A Global Atmospheric Model

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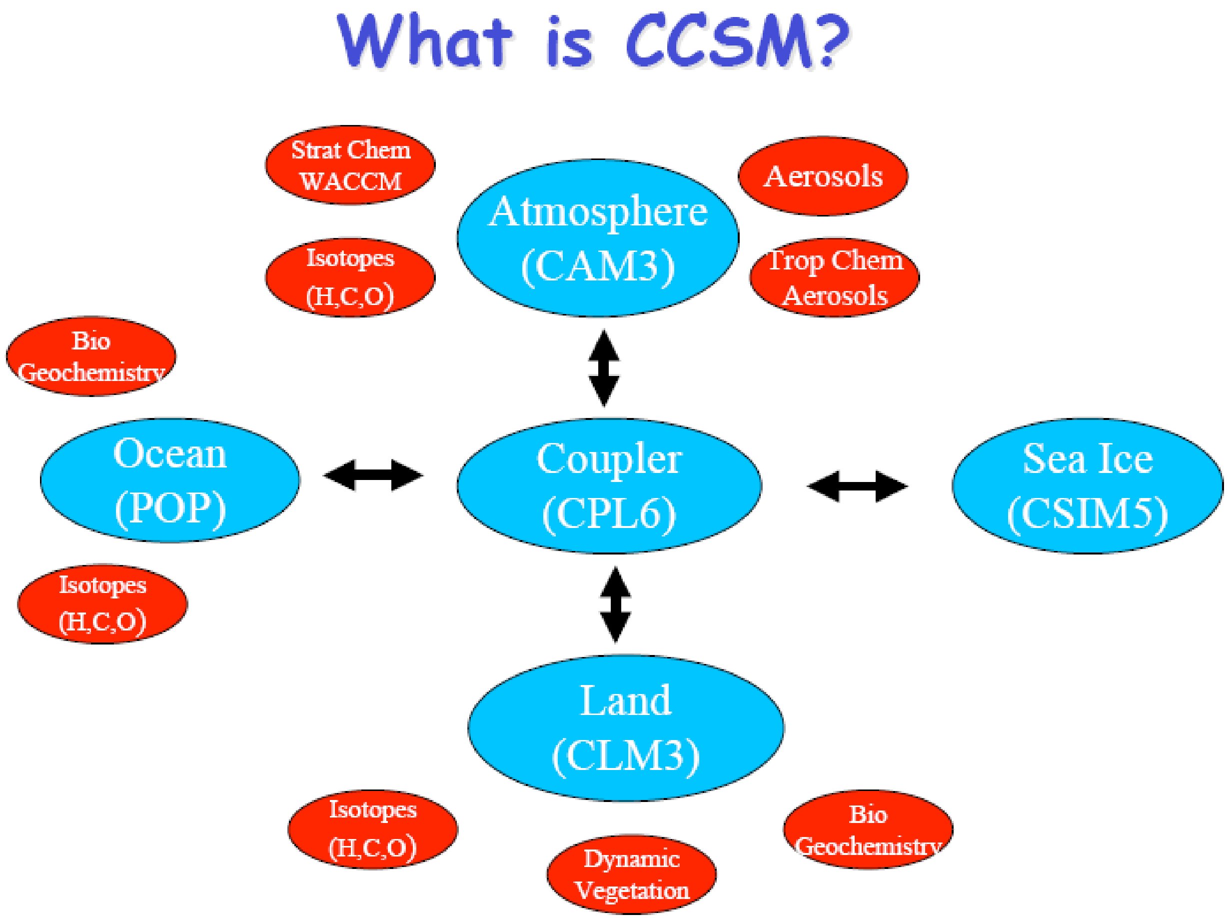
Outline

- Broad overview of what is in a global climate/weather model of the atmosphere
- Spectral dynamical core
- Some results-climate and global NWP
- End of era- why and what’s next?

Atmosphere in Coupled Climate System Model



Dynamic equations in vorticity divergence form

$$\frac{\partial \zeta}{\partial t} = k \cdot \nabla \times (n/\cos\phi) + F_{\zeta_H}, \qquad (3.3)$$

$$\frac{\partial \delta}{\partial t} = \nabla \cdot (n/\cos\phi) - \nabla^2 (E + \Phi) + F_{\delta_H}, \qquad (3.4)$$

$$\frac{\partial T}{\partial t} = \frac{-1}{a\cos^2\phi}\left[\frac{\partial}{\partial\lambda}(UT) + \cos\phi\frac{\partial}{\partial\phi}(VT)\right] + T\delta - \dot{\eta}\frac{\partial T}{\partial \eta} + \frac{R}{c_p^*}T_v\frac{\omega}{p}$$
$$+Q + F_{T_H} + F_{F_H}, \qquad (3.5)$$

$$\frac{\partial q}{\partial t} = \frac{-1}{a\cos^2\phi}\left[\frac{\partial}{\partial\lambda}(Uq) + \cos\phi\frac{\partial}{\partial\phi}(Vq)\right] + q\delta - \dot{\eta}\frac{\partial q}{\partial \eta} + S, \qquad (3.6)$$

$$\frac{\partial \pi}{\partial t} = \int_1^{\eta_t} \nabla\cdot\left(\frac{\partial p}{\partial \eta}V\right) d\eta. \qquad (3.7)$$

where

$$n_U = +(\zeta + f)V - \dot{\eta}\frac{\partial U}{\partial \eta} R\frac{T_v}{p}\frac{1}{a} - \frac{\partial p}{\partial \lambda} + F_U, \qquad (3.8)$$

$$n_V = -(\zeta + f)U - \dot{\eta}\frac{\partial V}{\partial \eta} - R\frac{T_v}{p}\frac{\cos\phi}{a}\frac{\partial p}{\partial \phi} + F_V, \qquad (3.9)$$

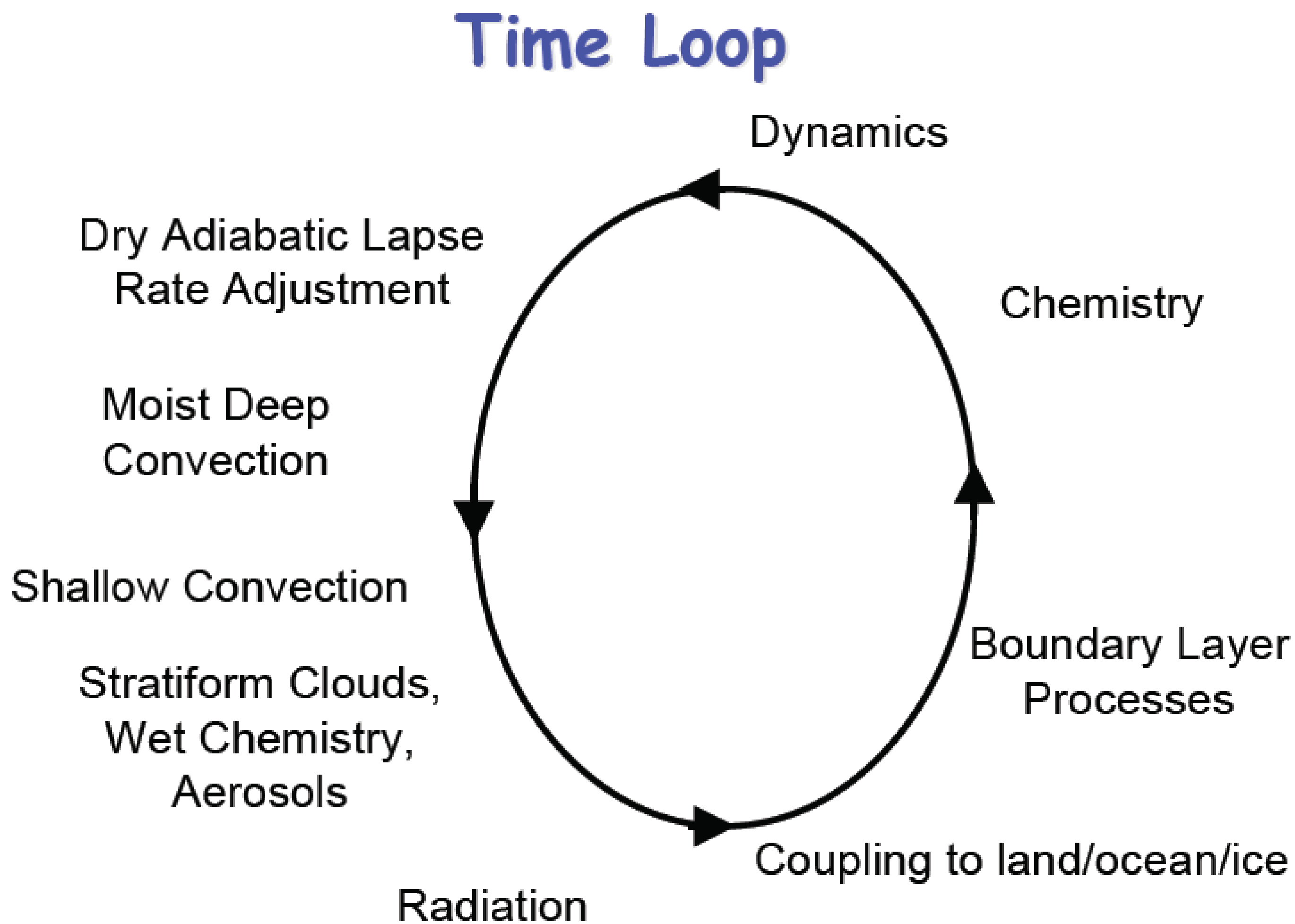
$$E = \frac{U^2 + V^2}{2\cos^2\phi}, \qquad (3.10)$$

$$(U, V) = (u, v)\cos\phi, \qquad (3.11)$$

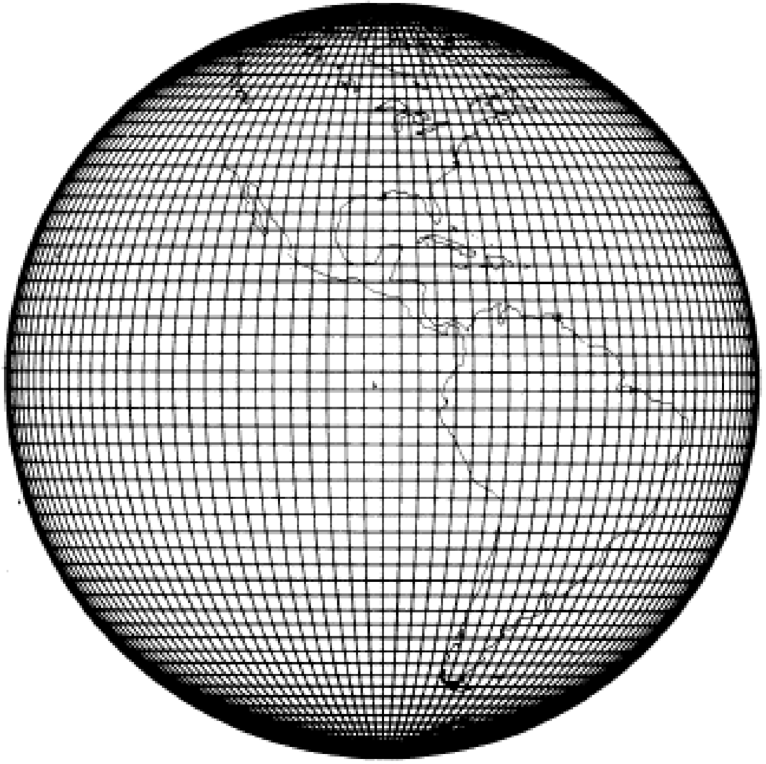
$$T_v = \left[1 + \left(\frac{R_v}{R} - 1\right)q\right]T, \qquad (3.12)$$

$$c_p^* = \left[1 + \left(\frac{c_{p_v}}{c_p} - 1\right)q\right]c_p. \qquad (3.13)$$

