# Turbulent Internal Wave Generation

...and other topics

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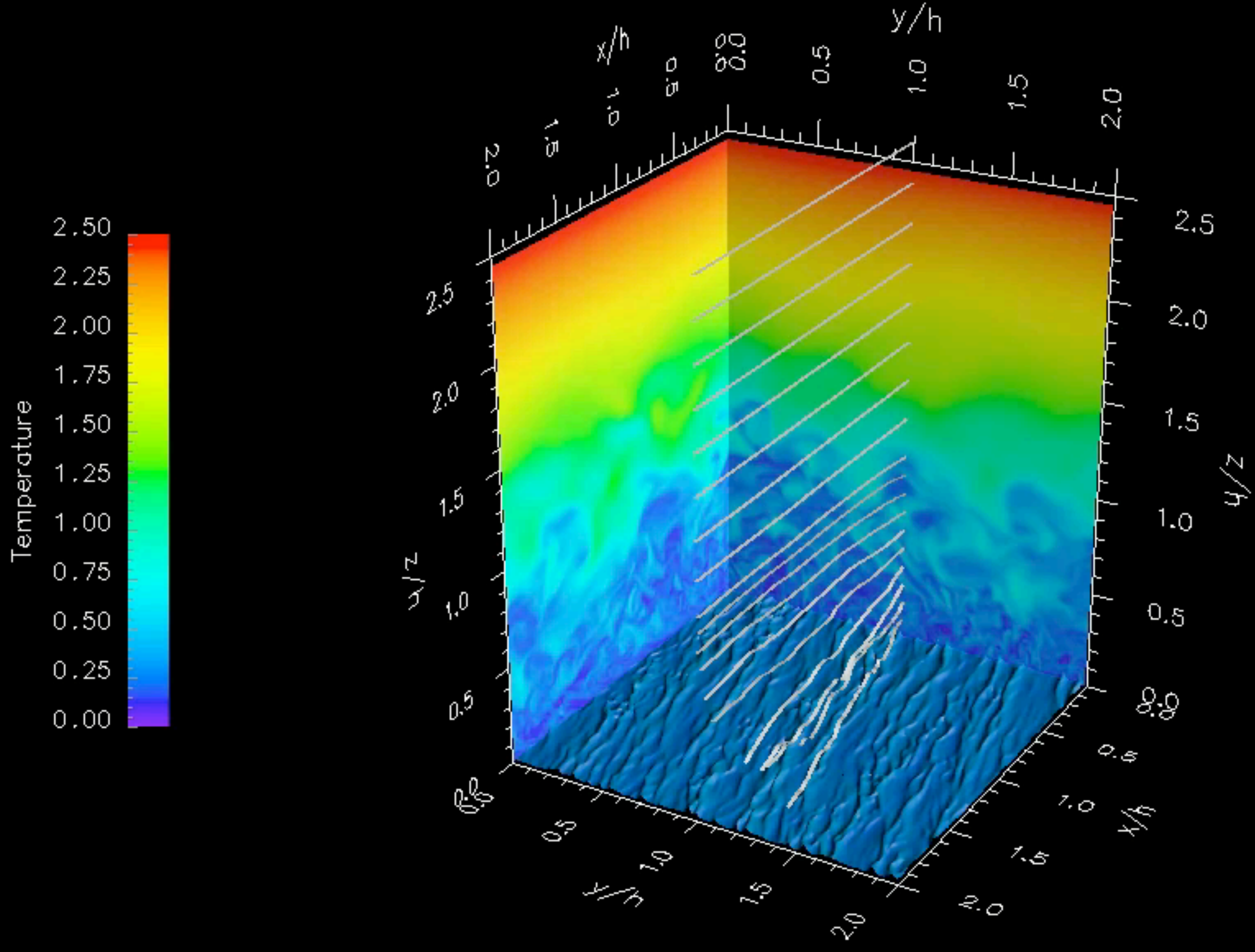
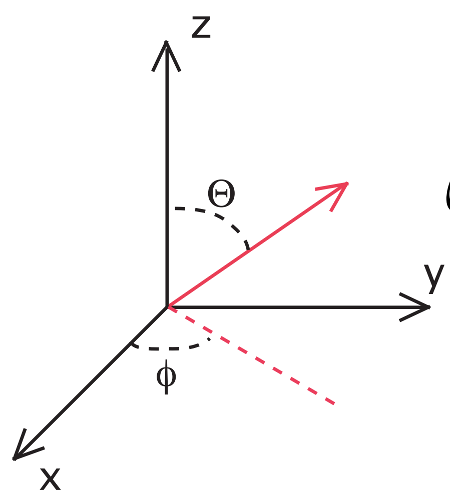
# Turbulent Ekman Layer

