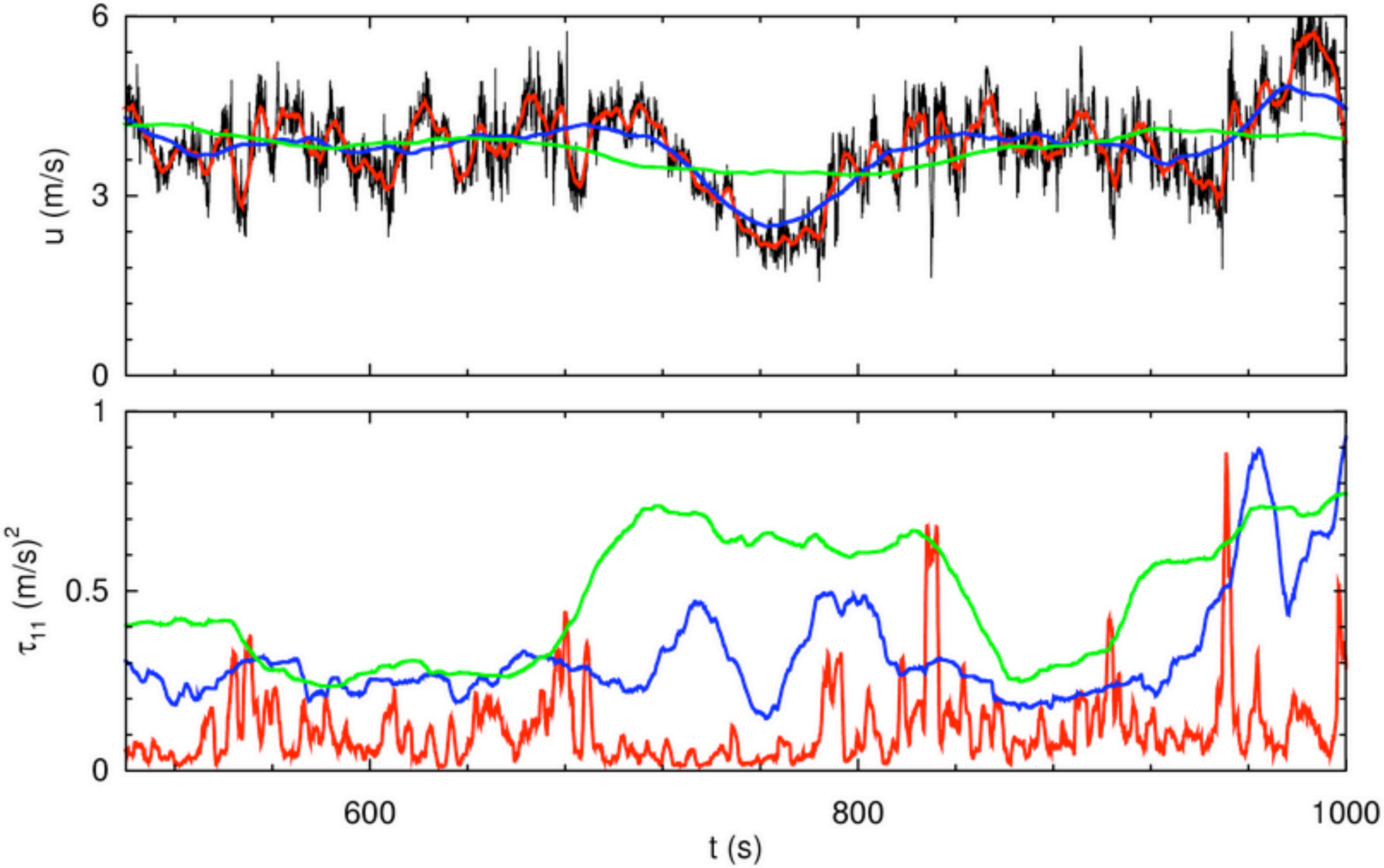

no filtering $\quad k_c = 0.3/m \quad k_c = 0.03/m \quad k_c = 0.01/m$

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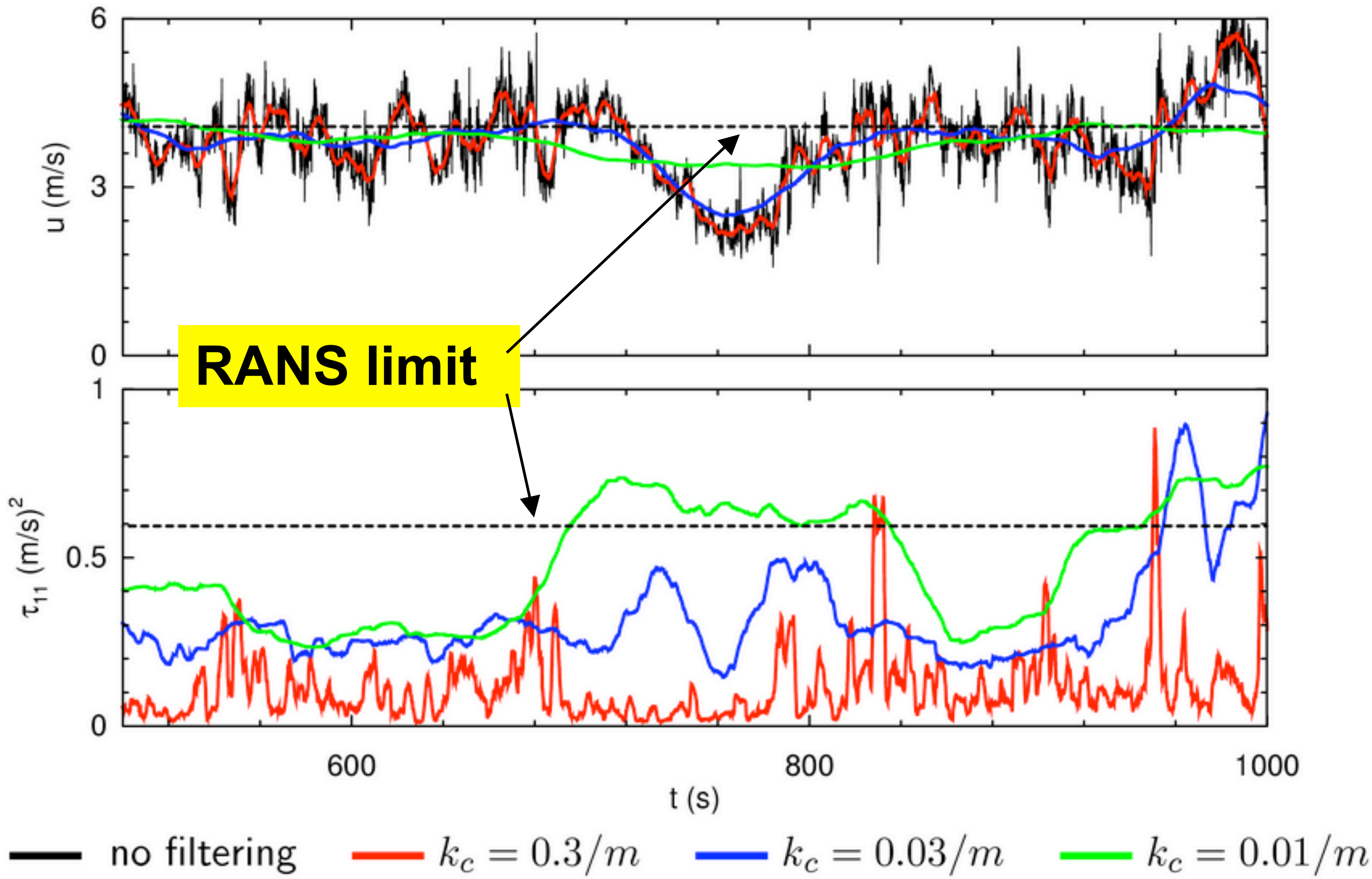


FLOW NEAR ROUGH BOUNDARIES

- Treatment of the lower boundary is the fundamental difference between Quasi-Direct Numerical Simulation (QDNS) [*Spalart et al.*(1997)]) and $1/Re \longrightarrow 0$ LES [*Deardorff*(1970)]
- Impossible to resolve all separation points and wakes (at high Re) at a complex boundary,$e.g.$, the boundary might not even be defined!
- Numerical commutation errors [*Berselli et al.*(2006)] are mixed up with physical modeling
- Typical outdoor LES uses simple near wall models
  - Based on ensemble average ideas (Monin-Obukhov similarity theory)
  - Generate spatial fluctuations by applying MO on a point-by-point basis or using a linearization of the quadratic drag formula [*Moeng*(1984)]
- **Sometimes you don't get to choose where $1/\triangle_f$ sits!**
- There's work to be done near rough boundaries

FLOW OVER A ROUGH BOUNDARY

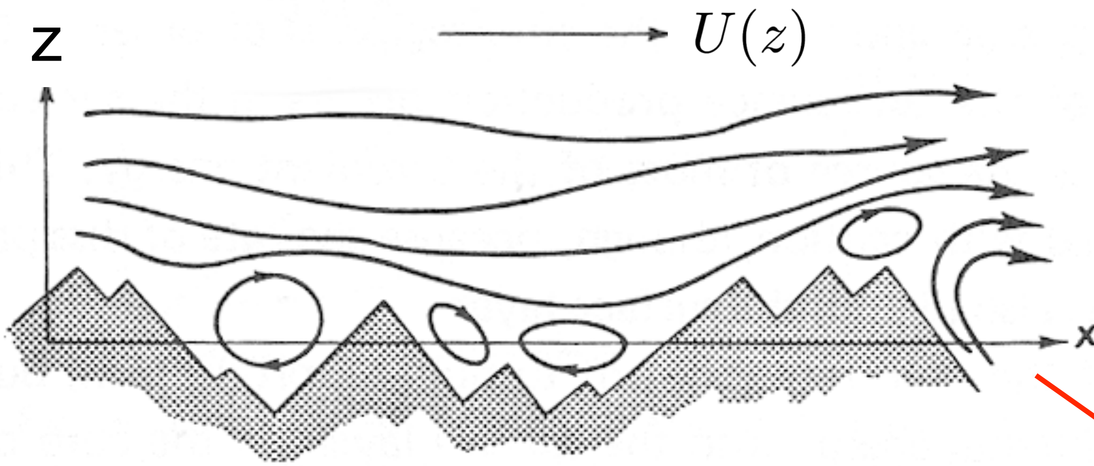
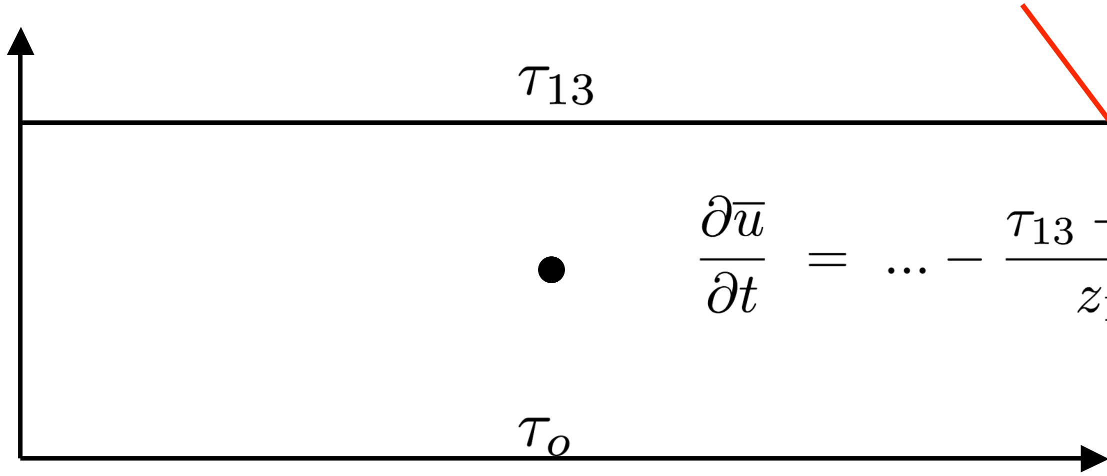