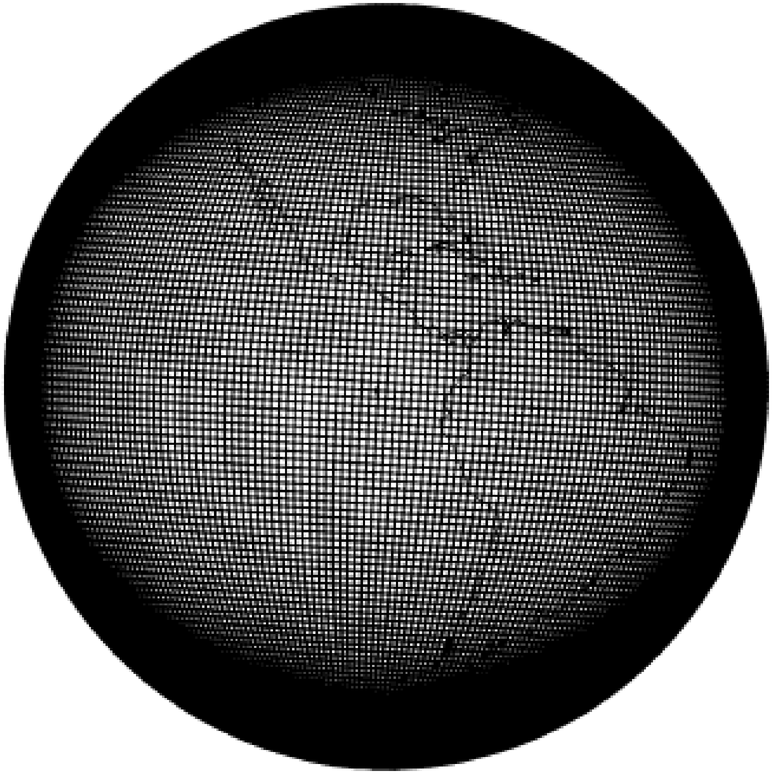
Atmospheric model = dynamics+physics



Horizontal Discretization



Typical Climate Application



Next Generation Climate Applications

Vertical resolution

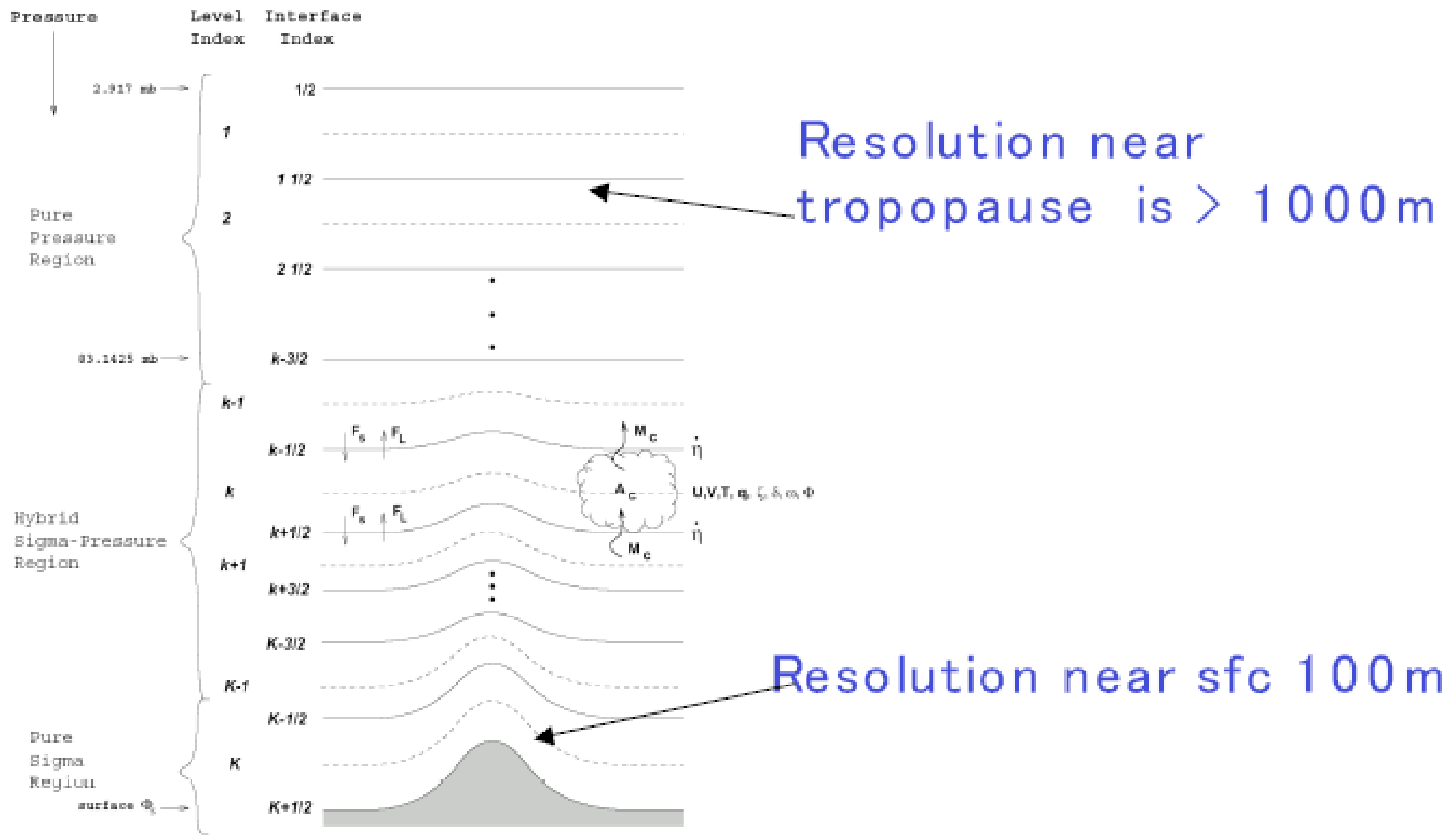


Figure 1. Vertical level structure of CCM

What is in *dynamics*? (CFD actually VLES)

* 3. Dynamics
    o 3.1 Eulerian Dynamical Core
        + 3.1.1 Generalized terrain-following vertical coordinates
        + 3.1.2 Conversion to final form
        + 3.1.3 Continuous equations using $\partial\ln(\pi)/\partial t$
        + 3.1.4 Semi-implicit formulation
        + 3.1.5 Energy conservation
        + 3.1.6 Horizontal diffusion
        + 3.1.7 Finite difference equations
        + 3.1.8 Time filter
        + 3.1.9 Spectral transform
        + 3.1.10 Spectral algorithm overview
        + 3.1.11 Combination of terms
        + 3.1.12 Transformation to spectral space
        + 3.1.13 Solution of semi-implicit equations
        + 3.1.14 Horizontal diffusion
        + 3.1.15 Initial divergence damping
        + 3.1.16 Transformation from spectral to physical space
        + 3.1.17 Horizontal diffusion correction
        + 3.1.18 Semi-Lagrangian Tracer Transport
        + 3.1.19 Mass fixers
        + 3.1.20 Energy Fixer
        + 3.1.21 Statistics Calculations
        + 3.1.22 Reduced grid

What is in *moist physics?*

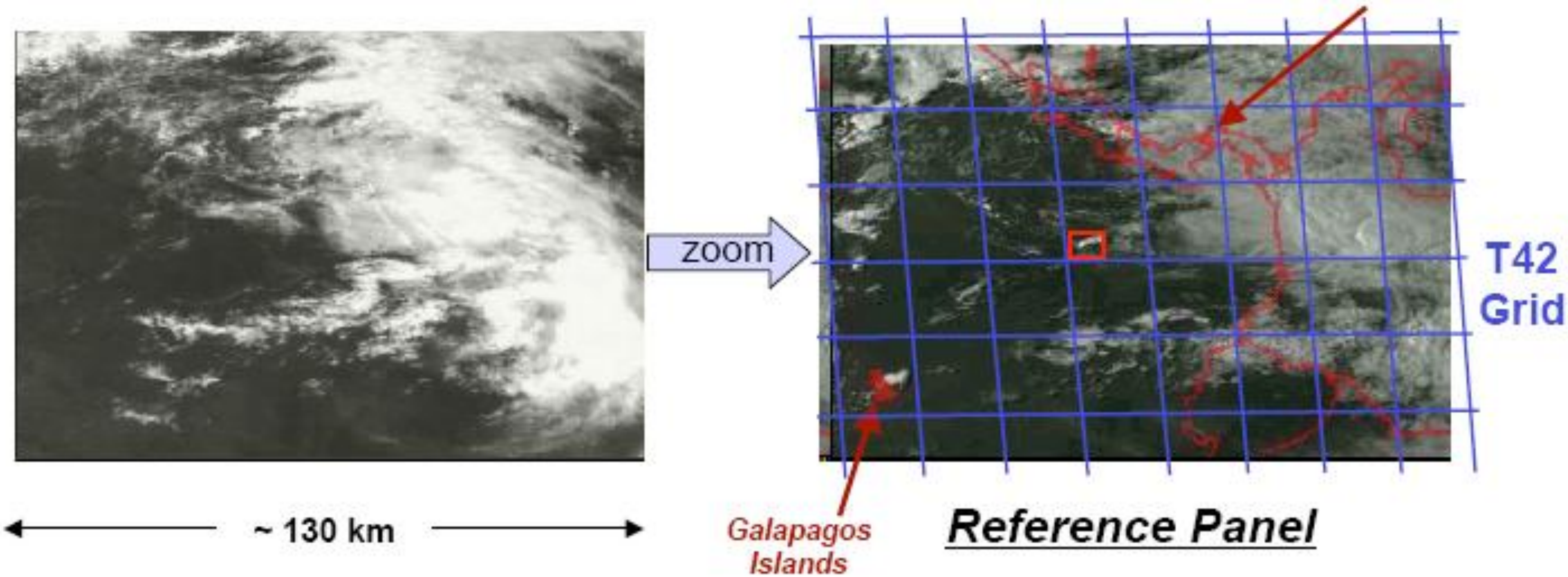
* 4. Model Physics
    - o 4.1 Deep Convection
        - + 4.1.1 Updraft Ensemble
        - + 4.1.2 Downdraft Ensemble
        - + 4.1.3 Closure
        - + 4.1.4 Numerical Approximations
        - + 4.1.5 Deep Convective Tracer Transport
    - o 4.2 Shallow/Middle Tropospheric Moist Convection
    - o 4.3 Evaporation of convective precipitation
    - o 4.4 Conversion to and from dry and wet mixing ratios for trace constituents in the model
    - o 4.5 Prognostic Condensate and Precipitation Parameterization
        - + 4.5.1 Introductory comments
        - + 4.5.2 Description of the macroscale component
        - + 4.5.3 Description of the microscale component
    - o 4.6 Dry Adiabatic Adjustment

What is in *SW radiation physics and clouds?*

What is in *LW radiation and PBL?*

- o 4.9 Parameterization of Longwave Radiation
  - + 4.9.1 Major absorbers
  - + 4.9.2 Water vapor
  - + 4.9.3 Trace gas parameterizations
  - + 4.9.4 Mixing ratio of trace gases
  - + 4.9.5 Cloud emissivity
  - + 4.9.6 Numerical algorithms and cloud overlap
- o 4.10 Surface Exchange Formulations
  - + 4.10.1 Land
    - # 4.10.1.1 Roughness lengths and zero-plane displacement
    - # 4.10.1.2 Monin-Obukhov similarity theory
  - + 4.10.2 Ocean
  - + 4.10.3 Sea Ice
- o 4.11 Vertical Diffusion and Boundary Layer Processes
  - + 4.11.1 Free atmosphere turbulent diffusivities
  - + 4.11.2 ``Non-local'' atmospheric boundary layer scheme

At current climate resolutions most physics is subgrid scale



Courtesy, NASA Goddard Space Flight Center Scientific Visualization Studio

Spectral dynamics-what and why

SPECTRAL METHOD (1 DIMENSION)

$$q(x_j, t) = \sum_{k=-K}^{K} q_k(t) e^{ikx_j}$$

$q_{-k} = q_k^*$

$q(x_j, t)$ and $q_k(t)$ equivalent if $K = J/2$

$$\frac{1}{J} \sum_{j=1}^{J} e^{ikx_j} e^{-ilx_j} = \delta_{kl}$$

$$\frac{1}{J} \sum_{j=1}^{J} q(x_j, t) e^{-ikx_j} = q_k(t)$$

ADVECTION, CONSTANT U

$$\frac{\partial q}{\partial t} + U\frac{\partial q}{\partial x} = 0$$

$$q(x_j, t) = \sum_{k=-K}^{K} q_k(t) e^{ikx_j}$$

$$\frac{1}{2\Delta t}\sum_{k=-K}^{K}(q_k^{n+1} - q_k^{n-1})e^{ikx_j} + U\sum_{k=-K}^{K} ikq_k^n e^{ikx_j} = 0$$

Multiply by e^{-ikx_j} and sum over the x_j from 1 to J

$$\frac{1}{2\Delta t}(q_k^{n+1} - q_k^{n-1}) + Uikq_k^n = 0$$

NO SPATIAL TRUNCATION ERROR = NO FALSE DISPERSION

Non-uniform Advection [U=U(x,t)]

SPECTRAL TRANSFORM METHOD

$$U(x_j, n\Delta t) = \sum_{k=-K}^{K} U_k^n e^{ikx_j}$$

$$\frac{\partial q(x_j, n\Delta t)}{\partial x} = \sum_{k=-K}^{K} ikq_k^n e^{ikx_j}$$