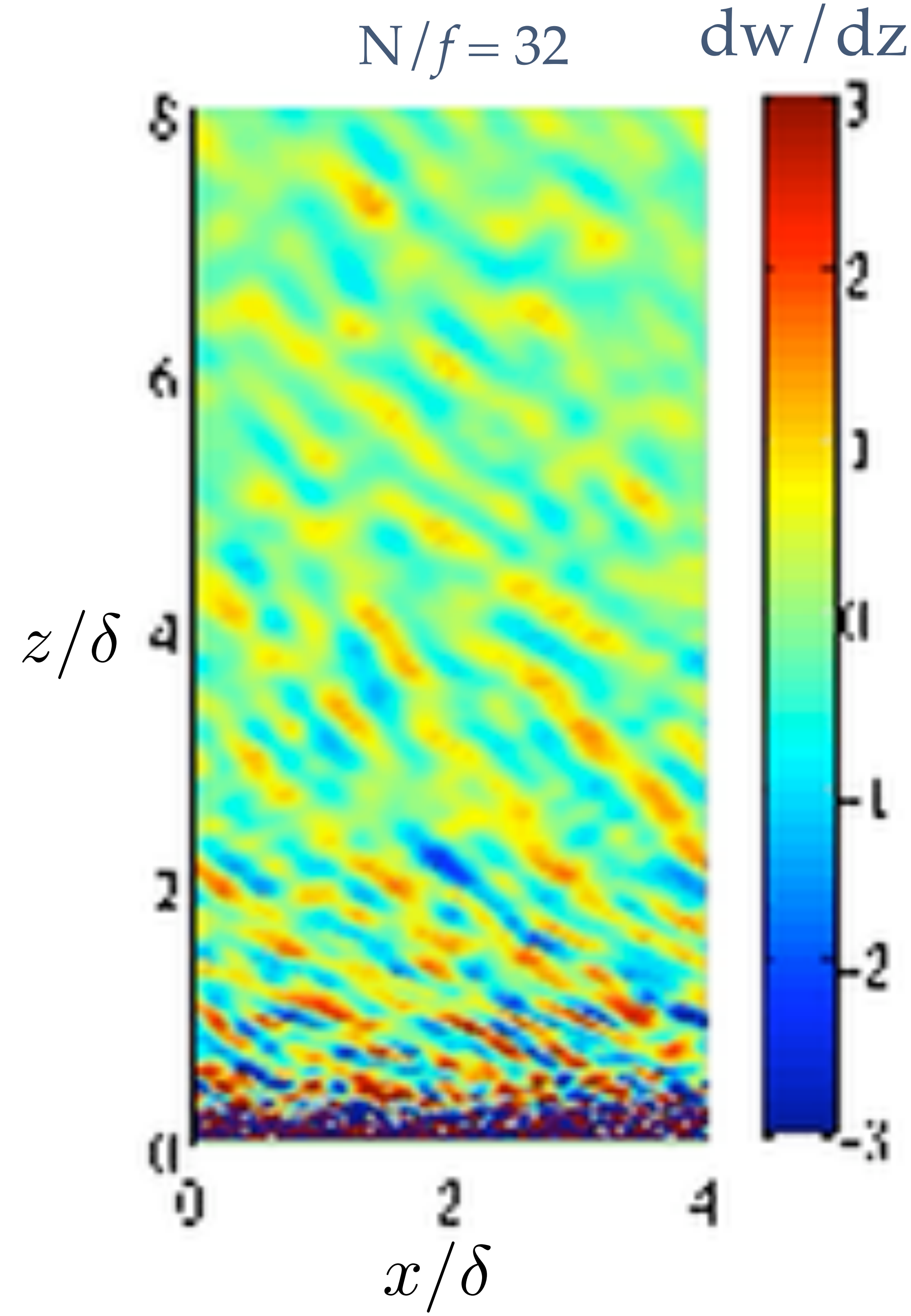


# "The mystery of the 45° waves"

| Previous Studies: | $\Theta$ |
|---|---|
| • Grid-generated Turbulence (Linden 1975, E&Hopfinger 1986, Dohan+Sutherland 2003,2005, etc.) | 42-55° |
| • Wakes (Bonneton et al. 1993, Gourlay et al. 2001, Spedding 2002, Diamessis et al. 2005, etc.) | |
| • Shear Layers (Sutherland & Linden 1988, Sutherland et al. 1994, Basak & Sarkar 2006, etc.) | 45-60° |
| • Gravity Currents (Flynn & Sutherland 2004, etc.) | 41-64° |
| • Topographic Lee (Aguilar & Sutherland 2006, etc.) | 40-46° |