Figure 5.9. Flow over a rough surface.

$$\tau_o = f(z_o, \overline{u}, z_1, z_1/L)$$



$$\frac{\partial \overline{u}}{\partial t} = \ldots - \frac{\tau_{13} - \tau_o}{z_1}$$

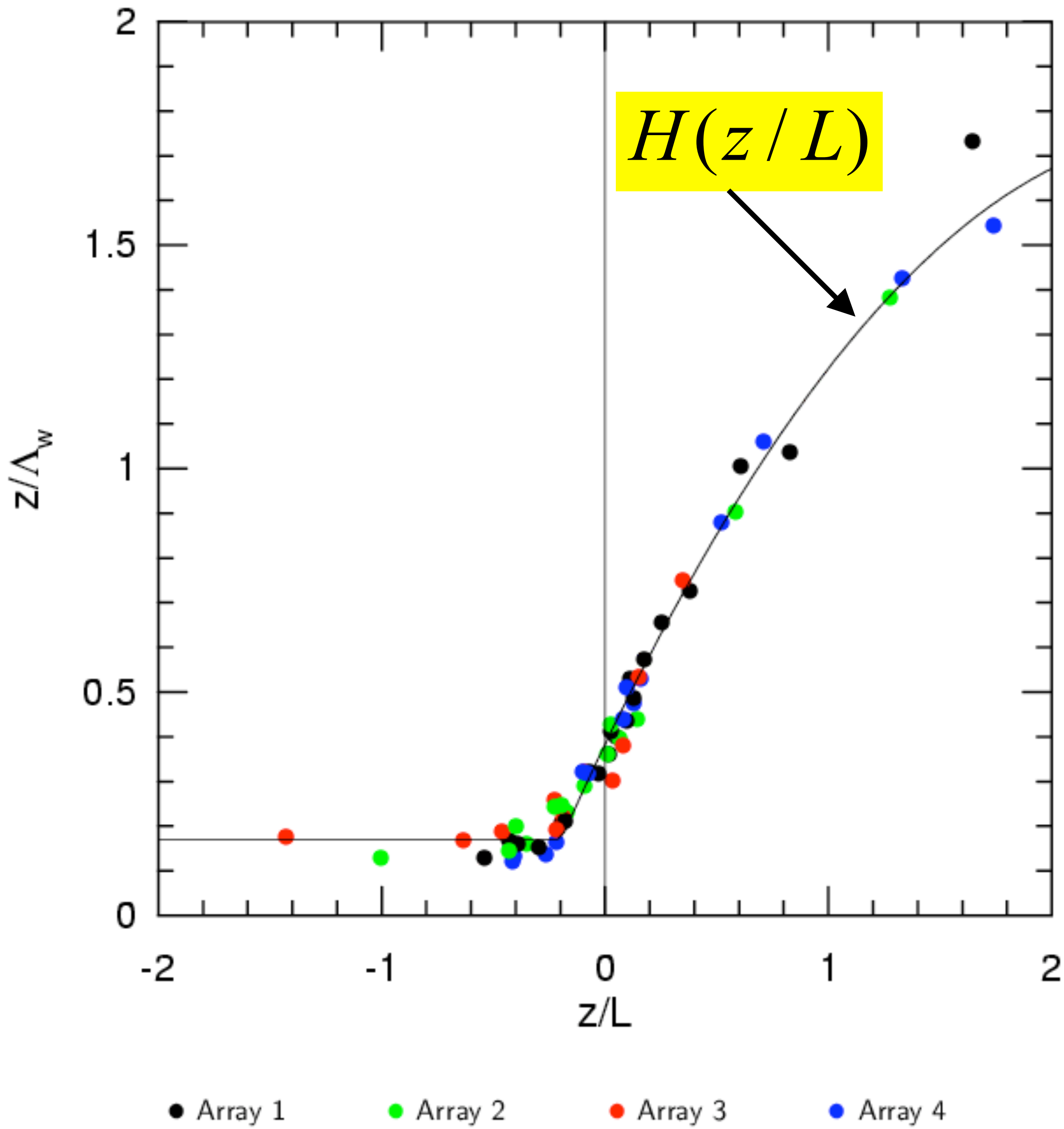
At a rough boundary all the flux is subgrid

SURFACE LAYER MEASUREMENTS AND LES: $\Lambda_w/\triangle_f$ AT FIRST GRID POINT OFF THE SURFACE

- $\Lambda_w \Longrightarrow$ horizontal wavelength of the peak in the vertical velocity spectrum
  - w is least resolved in LES
  - Obeys MO scaling, $i.e.$, , depends on (z, L)

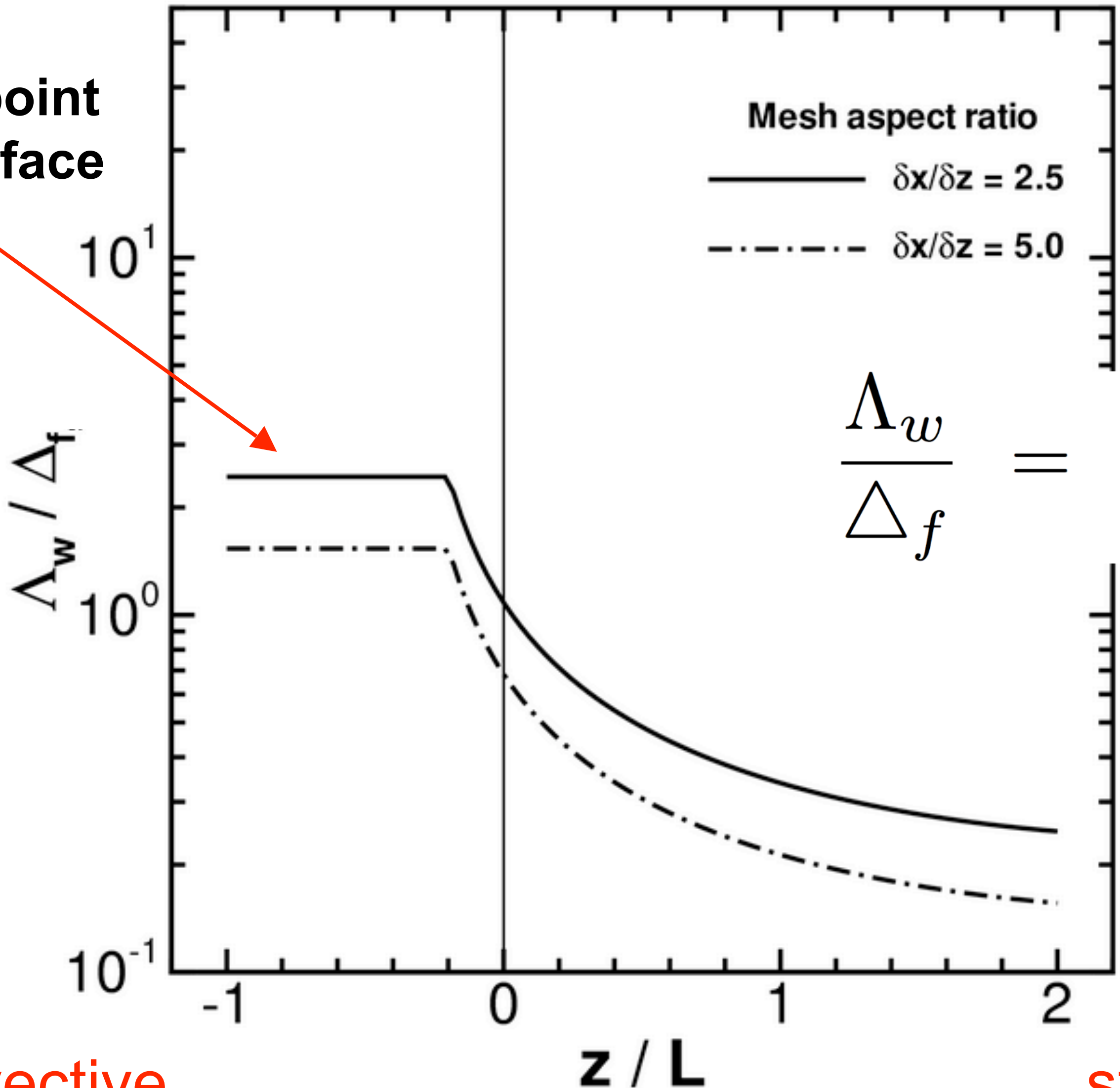
- $\triangle_f \sim (\delta_x \delta_y \delta_z)^{1/3}$ the cell averaging volume

Dependence of Peak Wavelength on Stratification



RATIO OF TURBULENCE LENGTH SCALE TO FILTER WIDTH AT FIRST LES GRIDPOINT $z = \delta_z$

First gridpoint off the surface



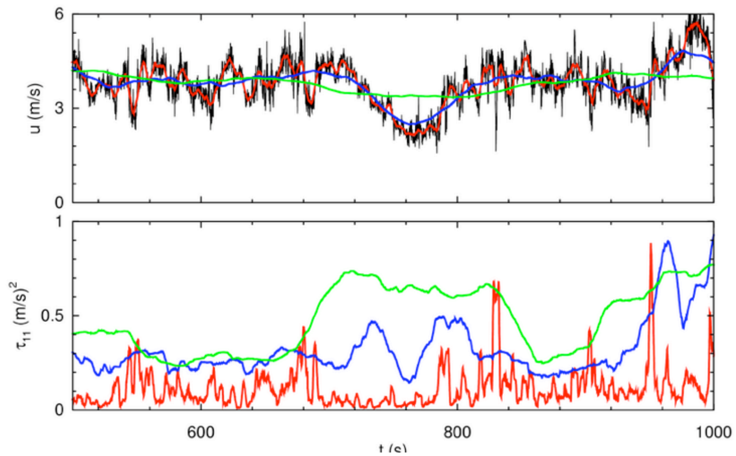
$$\frac{\Lambda_w}{\triangle_f} = \frac{A}{H(\delta z/L)}$$

convective

stable

Can we use targeted observations to provide insight as to the nature of SGS motions in high Re PBLs?