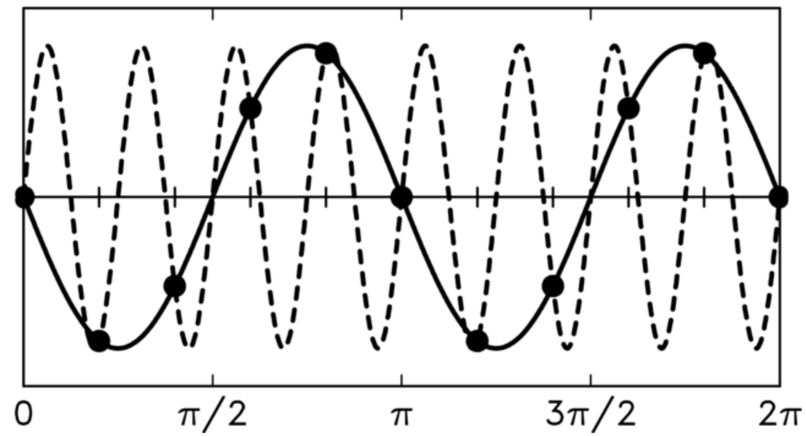
$U(x_j)\frac{\partial q}{\partial x}(x_j)$ calculated on the grid

$$\frac{\partial q_k^n}{\partial t} = -\left(U\frac{\partial q}{\partial x}\right)_k^n = -\frac{1}{J}\sum_{j=1}^{J}\left[U(x_j, n\Delta t)\frac{\partial q(x_j, n\Delta t)}{\partial x}\right] e^{-ikx_j}$$

$K \leq J/2$

$$\frac{1}{2\Delta t}(q_k^{n+1} - q_k^{n-1}) + \frac{1}{J}\sum_{j=1}^{J}\left[U(x_j, n\Delta t)\frac{\partial q(x_j, n\Delta t)}{\partial x}\right] e^{-ikx_j} = 0$$

ALIASING



$$e^{ilx_j} e^{imx_j} = e^{i(l+m)x_j}$$

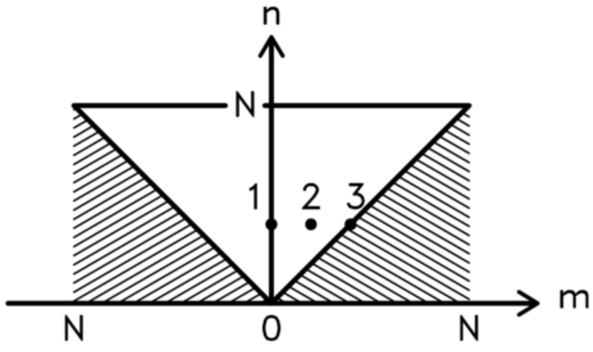
$$\sum_{l=-K}^{K} U_l e^{ilx_j} \sum_{m=-K}^{K} imq_m e^{imx_j} = \sum_{l=-K}^{K} \sum_{m=-K}^{K} imq_m U_l e^{i(l+m)x_j}$$

if $(l+m) > J/2$, then appears as $k < J/2$

choose K so aliased waves fall in range$(K, J/2)$

instead of $K = J/2$, $K \leq (J-1)/3$

On the sphere use the natural basis-avoids polar singularity

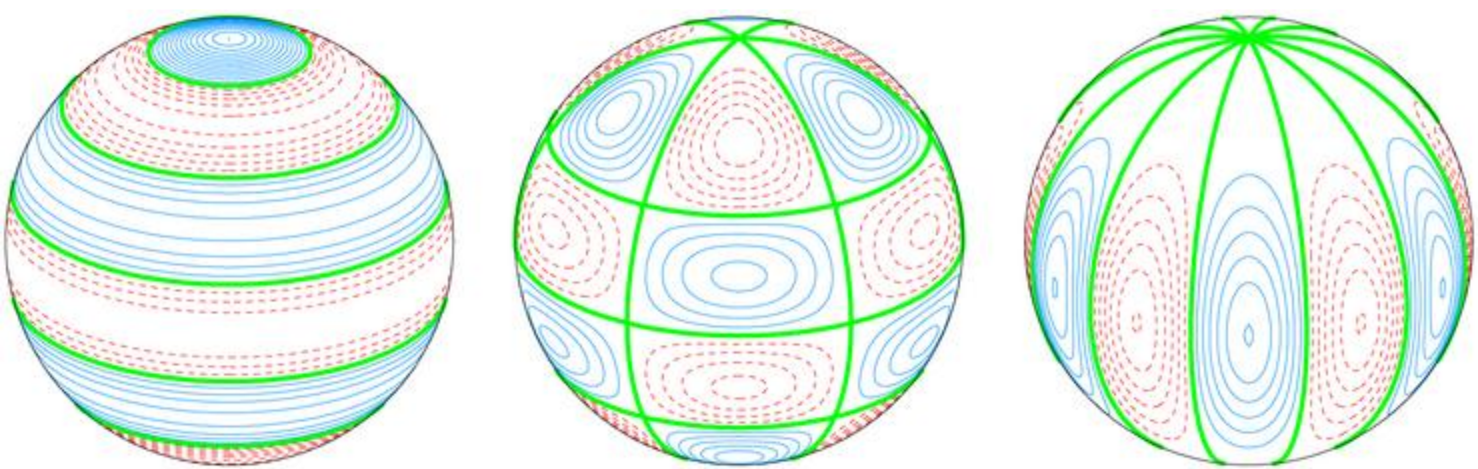


SPHERICAL HARMONICS

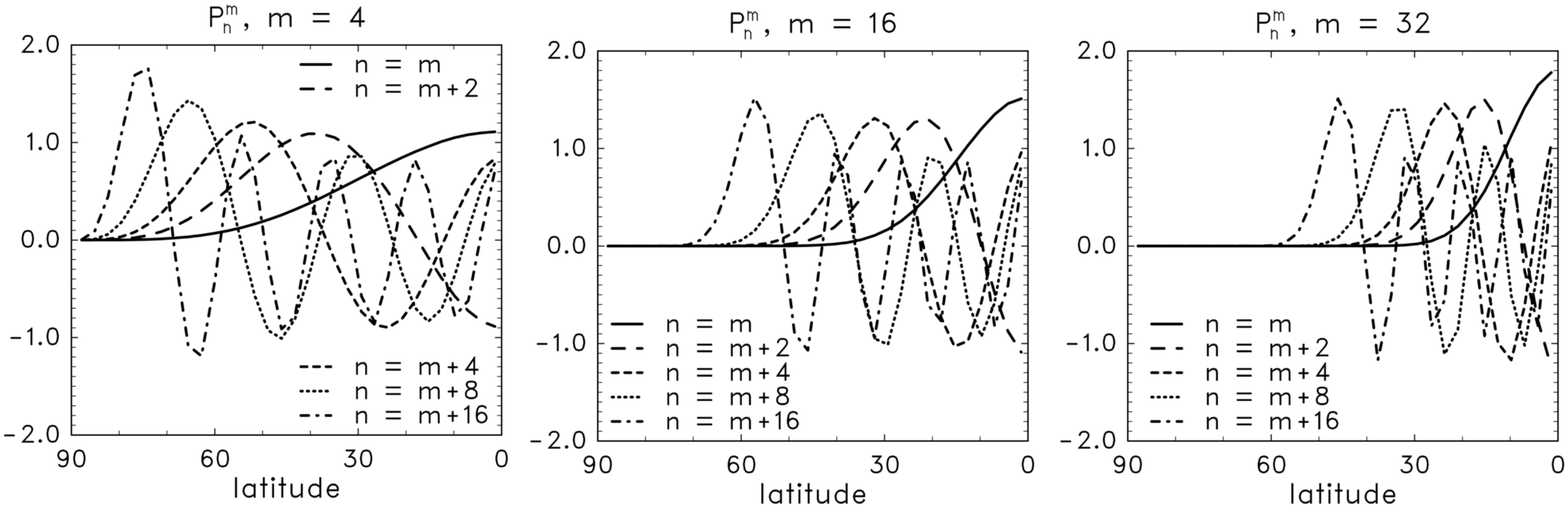
$$q(\lambda,\varphi,t)=\sum_{n=0}^{N}\sum_{m=-n}^{n} q_n^m(t)Y_n^m(\lambda,\mu)$$

$$Y_n^m(\lambda,\mu)=P_n^m(\mu)e^{im\lambda}$$

$$\mu=\sin\varphi$$



Fourier series in longitude and associated Legendre functions in latitude



FFT in longitude
Gaussian quadrature in latitude

$$q(\lambda, \varphi, t) = \sum_{n=0}^{N} \sum_{m=-n}^{n} q_n^m(t) Y_n^m(\lambda, \mu)$$

$$q_n^m = \int_{-1}^{1} \frac{1}{2\pi} \int_0^{2\pi} q(\lambda, \mu) e^{-im\lambda} d\lambda P_n^m(\mu) d\mu$$

$$q^m(\mu) = \frac{1}{2\pi} \int_0^{2\pi} q(\lambda, \mu) e^{-im\lambda} d\lambda$$

$$q_n^m = \sum_{j=1}^{J} q^m(\mu_j) P_n^m(\mu_j) w_j$$

$$\mu_j : \ J \text{ roots of } P_J(\mu) \ , \quad w_j = \frac{2(1-\mu_j^2)}{[J\ P_{J-1}(\mu_j)]^2} \ , \quad \sum_{j=1}^{J} w_j = 2.0$$

PROPERTIES OF SPHERICAL HARMONICS

$$Y_n^m(\lambda, \mu) = P_n^m(\mu) e^{im\lambda}$$

$$\nabla q = \frac{1}{a}\left(\frac{1}{\cos\varphi}\frac{\partial q}{\partial \lambda}\hat{\mathbf{i}}\ , \frac{\partial q}{\partial \varphi}\hat{\mathbf{j}}\right)$$

$$\frac{\partial Y_n^m}{\partial \lambda} = imY_n^m$$

$$\cos\varphi\frac{\partial Y_n^m}{\partial \varphi} = (n+1)\epsilon_n^m Y_{n-1}^m - n\epsilon_{n+1}^m Y_{n+1}^m \quad , \qquad \epsilon_n^m = \left(\frac{n^2 - m^2}{4n^4 - 1}\right)^{\frac{1}{2}}$$

$$\nabla^2 q = \frac{1}{a^2}\left[\frac{1}{\cos^2\varphi}\frac{\partial^2 q}{\partial \lambda^2} + \frac{1}{\cos\varphi}\frac{\partial}{\partial \varphi}\left(\cos\varphi\frac{\partial q}{\partial \varphi}\right)\right]$$

$$\nabla^2 Y_n^m = \frac{-n(n+1)}{a^2} Y_n^m$$

SHALLOW WATER EXAMPLE

$$\zeta = \frac{1}{a\cos\varphi}\left[\frac{\partial v}{\partial\lambda} - \frac{\partial}{\partial\varphi}(u\cos\varphi)\right] \quad , \quad \delta = \frac{1}{a\cos\varphi}\left[\frac{\partial u}{\partial\lambda} + \frac{\partial}{\partial\varphi}(v\cos\varphi)\right]$$

$$U = u\cos\varphi \quad , \quad V = v\cos\varphi$$

$$\frac{\partial\zeta}{\partial t} = -\frac{1}{a\cos^2\varphi}\frac{\partial}{\partial\lambda}\left[(\zeta + f)\,U\right] - \frac{1}{a\cos\varphi}\frac{\partial}{\partial\varphi}\left[(\zeta + f)\,V\right]$$

$$\frac{\partial\delta}{\partial t} = \frac{1}{a\cos^2\varphi}\frac{\partial}{\partial\lambda}\left[(\zeta + f)\,V\right] - \frac{1}{a\cos\varphi}\frac{\partial}{\partial\varphi}\left[(\zeta + f)\,U\right] - \nabla^2\left[gh + \frac{U^2 + V^2}{2\cos^2\varphi}\right]$$

$$\frac{\partial h}{\partial t} = -\frac{1}{a\cos^2\varphi}\frac{\partial}{\partial\lambda}(hU) - \frac{1}{a\cos\varphi}\frac{\partial}{\partial\varphi}(hV)$$