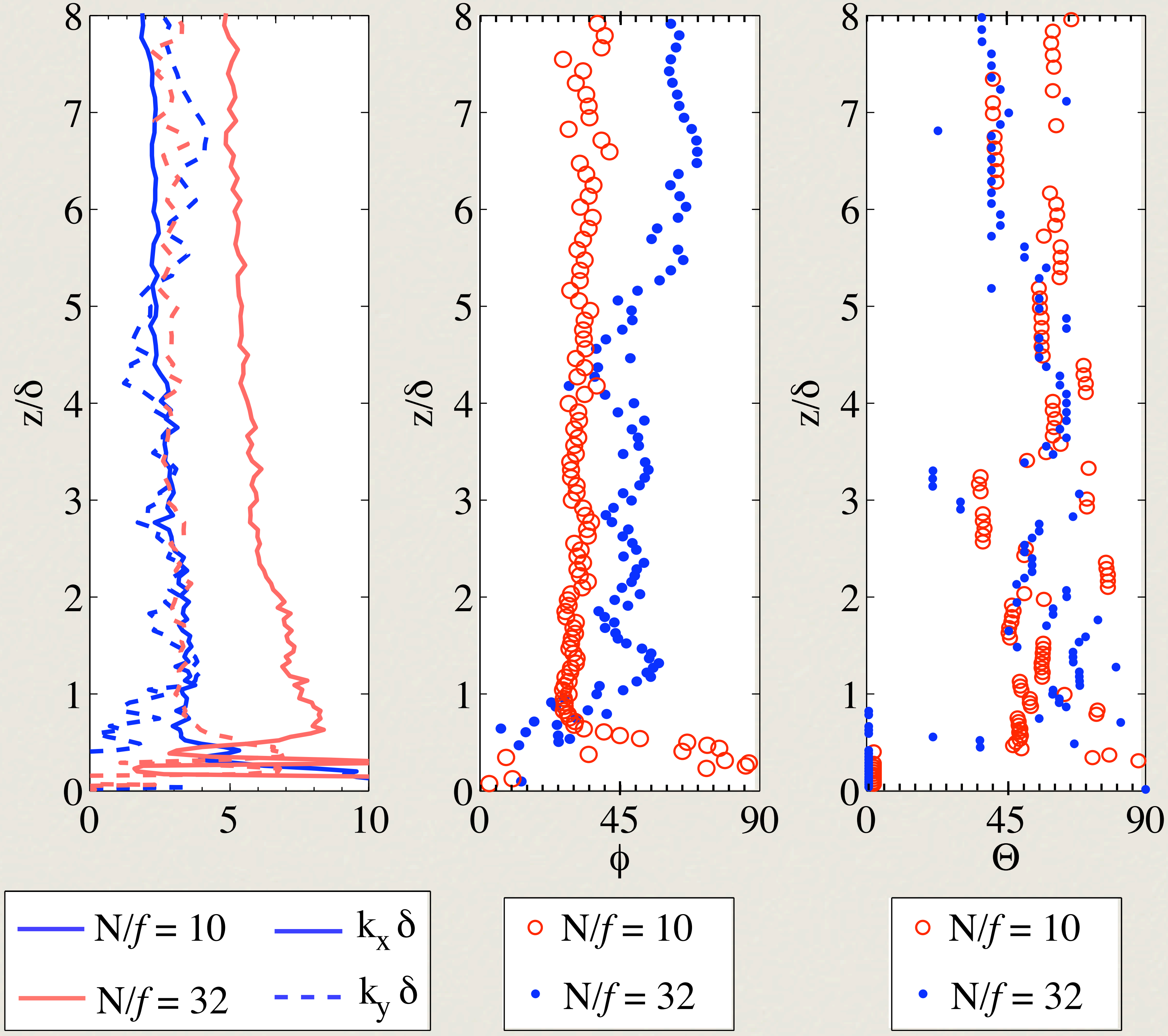



$$\omega = N cos(\Theta)$$

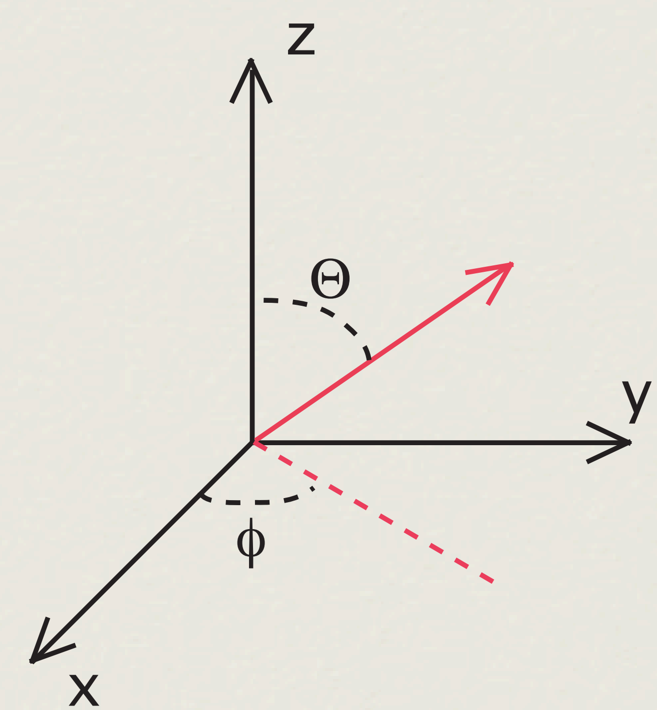


# Internal Wave Characteristics

## For waves with largest dw / dz




$$\frac{\omega}{N} = cos(\Theta)$$



# Viscous Decay Model

Kinetic and Potential Energy Eqns. for linear waves:

$$\frac{\partial K}{\partial t} = -\frac{1}{\rho_0}\nabla\cdot < \mathbf{u}'p' > -\frac{g}{\rho_0} < \rho'w' > +\nu\nabla^2 K - \nu < \frac{\partial u_i'}{\partial x_j}\frac{\partial u_i'}{\partial x_j} >,$$

$$\frac{\partial P}{\partial t} = \frac{g}{\rho_0} < \rho'w' > +\kappa\nabla^2 P - \frac{g}{\rho_0 d < \rho > /dz}\kappa < \frac{\partial \rho'}{\partial x_j}\frac{\partial \rho'}{\partial x_j} >$$

Combine using wave energy, $W=K+P$

$$\longrightarrow \quad \frac{\partial W}{\partial t} + \nabla\cdot(W\mathbf{c}_g) = \nu\frac{d^2W}{dz^2} - 2\nu|\mathbf{k}|^2 W$$

(using: $< p'\mathbf{u}' >= \mathbf{c}_g W$)

# Viscous Decay Model

$$\frac{\partial W}{\partial t} + \nabla \cdot (W\mathbf{c}_g) = \nu \frac{d^2 W}{dz^2} - 2\nu |\mathbf{k}|^2 W$$

For waves in a slowly varying medium $\nabla \cdot \mathbf{c}_g \approx 0$
and neglecting viscous diffusion:

$$\frac{DW}{Dt} = -2\nu |\mathbf{k}|^2 W$$

where D/Dt is time derivative following $c_g$

In terms of the vertical velocity amplitude (A):

$$\frac{D}{Dt}(|\mathbf{k}|^2 A^2) = -2\nu |\mathbf{k}|^4 A^2$$

# Viscous Decay Model

From the dispersion relation for internal waves:

