A PBL Turbulence Problem?

Courtesy NOAA
Maximum azimuthal wind speed (Emanuel, 2004)

$V_{max}^2 = \frac{T_s - T_o}{T_o} C_k \left( k_s^* - k \right)$
Before Bonnie

SST Variations, Ocean Mixing and Biology

Shuyi Chen, RSMAS
Before Bonnie

After Bonnie

SST Variations, Ocean Mixing and Biology

Shuyi Chen, RSMAS
“... We know what the exact equations for the Planetary Boundary Layer are ... let’s just use DNS?”
LENGTH AND TIME SCALES OF HIGH-$Re$
CONVECTIVE PBL TURBULENCE

• Energy-containing (large) eddies $\mathcal{L} \sim \mathcal{O}(z_i)$; $z_i \sim 1000\text{m}$

• Velocity scale of the (large) eddies $\mathcal{U} \sim \left(gQ_\star z_i/\Theta_o\right)^{1/3}$; $\mathcal{U} \sim 1\text{m/s}$

• Large eddy turnover time $\mathcal{T} = \mathcal{L}/\mathcal{U}$; $\mathcal{T} = 1000\text{s}$

• Kolmogorov microscale $\eta = \left(\nu^3/\epsilon\right)^{1/4}$; $\eta \sim 1\text{mm}$

• Reynolds number $Re_\mathcal{L} = \mathcal{U}\mathcal{L}/\nu = 10^6$
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Amount of work (number of mode-steps) is

\[
N^3 \cdot N_s \approx Re_{\mathcal{L}}^3
\]

Therefore

\[
N^3 \cdot N_s \approx 10^{18} \quad \text{for} \quad Re_{\mathcal{L}} = 10^6
\]
“... DNS is mighty useful, however we don’t have enough IBM SP’s to get to a PBL Reynolds number and resolve all scales of a rough wall ... We need something else ...”
HOW DO WE GET TO OUTDOOR LES

- Spatially filter the full governing equations to eliminate small scale fluctuations

- Subgrid-scale (SGS) challenge
  - Spatial correlations of small scale fluctuations \( \neq 0 \)
  - High \( Re \) limit, universality of small scales
  - Building equations for SGS correlations

- Coping with a rough wall boundary

See Tom Lund discussion later for details!
SPATIALLY FILTERED EQUATIONS FOR DRY BOUSSINESQ ROTATING ATMOSPHERIC PBL

\[
\frac{\partial \overline{u}}{\partial t} + (\overline{u} \cdot \nabla) \overline{u} = -f \times \overline{u} - \nabla \overline{\pi} + \frac{\hat{z}g}{\theta_*} \overline{\theta'} + \nu \nabla^2 \overline{u} - \nabla \cdot \mathbf{T}
\]

\[
\frac{\partial \overline{\theta}}{\partial t} + (\overline{u} \cdot \nabla) \overline{\theta} = \alpha \nabla^2 \overline{\theta} - \nabla \cdot \mathbf{B}
\]

\[\nabla \cdot \overline{u} = 0 \implies \nabla^2 \overline{\pi} = \overline{s}\]

New terms! Subgrid-scale momentum and scalar fluxes

\[
\mathbf{T} = \overline{u_i u_j} - \overline{u_i \overline{u_j}}
\]

\[
\mathbf{B} = \overline{u_i \theta} - \overline{u_i \overline{\theta}}
\]

COMMENTS ON THE LES EQUATIONS AND THE SUBGRID SCALE STRESS TENSOR

\[
\begin{align*}
\frac{\partial \bar{u}_i}{\partial t} + \frac{\partial \bar{u}_i \bar{u}_j}{\partial x_j} &= \ - \frac{1}{\rho} \frac{\partial p}{\partial x_i} - \frac{\partial}{\partial x_j} \left( \mathcal{T}_{ij} - \nu \frac{\partial \bar{u}_i}{\partial x_j} \right) \\
\mathcal{T}_{ij} &= \bar{u}_i \bar{u}_j - \bar{u}_i \bar{u}_j
\end{align*}
\]

- The LES equations contain two parameters, \( Re \) and the filter properties (loosely the shape of the filter and its cutoff \( k_c \))

- Solutions of the LES equations are stochastic, i.e., \( \bar{u}_i \) is a random variable in \((x,t)\)

- \( \mathcal{T}_{ij} \) is unknown! It needs to be expressed in terms of known resolved fields \( \overline{u}_i \)