EXPLICIT APPROXIMATIONS $\quad \zeta^n_{i,j} \to \zeta^n$

$$\frac{\partial \zeta}{\partial t} = -\frac{1}{a\cos^2\varphi}\frac{\partial}{\partial \lambda}\left[(\zeta + f)\,U\right] - \frac{1}{a\cos\varphi}\frac{\partial}{\partial \varphi}\left[(\zeta + f)\,V\right]$$

$$\zeta^{n+1} = \zeta^{n-1} - \frac{2\Delta t}{a\cos^2\varphi}\frac{\partial}{\partial \lambda}\left[(\zeta^n + f)\,U^n\right] - \frac{2\Delta t}{a\cos\varphi}\frac{\partial}{\partial \varphi}\left[(\zeta^n + f)\,V^n\right]$$

$$\frac{\partial \delta}{\partial t} = \frac{1}{a\cos^2\varphi}\frac{\partial}{\partial \lambda}\left[(\zeta + f)\,V\right] - \frac{1}{a\cos\varphi}\frac{\partial}{\partial \varphi}\left[(\zeta + f)\,U\right] \\ -\nabla^2\left[gh + \frac{U^2 + V^2}{2\cos^2\varphi}\right]$$

$$\delta^{n+1} = \delta^{n-1} + \frac{2\Delta t}{a\cos^2\varphi}\frac{\partial}{\partial \lambda}\left[(\zeta^n + f)\,V^n\right] - \frac{2\Delta t}{a\cos\varphi}\frac{\partial}{\partial \varphi}\left[(\zeta^n + f)\,U^n\right] \\ -2\Delta t\nabla^2\left[gh^n + \frac{(U^n)^2 + (V^n)^2}{2\cos^2\varphi}\right]$$

$$\frac{\partial h}{\partial t} = -\frac{1}{a\cos^2\varphi}\frac{\partial}{\partial \lambda}(hU) - \frac{1}{a\cos\varphi}\frac{\partial}{\partial \varphi}(hV)$$

$$h^{n+1} = h^{n-1} - \frac{2\Delta t}{a\cos^2\varphi}\frac{\partial}{\partial \lambda}(h^n U^n) - \frac{2\Delta t}{a\cos\varphi}\frac{\partial}{\partial \varphi}(h^n V^n)$$

For nonlinear terms grid to spectral: FFT then Legendre

$$
\begin{aligned}
\left\{\frac{\partial}{\partial\lambda}\left[(\zeta+f)V\right]\right\}^m &= \frac{1}{2\pi}\int_o^{2\pi}\frac{\partial\left[(\zeta+f)V\right]}{\partial\lambda}e^{-im\lambda}d\lambda \\
&= -\frac{1}{2\pi}\int_o^{2\pi}\left[(\zeta+f)V\right]\frac{\partial\left(e^{-im\lambda}\right)}{\partial\lambda}d\lambda \\
&= im\frac{1}{2\pi}\int_o^{2\pi}\left[(\zeta+f)V\right]e^{-im\lambda}d\lambda
\end{aligned}
$$

$$
\left\{\frac{\partial}{\partial\lambda}\left[(\zeta+f)V\right]\right\}_n^m = im\sum_{j=1}^{J}\left[(\zeta+f)V\right]_j^m P_n^m(\mu_j)w_j
$$

Latitudinal derivatives need care

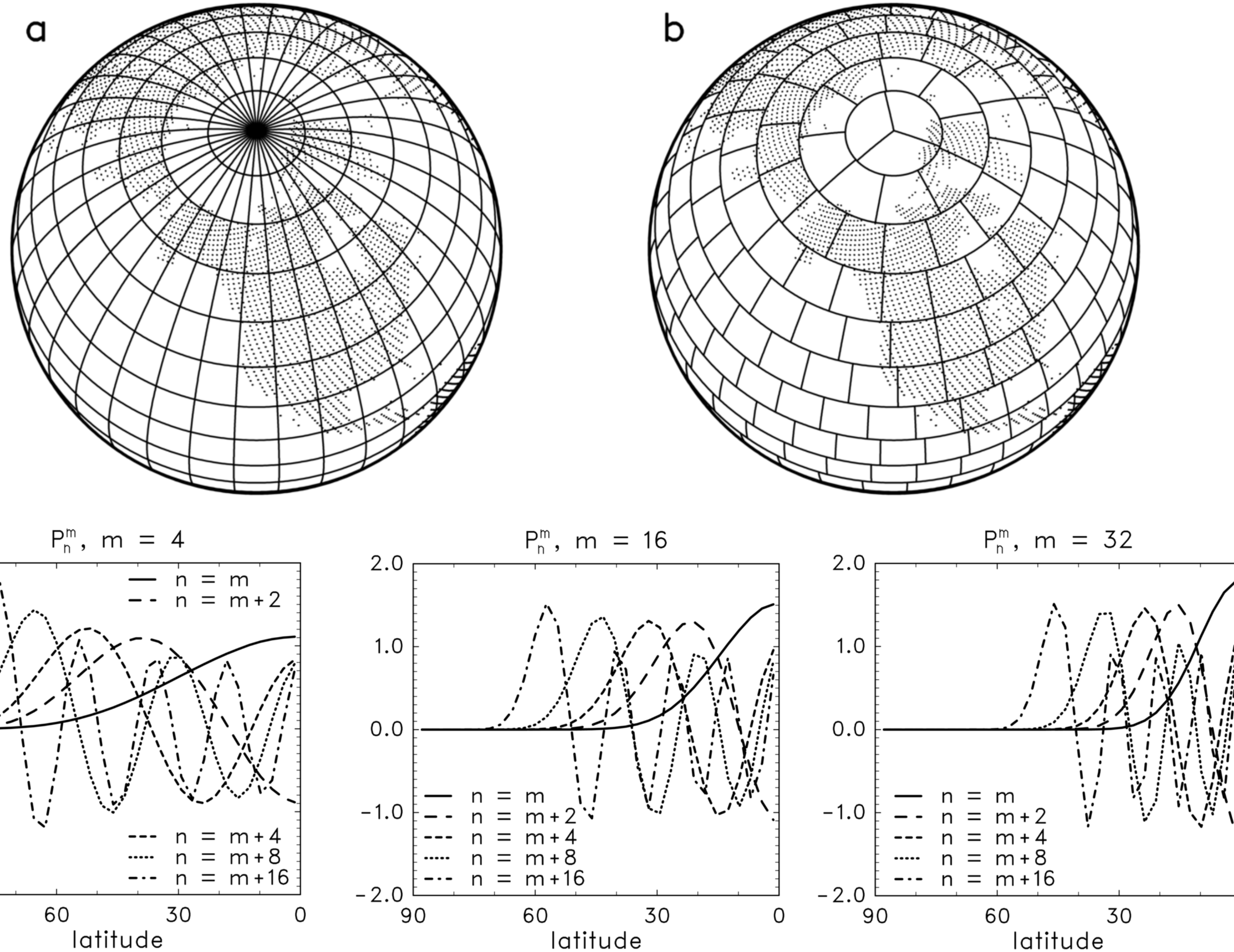
$$\left\{\frac{\partial}{\partial\mu}\left[(\zeta+f)U\right]\right\}_n^m = \int_{-1}^{1}\frac{\partial}{\partial\mu}\left[(\zeta+f)U\right]^m P_n^m d\mu$$
$$= -\int_{-1}^{1}\left[(\zeta+f)U\right]^m \frac{dP_n^m}{d\mu}d\mu$$

$$H_n^m = (1-\mu^2)\frac{dP_n^m}{d\mu}$$

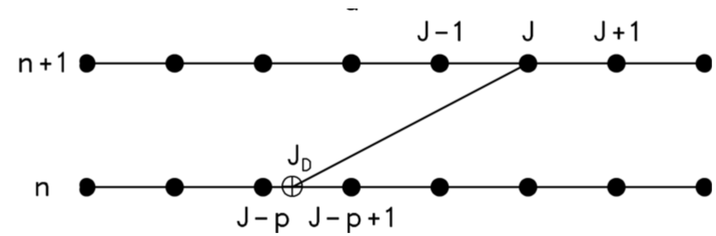
$$\left\{\frac{\partial}{\partial\mu}\left[(\zeta+f)U\right]\right\}_n^m = -\sum_{j=1}^{J}\left[(\zeta+f)U\right]_j^m \frac{H_n^m(\mu_j)}{(1-\mu_j^2)}w_j$$

$$\left\{\nabla^2\left[gh+\frac{(U)^2+(V)^2}{2\cos^2\varphi}\right]\right\}_n^m =$$
$$\frac{-n(n+1)}{a^2}\sum_{j=1}^{J}\left[gh+\frac{(U)^2+(V)^2}{2\cos^2\varphi}\right]_j^m P_n^m(\mu_j)w_j$$

REDUCED GRID



SEMI-LAGRANGIAN METHOD



$$\frac{\partial q}{\partial t} + U\frac{\partial q}{\partial x} = S$$

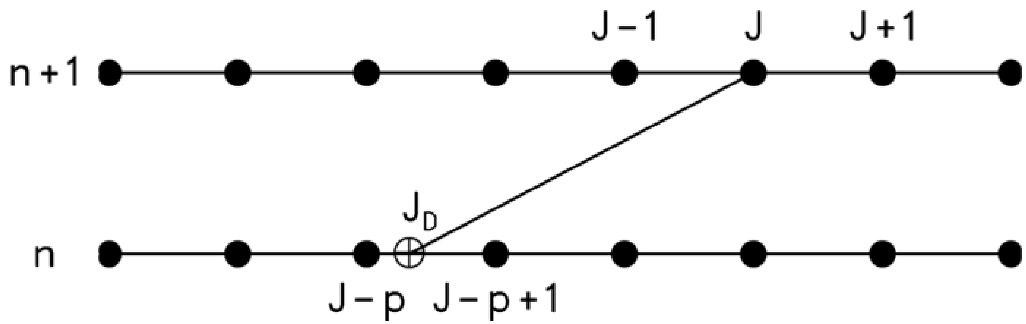
$$\frac{dq}{dt} = S(x,t) \quad , \quad \frac{dx}{dt} = U(x,t)$$

$$q_j^{n+1} - q_{j_D}^n = \int_{(x_{j_D},t^n)}^{(x_j,t^{n+1})} S(x,t)ds \quad , \quad x_j - x_{j_D} = \int_{(x_{j_D},t^n)}^{(x_j,t^{n+1})} U(x,t)ds$$

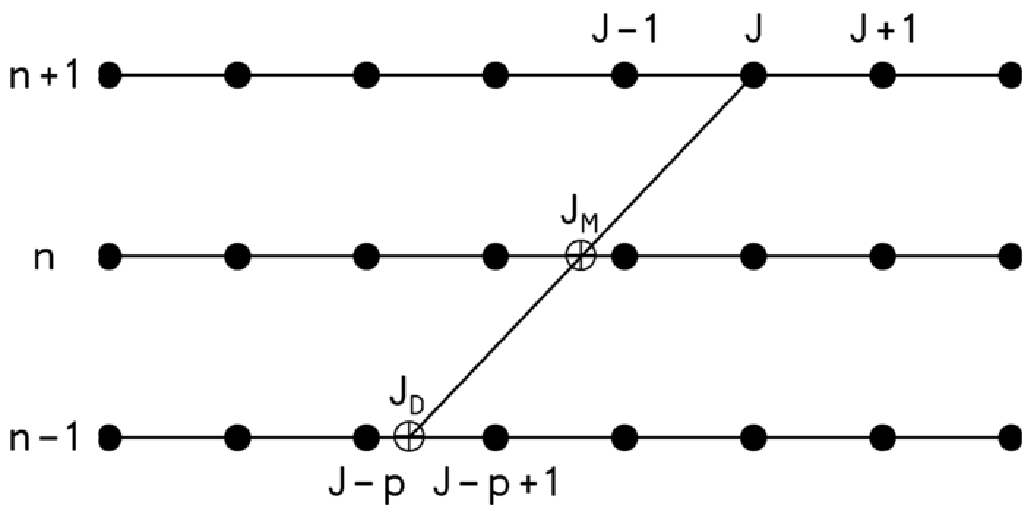
$$q_j^{n+1} = q_{j_D}^n + \Delta t S_{j_M}^{n+1/2} \quad , \quad x_{j_D} = x_j - U\Delta t$$

$$q_{_A}^{n+1} = q_{_D}^n + \Delta t S_{_M}^{n+1/2} \quad , \quad x_{_D} = x_{_A} - U\Delta t$$

TWO-TIME LEVEL



THREE-TIME LEVEL



Staying power of spectral dynamical core

EFFICIENCY GAINS SINCE 1987

	NWP	CLIMATE
Eulerian → semi-Lagrangian	5	1.2
3-time-level → 2-time-level	1.8	1.3
Full grid → Reduced grid	1.4	1.4
Quadratic → Linear grid	3.4	3.4
Troposphere → Stratosphere Thinner PBL levels	1.7	1.7
	73	13