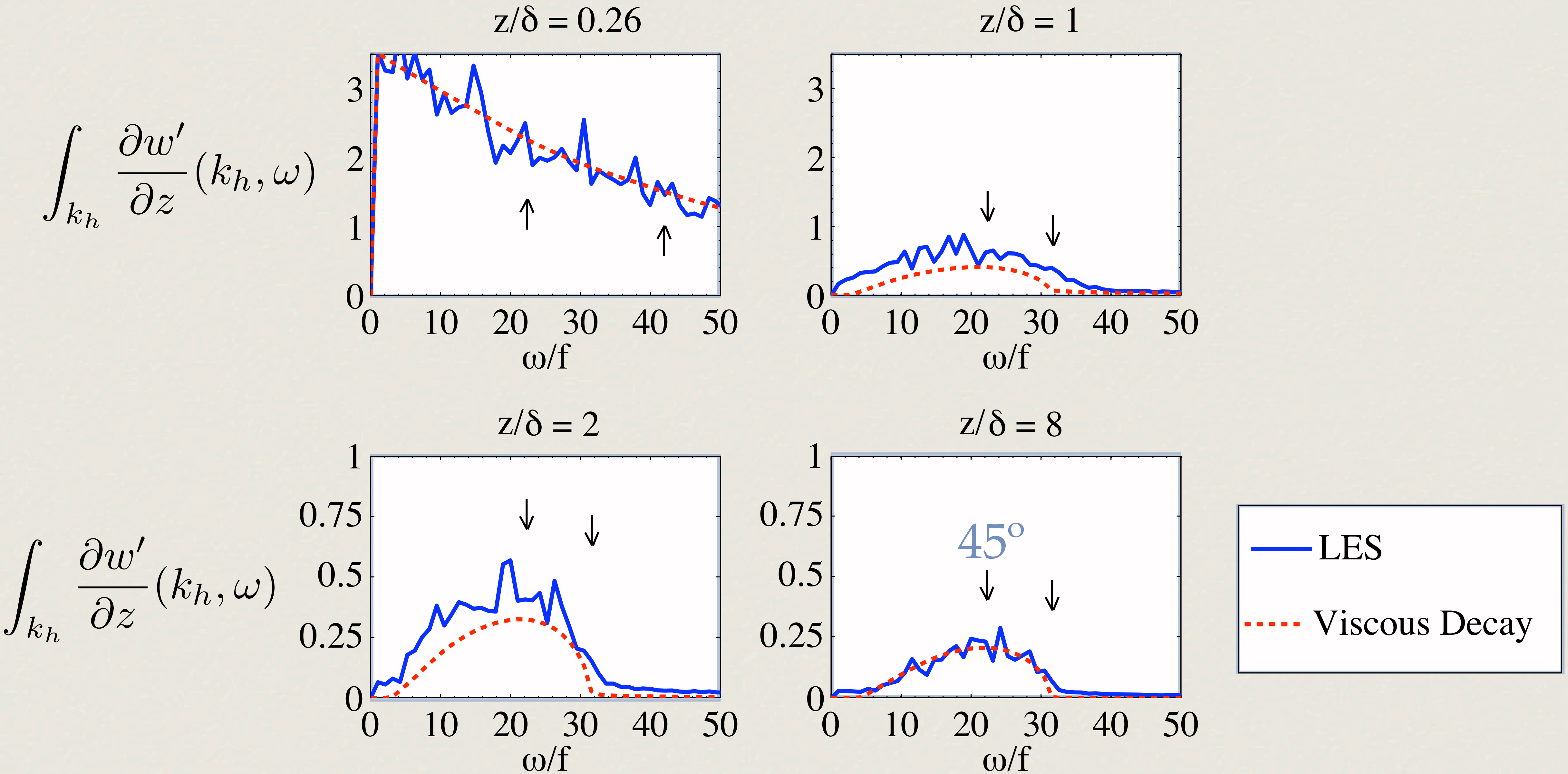
$$c_{gz} = \frac{k_h}{\omega|\mathbf{k}|^2}(\omega^2 - f^2)^{1/2}(N^2 - \omega^2)^{1/2} = \frac{dz}{dt}$$

The amplitude of the vertical velocity in a lab reference frame is:

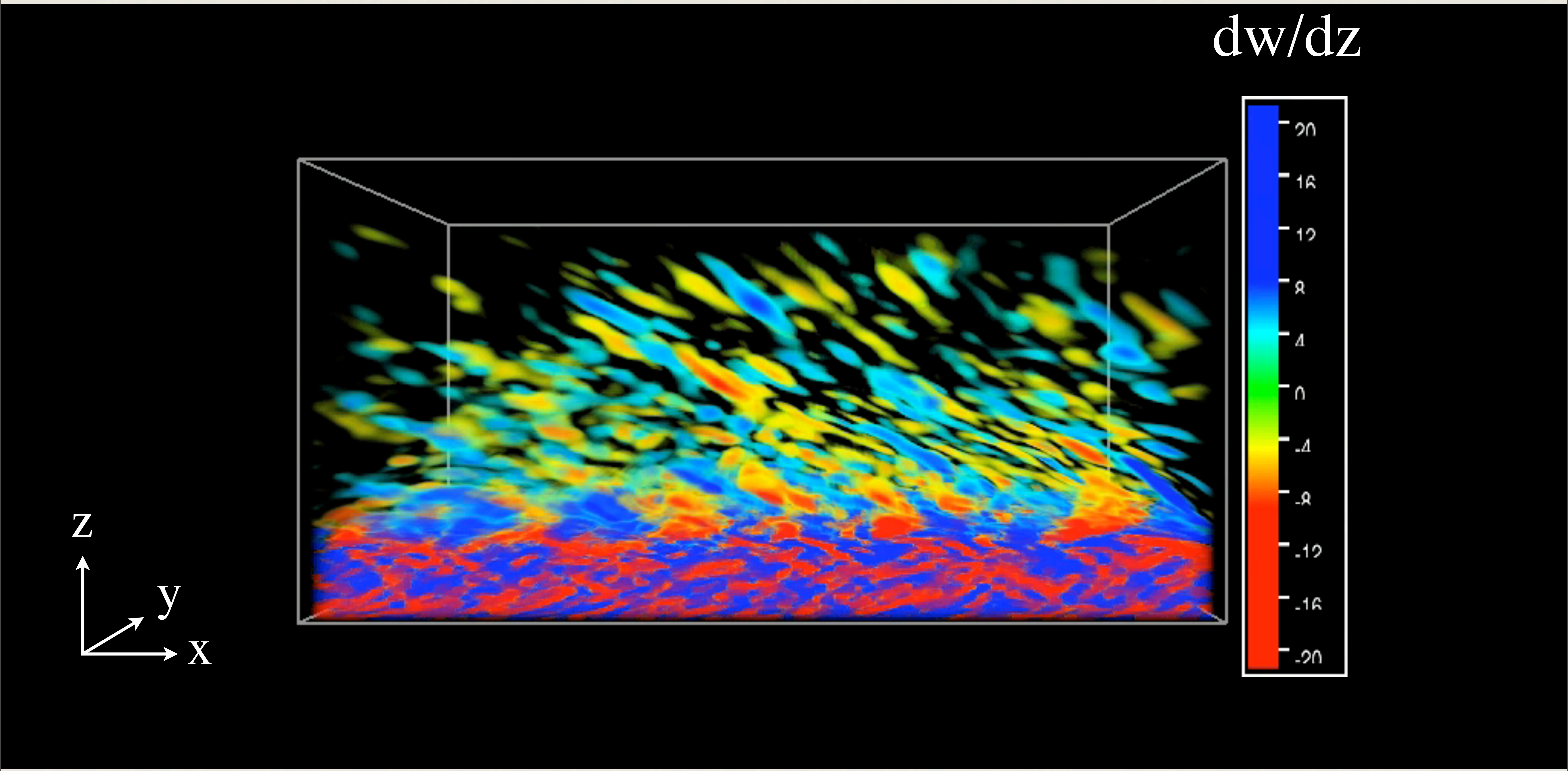
$$A(z) = A_0 \frac{|\mathbf{k}_0|}{|\mathbf{k}|} exp[\frac{-\nu\omega}{k_h}(\omega^2 - f^2)^{-1/2} \int_0^z |\mathbf{k}|^4 (N^2 - \omega^2)^{-1/2} dz']$$