HIGH REYNOLDS NUMBER OBSERVATIONS AND LES



- **SINGLE-POINT MEASUREMENTS**
  - Cannot be used directly to improve LES

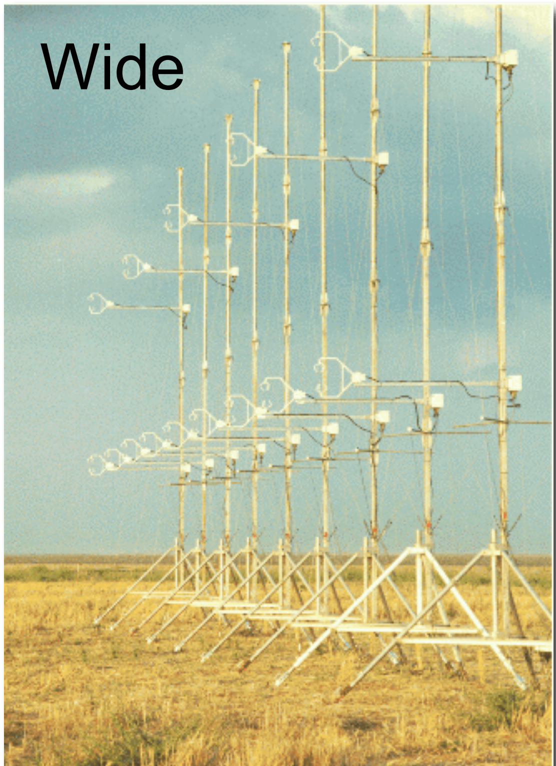
- **MULTI-POINT MEASUREMENTS**
  - Span a range of filter widths, $e.g.$, $\mathcal{O}$(m) to $\mathcal{O}$(100m)
  - Ideally 3-D, time varying "volume" of turbulence and scalars in canonical flows with shear, stratification, near boundaries, ...
  - Horizontal Array Turbulence Study field campaigns, HATS (2000), OHATS (2004), CHATS (2007), AHATS (2008)

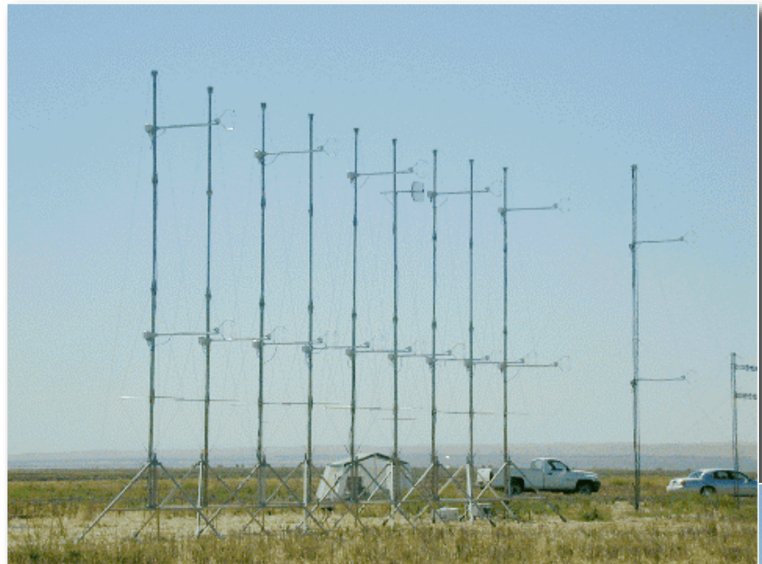
HATS CONFIGURATIONS

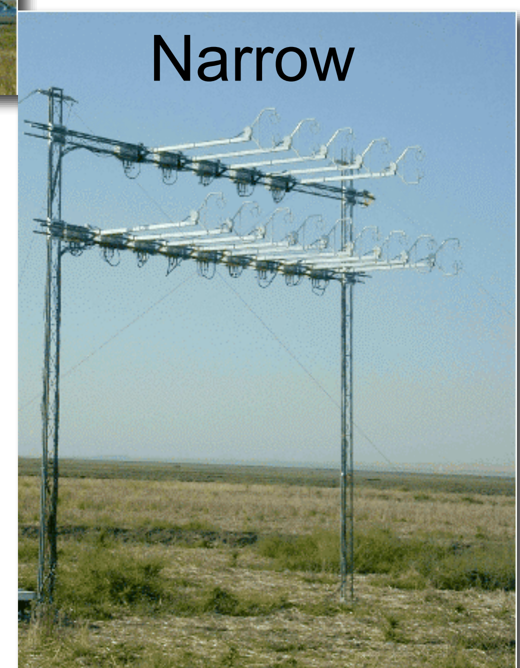
$\sim 36 \; cases$

$-1.2 < z/L < 1.6$

$0.15 < \Lambda_w/\triangle_f < 15$







RATIONALE FOR EXPERIMENTAL DESIGN

- Allows *spatial* filtering of flow field and decomposition into resolved and subfilter scale velocities $(\overline{U_i}, u_i)$:

$$U_i = \overline{U_i} + u_i \equiv \int U(x'_j) G(x_i, x'_j) dx'_j + u_i$$

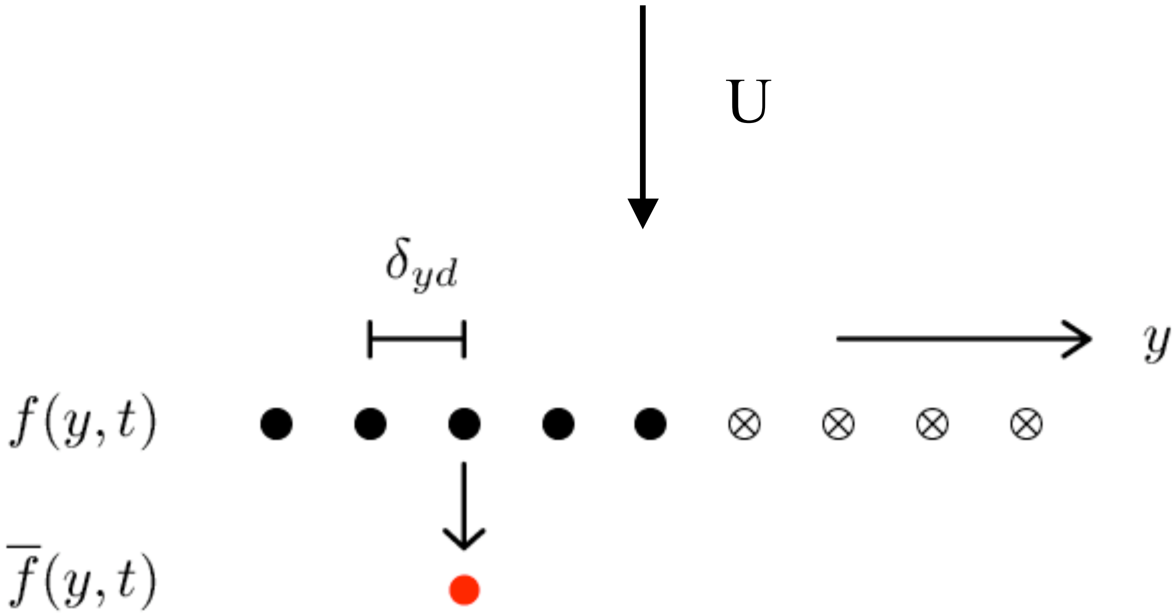
- Allows construction of SFS fluxes:

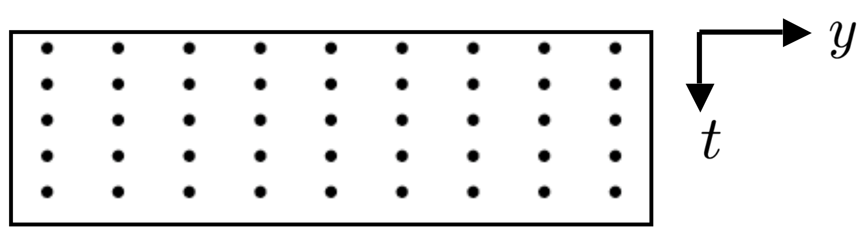
$$\mathcal{T}_{ij} = \overline{U_i U_j} - \overline{U_i}\;\overline{U_j}$$

- Allows measurement of resolved gradients $\partial\overline{U_i}/\partial x$, $\partial\overline{U_i}/\partial y$ and $\partial\overline{U_i}/\partial z$

- Allows expansion of SFS fluxes $\mathcal{T}_{ij}$ into Leonard, Cross, and Reynolds terms which requires *double* spatial filtering, *e.g.*, $\overline{\overline{U_i} u_j}$

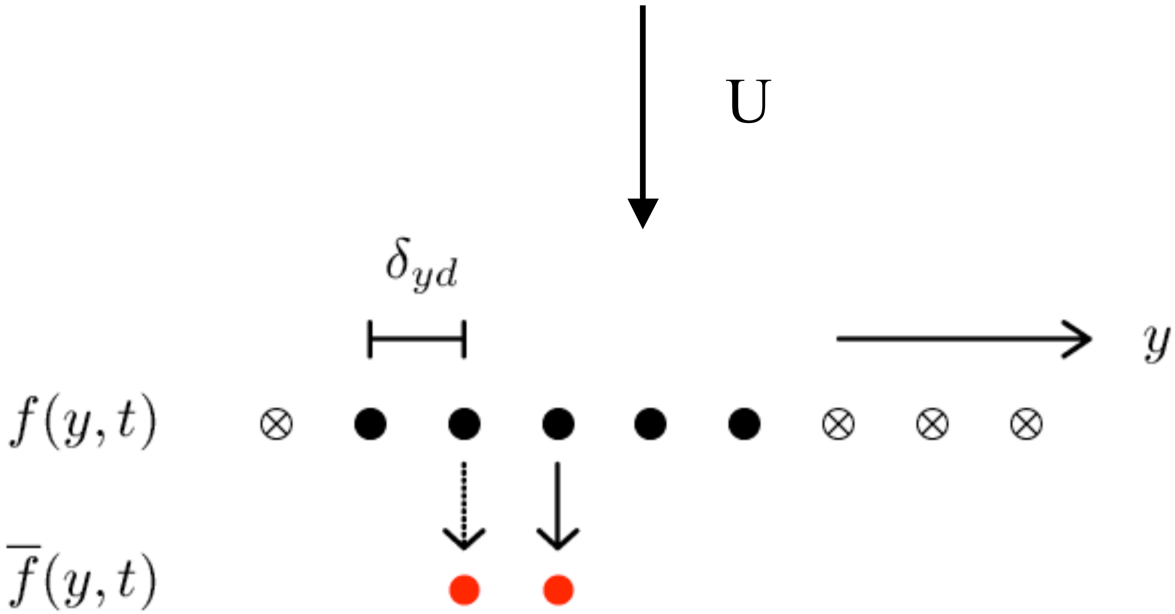
AN EXAMPLE OF LATERAL (Y) FILTERING





``2D plane of turbulence”