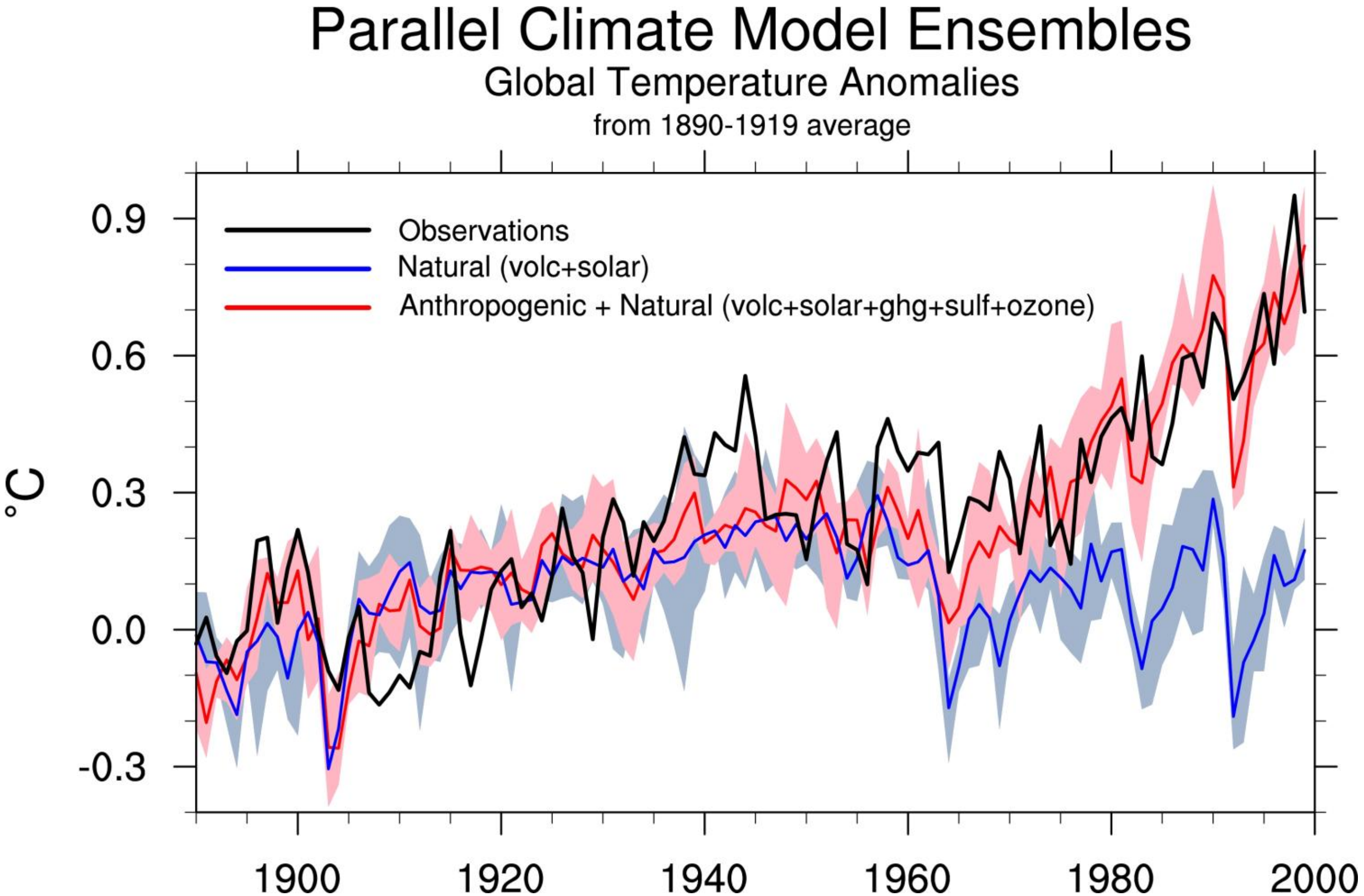
For climate: T75 for the cost of T42

Machenhauer, B., 1979: The spectral method. In A. Kasahara (ed.), *Numerical Methods Used in Atmospheric Models, Vol. 2*, GARP Publications Series No 17, WMO and ICSU, Geneva, pp. 121–275.

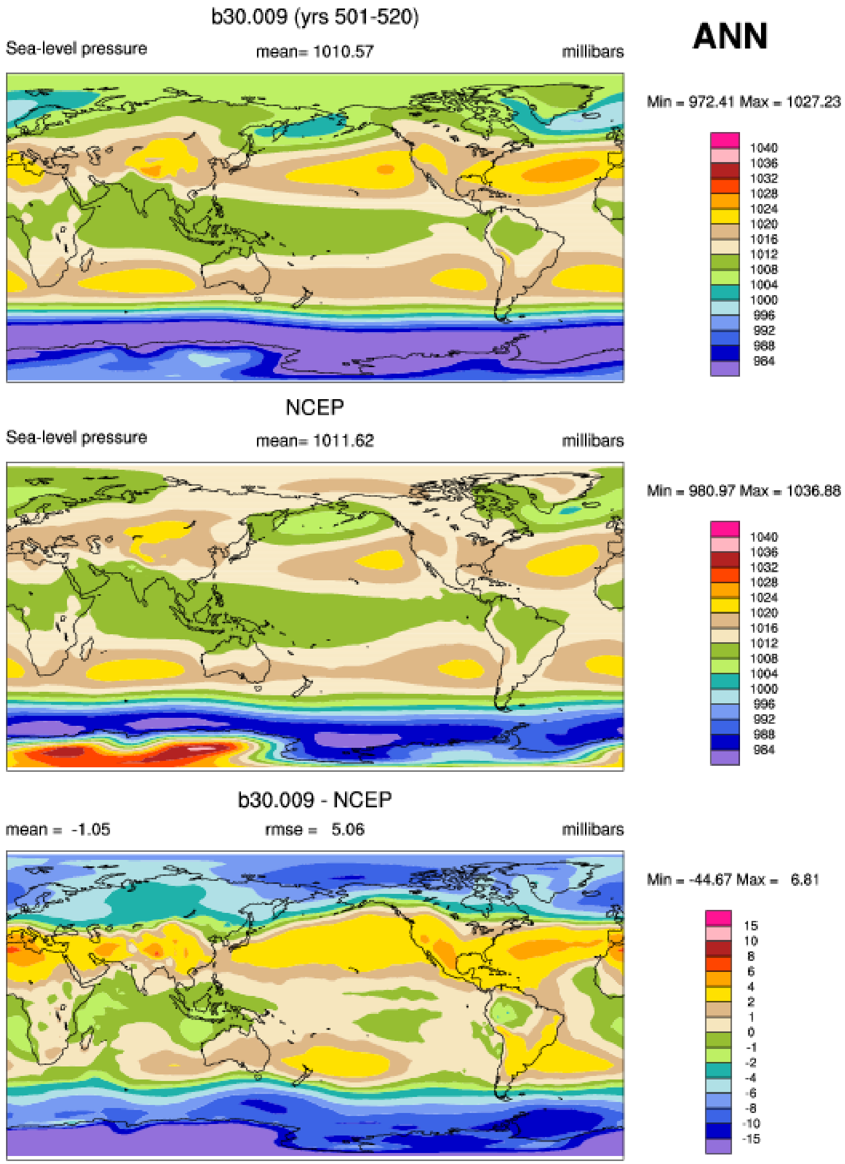
Williamson, D. L. and R. Laprise, 2000: Numerical Approximations for Atmospheric General Circulation Models. In P. Mote and A. O'Neill (eds.), *Numerical Modelling of the Global Atmosphere in the Climate System*, Kluwer Academic Publishers, Netherlands, 127–219.

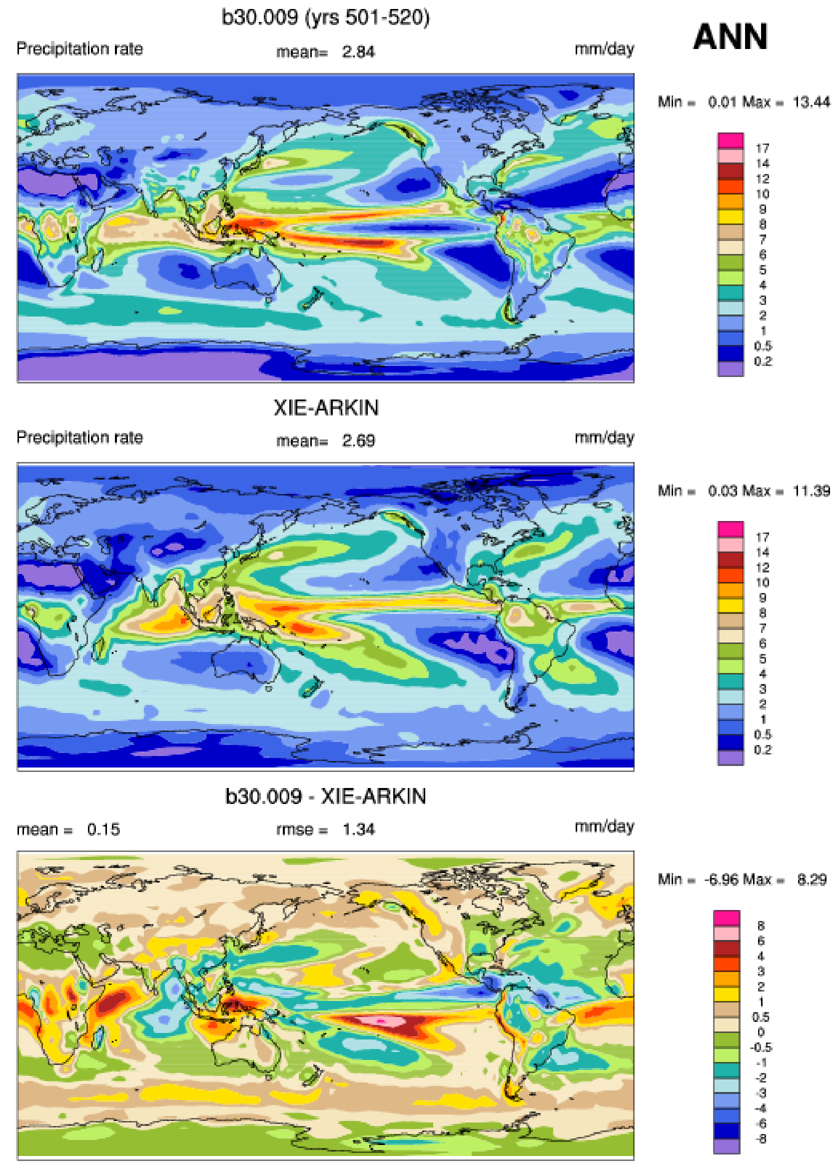
Staniforth, A. and Côté, J., 1991: Semi-Lagrangian integration schemes for atmospheric models - A Review, *Mon. Wea. Rev.*, **119**, 2206–2223.

