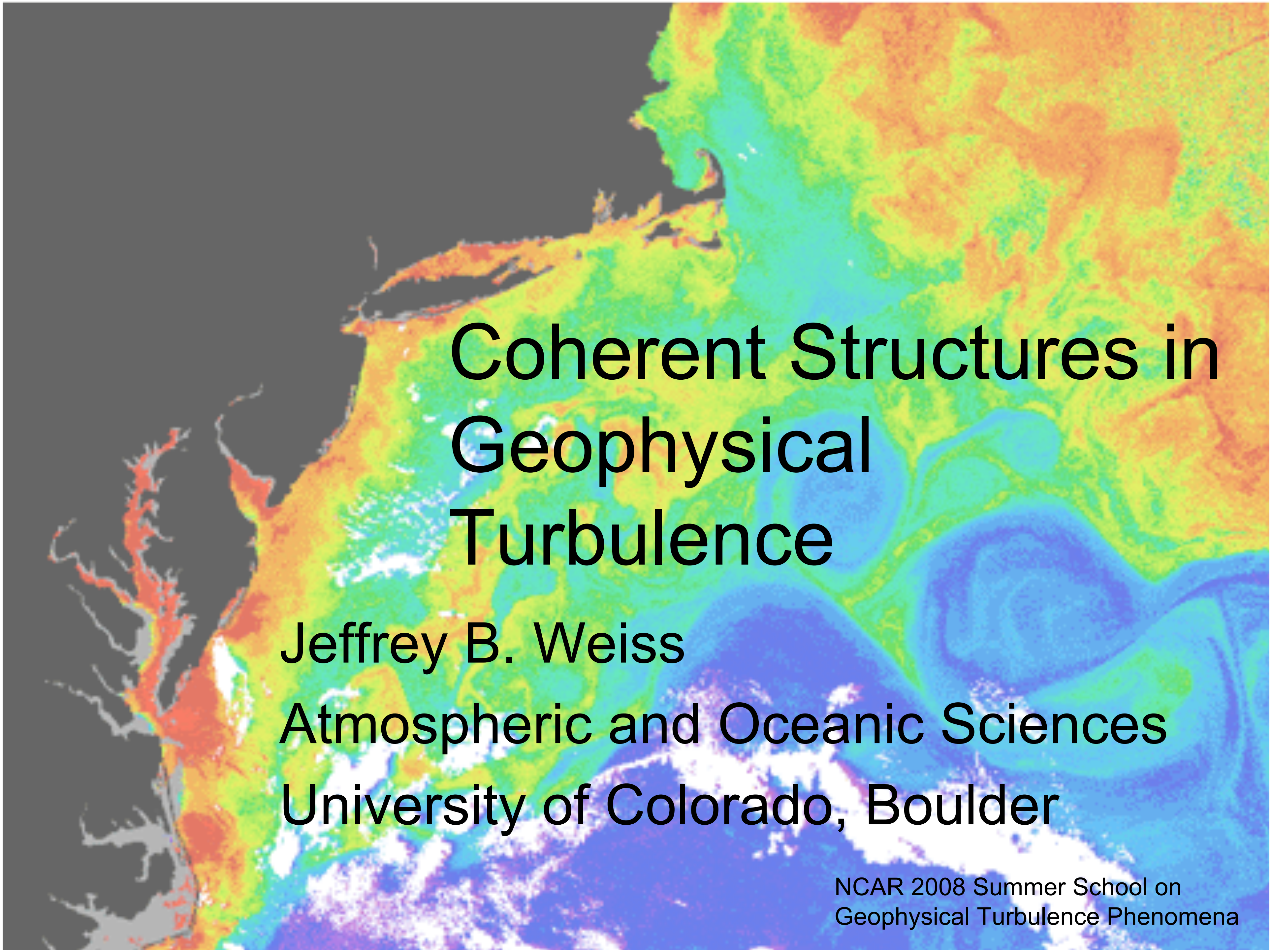

Coherent Structures in Geophysical Turbulence

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NCAR 2008 Summer School on
Geophysical Turbulence Phenomena

Outline

- structured view of turbulence
- components of 3d QG turbulence

  (Petersen, Julien, Weiss, 2006)

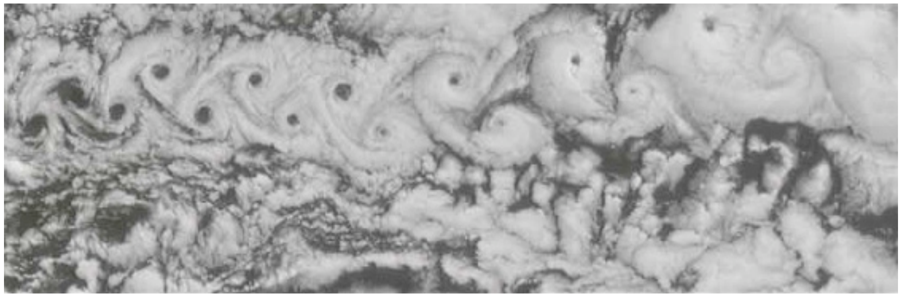
- vortex interactions in 3d QG turbulence

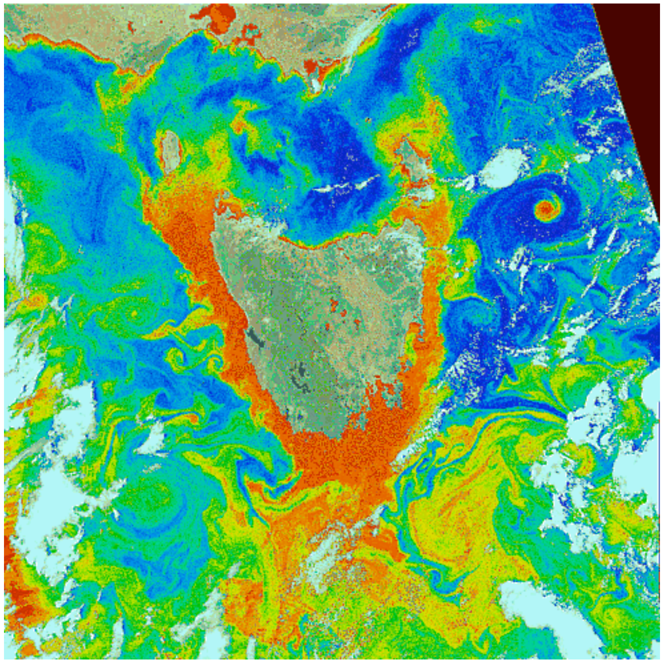
  (Martinsen-Burrell, Julien, Petersen, Weiss, 2006)

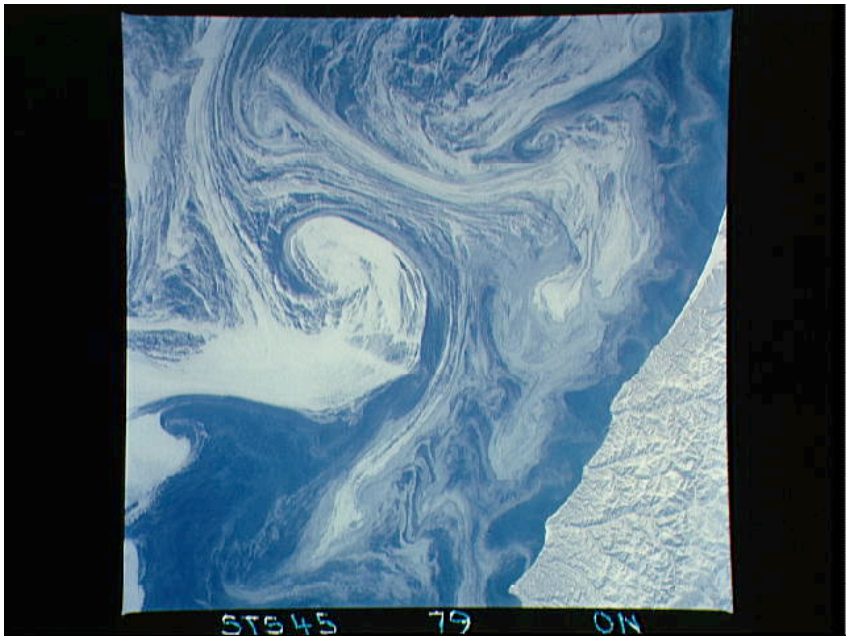
- reaction enhancement by vortex stirring