AN EXAMPLE OF LATERAL (Y) FILTERING



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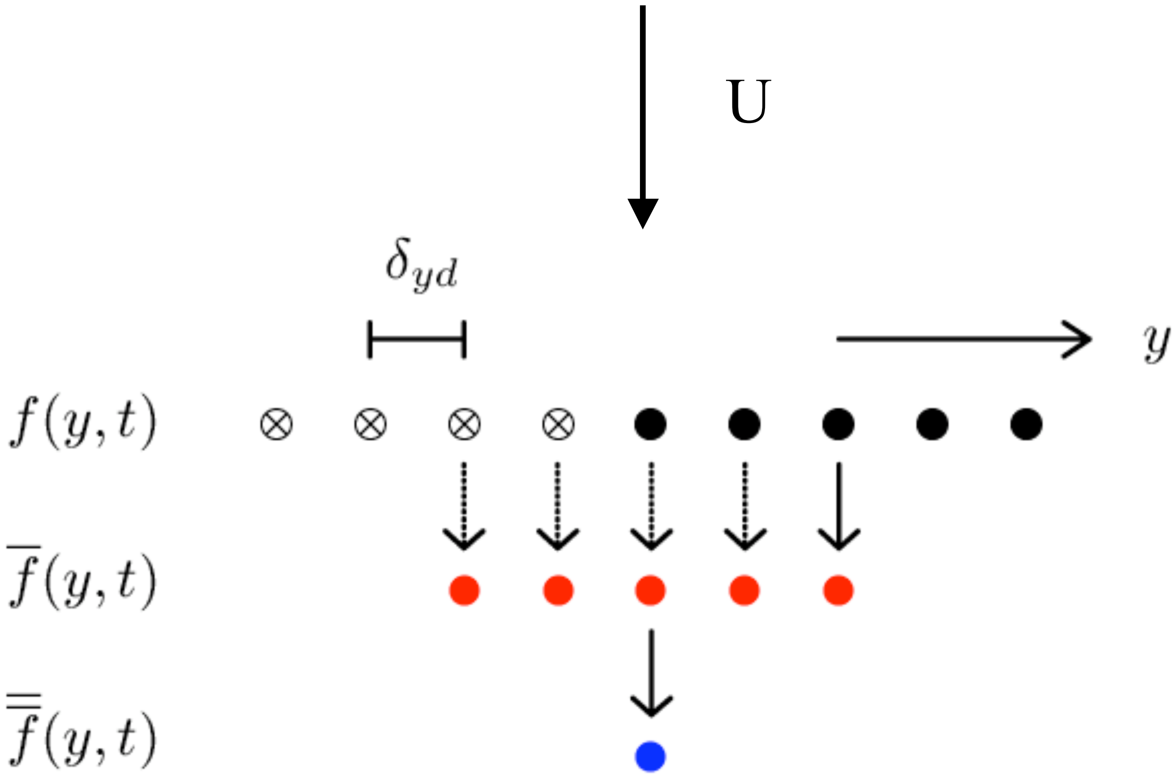
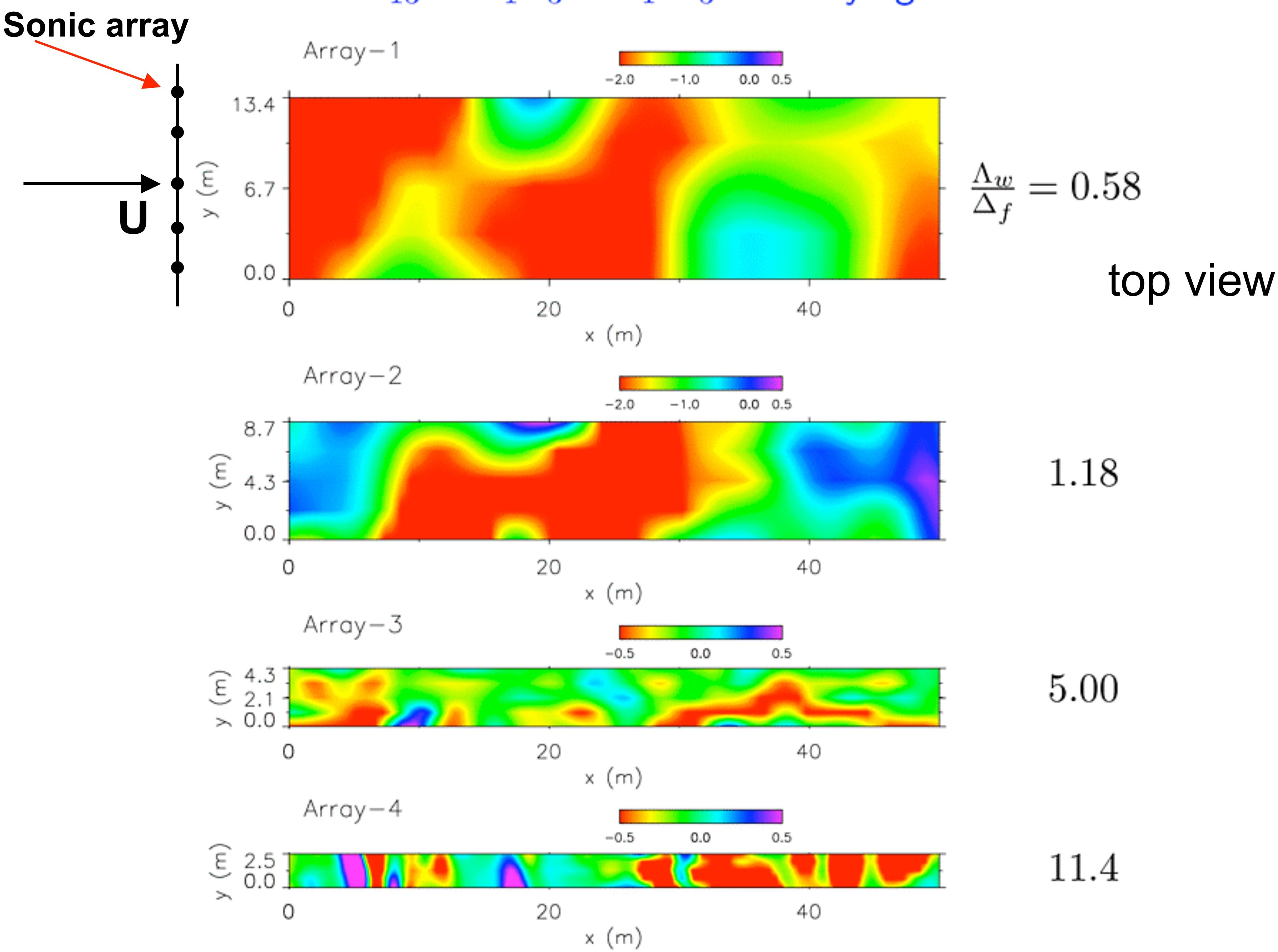
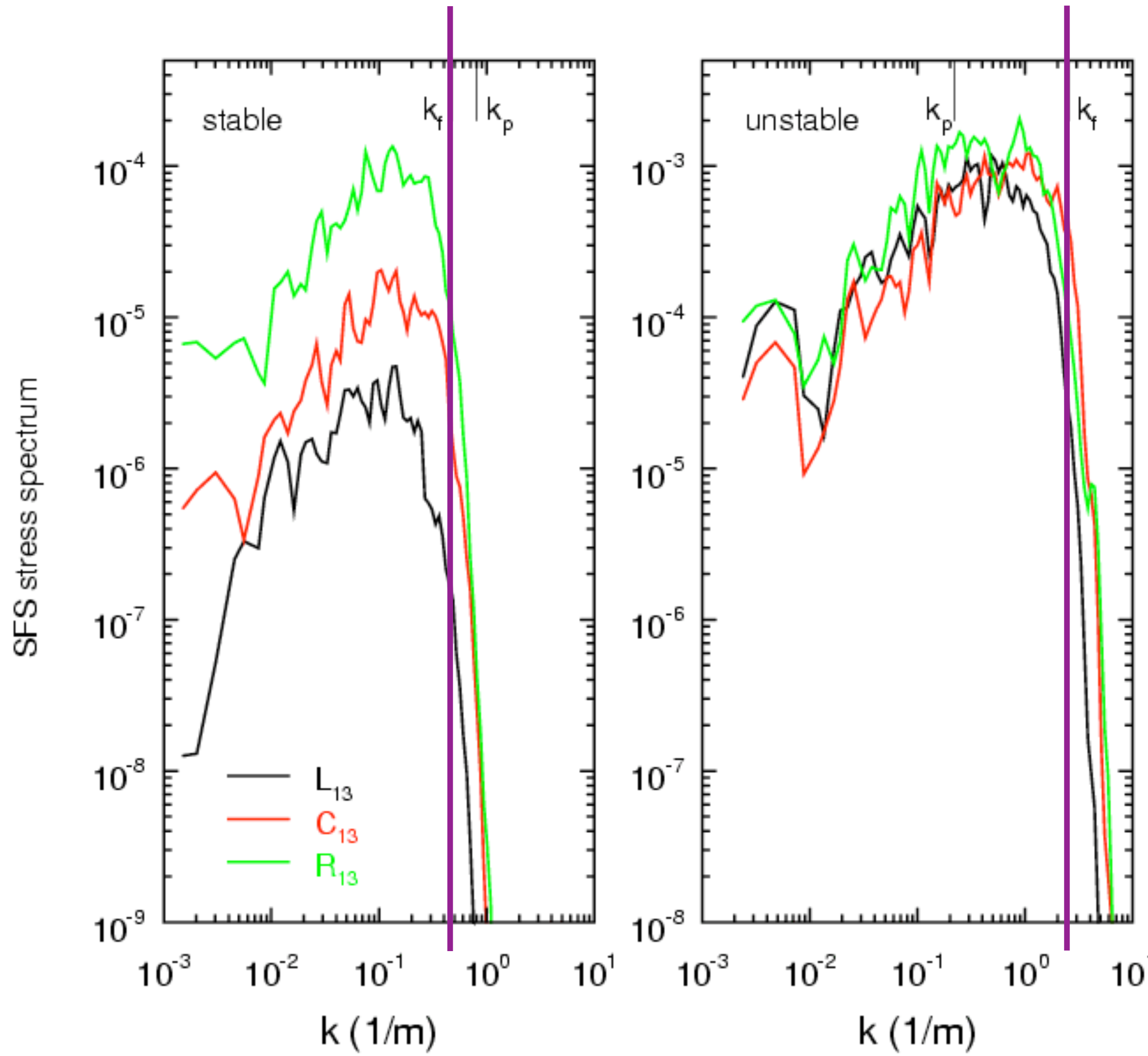