Calibrate with 20th century and test hypotheses

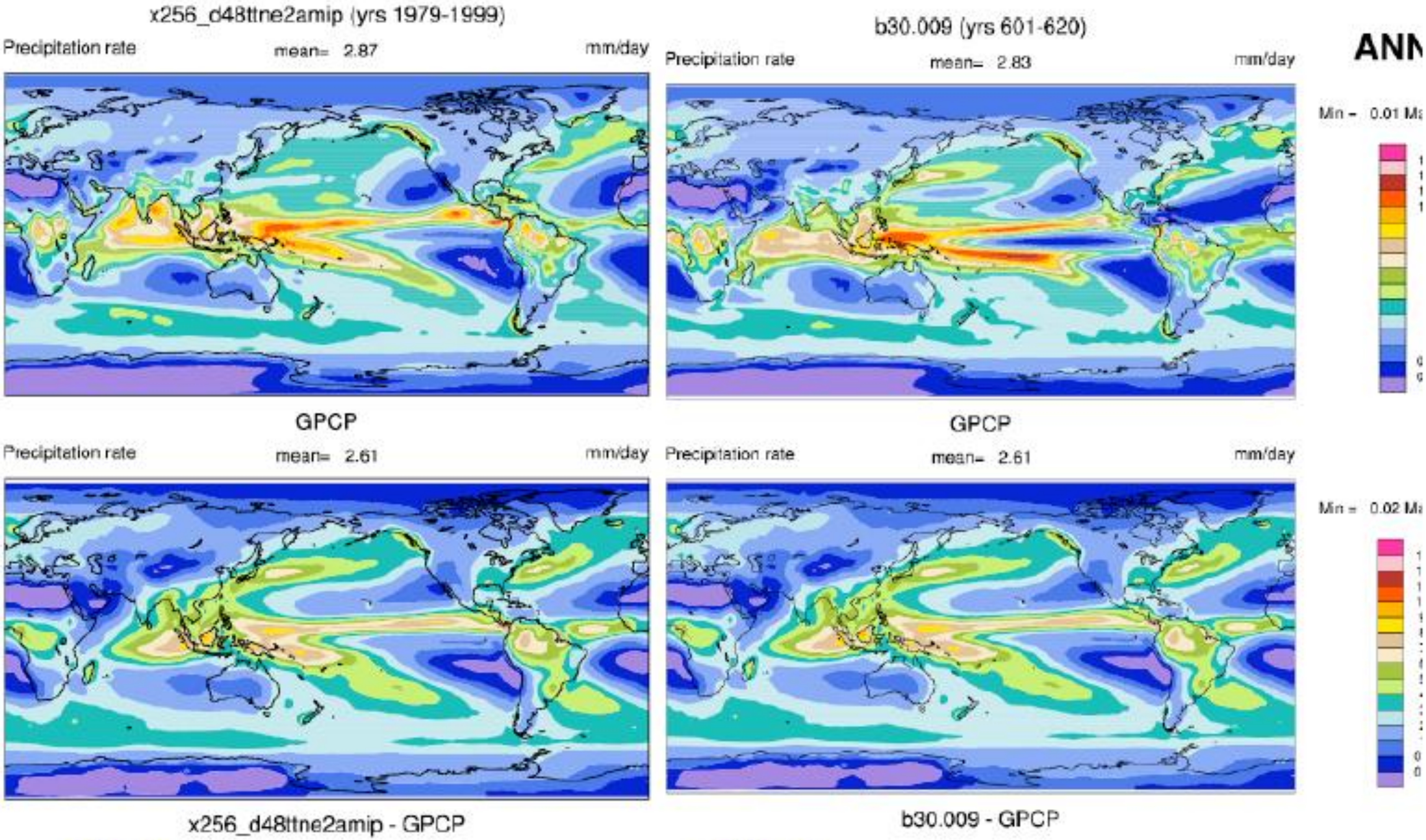


Climate results for coupled system





Coupled Models allow biases to grow



CAM

CCSM

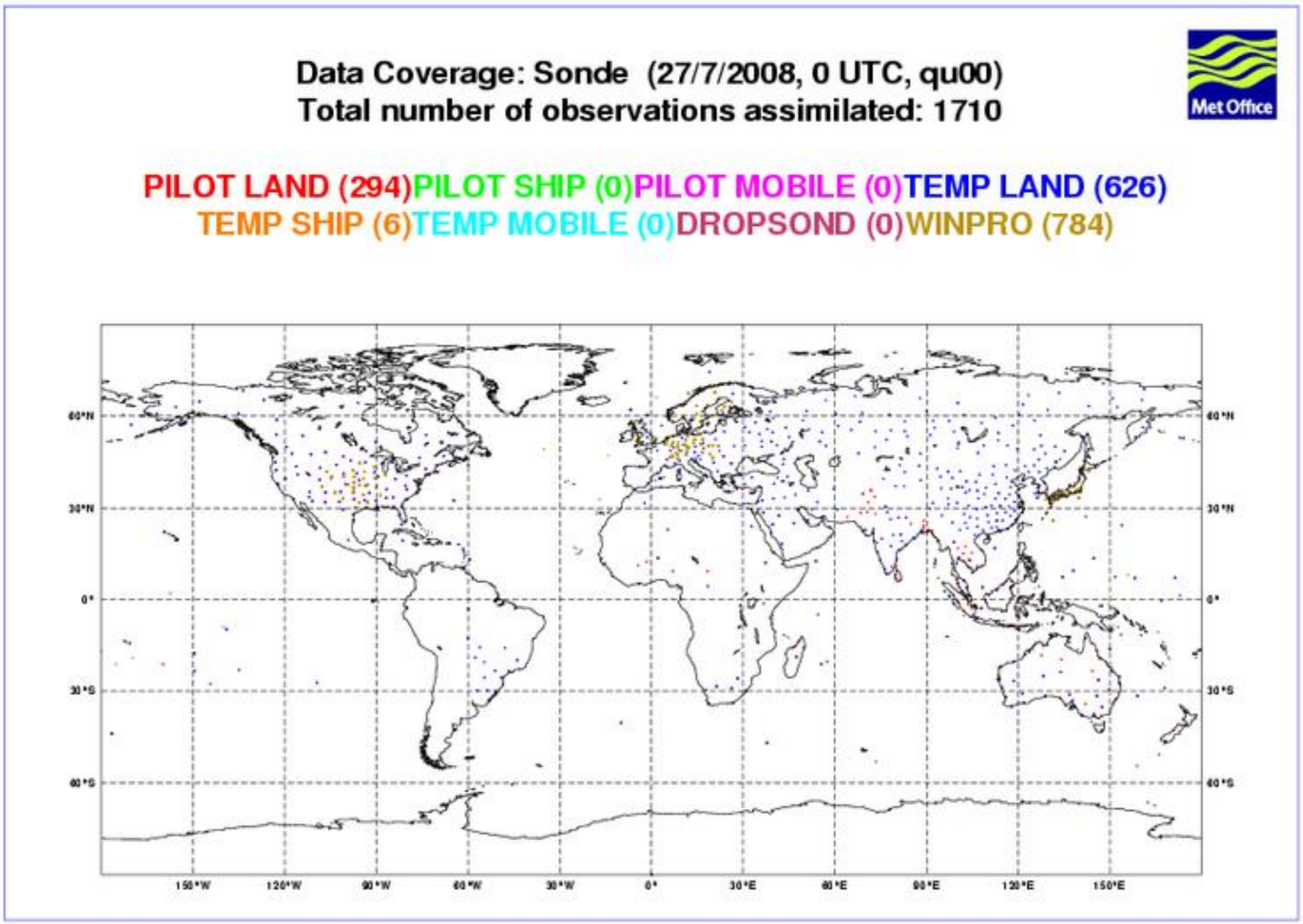
Prediction is an initial value problem

How do we get

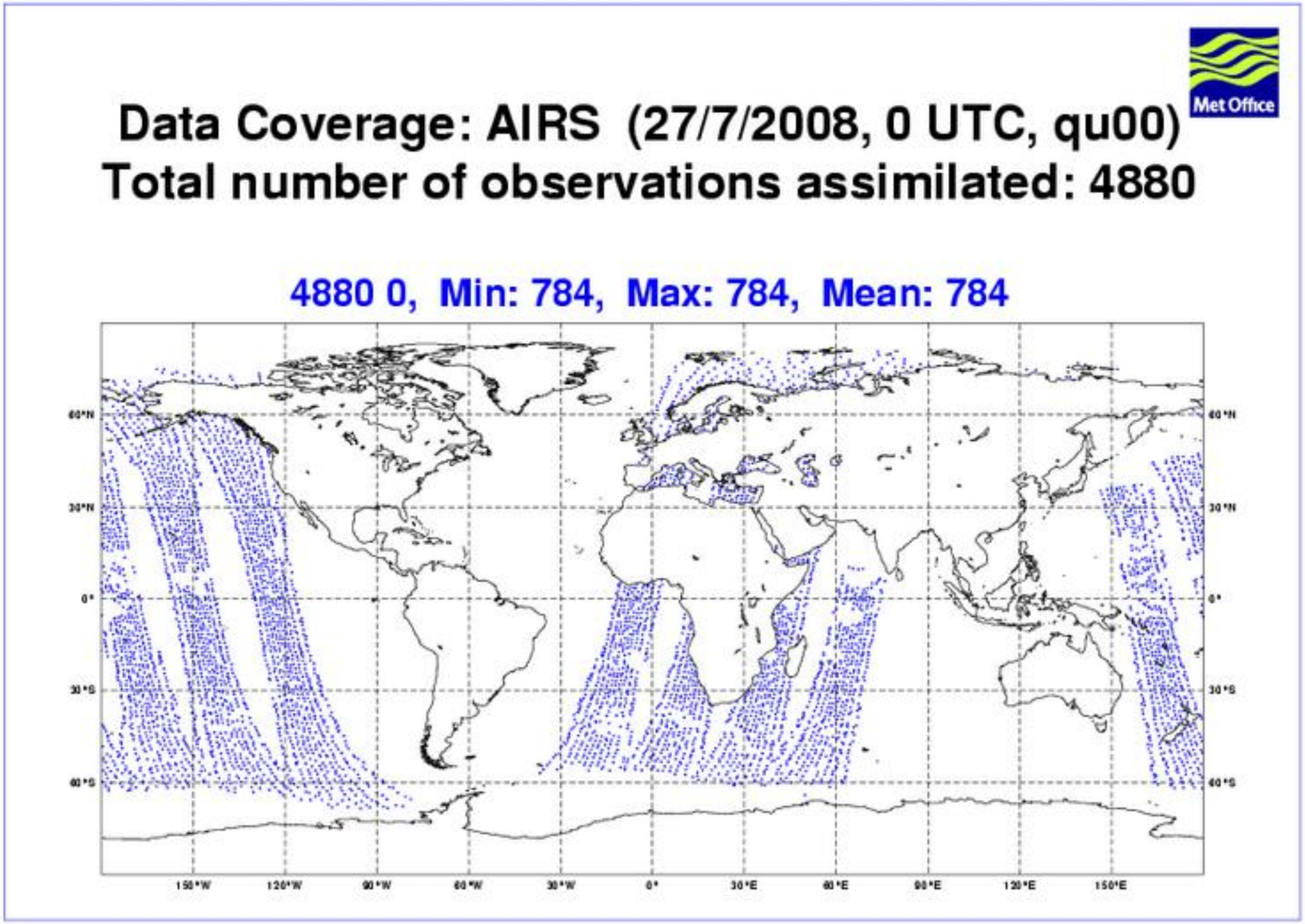
Initial

Conditions?

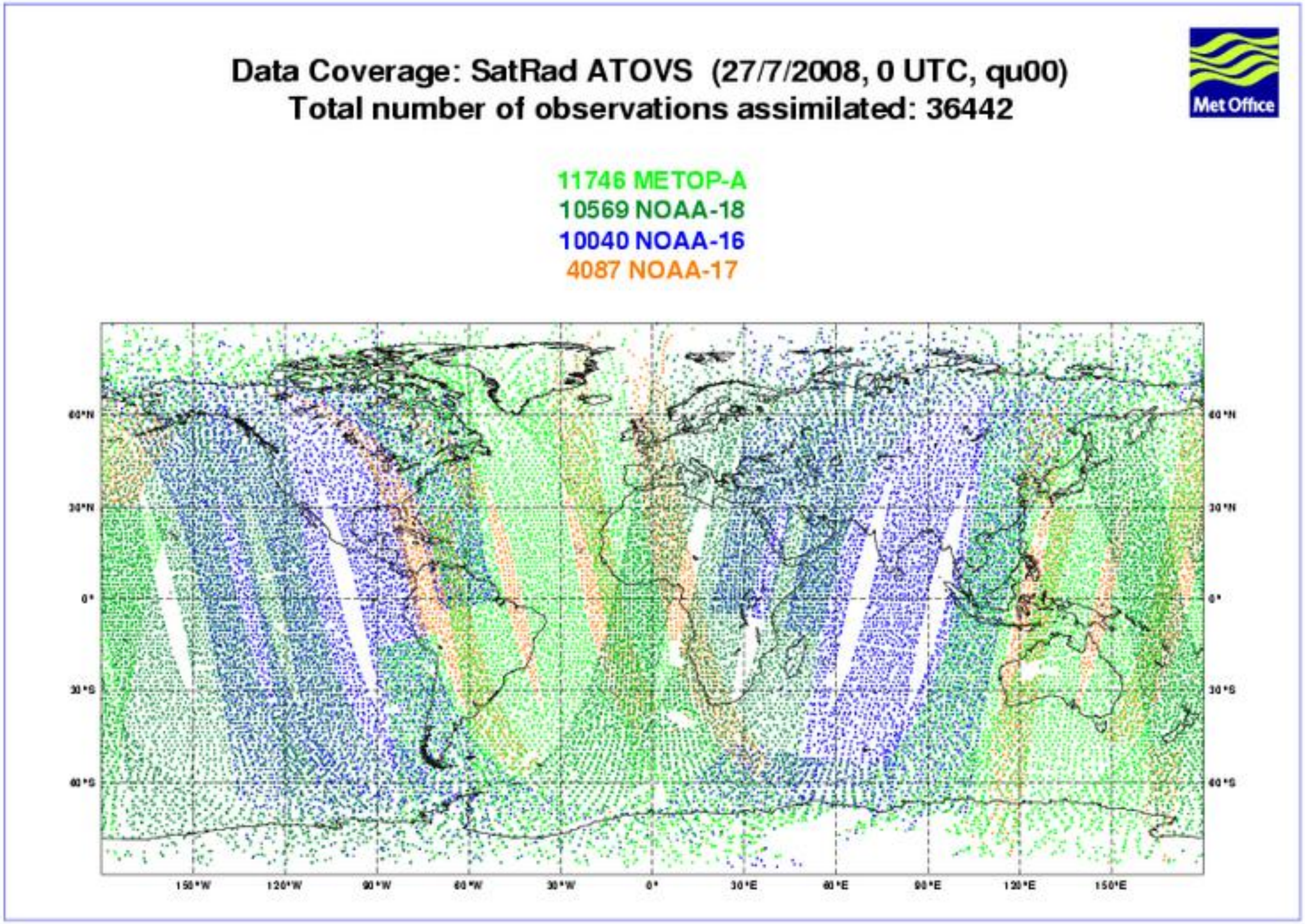
Observations for prediction: radiosondes



Observations for prediction: satellite radiances



Observations for prediction: satellite radiances



Data Assimilation: A Bayesian perspective

Bayes theorem relates conditional probabilities
Denominator is normalization
p(x|y) ~ product of probability densities

$$p(x \mid y) = \frac{p(y \mid x)\, p(x)}{p(y)}$$

$$p(y) = \int p(y \mid x)\, p(x) dx$$

$$p(x \mid y) \propto p(y \mid x)\, p(x)$$

For use in data assimilation framework think of RHS as product of the pdf of the prior p(x) times the pdf of the probability of an observation y given the prior x
LHS is then the probability of the state x given an obs y

NB:If RHS is product of Gaussian pdf's LHS Gaussian

Example: Ensemble Kalman Filtering

Standard KF is prohibitively expensive because covariance prediction is NxN. Ensemble prediction methods ‘successfully’ predict forecast uncertainty. Use ensemble to predict the covariance.