Given $A_0(k_h, \omega)$ we can then predict the wave amplitude, *A(z)* using the viscous decay rate

# Viscous Decay Model



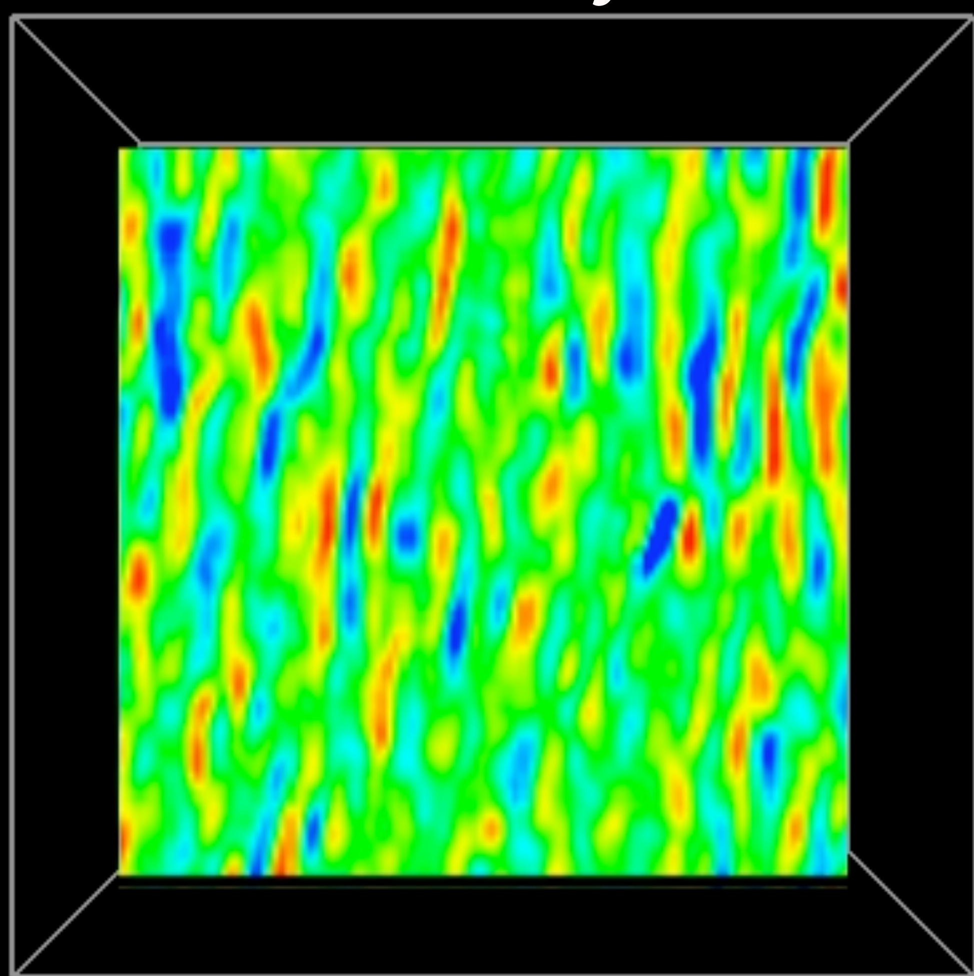
# 3D Visualization



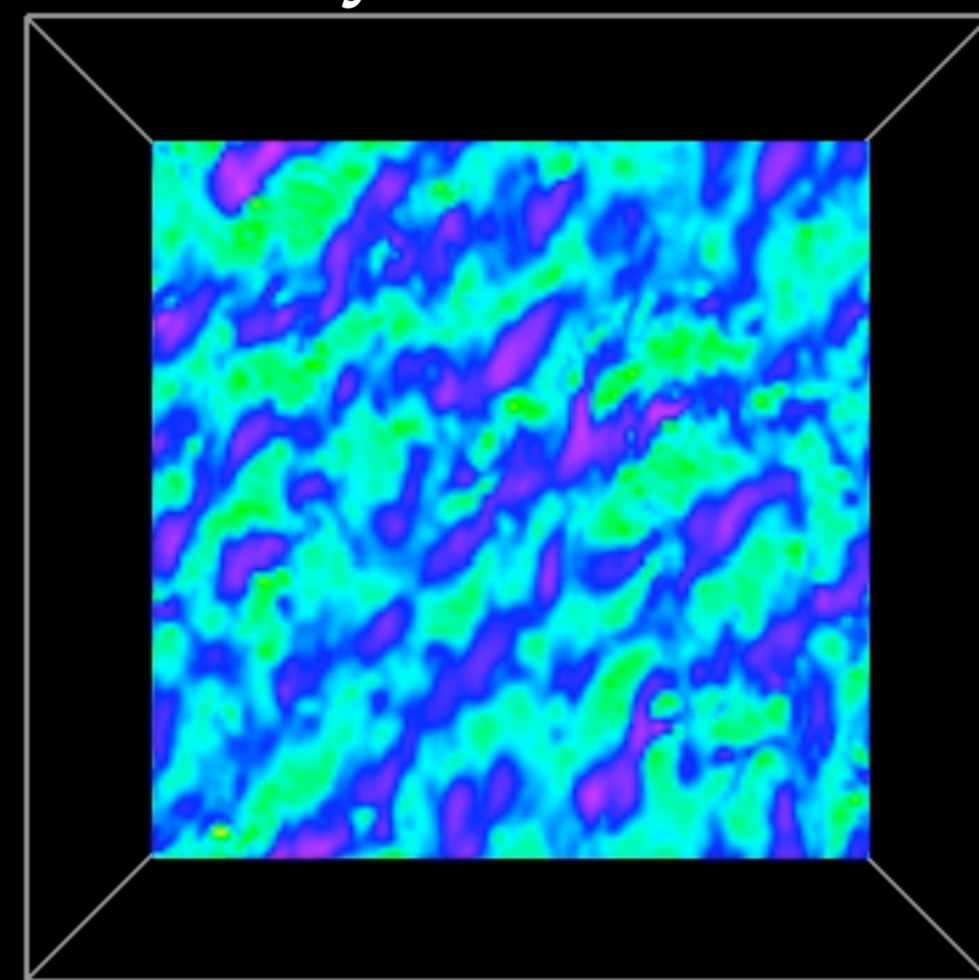