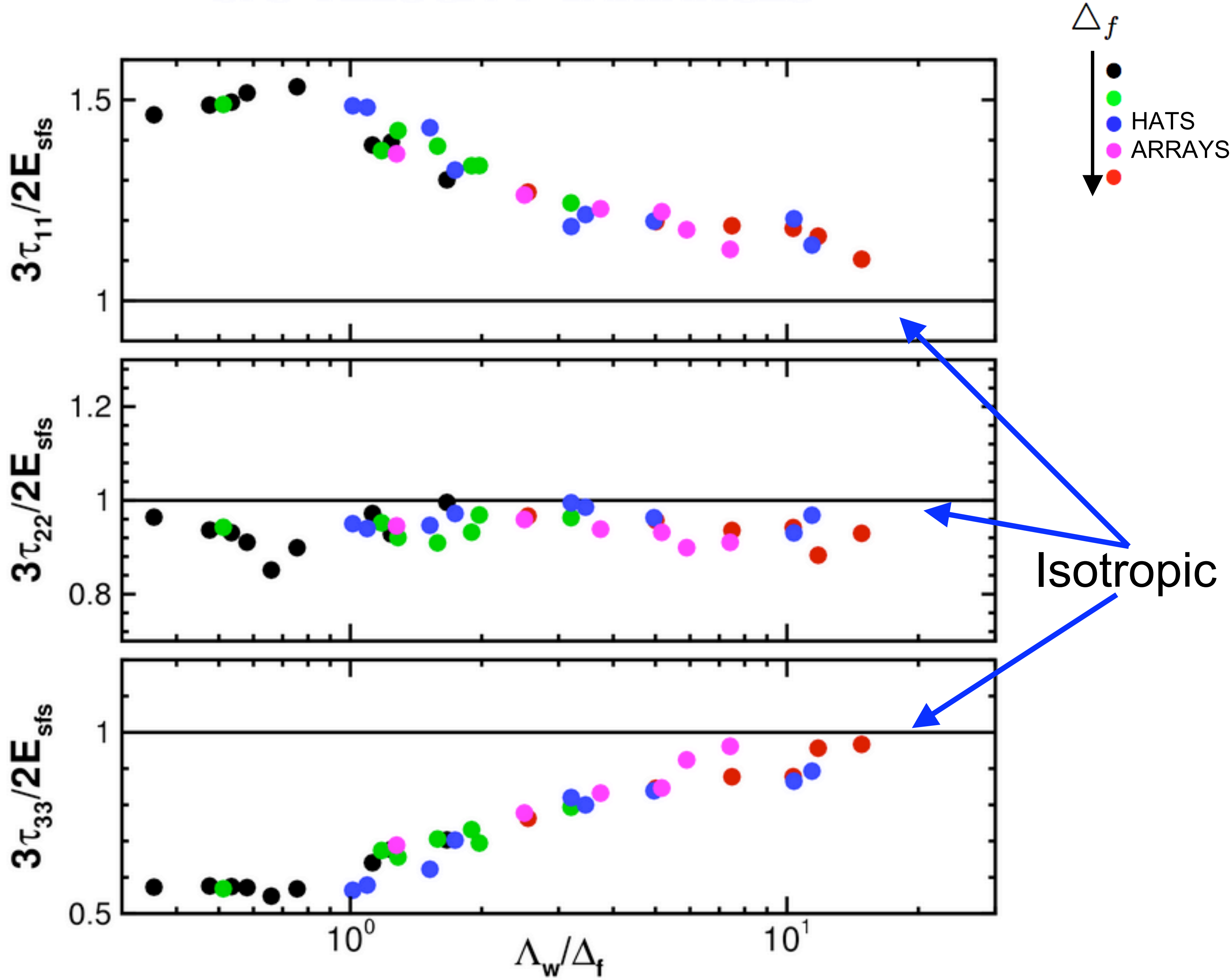
SFS Flux $\tau_{13} = \overline{U_1 U_3} - \overline{U_1}\ \overline{U_3}$ for Varying Filter Widths



Spectra of Leonard, Cross, Reynolds Terms for (1,3) Component



SFS VELOCITY VARIANCES



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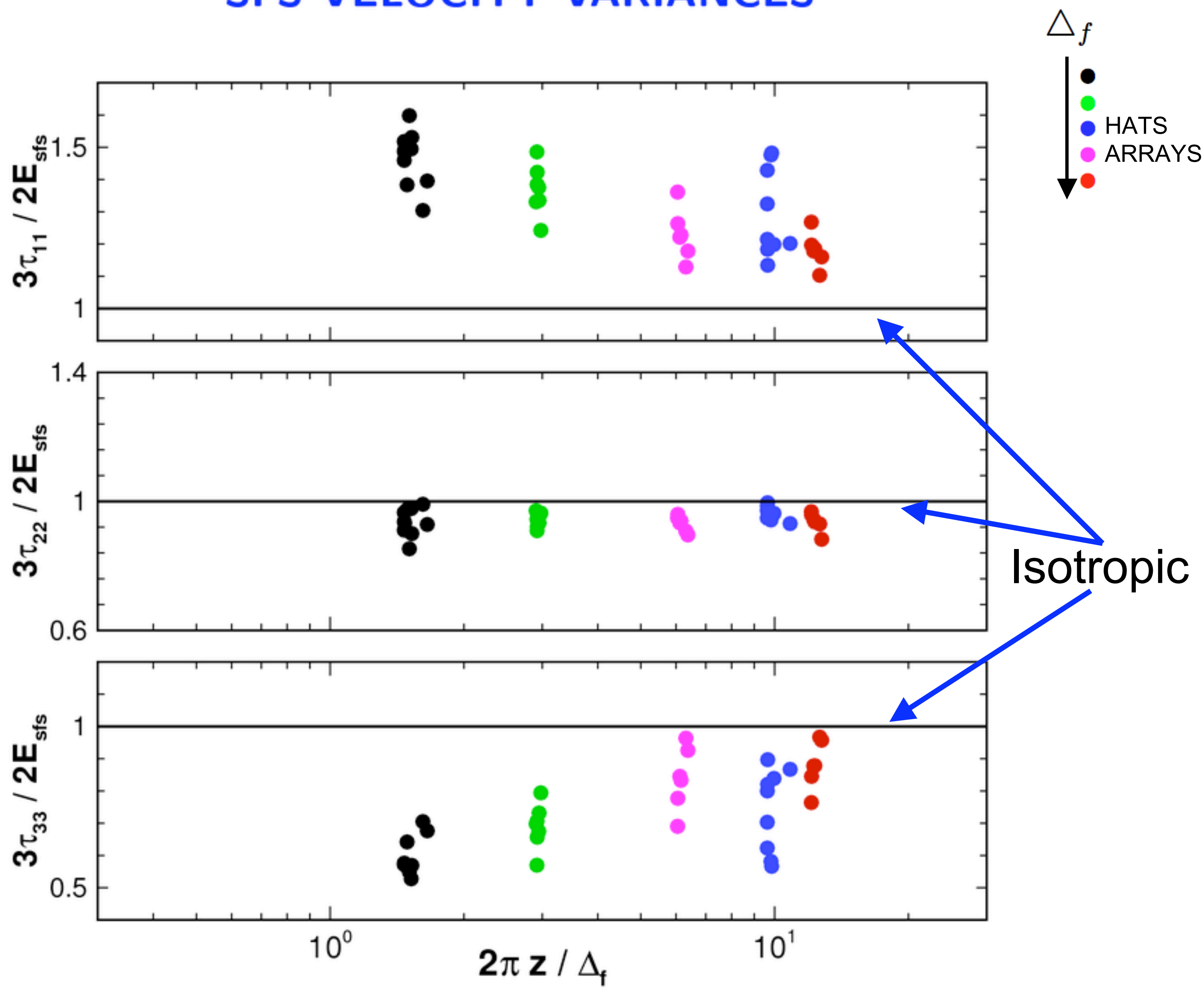
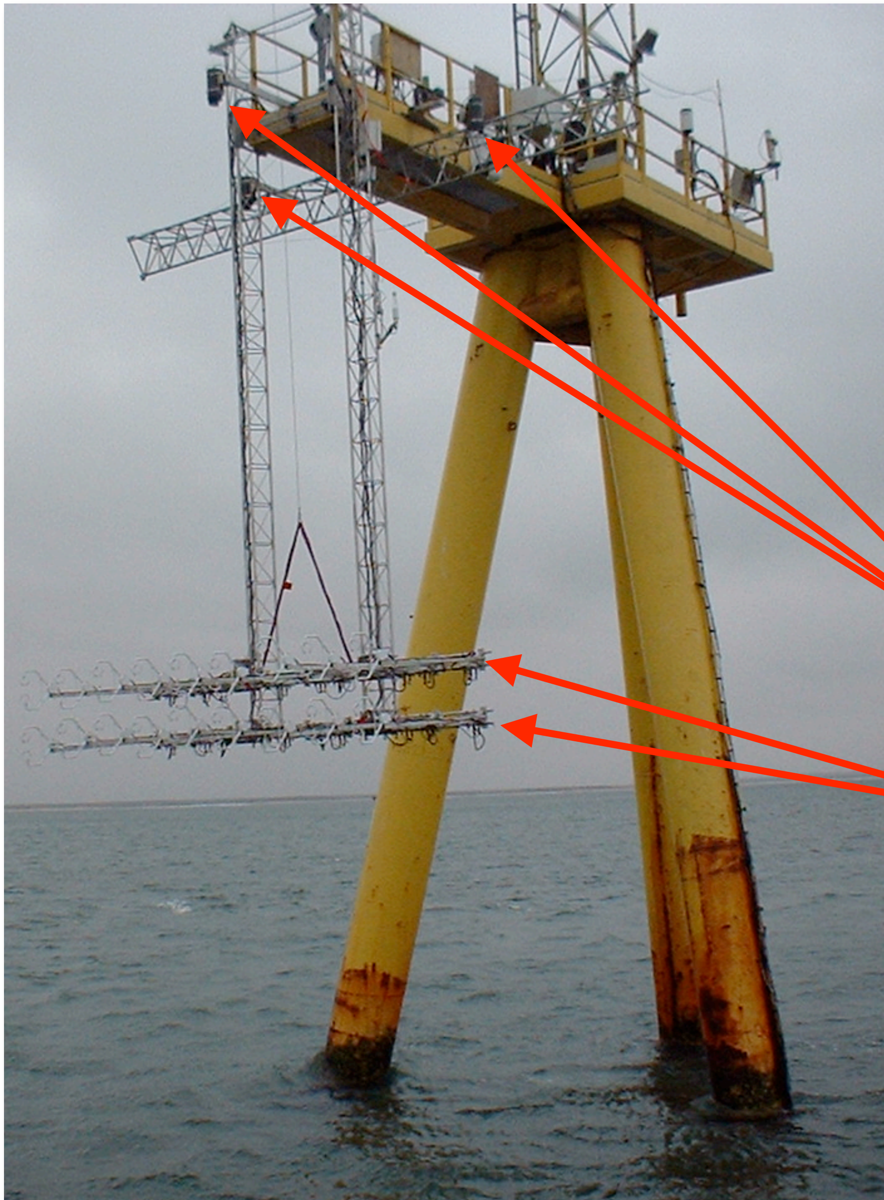
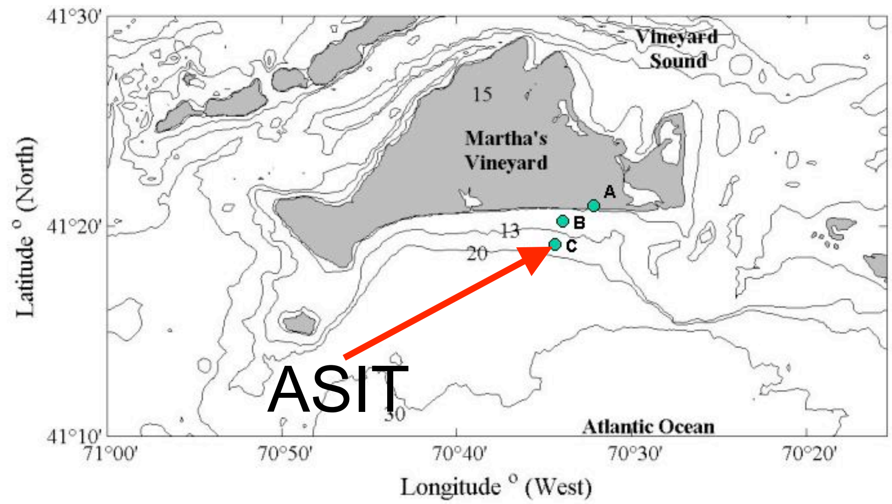


