Geophysical Fluid Dynamics of the Earth

Jeffrey B. Weiss
University of Colorado, Boulder
The Earth is a spinning sphere

- Coriolis force depends on latitude
  \[ f = 2\Omega \sin \theta \]
- Solar flux depends on latitude

Michael Ritter,
http://www.uwsp.edu/geo/faculty/ritter/geog101/textbook/title_page.html
Atmosphere is turbulent

• global scale forcing:
  ▪ \( L \sim 10^7 \text{ m} \)

• dissipation:
  ▪ \( \nu_{\text{air}} \sim 10^{-5} \text{ m}^2/\text{s} \)

• jet stream velocity:
  ▪ \( U \sim 10^2 \text{ m/s} \)

• \( R_{\text{atm}} = 10^{14} \)
Direct simulation impossible

- Kolmogorov scale
  - $\eta = 1 \text{ mm}$
- current global models:
  - $\sim 1^\circ$ resolution $\sim 100 \text{ km}$
- improvement needed: $10^8$
- computer speed-up: $10^{32} \sim 2^{100}$
- Moore’s Law: 200 years
- need to be smarter (or very patient)
Climate system self-organizes

- layers:
  - troposphere
- circulations
  - Hadley cell
- spatially localized organization
  - ocean vortices
- spatio-temporal organization
  - El-Niño
Radiative forcing

IPCC
Atmospheric Layers

- troposphere is weather layer
- height governed by
  - radiation
  - convection
  - dynamics

- tropopause dynamics interesting
- stratosphere stably stratified
planetary boundary layer

- directly feels surface
- 3d turbulence
- strong diurnal cycle
- surface layer
  - lower 10%
  - log wind profile

- free atmosphere
  - affected by rotation/stratification

Arya, S.P. Air Pollution Meteorology and Dispersion
ocean mixed layer

- directly feels atmosphere
- mixed by wind
- stabilized by solar heating
deep ocean

- thermocline
  - 50 - 1000 m
- below stably stratified
- slow currents

http://www.windows.ucar.edu/tour
Large scale circulations

- 3-cell model
  - trades
  - westerlies
  - ITCZ
  - deserts
  - frontal zone
- shifts seasonally
wind-driven ocean circulation

- winds + basins = gyres
- horizontal
- large geostrophic component
- timescales: months
serendipitous tracers

- 1990: 20,000 Nike shoes washed into Pacific Gyre
- 1992: 29,000 bathtub toys
- locations from beachcombers

thermohaline circulation

- density driven vertical overturning
- forcing:
  - small scale intermittent convection
  - uncertain vertical mixing
- timescale: 1000 yrs
- large climate impact
spatially localized features

• Coherent structures
  - vortices, jets, fronts
• rotation and stratification suppress vortex stretching
• inverse cascade to large scales

http://eosweb.larc.nasa.gov/HPDOCS/misr/misr_images/arctic_vortex.jpg
Spatio-temporal phenomena

- El Niño, Monsoons, NAO, MJO, PDO, ...
- As records lengthen, expect to see more
- can have extreme multi-scale components
Monsoon

• seasonal circulation associated with land-sea contrast
• complex intermittency
• experience the North American Monsoon

Balances

- often find balances between subset of forces
- provide insight into dynamics
- departures important even if small
- asymptotic analysis gives reduced models
Hydrostatic Balance

- aspect ratio $\alpha = H/L$
- Froude number $Fr = U/NH$
- non-rotating Boussinesq equations for departures about basic state $b(z)$
- vertical momentum eqn
  $$Fr^2 \alpha^2 \frac{Dw}{Dt} = -\frac{\partial \phi}{\partial z} + b'$$

- $\alpha^2 Fr^2 \ll 1$ gives hydrostatic balance
  - $U^2/L^2 N^2 \ll 1$
  - $t_{buoy}^2 \ll t_{adv}^2$
• large scale troposphere
  ▪ $N \sim 10^{-2}/s$, $H \sim 10$ km, $L \sim 1000$ km, $U \sim 10$ m/s
  ▪ $Fr \sim 0.1$    $\alpha \sim 0.01$
  ▪ $\alpha^2 Fr^2 \sim 10^{-6}$
  ▪ fails at fronts and convection