Step 1:
Estimate mean and covariance using forecast ensemble

$$\overline{x} = \frac{1}{N}\sum_{k=1}^{N} x_k$$

$$\overline{C} = \frac{1}{N-1}\sum_{k=1}^{N} (x_k - \overline{x})(x_k - \overline{x})^T$$

$$\overline{C} = P^f$$

Example:EnKF

Step2: Make an observation y=H(x)

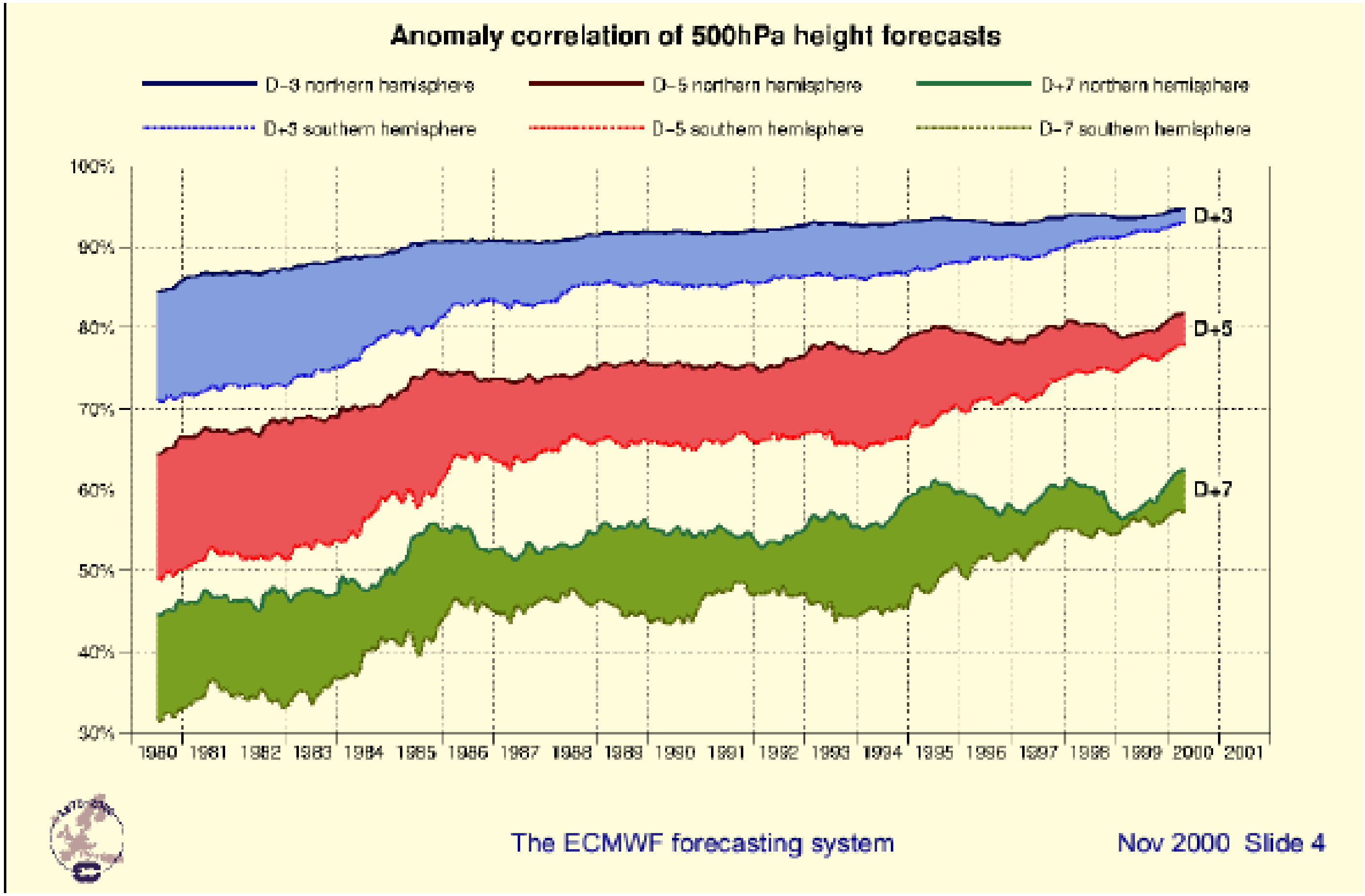
Simplest example is a single observation
e.g. H=(1 0) in two dimensions. Use ensemble
correlations to regress changes in x.

Step 3: Multiply pdf's and compute
posterior distribution

Step 4: Adjust ensemble

Step 5: Gaussian: mean= most likely state

Improvement in forecast skill due to advances in modeling AND data assimilation techniques



Ensemble forecasting Success

European Storm of the Century

Lothar 12/24/99

Ensemble prediction gives hint of storm

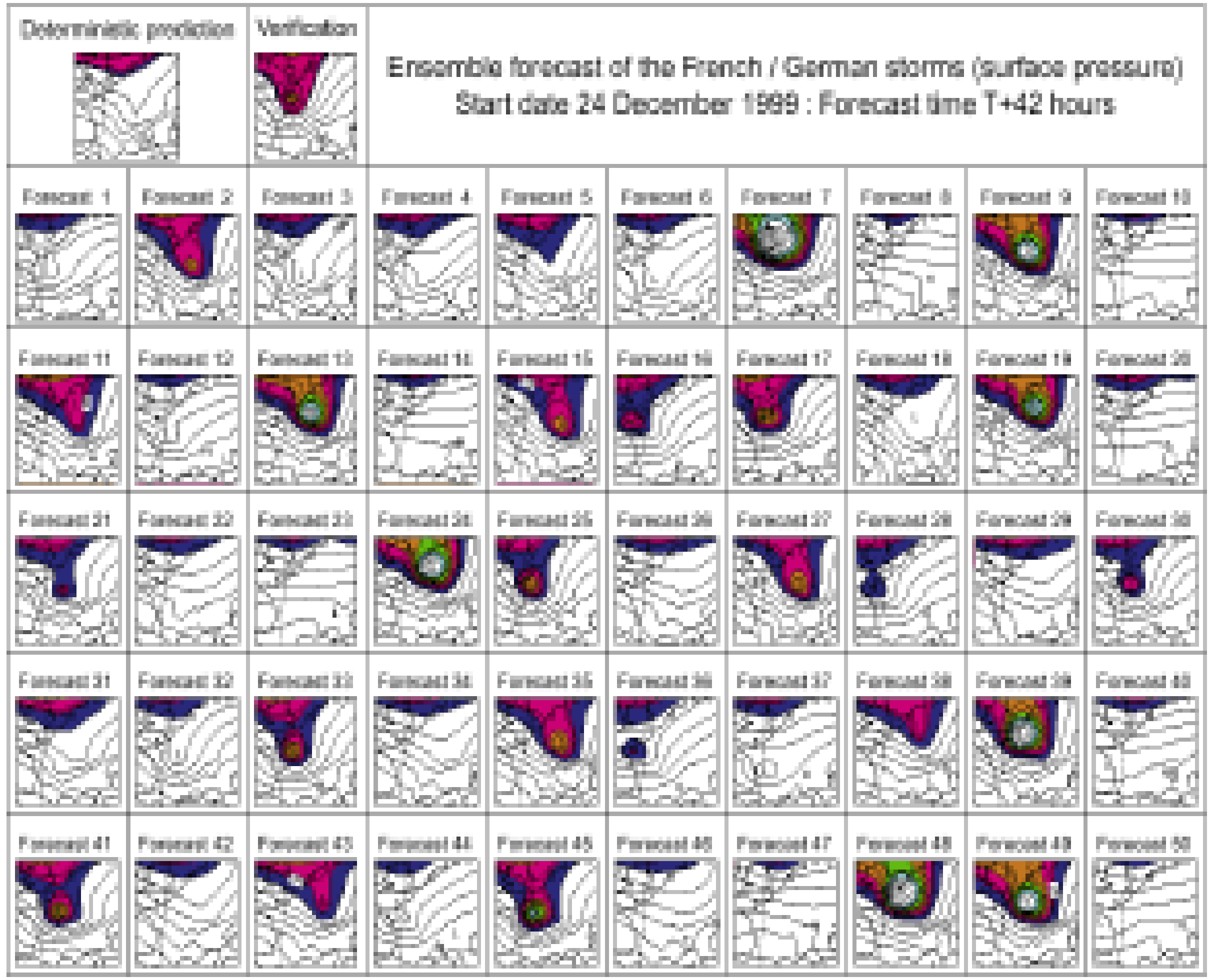


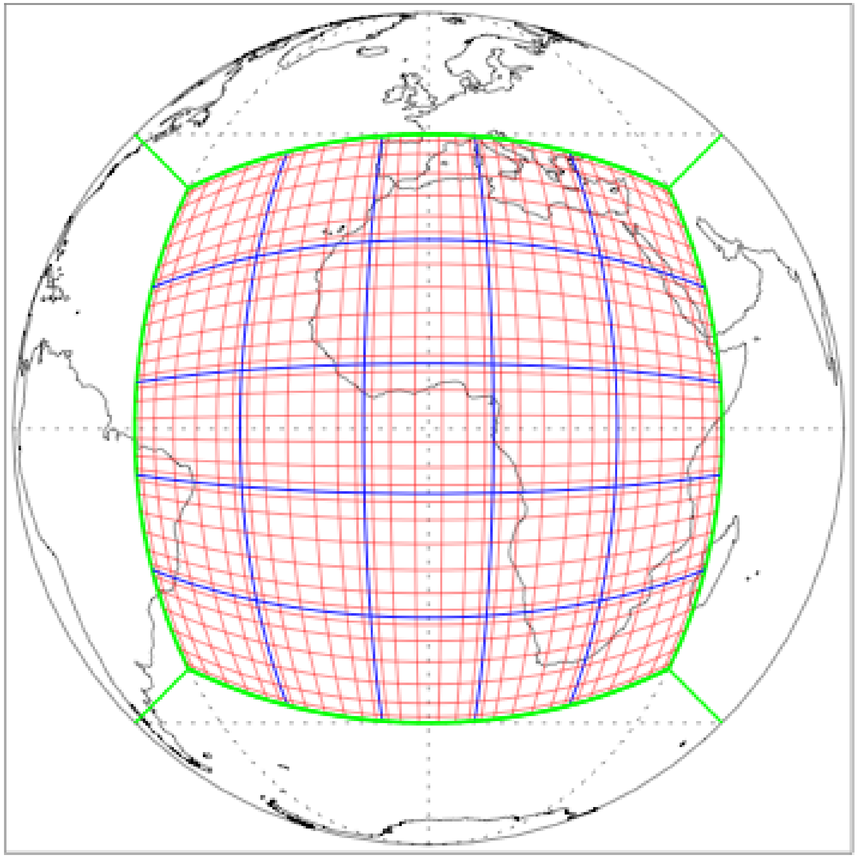
Fig 2.9: 42-h Ensemble forecast for the destructive French/German wind storm "Lothar" (Fig. 1.1) from the European Centre for Medium-range Weather Forecasts (ECMWF), TL255 runs of the operational EPS, verifying a 1200 UTC 26 December 1999. Mean seal-level pressure (lines and shaded; 4-mb interval). *Upper 2 panels:* deterministic prediction (left) and verification analysis (right). *Lower 50 panels:* individual ensemble members. Note that though the deterministic forecast does not capture this extreme event, 14 of the ensemble members predict a storm of equal or greater intensity than the verification analysis (courtesy of Federico Grazzini ECMWF).