OHATS FIELD CAMPAIGN



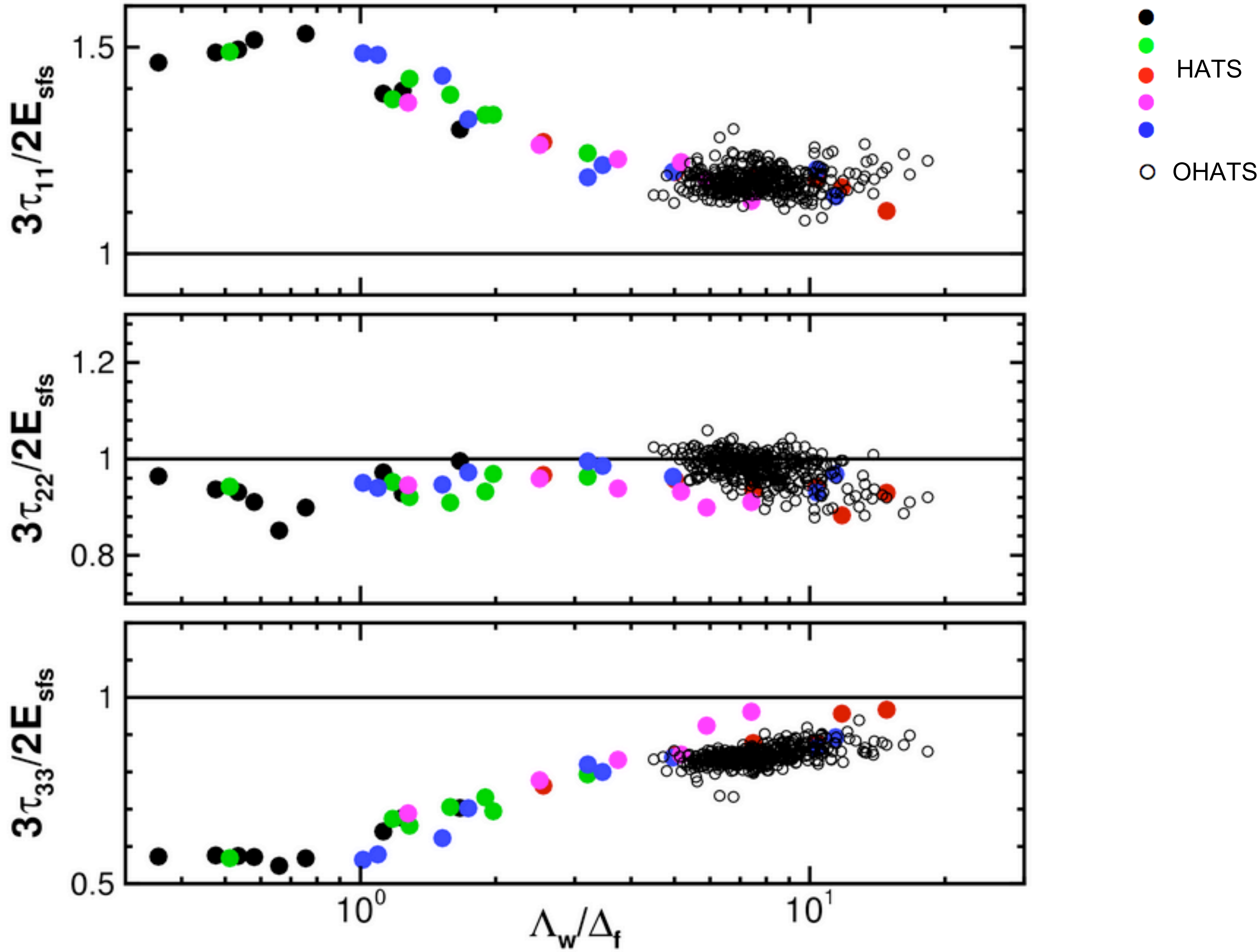


Laser altimeters

18 CSATS

275 hours ``12 days of data” analyzed

SFS VELOCITY VARIANCES



RATE EQUATIONS FOR SUBGRID DEVIATORIC STRESS

- **What are the parent equations for the Smagorinsky model?**

RATE EQUATIONS FOR SUBGRID DEVIATORIC STRESS

The SGS stress is

$$\tau_{ij} = \overline{u_i u_j} - \overline{u}_i \overline{u}_j$$

To get the "rate equation" for SGS τ_{ij}

$$\frac{\partial \tau_{ij}}{\partial t} = \left[\overline{u_j \frac{\partial u_i}{\partial t}} - \overline{u_j} \frac{\partial \overline{u}_i}{\partial t} \right]$$

Substitution steps:

$$u_j \frac{\partial u_i}{\partial t} = u_j \mathcal{R}_i \qquad \overline{u}_j \frac{\partial \overline{u}_i}{\partial t} = \overline{u}_j \overline{\mathcal{R}}_i$$

The difference is now τ_{ij} is the deviatoric stress, *i.e.*, $-2/3 e \delta_{ij}$

Considerable Algebra !

RATE EQUATIONS FOR SUBGRID DEVIATORIC STRESS

- **What are the parent equations for the Smagorinsky model?**
  - Lilly (1967), Deardorff (1973), Wyngaard (2004), Hatlee & Wyngaard (2007)

$$
\begin{aligned}
\frac{D\tau_{ij}}{Dt} \quad = \quad & \frac{2}{3}e\left(\frac{\partial \overline{u}_i}{\partial x_j} + \frac{\partial \overline{u}_j}{\partial x_i}\right) \\
& -\left[\tau_{ik}\frac{\partial \overline{u}_j}{\partial x_k} + \tau_{jk}\frac{\partial \overline{u}_i}{\partial x_k} - \frac{1}{3}\delta_{ij}\tau_{kl}\left(\frac{\partial \overline{u}_k}{\partial x_l} + \frac{\partial \overline{u}_l}{\partial x_k}\right)\right] \\
& -\frac{1}{\rho}\left[\overline{p\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)} - \overline{p}\left(\frac{\partial \overline{u}_i}{\partial x_j} + \frac{\partial \overline{u}_j}{\partial x_i}\right)\right] \\
& + \text{transport} \; + \; \text{buoyancy production}
\end{aligned}
$$

Isotropic production

Pressure destruction

Anisotropic deviatoric production

RATE EQUATIONS FOR SUBGRID DEVIATORIC STRESS

- **What are the parent equations for the Smagorinsky model?**
  - Lilly (1967), Deardorff (1973), Wyngaard (2004), Hatlee & Wyngaard (2007)

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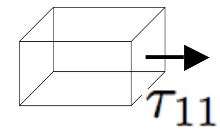
Rotta model

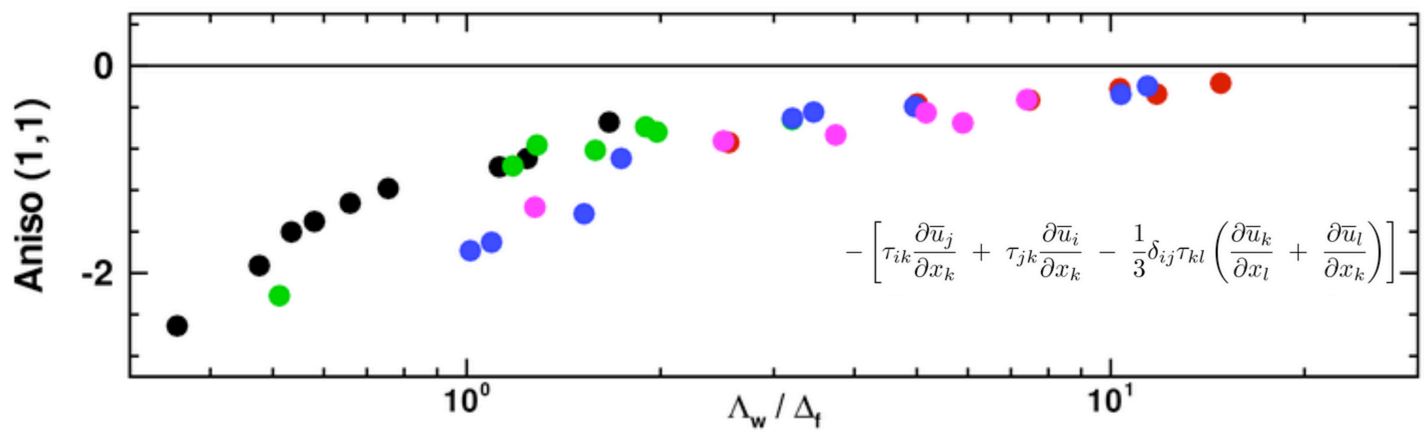
$$\boxed{\frac{\tau_{ij}}{T} = \frac{2}{3}e\left(\frac{\partial \overline{u}_i}{\partial x_j} + \frac{\partial \overline{u}_j}{\partial x_i}\right)}$$