• large scale ocean
  ▪ $N \sim 10^{-2}/s$, $H \sim 1$ km, $L \sim 1000$ km, $U \sim 0.1$m/s
  ▪ $Fr \sim 0.01$    $\alpha \sim 0.001$
  ▪ $\alpha^2 Fr^2 \sim 10^{-10}$
  ▪ hydrostatic down to small horizontal scales
  ▪ fails at deep convection sites
    • preconditioning: Fr near unity
    • plumes: $H/L$ large
Geostrophic balance

• Rossby number: Ro = U/ f L

• horizontal momentum eqn

\[
\frac{Du}{Dt} + f \times u = -\frac{1}{\rho} \nabla p
\]

scaling: \[Ro f U \quad fU\]

• Ro \ll 1: geostrophy
  • flow along isobars
  • diagnostic relation
upper air map
thermal wind

• combine geostrophic and hydrostatic
• obtain wind shear

\[
\mathbf{f} \times \mathbf{u} = -\nabla_h \phi \quad \frac{\partial \phi}{\partial z} = b'
\]

\[
f \times \frac{\partial \mathbf{u}}{\partial z} = -\nabla_h b'
\]

• midlatitudes
  ▪ heating decreases with latitude
  ▪ eastward wind increases with height
  ▪ jet intensification
turbulent cascades

- compare homogeneous isotropic 3d and 2d
- energy injection at large scale
- dissipation at small scale
- intermediate: inertial scales, only depend on energy flux $\varepsilon$
- focus on spectra: $E = \int E(k) dk$
- assume local in $k$ and universal

$E(k) = g(\varepsilon, k)$
• dimensional analysis

\[ k \sim \frac{1}{L} \]

\[ E \sim \frac{L^2}{T^2} \]

\[ E(k) \sim EL \sim \frac{L^3}{T^2} \]

\[ \varepsilon \sim \frac{E}{T} \sim \frac{L^2}{T^3} \]

\[ E(k) = g(\varepsilon, k) = \mathcal{K}\varepsilon^\alpha k^\beta \]

• T requires \( \alpha = \frac{2}{3} \)

• L requires \( \beta = -\frac{5}{3} \)

Vallis, Atmospheric and Oceanic Fluid Dynamics, 2006
2d cascades

• In 2d, two inviscid invariants
  ▪ energy $E$ and enstrophy $Z$
  ▪ in 3d vortex stretching creates enstrophy
  ▪ $Z(k) = k^2 E(k)$
• centroid wavenumber $k_c = \int k E(k), \quad E = 1$
• width $W = \int (k - k_c)^2 E(k) = Z - k_c^2$
• initial narrow $E(k)$ spreads
  $$dW/dt > 0 \Rightarrow dk_c^2/dt < 0$$
• energy cascades to large scales: inverse

Vallis, Atmospheric and Oceanic Fluid Dynamics, 2006

• similar argument: enstrophy to small scales: direct
• energy inertial range to large scales
• enstrophy inertial range to small scales
• energy inertial ranges same scaling as 3d
• need mechanism to remove energy at large scales, e.g. Ekman drag

• enstrophy flux
  • $\eta \sim Z/T \sim 1/T^3$
  • $E(k) \sim \eta^{2/3} k^{-3}$

Vallis, Atmospheric and Oceanic Fluid Dynamics, 2006
cascades on 2d $\beta$-plane

- vorticity equation on 2d $\beta$-plane
  - $f = \beta y$
  
  \[
  \frac{\partial \zeta}{\partial t} + \mathbf{u} \cdot \nabla \zeta + \beta v = 0
  \]

- Rossby waves
  
  \[
  \zeta = A e^{i(k \cdot \mathbf{x} - \omega t)}, \quad A \ll 1
  \]

- dispersion relation
  
  \[
  \omega = \frac{-\beta k_x}{k_x^2 + k_y^2}
  \]
• scaling
\[ \frac{\partial \zeta}{\partial t} + \mathbf{u} \cdot \nabla \zeta + \beta v = 0 \]
\[ \frac{U^2}{L^2} \quad \beta U \]

• small scales
  ▪ advection and turbulence dominate

• crossover: Rhines scale
\[ L_R \sim \sqrt{U/\beta} \]

• large scales, $\beta$ dominates
  ▪ waves inhibit cascades
• anisotropy of Rossby waves
  • advection freq $U_k \sim$ Rossby wave freq $\omega$

$$k_x = k_R \cos^{3/2} \theta, \quad k_y = k_R \sin \theta \cos^{1/2} \theta$$
ocean jets

- zonal jets observed to populate ocean

Maximenko, et al 2005