- Subgrid scale \( \mathcal{T}_{ij} \) is stochastic and depends on the filter. This makes \( \overline{u}_i \) also filter dependent.
SIMPLE (CHEAP) FILTERING EXAMPLE

\[ \tau_{11} = \overline{u_1 u_1} - \overline{u_1} \overline{u_1} \]
What happens to $\bar{u}_i$ and $T_{ij}$ as we vary the filter cutoff $k_c$?
What happens to $\bar{u}_i$ and $T_{ij}$ as we vary the filter cutoff $k_c$?
MOVING BETWEEN DNS ↔ LES ↔ RANS

What happens to $\bar{u}_i$ and $T_{ij}$ as we vary the filter cutoff $k_c$?

![Graph showing the variation of $u$ and $\tau_{11}$ with time for different filter cutoffs.]

- Black: no filtering
- Red: $k_c = 0.3/m$
- Blue: $k_c = 0.03/m$
- Green: $k_c = 0.01/m$
MOVING BETWEEN DNS $\leftrightarrow$ LES $\leftrightarrow$ RANS

What happens to $\bar{u}_i$ and $T_{ij}$ as we vary the filter cutoff $k_c$?
What happens to $\bar{u}_i$ and $T_{ij}$ as we vary the filter cutoff $k_c$?
FLOW NEAR ROUGH BOUNDARIES

• Treatment of the lower boundary is the fundamental difference between Quasi-Direct Numerical Simulation (QDNS) [Spalart et al. (1997)] and $1/Re \rightarrow 0$ LES [Deardorff (1970)]

• Impossible to resolve all separation points and wakes (at high $Re$) at a complex boundary, e.g., the boundary might not even be defined!

• Numerical commutation errors [Berselli et al. (2006)] are mixed up with physical modeling

• Typical outdoor LES uses simple near wall models
  – Based on ensemble average ideas (Monin-Obukhov similarity theory)
  – Generate spatial fluctuations by applying MO on a point-by-point basis or using a linearization of the quadratic drag formula [Moeng (1984)]

• Sometimes you don’t get to choose where $1/\Delta_f$ sits!

• There’s work to be done near rough boundaries
FLOW OVER A ROUGH BOUNDARY

At a rough boundary all the flux is subgrid

\[ \tau_o = f(z_o, \bar{u}, z_1, z_1/L) \]

\[ \frac{\partial \bar{u}}{\partial t} = \ldots - \frac{\tau_{13} - \tau_o}{z_1} \]
SURFACE LAYER MEASUREMENTS AND LES:
\( \Lambda_w / \Delta_f \) AT FIRST GRID POINT OFF THE SURFACE

- \( \Lambda_w \rightarrow \) horizontal wavelength of the peak in the vertical velocity spectrum
  - \( w \) is least resolved in LES
  - Obeys MO scaling, i.e., depends on \((z, L)\)

- \( \Delta_f \sim (\delta_x \delta_y \delta_z)^{1/3} \) the cell averaging volume
Dependence of Peak Wavelength on Stratification

$H(z/L)$

$\frac{z}{\Lambda_w}$ vs. $\frac{z}{L}$

- Black circles: Array 1
- Green circles: Array 2
- Red circles: Array 3
- Blue circles: Array 4
RATIO OF TURBULENCE LENGTH SCALE TO FILTER WIDTH AT FIRST LES GRIDPOINT $z = \delta_z$

First gridpoint off the surface

Convective

Stable

\[
\frac{\Lambda_w}{\Delta_f} = \frac{A}{H(\delta z/L)}
\]
Can we use targeted observations to provide insight as to the nature of SGS motions in high Re PBLs?
HIGH REYNOLDS NUMBER OBSERVATIONS AND LES

- **SINGLE-POINT MEASUREMENTS**
  - Cannot be used directly to improve LES

- **MULTI-POINT MEASUREMENTS**
  - Span a range of filter widths, *e.g.*, $\mathcal{O}(m)$ to $\mathcal{O}(100m)$
  - Ideally 3-D, time varying “volume” of turbulence and scalars in canonical flows with shear, stratification, near boundaries, ...
HATS CONFIGURATIONS

Wide

Narrow

$\sim 36$ cases

$-1.2 < \frac{z}{L} < 1.6$

$0.15 < \frac{\Lambda_w}{\Delta f} < 15$
RATIONALE FOR EXPERIMENTAL DESIGN

- Allows *spatial* filtering of flow field and decomposition into resolved and subfilter scale velocities \( (\overline{U_i}, u_i) \):

\[
\overline{U_i} = \overline{U_i} + u_i = \int U(x_j') G(x_i, x_j') dx_j' + u_i
\]