Time scale

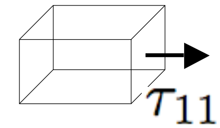
$$T = c\,\frac{\Delta_f}{\sqrt{e}}$$

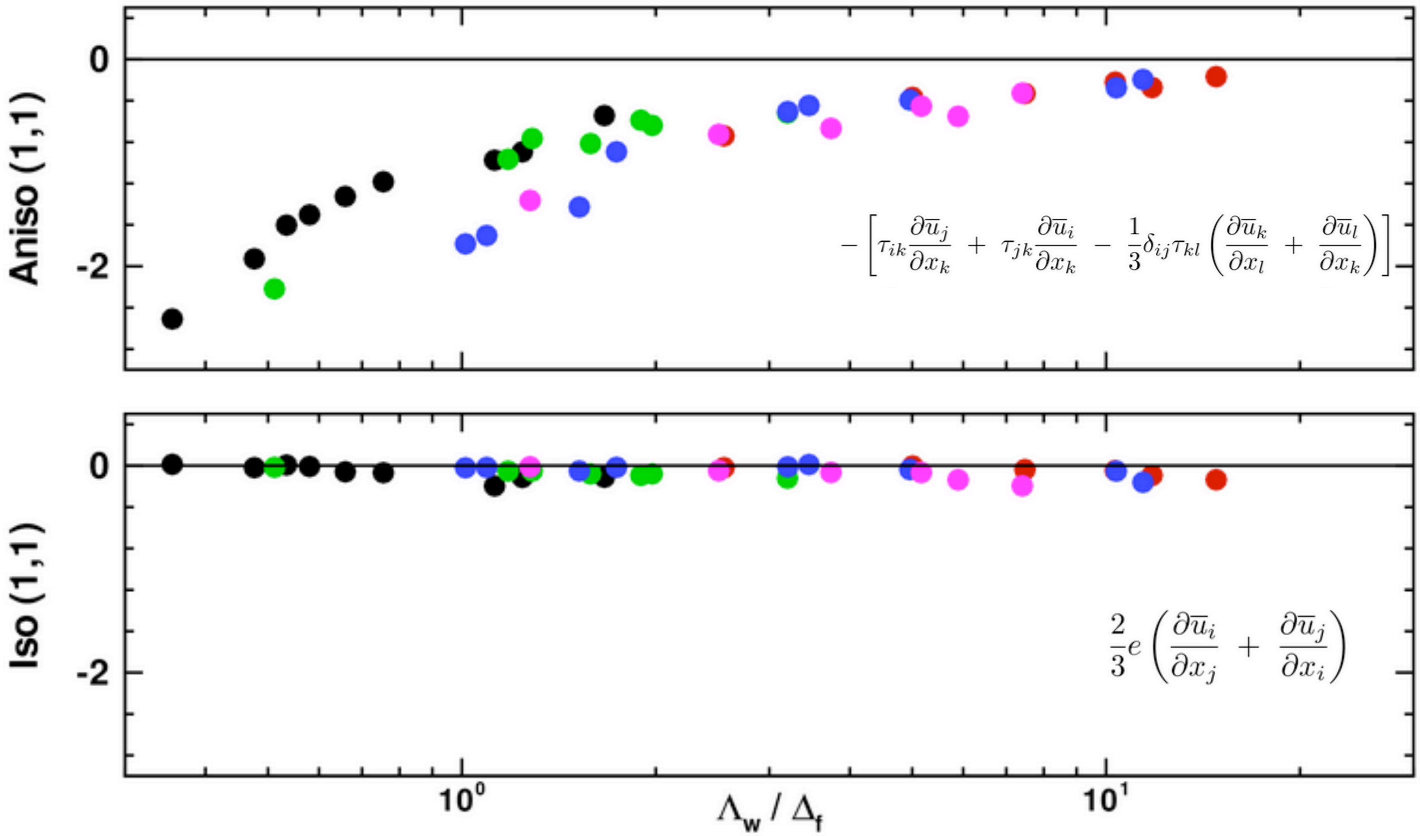
PRODUCTION OF SUBFILTER SCALE FLUX τ_{11}



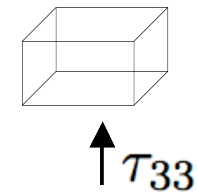


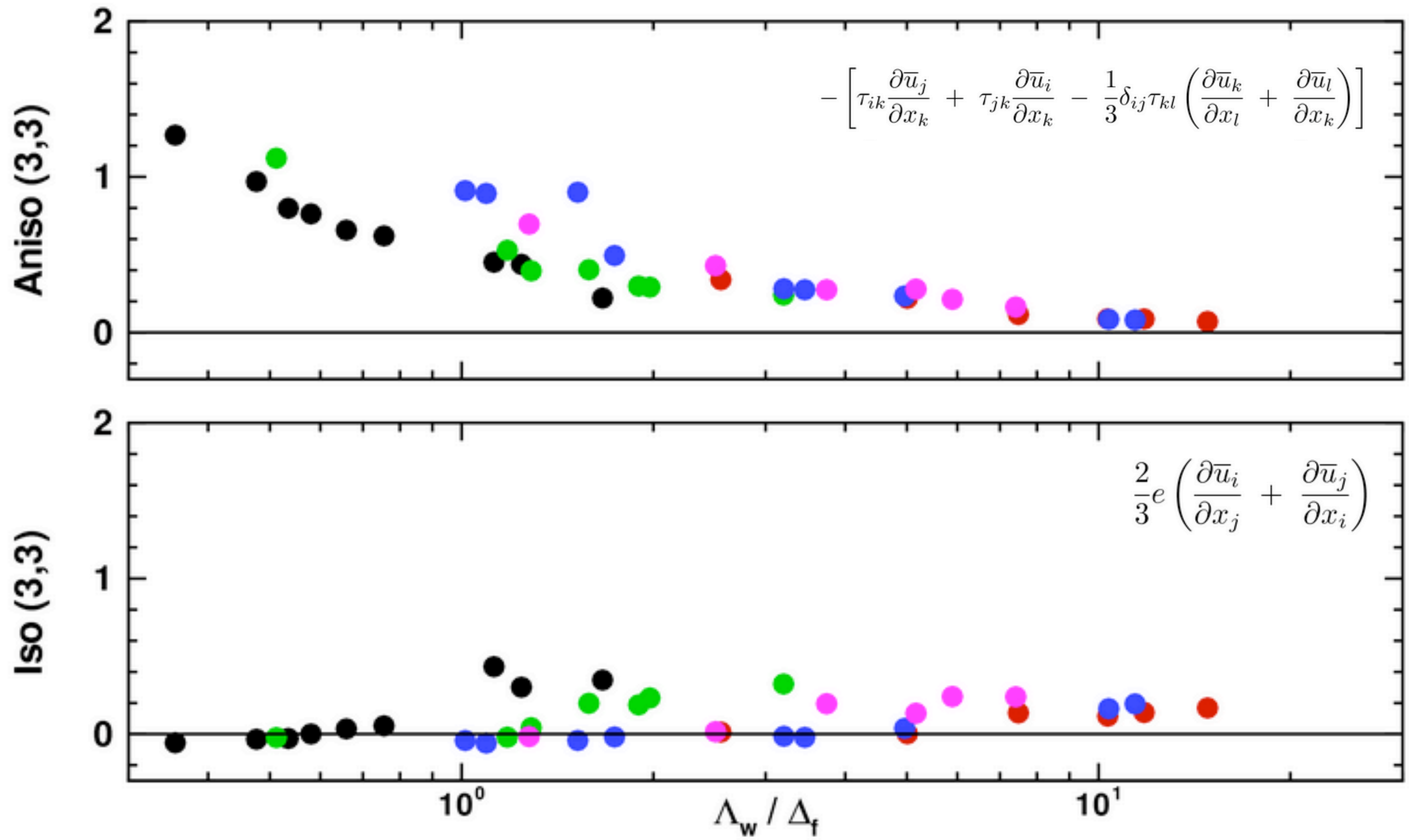
PRODUCTION OF SUBFILTER SCALE FLUX τ_{11}



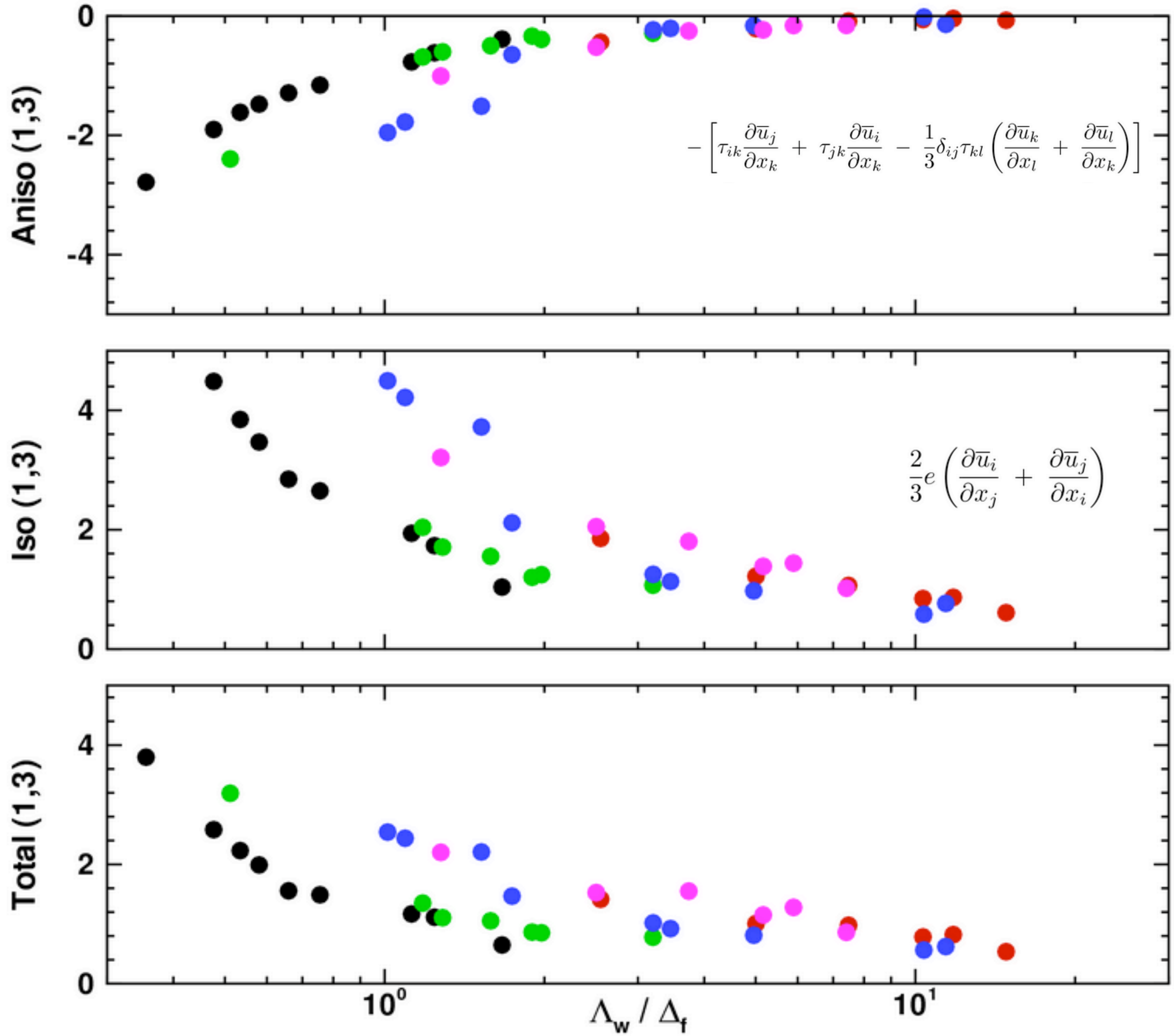


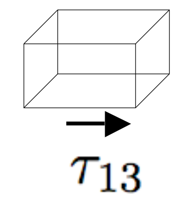
PRODUCTION OF SUBFILTER SCALE FLUX τ_{33}





PRODUCTION OF SUBFILTER SCALE FLUX τ_{13}





VARIATION OF DEVIATORIC STRESS IN LIMIT $\Lambda_w/\triangle_f \to 0$

$\langle\tau_{11}\rangle = T\left(-2\langle\tau_{13}\rangle\frac{\partial U}{\partial z} + \frac{2}{3}\epsilon\right)$	$\langle\tau_{11}\rangle = 0$
$\langle\tau_{22}\rangle = T\left(\frac{2}{3}\epsilon\right)$	$\langle\tau_{22}\rangle = 0$
$\langle\tau_{33}\rangle = T\left(\frac{2}{3}\epsilon\right)$	$\langle\tau_{33}\rangle = 0$
$\langle\tau_{13}\rangle = T\left(\frac{2}{3}e\frac{\partial U}{\partial z} - \langle\tau_{33}\rangle\frac{\partial U}{\partial z}\right)$	$\langle\tau_{13}\rangle = T\left(\frac{2}{3}e\frac{\partial U}{\partial z}\right)$

Steady-state rate equations

Smagorinsky (eddy viscosity) model

WHAT ABOUT SCALARS?