  (Crimaldi, Hartford, Weiss, 2006; Crimaldi, Cadwell, Weiss, 2008)

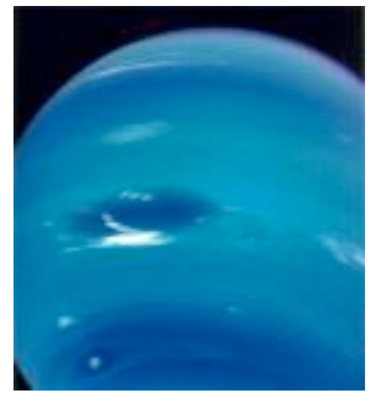
structures ubiquitous

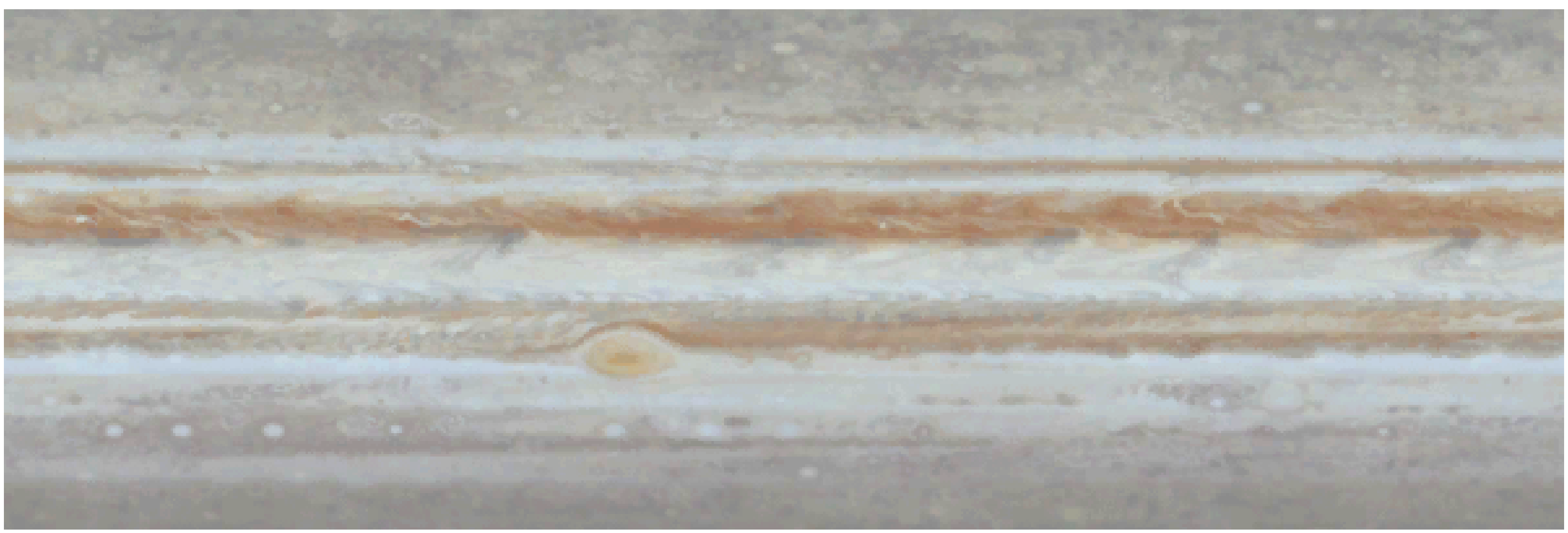




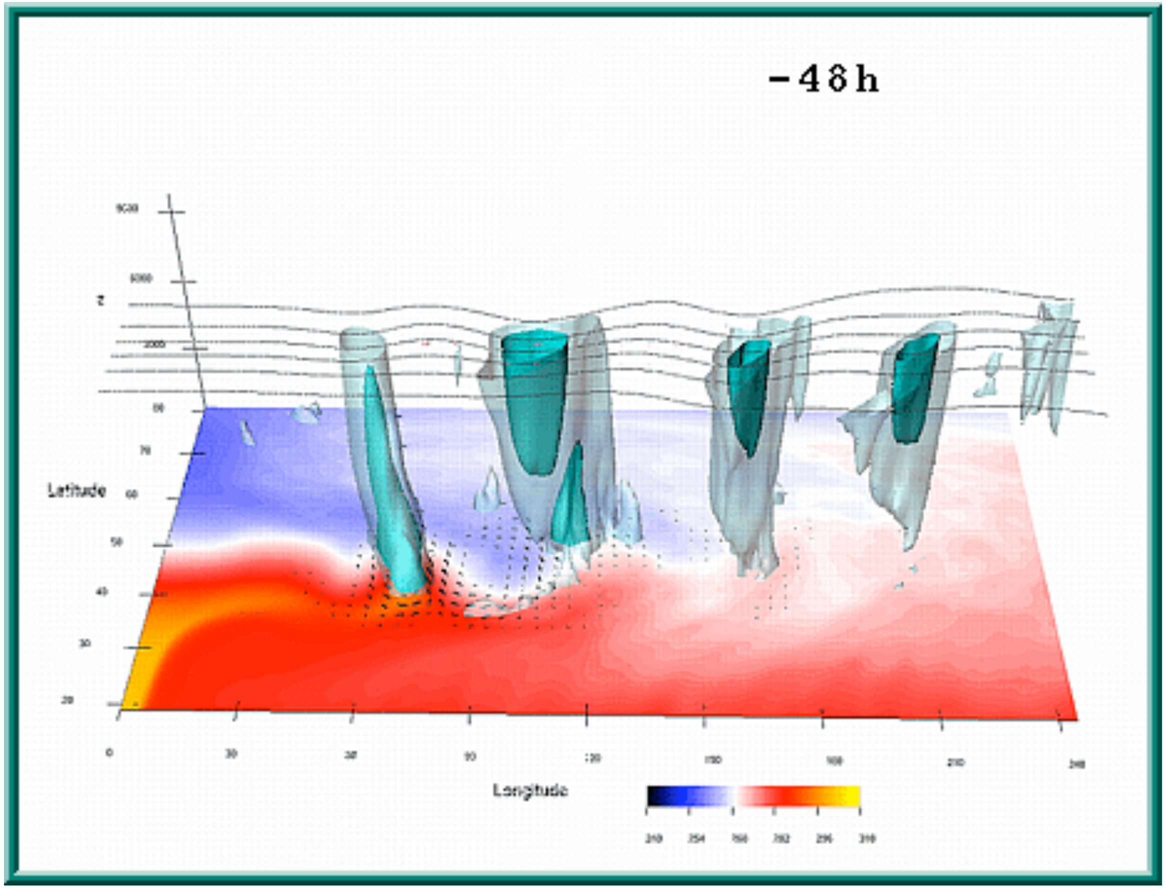


atmospheric vortices



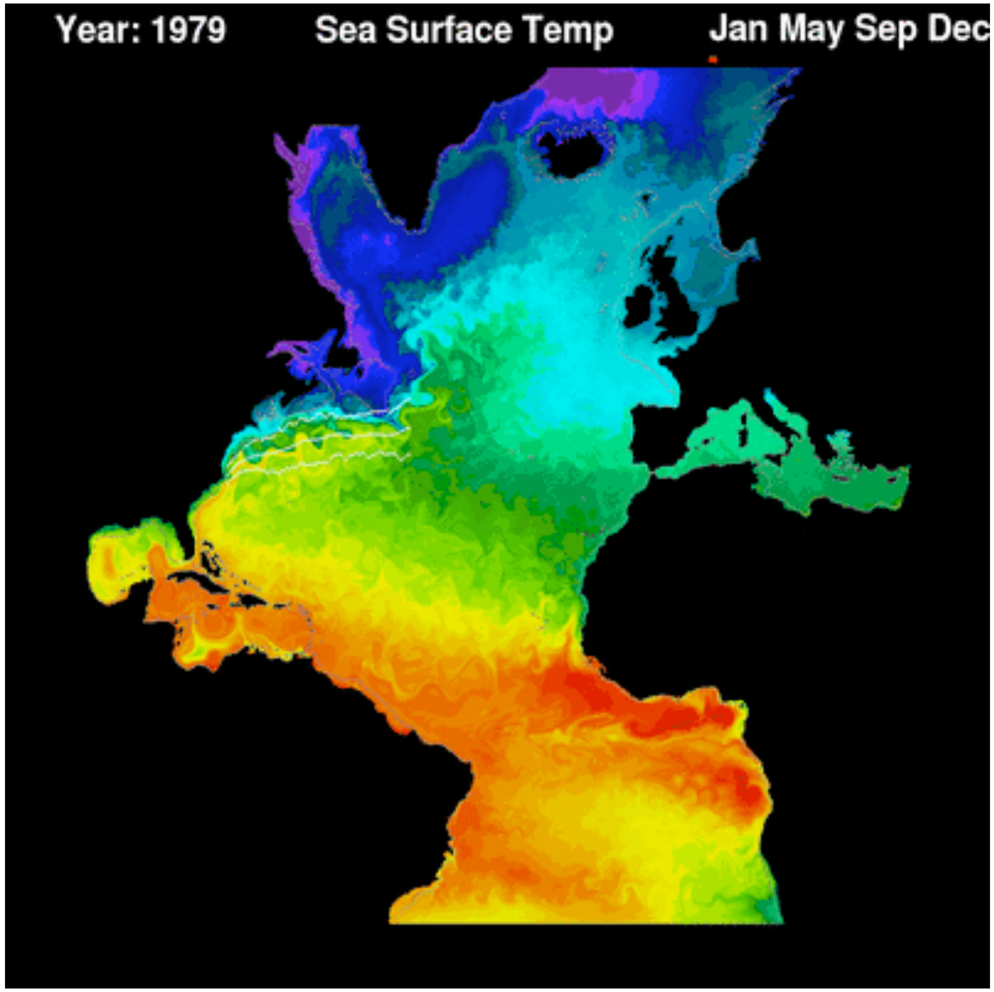


baroclinic lifecycle



Orlanski and Gross, 2000

ocean jets and vortices

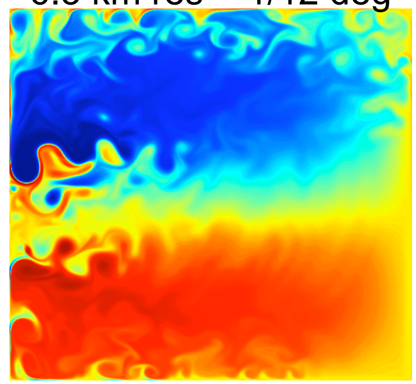


MICOM Ocean GCM

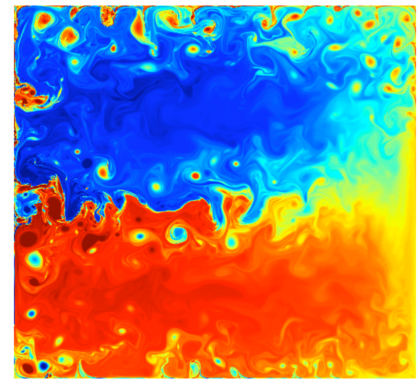
ocean coherent vortices

- Evidence for an “explosion” in coherent vortex population as Re increases
- QG ocean gyre

6.3 km res ~ 1/12 deg



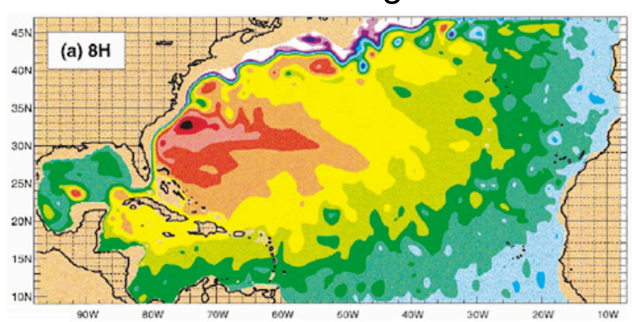
1.6 km res ~ 1/48 deg



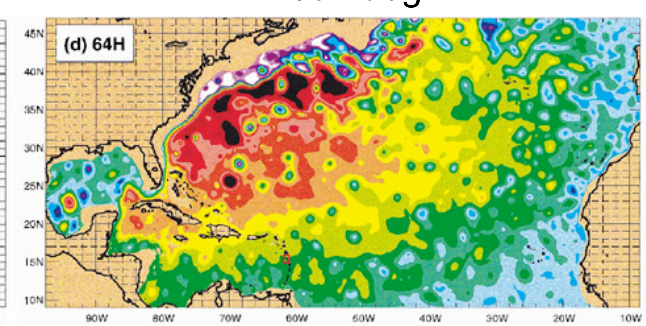
(Siegel, et al, 2001)

- NRL NLOM

1/8 deg



1/64 deg



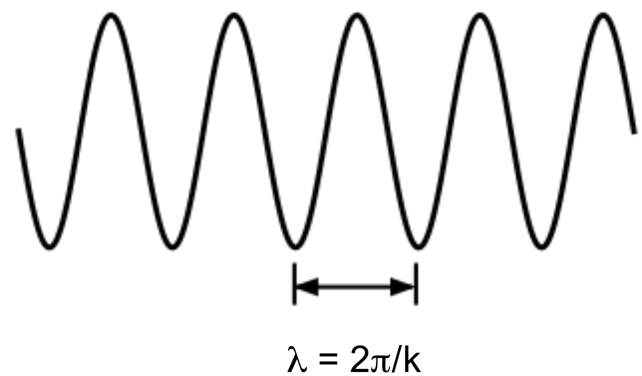
“a basin-wide explosion in the number and strength of mesoscale eddies”(Hurlbert and Hogan, 2000)

what is a structure?

- know it when you see it
- recurrent
- spatially localized $\Rightarrow$ not a wave
- spatially isolated
- long lived in a Lagrangian frame

traditional turbulence theory

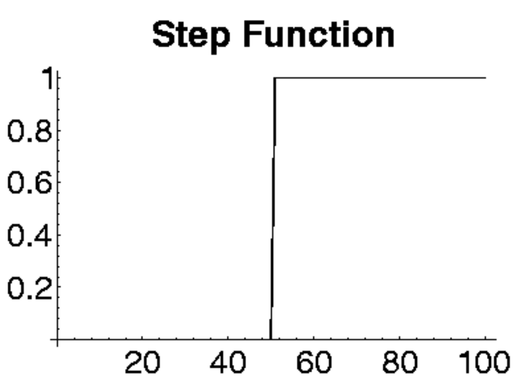
- treats fluid as random