# Top-Down View

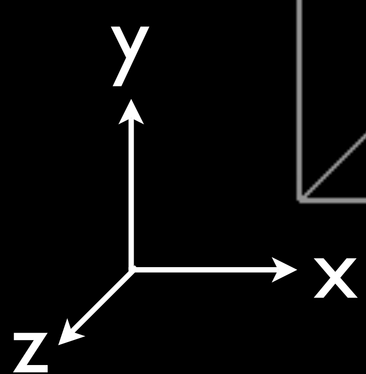
*N*/*f* = 75

Outer Layer



Pycnocline



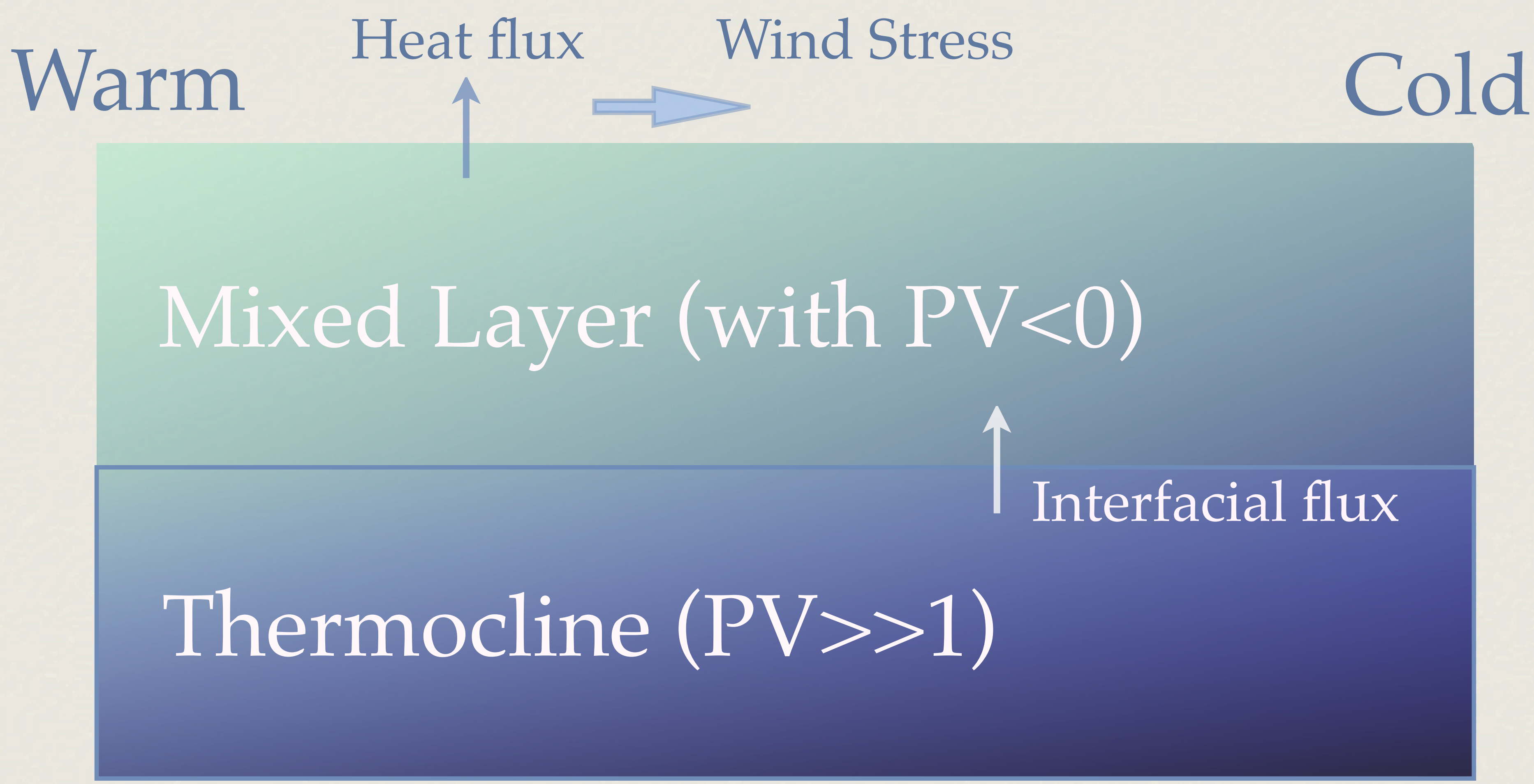



$\partial w/\partial z, z/\delta = 0.6$

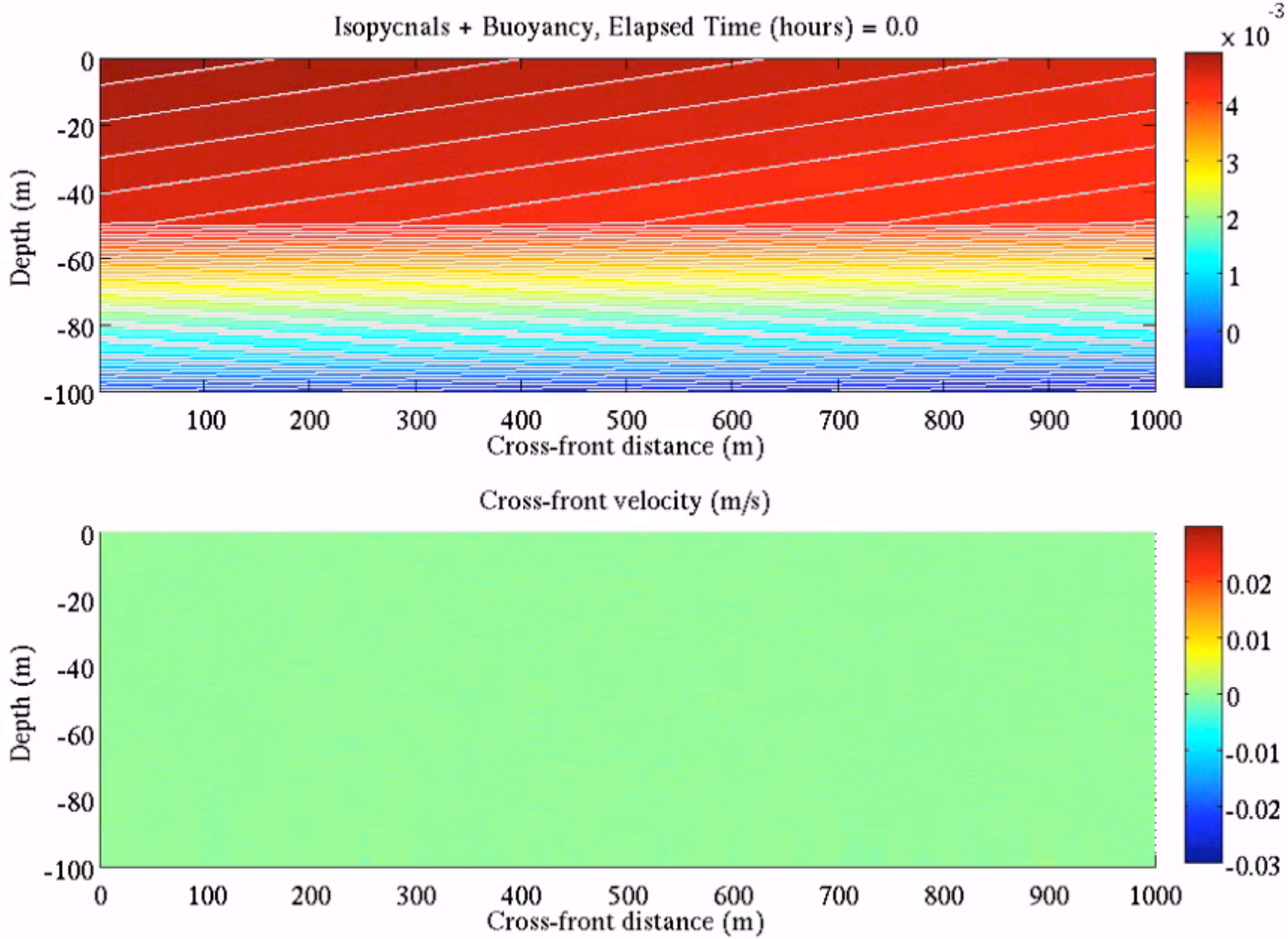
$\theta', z/\delta = 0.17$

# Symmetric Instability

- Occurs at very strong fronts when the potential vorticity is negative (PV < 0)
- Since PV is conserved, an isolated instability cannot arrest itself.