RATE EQUATIONS FOR SUBGRID SCALAR FLUX

- **What are the parent equations for subgrid-scale scalar flux?**

$$f_i = \overline{u_i c} - \overline{u}_i \overline{c}$$

$$\begin{aligned}
\frac{Df_i}{Dt} = \; & -\frac{2}{3} e \frac{\partial \overline{c}}{\partial x_i} \\
& -f_j \frac{\partial \overline{u}_i}{\partial x_j} + \tau_{ij} \frac{\partial \overline{c}}{\partial x_j} \\
& +\frac{1}{\rho}\left(\overline{p \frac{\partial c}{\partial x_i}} - \overline{p} \frac{\partial \overline{c}}{\partial x_i} \right) \\
& + \text{transport} + \text{buoyancy}
\end{aligned}$$

Isotropic production

Pressure destruction

Anisotropic production

RATE EQUATIONS FOR SUBGRID SCALAR FLUX

- **What are the parent equations for subgrid-scale scalar flux?**

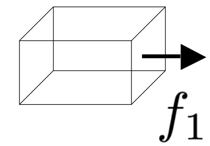
$$f_i = \overline{u_i c} - \overline{u}_i \overline{c}$$

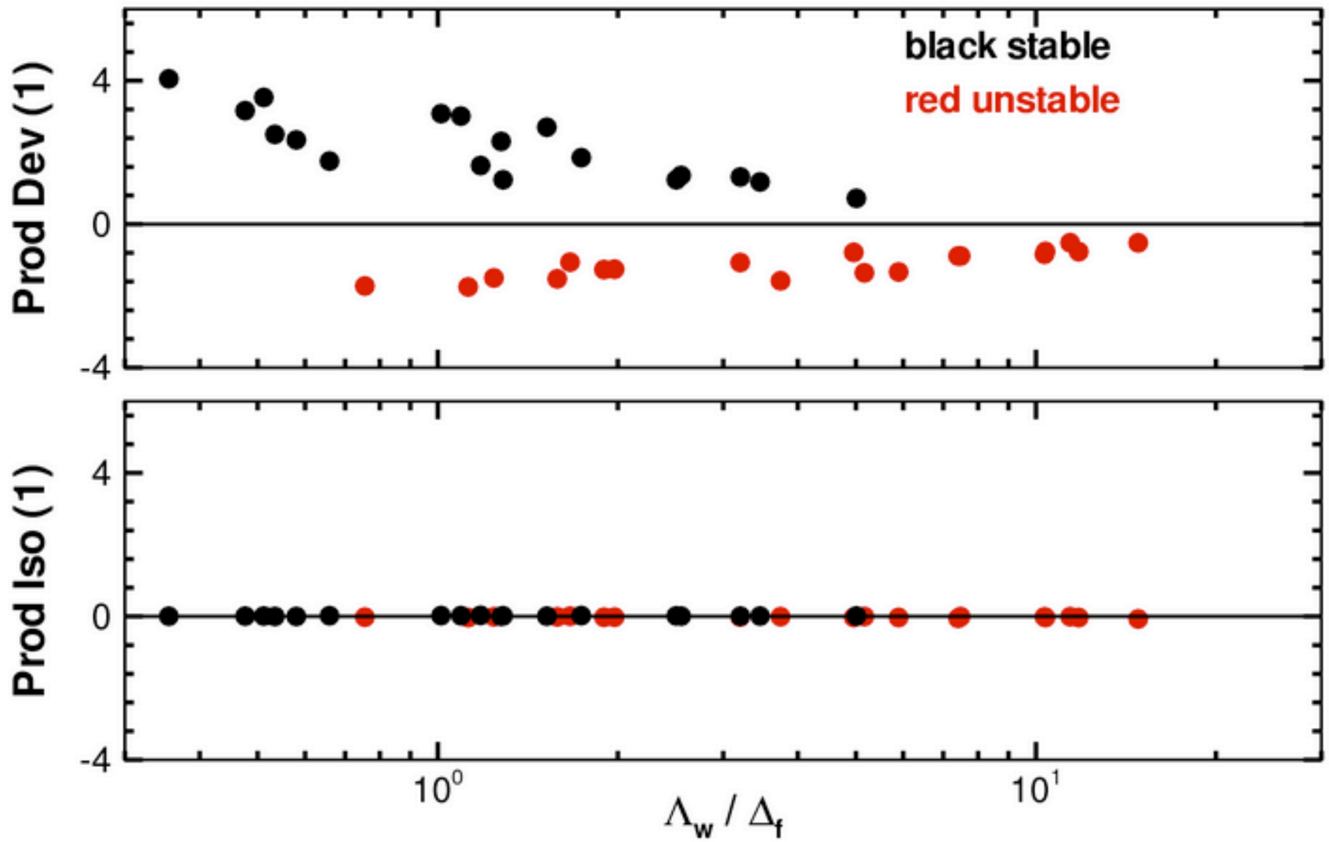
$$\begin{aligned}
\cancelto{0}{\frac{D f_i}{Dt}} = & -\frac{2}{3} e \frac{\partial \overline{c}}{\partial x_i} \\
& \cancelto{0}{-f_j \frac{\partial \overline{u}_i}{\partial x_j} + \tau_{ij} \frac{\partial \overline{c}}{\partial x_j}} \\
& + \frac{1}{\rho}\left(\overline{p \frac{\partial c}{\partial x_i}} - \overline{p} \frac{\partial \overline{c}}{\partial x_i} \right) \\
& + \cancelto{0}{\text{transport}} + \cancelto{0}{\text{buoyancy}}
\end{aligned}$$

Eddy viscosity model

$$\boxed{f_i = -\nu_h \frac{\partial \overline{c}}{\partial x_i} \qquad \nu_h = \frac{2 c_h \triangle_f \sqrt{e}}{3}}$$

PRODUCTION OF SUBFILTER SCALE SCALAR FLUX f_1





SUBGRID-SCALE SCALAR FLUX

Comments:

- Net horizontal scalar flux $f_1 = \langle \overline{uc} - \overline{u}\,\overline{c} \rangle \neq 0$ even for horizontally homogeneous PBLs, $i.e.,$ $\dfrac{\partial}{\partial x}\langle C \rangle = 0$
- Tilting of vertical flux by vertical shear is important $f_1 \sim -f_3 \dfrac{\partial \overline{u}}{\partial z} T$
- No eddy viscosity model can capture anisotropic production

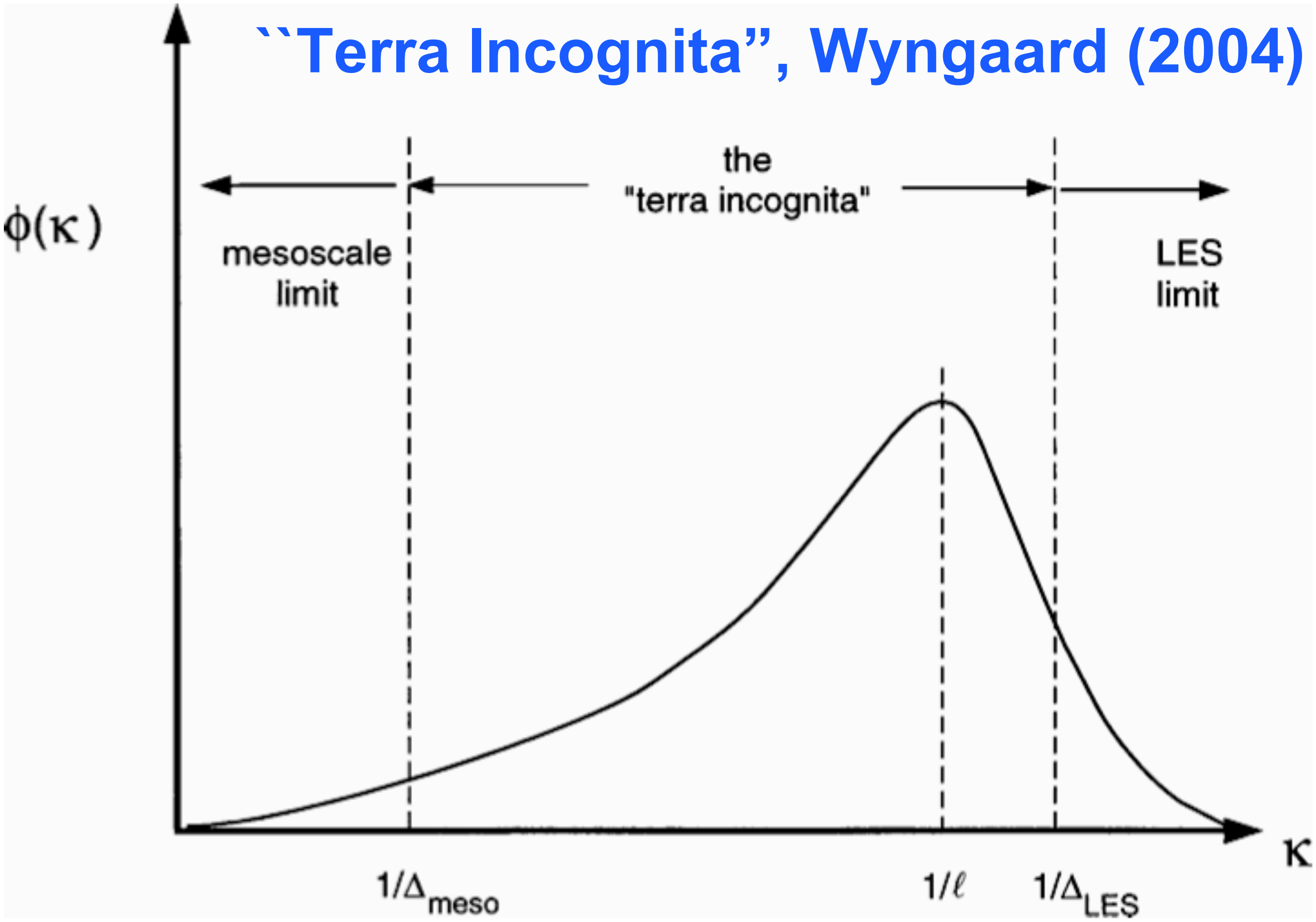


FIG. 1. A schematic of the turbulence spectrum $\phi(\kappa)$ in the horizontal plane as a function of the horizontal wavenumber magnitude κ. Its peak is at $\kappa \sim 1/l$, with l the length scale of the energetic eddies; Δ is the scale of the smoothing filter. In the mesoscale limit (left), $\Delta_{meso} \gg l$ and none of the turbulence is resolved. In the LES limit (right), $\Delta_{LES} \ll l$ and the energy-containing turbulence is resolved.

Where is your ``LES" ?

SUMMARY

- Turbulent stratified PBLs impact climate, weather, and applications

- LES in combination with parallel computing is a powerful technique for modeling atmospheric and oceanic boundary layers, $e.g.$,
  - Convection, stable boundary layers and flows with complex surface layers
  - We are developing algorithms to allow modestly complex terrain

- Multi-point measurements from the HATS field campaigns compliment our ability to compute
  - Evaluation of subgrid scale models with high Re data
  - Rate equations provide insight into SGS dynamics
  - Importance of anisotropic production for stress and scalar especially for $\Lambda_w/\Delta_f \sim \mathcal{O}(1)$ or less
  - Data highlights the shortcomings of an eddy viscosity approach

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