- focus on Fourier space
  - "eddies" with scale k ~ cos(kx)
  - but waves are not structures

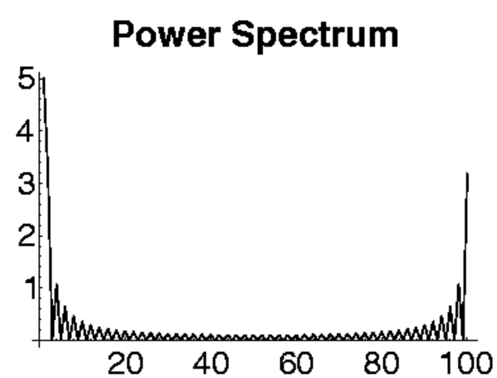


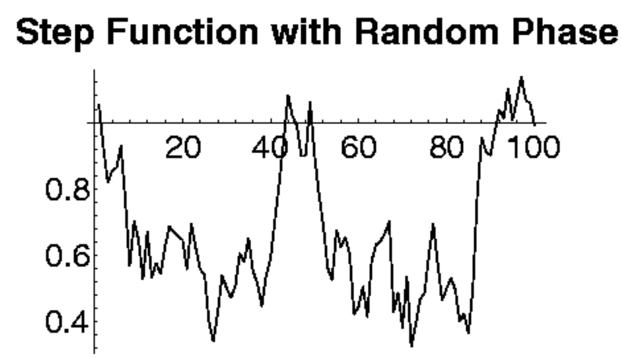
- main concern is spectra
  - $E(k) \sim k^{-5/3}$ in 3d homogeneous isotropic
  - $E(k) \sim k^{-3}$ in 2d

phases are not random

- traditional theories of turbulence:
  - wavenumber space: $A(k)e^{i\phi(k)}$
  - random phase approx. common
- structures:
  - local in physical space
  - random phase approximation fails







turbulence in atmospheric and oceanic models

- subgrid-scale turbulence
  - need parameterization
  - eddy diffusion often fails
- resolved turbulence
  - predictability of structures
    e.g. number and location of storms and jets

structures and human impact

- structures have enormous human impact
  - hurricanes
  - tornados
  - storms
  - jet stream path
  - Gulf stream

theory of structured turbulence

- Goal 1: predict statistical properties of structures
  - e.g. number of hurricanes, not slope of spectrum
  - partial success in 2d and 3d QG
- Goal 2: construct models that capture structure dynamics
  - success in 2d
- Goal 3: construct SGS parameterizations of transport