- Allows construction of SFS fluxes:

\[
\mathbf{T}_{ij} = \overline{U_i U_j} - \overline{U_i} \overline{U_j}
\]

- Allows measurement of resolved gradients \( \partial \overline{U_i} / \partial x \), \( \partial \overline{U_i} / \partial y \) and \( \partial \overline{U_i} / \partial z \)

- Allows expansion of SFS fluxes \( \mathbf{T}_{ij} \) into Leonard, Cross, and Reynolds terms which requires *double* spatial filtering, e.g., \( \overline{U_i u_j} \)
AN EXAMPLE OF LATERAL (Y) FILTERING

\[ f(y, t) \]

\[ \bar{f}(y, t) \]

\[ \delta_{yd} \]

\[ U \]

\[ y \]

``2D plane of turbulence``
AN EXAMPLE OF LATERAL (Y) FILTERING

\[ f(y, t) \]

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\[ \delta_{yd} \]

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\[ y \]
AN EXAMPLE OF LATERAL (Y) FILTERING

\[ f(y, t) \quad \otimes \quad \otimes \quad \otimes \quad \otimes \quad \bullet \quad \bullet \quad \bullet \quad \bullet \quad \bullet \quad \bullet \quad \bullet \quad \bullet \]

\[ \bar{f}(y, t) \quad \bullet \quad \bullet \quad \bullet \quad \bullet \quad \bullet \quad \bullet \quad \bullet \quad \bullet \quad \bullet \]

\[ \overline{\bar{f}(y, t)} \quad \downarrow \quad \downarrow \quad \downarrow \quad \downarrow \quad \downarrow \quad \downarrow \quad \downarrow \quad \downarrow \quad \downarrow \quad \downarrow \quad \downarrow \quad \downarrow \]

\[ U \quad \delta_{yd} \quad \Rightarrow \quad y \]
SFS Flux $\tau_{13} = \overline{U_1 U_3} - \overline{U_1} \overline{U_3}$ for Varying Filter Widths

$\frac{\Lambda_w}{\Delta f} = 0.58$

top view

$1.18$

$5.00$

$11.4$
Spectra of Leonard, Cross, Reynolds Terms for (1,3) Component
SFS VELOCITY VARIANCES

$3 \tau_{11} / 2E_{sfs}$

$3 \tau_{22} / 2E_{sfs}$

$3 \tau_{33} / 2E_{sfs}$

$2\pi z / \Delta f$

Isotropic

$\Delta f$

HATS

ARRAYS
OHATS FIELD CAMPAIGN

ASIT

Laser altimeters
18 CSATS

275 hours "12 days of data" analyzed
RATE EQUATIONS FOR SUBGRID DEVIATORIC STRESS

- What are the parent equations for the Smagorinsky model?
RATE EQUATIONS FOR SUBGRID DEVIATORIC STRESS

The SGS stress is

$$\tau_{ij} = \bar{u}_i \bar{u}_j - \bar{u}_i \bar{u}_j$$

To get the “rate equation” for SGS $\tau_{ij}$

$$\frac{\partial \tau_{ij}}{\partial t} = \left[ u_j \frac{\partial u_i}{\partial t} - \frac{\partial u_i}{\partial t} \bar{u}_j \right]$$

Substitution steps:

$$u_j \frac{\partial u_i}{\partial t} = u_j R_i$$

$$\bar{u}_j \frac{\partial \bar{u}_i}{\partial t} = \bar{u}_j \bar{R}_i$$

The difference is now $\tau_{ij}$ is the deviatoric stress, i.e., $-2/3e \delta_{ij}$

Considerable Algebra!
RATE EQUATIONS FOR SUBGRID DEVIATORIC STRESS

- What are the parent equations for the Smagorinsky model?