# Secondary K-H Instability

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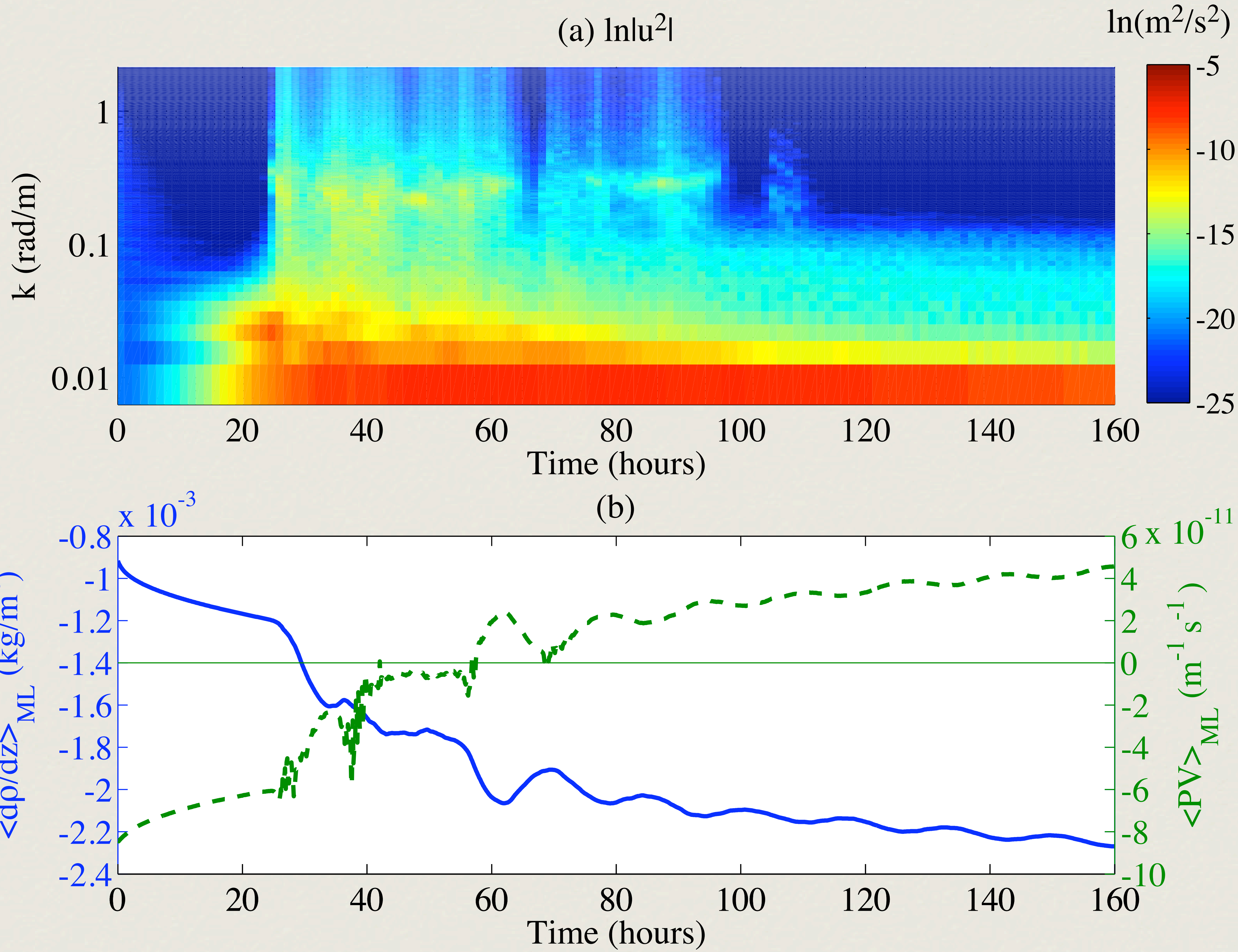
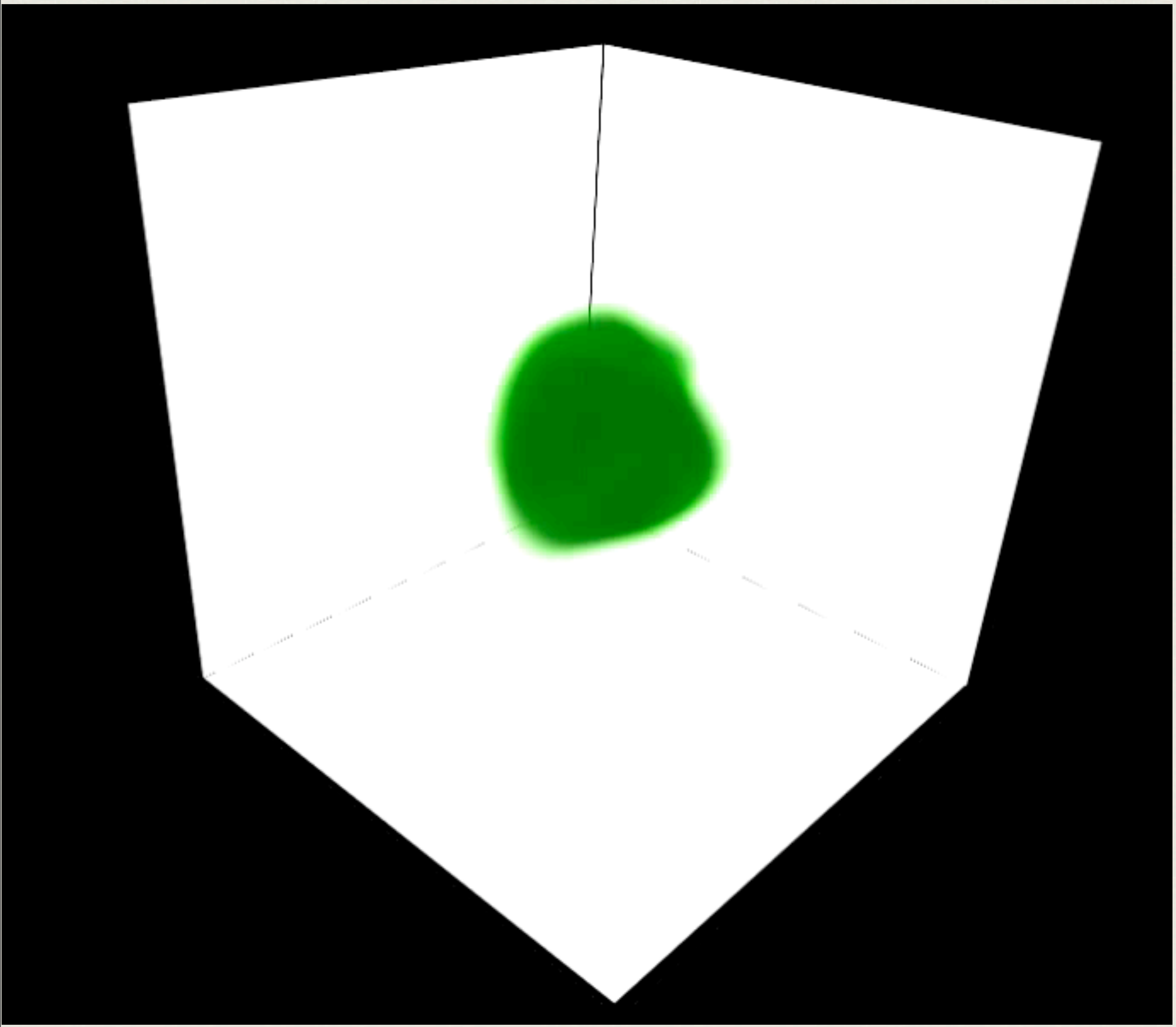
# Secondary K-H Instability



# Bacterial Advection



Green = Nutrient

Red = High bacterial concentration

Blue = Low bacterial concentration

Bacteria have the ability to swim up gradients (chemotaxis)