2d and QG are laboratories

- vorticity equation

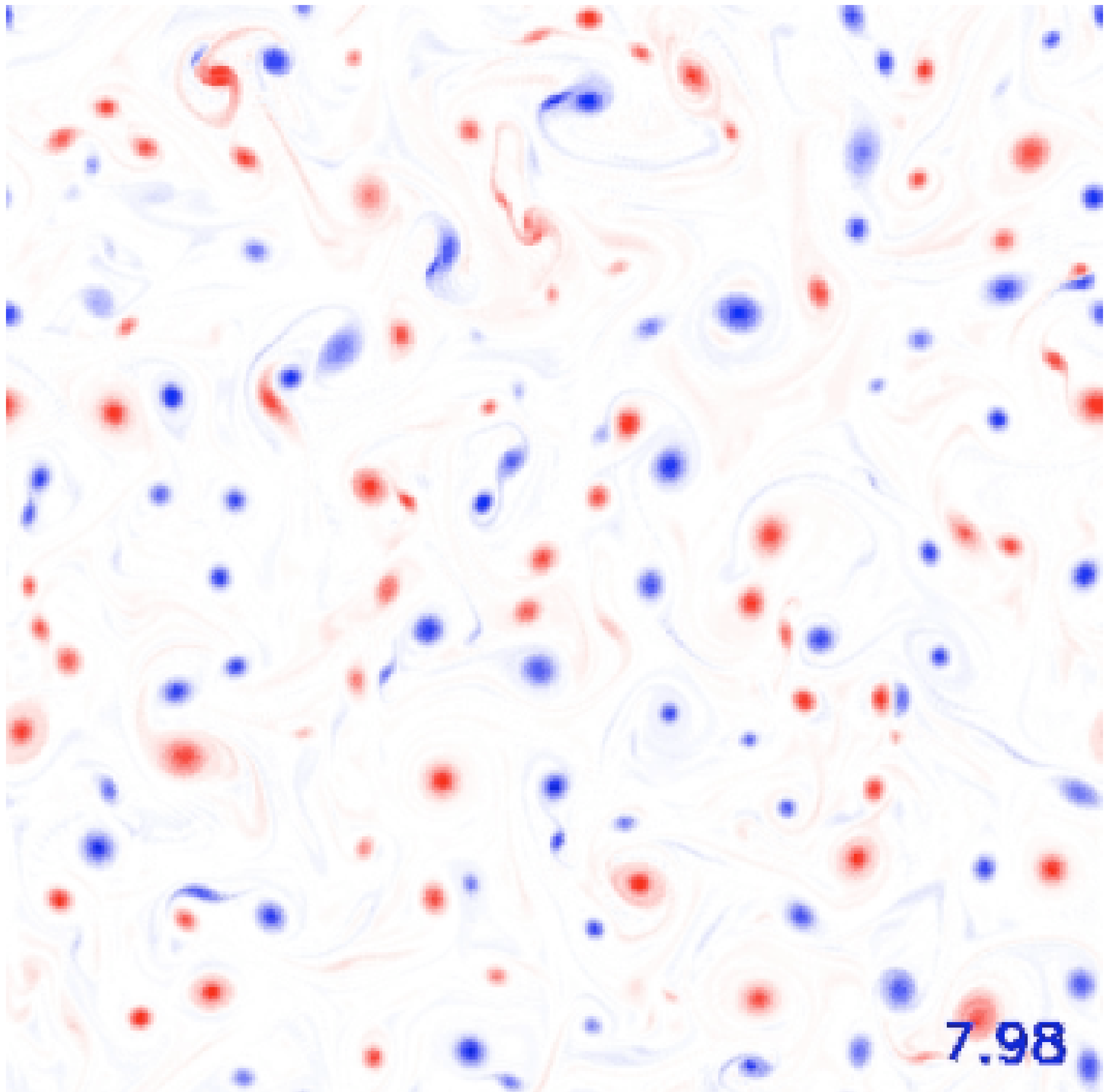
$$q_t + \psi_x q_y - \psi_y q_x = Dissipation + Forcing$$

- vorticity - streamfunction rel'n

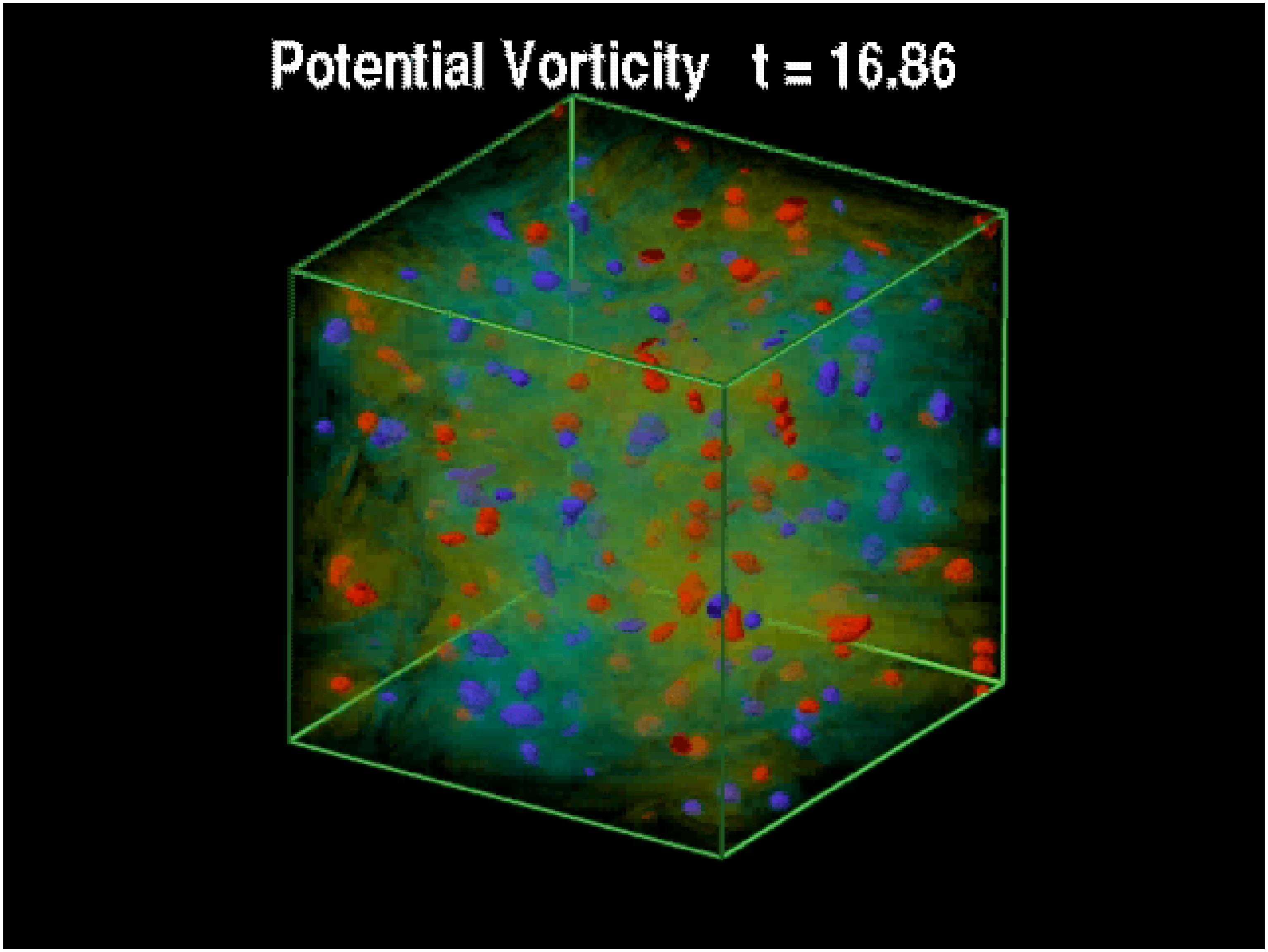
  - 2D $\quad q = \nabla^2_{2D}\psi$
  - 3D QG $\quad q = \nabla^2_{2D}\psi + \partial_z \frac{1}{S(z)} \partial_z \psi$