\[
\frac{D\tau_{ij}}{Dt} = \frac{2}{3} e \left( \frac{\partial \bar{u}_i}{\partial x_j} + \frac{\partial \bar{u}_j}{\partial x_i} \right)
- \left[ \tau_{ik} \frac{\partial \bar{u}_j}{\partial x_k} + \tau_{jk} \frac{\partial \bar{u}_i}{\partial x_k} - \frac{1}{3} \delta_{ij} \tau_{kl} \left( \frac{\partial \bar{u}_k}{\partial x_l} + \frac{\partial \bar{u}_l}{\partial x_k} \right) \right]
- \frac{1}{\rho} \left[ \rho \left( \frac{\partial \bar{u}_i}{\partial x_j} + \frac{\partial \bar{u}_j}{\partial x_i} \right) - \bar{p} \left( \frac{\partial \bar{u}_i}{\partial x_j} + \frac{\partial \bar{u}_j}{\partial x_i} \right) \right]
+ \text{transport} + \text{buoyancy production}
\]
RATE EQUATIONS FOR SUBGRID DEVIATORIC STRESS

- What are the parent equations for the Smagorinsky model?

\[
\frac{D\tau_{ij}}{Dt} = \frac{2}{3}e \left( \frac{\partial \bar{u}_i}{\partial x_j} + \frac{\partial \bar{u}_j}{\partial x_i} \right) - \tau_{ik} \frac{\partial \bar{u}_i}{\partial x_k} - \frac{1}{3} \delta_{ij} \tau_{kl} \left( \frac{\partial \bar{u}_k}{\partial x_l} + \frac{\partial \bar{u}_l}{\partial x_k} \right) - \frac{1}{\rho} \left[ p \left( \frac{\partial \bar{u}_i}{\partial x_j} + \frac{\partial \bar{u}_j}{\partial x_i} \right) - \bar{p} \left( \frac{\partial \bar{u}_i}{\partial x_j} + \frac{\partial \bar{u}_j}{\partial x_i} \right) \right] + \text{transport + buoyancy production}
\]

\[
\frac{\tau_{ij}}{T} = \frac{2}{3}e \left( \frac{\partial \bar{u}_i}{\partial x_j} + \frac{\partial \bar{u}_j}{\partial x_i} \right)
\]

\[
T = c \frac{\Delta f}{\sqrt{e}}
\]
PRODUCTION OF SUBFILTER SCALE FLUX $\tau_{11}$

$$ - \left[ \tau_{ik} \frac{\partial \overline{u}_i}{\partial x_k} + \tau_{jk} \frac{\partial \overline{u}_j}{\partial x_k} - \frac{1}{3} \delta_{ij} \tau_{kl} \left( \frac{\partial \overline{u}_k}{\partial x_i} + \frac{\partial \overline{u}_i}{\partial x_k} \right) \right] $$

[Graph showing the relationship between Aniso (1,1) and $\Lambda_w / \Delta_f$.]
PRODUCTION OF SUBFILTER SCALE FLUX $\tau_{11}$

\[
- \left[ \tau_{ik} \frac{\partial \bar{u}_i}{\partial x_k} + \tau_{jk} \frac{\partial \bar{u}_j}{\partial x_k} - \frac{1}{3} \delta_{ij} \tau_{kl} \left( \frac{\partial \bar{u}_k}{\partial x_i} + \frac{\partial \bar{u}_l}{\partial x_k} \right) \right] \\
\frac{2}{3} e \left( \frac{\partial \bar{u}_i}{\partial x_j} + \frac{\partial \bar{u}_j}{\partial x_i} \right)
\]
PRODUCTION OF SUBFILTER SCALE FLUX $\tau_{33}$

\[
- \left[ \tau_{ik} \frac{\partial \bar{u}_k}{\partial x_k} + \tau_{jk} \frac{\partial \bar{u}_k}{\partial x_k} - \frac{1}{3} \delta_{ij} \tau_{kl} \left( \frac{\partial \bar{u}_k}{\partial x_l} + \frac{\partial \bar{u}_k}{\partial x_i} \right) \right]
\]

\[
\frac{2}{3} e \left( \frac{\partial \bar{u}_i}{\partial x_j} + \frac{\partial \bar{u}_j}{\partial x_i} \right)
\]
PRODUCTION OF SUBFILTER SCALE FLUX $\tau_{13}$

\[
- \left[ \frac{\tau_{ik}}{\partial x_k} + \frac{\tau_{jk}}{\partial x_k} - \frac{1}{3} \delta_{ij} \tau_{kl} \left( \frac{\partial \bar{u}_k}{\partial x_l} + \frac{\partial \bar{u}_l}{\partial x_k} \right) \right]
\]

\[
\frac{2}{3} \varepsilon \left( \frac{\partial \bar{u}_i}{\partial x_j} + \frac{\partial \bar{u}_j}{\partial x_i} \right)
\]
VARIATION OF DEVIATORIC STRESS IN LIMIT $\Lambda_{\infty}/\Delta_f \to 0$