The Future (massively parallel): HOMME

- High-Order Methods Modeling Environment
- Spectral elements, Continuous or discontinuous Galerkin on cube sphere
- Explicit or semi-implicit time integration
- Proven highly scalable on MP systems O(90000) cpus
- Vertical discretization: finite-difference (CAM)
- Current FV core also a candidate

Avoid global communication and pole problem with cubed sphere

- Equiangular projection
- Sadourny (72), Rancic (96), Ronchi (96)
- Some models moving towards this approach



Metric tensor

$$g_{ij} = \frac{1}{r^4 \cos^2 x_1 \cos^2 x_2} \begin{bmatrix} 1 + \tan^2 x_1 & -\tan x_1 \tan x_2 \\ -\tan x_1 \tan x_2 & 1 + \tan^2 x_2 \end{bmatrix}.$$

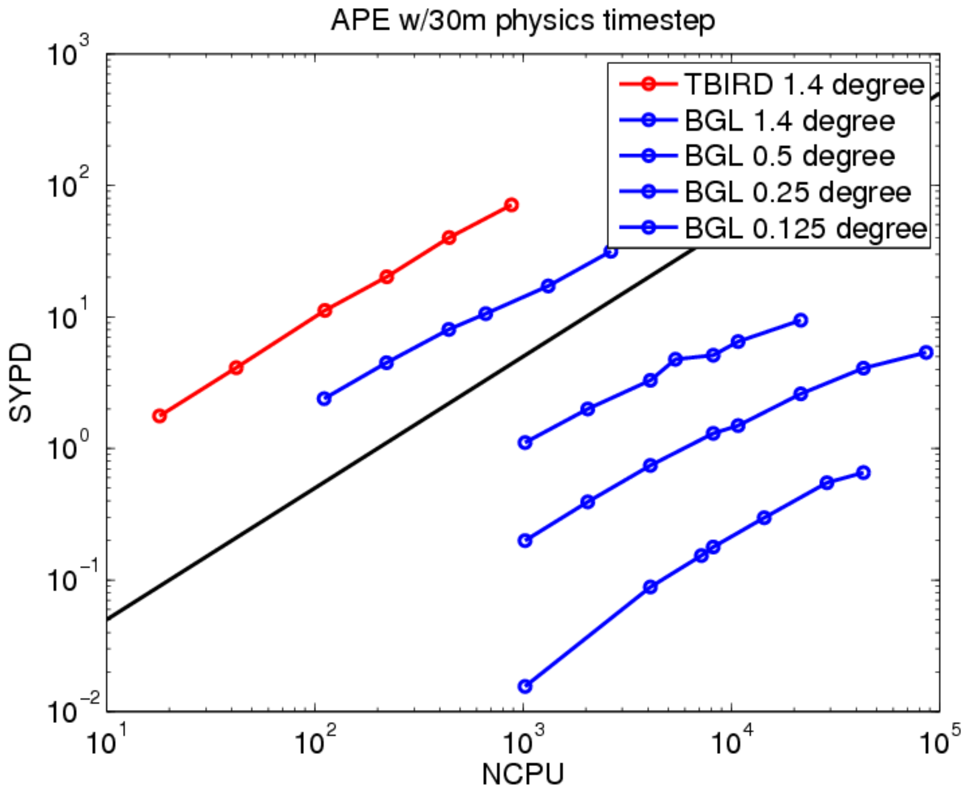
Rewrite div and vorticity

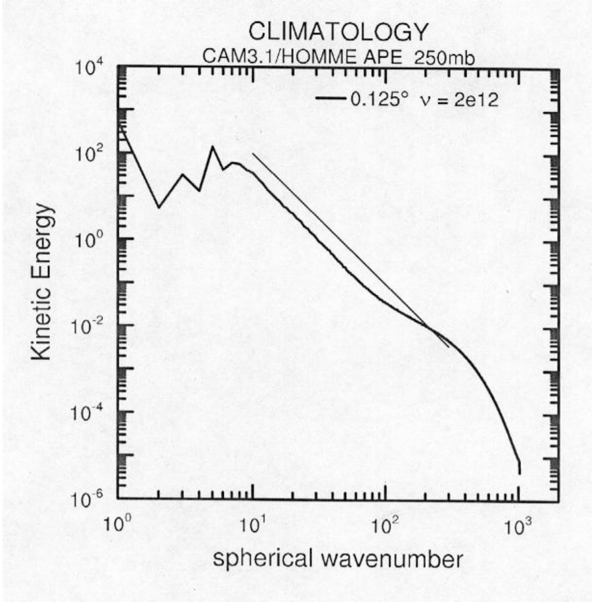
$$g \, \nabla \cdot \mathbf{v} = \frac{\partial}{\partial x^j} \left(g \, u^j \right), \quad g \, \zeta = \epsilon_{ij} \frac{\partial u_j}{\partial x^i}.$$

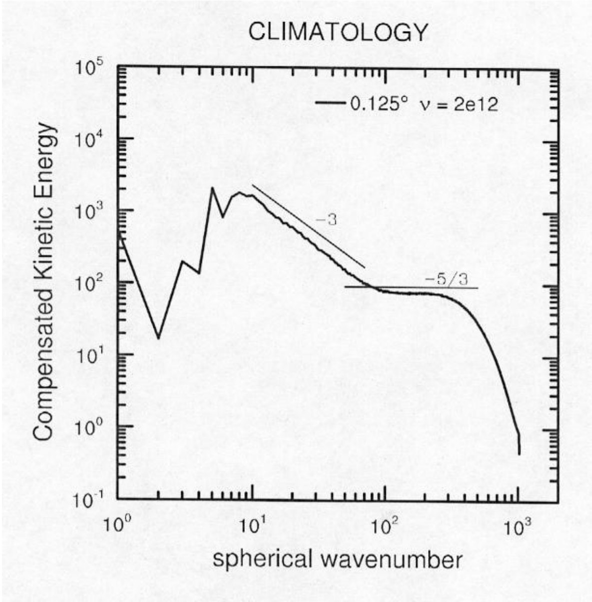
Taylor's result
HOMME Dycore

Aquaplanet simulation 1/8 deg

Scaling on 96,000 processors

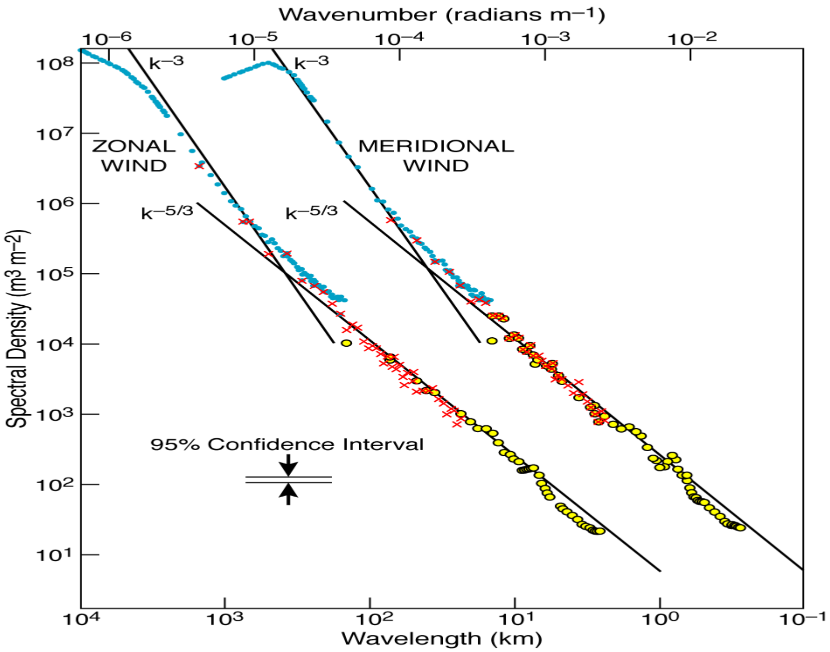






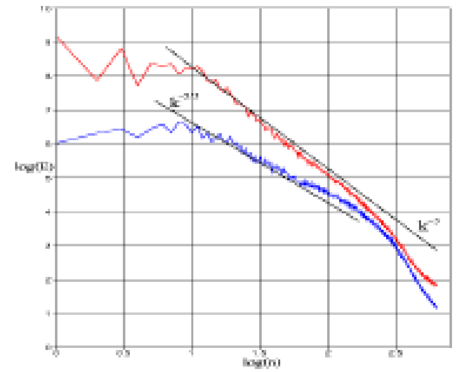
Observations and ECMWF

Nastrom –Gage spectrum

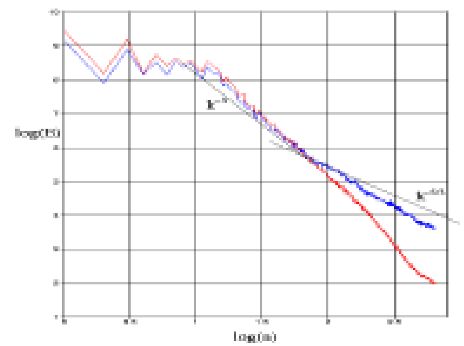


Observations

ECMWF spectrum day 10



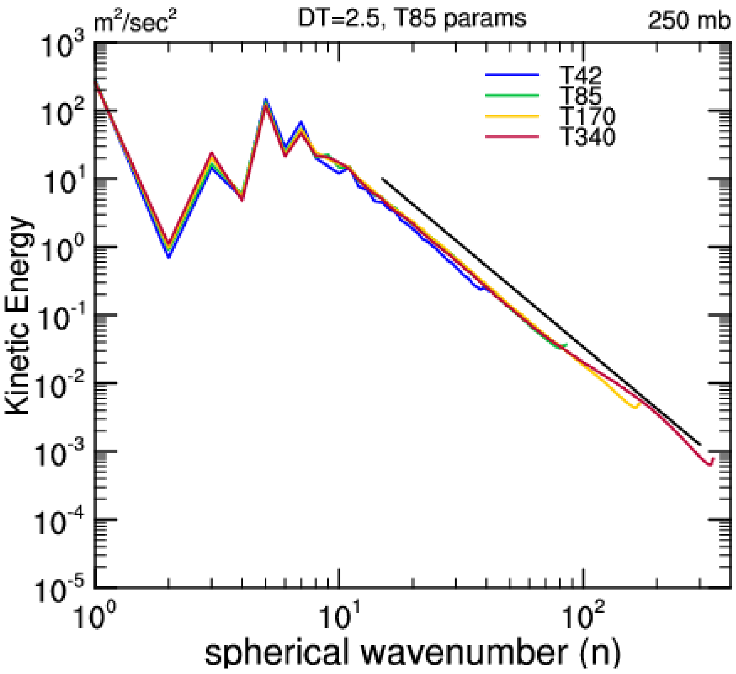
Standard Model T799



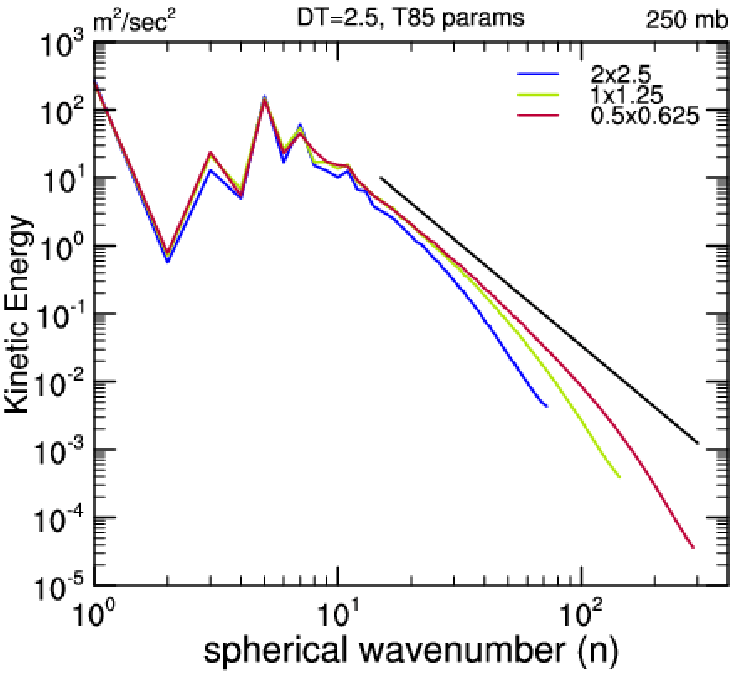
With Stochastic CA forcing

Current version of CAM has finite volume discretization-moving cubed sphere

EULERIAN SPECTRAL



FINITE VOLUME



The End

Questions ?