$$\xrightarrow[S=1]{} \nabla^2_{3D}\psi$$

self organizing vortices: 2d



self-organizing vortices: 3d QG



Structure Based Scaling Theory

- mean vortex theory
  - avg size, amplitude, …
  - global quantities due to vortex component
- assumes
  - algebraic evolution t^{α}
  - self-similar temporal evolution
  - a few exponents, predicts others

structure recognition

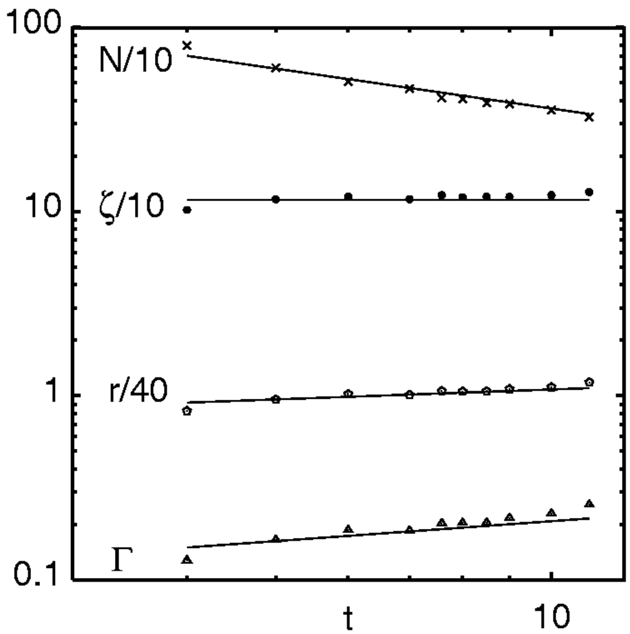
- verifying scaling theories requires recognition algorithms
- variety of algorithms exists
  - subjective algorithms

    (e.g. Weiss and McWilliams, 1994; 1999; Petersen et al 2006)
  - wavelet-based algorithms

    (e.g. Siegel and Weiss, 1997; Whitcher et al 2003; 2006)
- easy in simple systems, seek algorithms that work in more complex cases

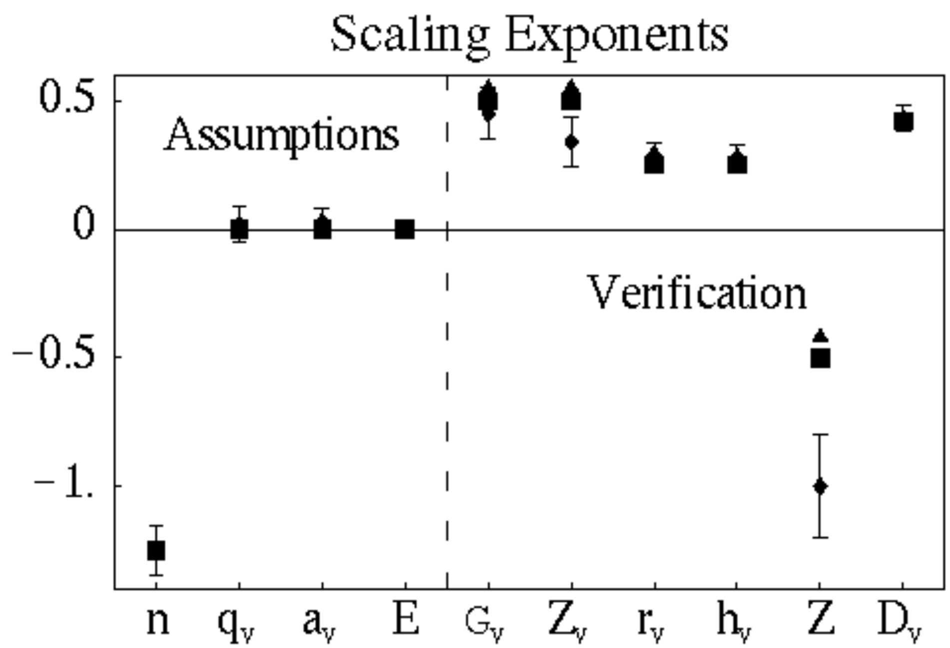
- scaling theory works well in 2D
- 3D QG needs higher Re

2D



(*Bracco, et al, 2000*)

3D QG



(*McWilliams, et al, 1999*)

reduced dynamical models

- reduced models allow computation of
  - predictability
  - transports
- models require
  - partition into structures
  - conservative structure dynamics
  - transformation dynamics
- successful in 2d *(Weiss and McWilliams, 1993)*

2. Components of QG turbulence

Petersen et al 2006

- structured turbulence as a multi-component fluid
- 2d: vortices, circulation cells, and background
  - circulation cells: regions of high-kinetic energy just outside vortices; often lumped with vortices
- separation is in vorticity
- vorticity induces velocity through Green's fn
  - velocity is global, even if vorticity component is local
- variety of techniques for identifying components

Identifying components

- use so-called Okubo-Weiss field

  (Okubo 1970; John Weiss 1991)

  $Q = \text{strain}^2 - \text{vorticity}^2$

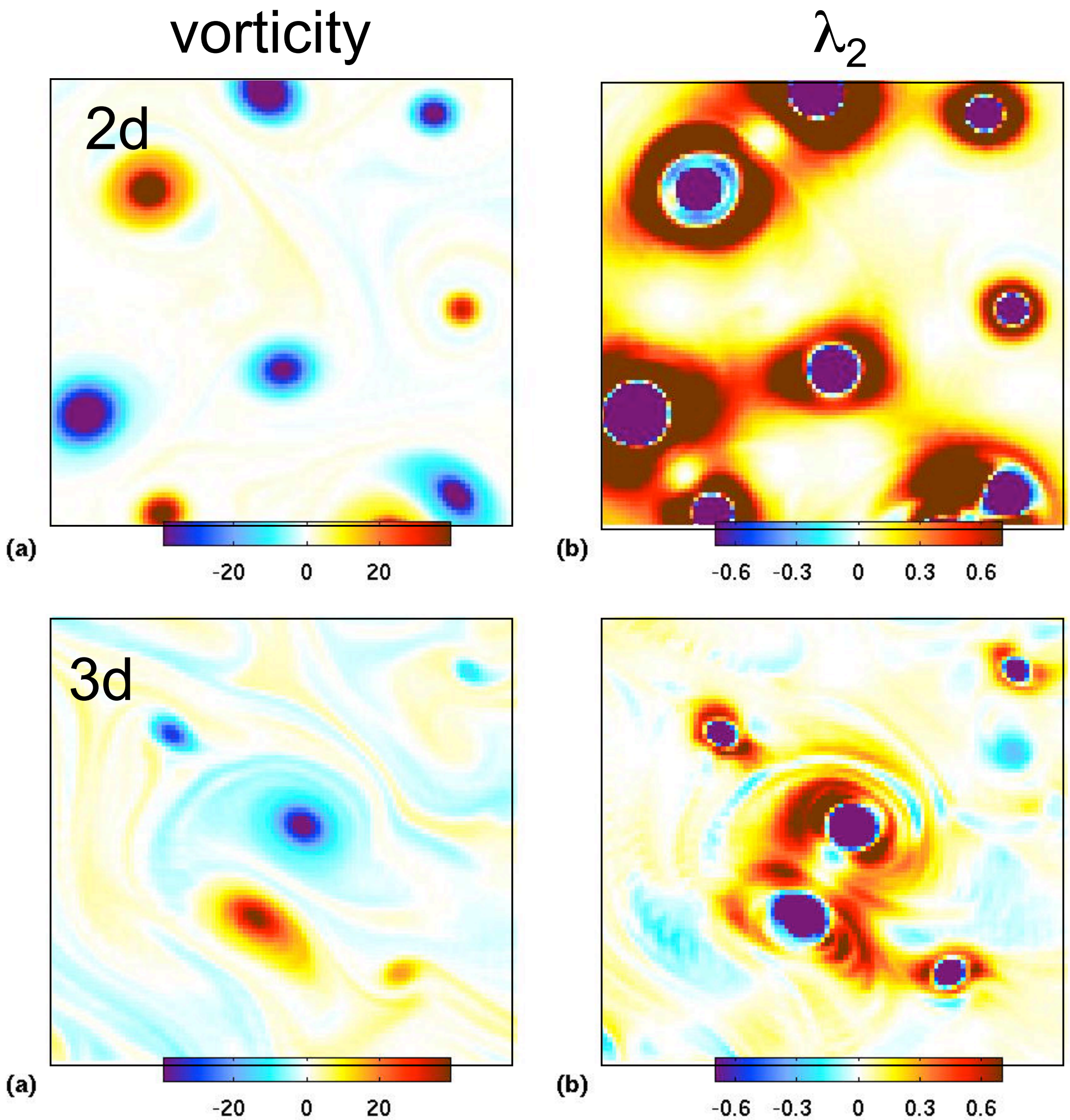
- in 3d homog-isotropic, λ_2 often used
  - middle eigenvalue of matrix related to velocity gradient tensor *(Jeong and Hussain, 1995)*
- in 2d and 3d QG, $\lambda_2 = Q/4$
- use simple criterion based on λ_2 threshold
- results relatively insensitive to threshold choice