\[ \langle \tau_{11} \rangle = T \left( -2 \langle \tau_{13} \rangle \frac{\partial U}{\partial z} + \frac{2}{3} \epsilon \right) \]
\[ \langle \tau_{22} \rangle = T \left( \frac{2}{3} \epsilon \right) \]
\[ \langle \tau_{33} \rangle = T \left( \frac{2}{3} \epsilon \right) \]
\[ \langle \tau_{13} \rangle = T \left( \frac{2}{3} \epsilon \frac{\partial U}{\partial z} - \langle \tau_{33} \rangle \frac{\partial U}{\partial z} \right) \]

Steady-state rate equations
Smagorinsky (eddy viscosity) model
WHAT ABOUT SCALARS?
What are the parent equations for subgrid-scale scalar flux?

\[ f_i = \bar{u}_i \bar{c} - \bar{u}_i \bar{c} \]

\[
\frac{Df_i}{Dt} = -\frac{2}{3} e \frac{\partial \bar{c}}{\partial x_i} - f_j \frac{\partial \bar{u}_i}{\partial x_j} + \tau_{ij} \frac{\partial \bar{c}}{\partial x_j} + \frac{1}{\rho} \left( p \frac{\partial \bar{c}}{\partial x_i} - \bar{p} \frac{\partial \bar{c}}{\partial x_i} \right) + \text{transport + buoyancy}
\]

- **Isotropic production**
- **Pressure destruction**
- **Anisotropic production**
RATE EQUATIONS FOR SUBGRID SCALAR FLUX

- What are the parent equations for subgrid-scale scalar flux?

\[ f_i = \bar{u}_i \bar{c} - \bar{u}_i \bar{c} \]

\[ \frac{D f_i}{D t} = -\frac{2}{3} e \frac{\partial \bar{c}}{\partial x_i} + \frac{\partial \bar{u}_i}{\partial x_j} \frac{\partial \bar{c}}{\partial x_j} + \tau_{ij} \frac{\partial \bar{c}}{\partial x_j} + \frac{1}{\rho} \left( \bar{p} \frac{\partial \bar{c}}{\partial x_i} - \bar{\rho} \frac{\partial \bar{c}}{\partial x_i} \right) + \text{transport} + \text{buoyancy} \]

Eddy viscosity model

\[ f_i = -\nu_h \frac{\partial \bar{c}}{\partial x_i} \]

\[ \nu_h = \frac{2c_h \Delta f \sqrt{e}}{3} \]
PRODUCTION OF SUBFILTER SCALE SCALAR FLUX $f_1$
Comments:

- Net horizontal scalar flux $f_1 = \langle \bar{u} \bar{c} - \bar{u} \bar{c} \rangle \neq 0$ even for horizontally homogeneous PBLs, i.e., $\frac{\partial}{\partial x} \langle C' \rangle = 0$

- Tilting of vertical flux by vertical shear is important $f_1 \sim -f_3 \frac{\partial \bar{u}}{\partial z} T$

- No eddy viscosity model can capture anisotropic production
Where is your "LES"?

Fig. 1. A schematic of the turbulence spectrum $\phi(\kappa)$ in the horizontal plane as a function of the horizontal wavenumber magnitude $\kappa$. Its peak is at $\kappa \sim 1/l$, with $l$ the length scale of the energetic eddies; $\Delta$ is the scale of the smoothing filter. In the mesoscale limit (left), $\Delta_{meso} \gg l$ and none of the turbulence is resolved. In the LES limit (right), $\Delta_{LES} \ll l$ and the energy-containing turbulence is resolved.
SUMMARY

- Turbulent stratified PBLs impact climate, weather, and applications

- LES in combination with parallel computing is a powerful technique for modeling atmospheric and oceanic boundary layers, e.g.,
  - Convection, stable boundary layers and flows with complex surface layers
  - We are developing algorithms to allow modestly complex terrain

- Multi-point measurements from the HATS field campaigns compliment our ability to compute
  - Evaluation of subgrid scale models with high $Re$ data
  - Rate equations provide insight into SGS dynamics
  - Importance of anisotropic production for stress and scalar especially for $\Lambda_w/\Delta_f \sim \mathcal{O}(1)$ or less
  - Data highlights the shortcomings of an eddy viscosity approach
References


References