The λ_2 field

- vortex cores:
  vorticity dominates
  large negative λ_2

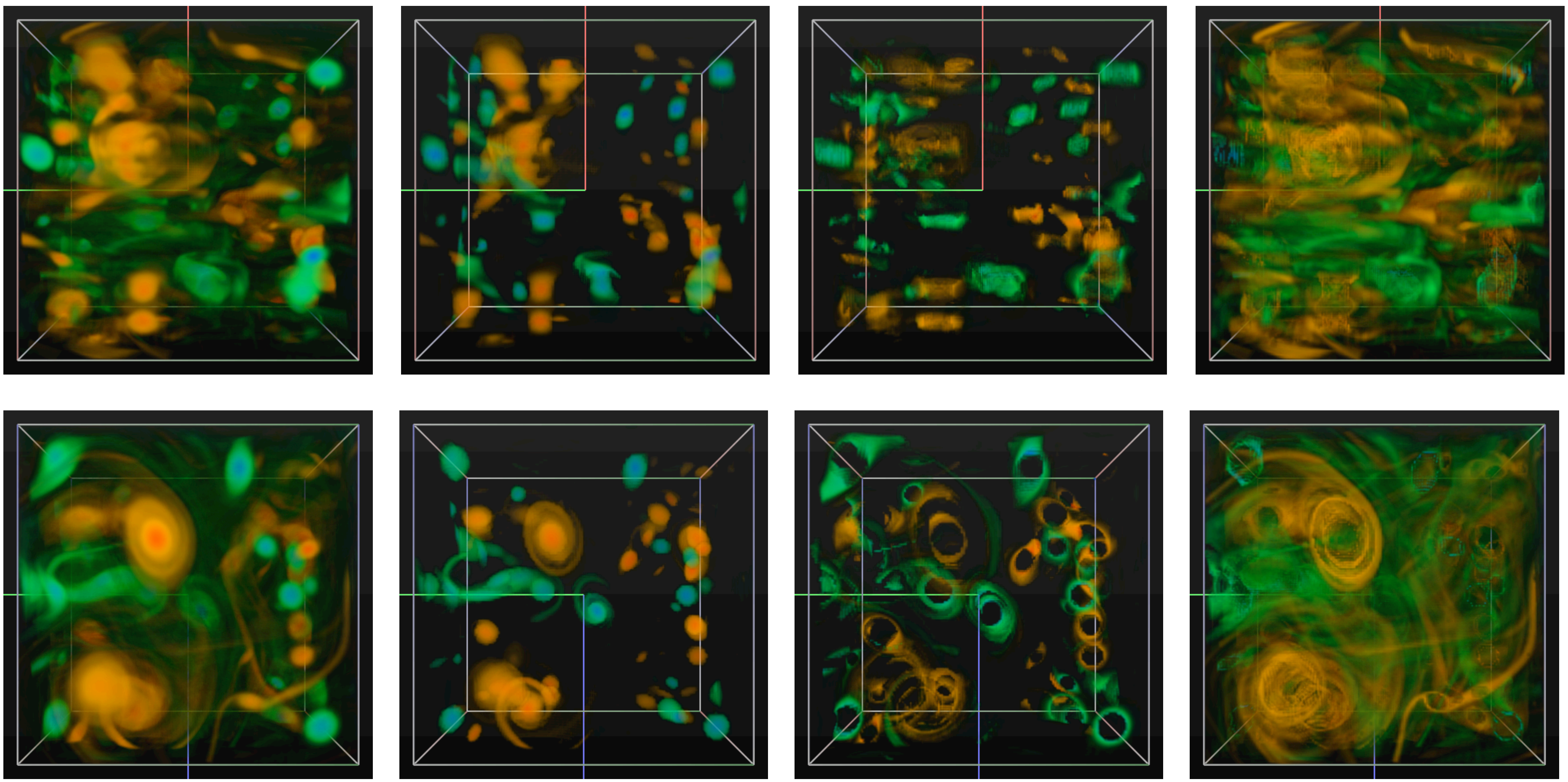
- circulation cells:
  strain dominates
  large positive λ_2

- background:
  λ_2 near zero

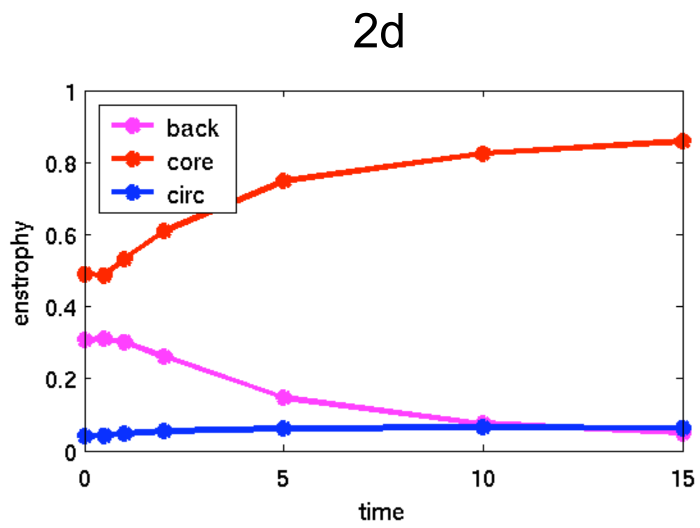


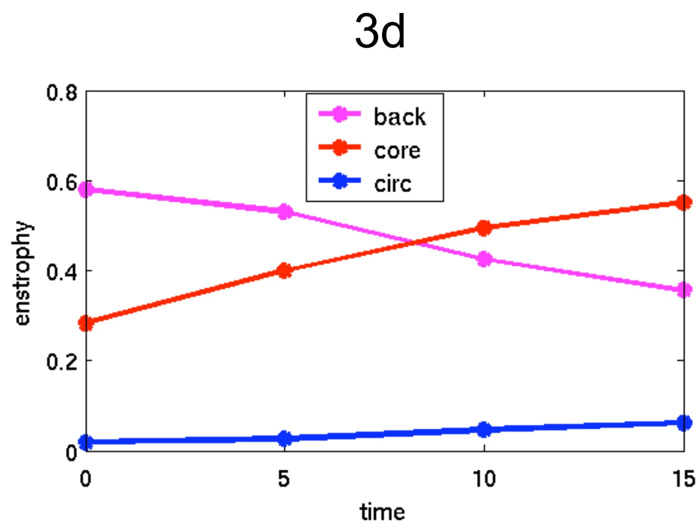
Components in 3d

full field | cores | circulation cells | background



Enstrophy in components

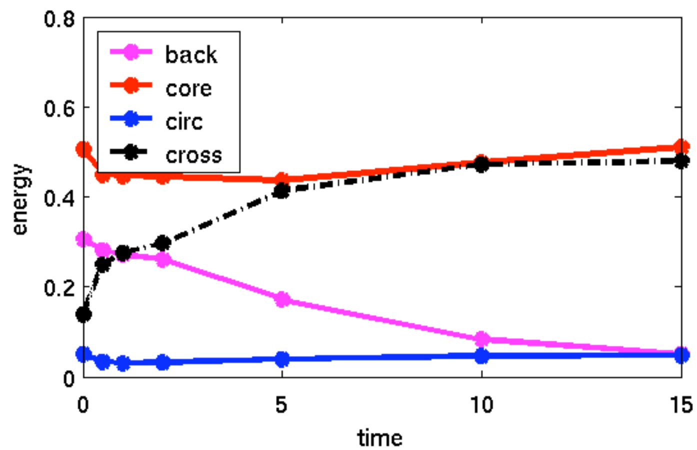




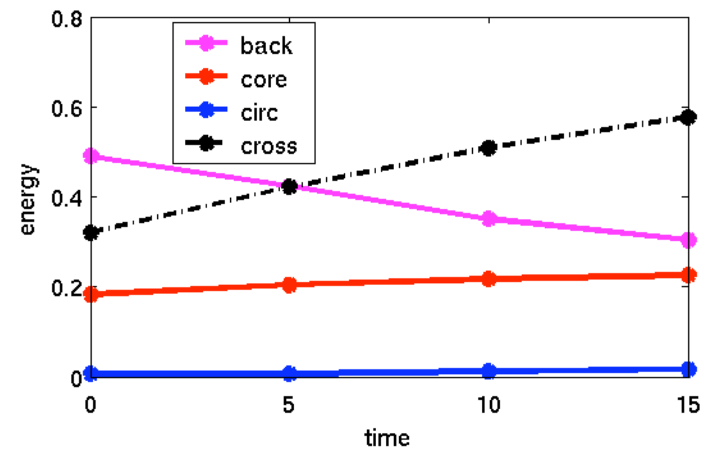
- enstrophy in cores eventually dominates
- 3d: more enstrophy in background = more filaments

Kinetic Energy induced by components

2d



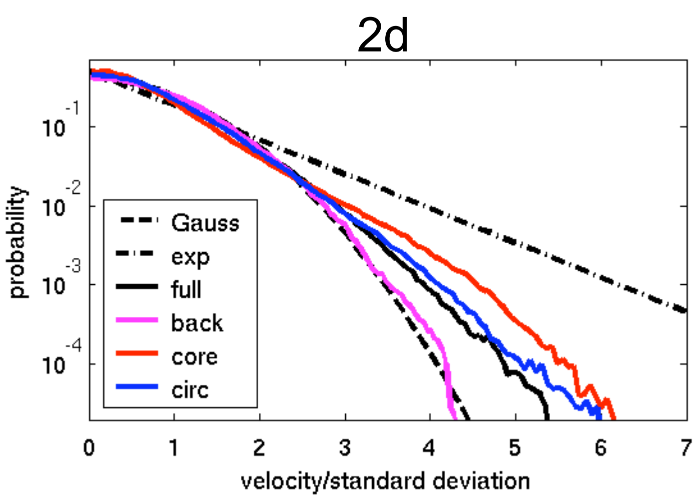
3d

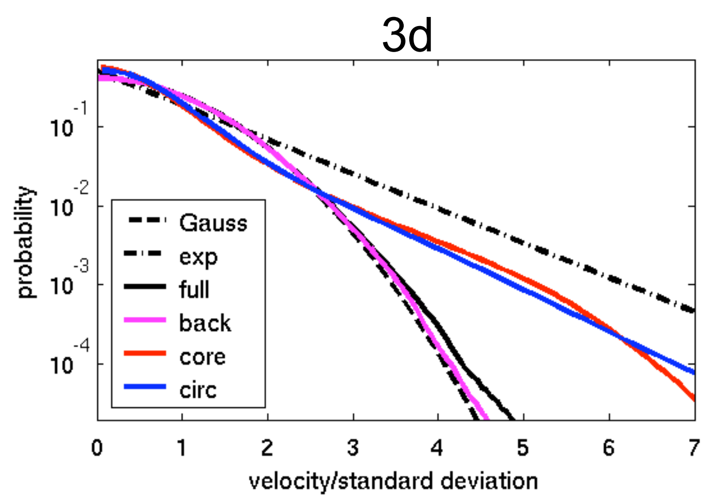


- much more energy due to background in 3d
- even more than due to cores

Velocity pdfs

- non-Gaussian velocity pdfs in 2d
  - due to vortex component
  - background is Gaussian
- 3d more Gaussian than 2d due to stronger background

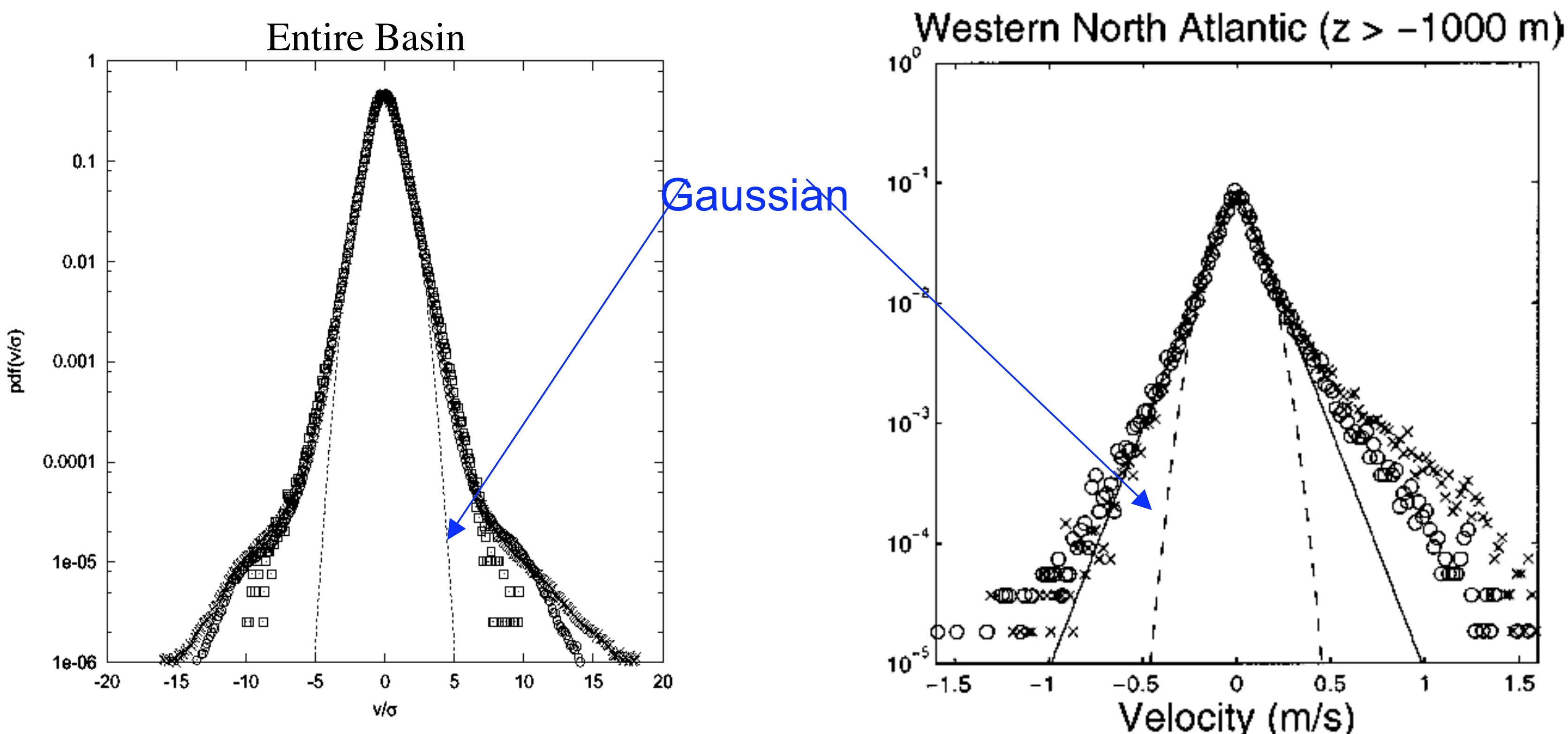




Ocean Basin Velocity PDF’s

(Bracco, et al, 2000)

- long tails in models and observations
- due to coherent vortices?
- more like 2d than 3d?



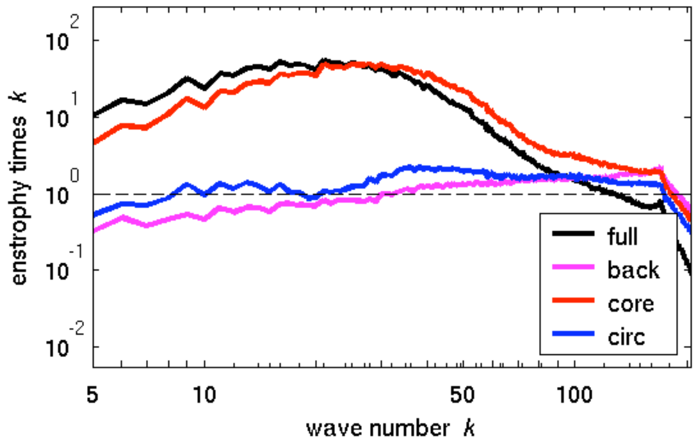
QG Model and Ocean GCM

Ocean Floats

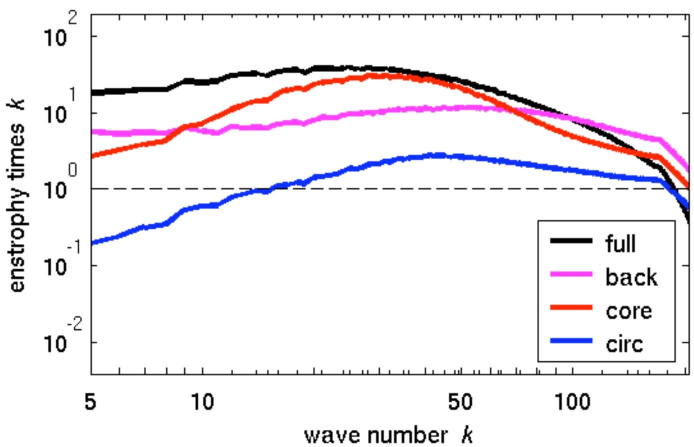
Spectra

- Kraichnan-Batchelor: enstrophy ~ k^{-1}

2d



3d



- cores steeper
- background almost k^{-1}
- closer in 2d

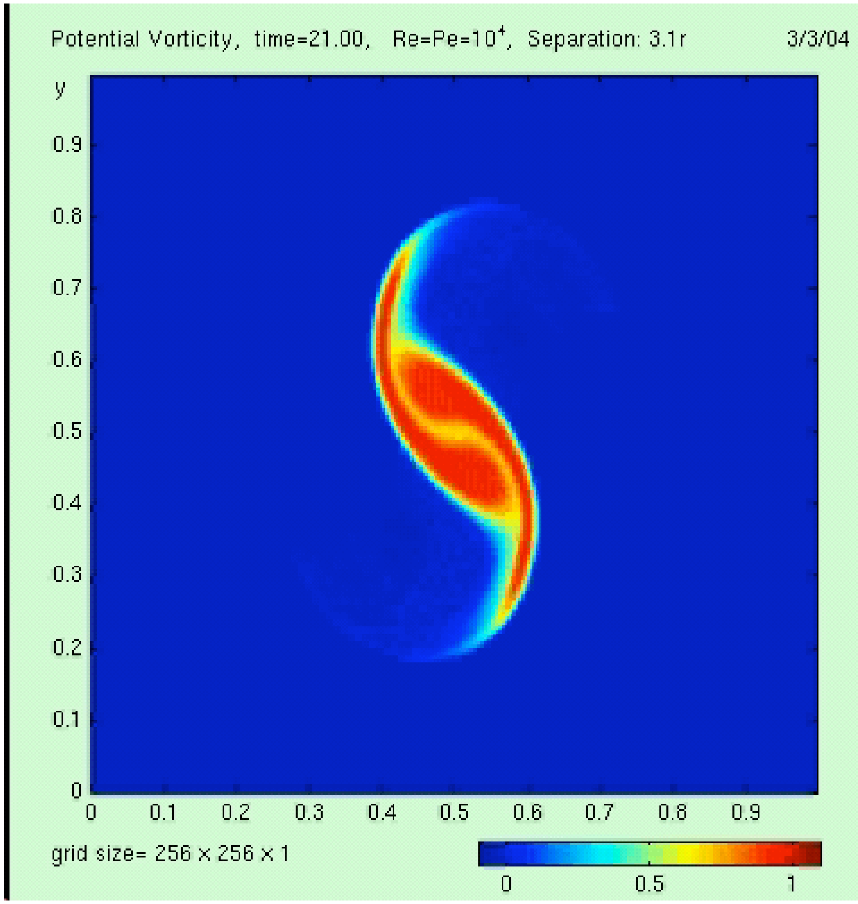
3. QG Vortex interactions

(Martinsen-Burrell et al 2006)

- 2d: vortex merger is the dominant evolutionary mechanism
  - critical merger distance = 3.3 radius
  - understood in term of
    - V-states *(Deem and Zabusky, 1978)*
    - Lagrangian manifold structure *(Velasco Fuentes, 2001)*
    - chaos in elliptical model *(Weiss and McWilliams, 1993)*
- 3d: merger and alignment are dominant
  - alignment is controversial

2d vortex merger

- vortices merge if close enough
  - inverse and direct cascade

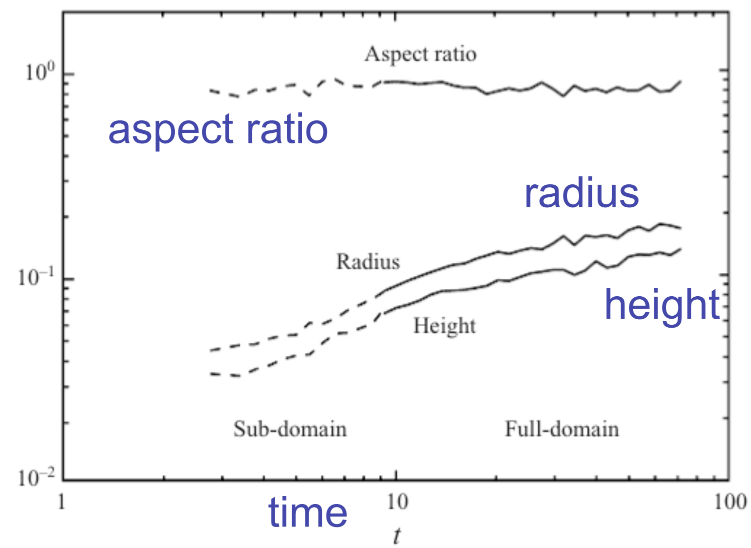


3d QG vortex merger

- previous studies show merger similar to 2d when vertical separation is small to moderate
  (von Hardenberg, et al 2000; Reinaud and Dritschel 2002)
- elliptical moment model used for tall vortices
  (Miyazaki et al 2001; 2002)
- here, use elliptical moment model to study alignment and merger
  - model obtained from Hamiltonian reduction
  - keeps moments through 2nd order
  - ten degrees of freedom per vortex
  - invariants reduce dimensionality
  - N vortices: 6N-5 energy surface in 6N-4 phase space

vortex aspect ratio

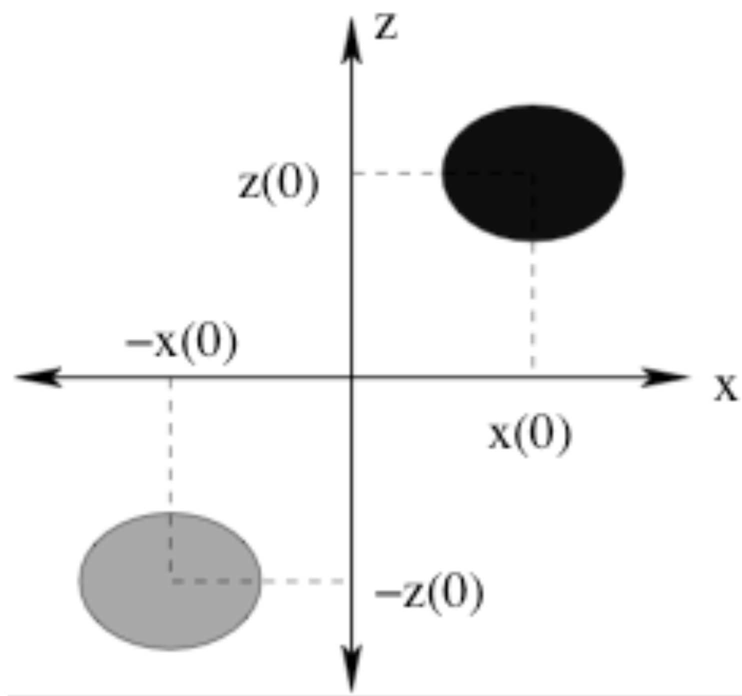
- vortices in 3d QG turbulence have preferred aspect ratio of 0.8
- may be due to vortex stability in strain *(Reinaud et al 2003)*



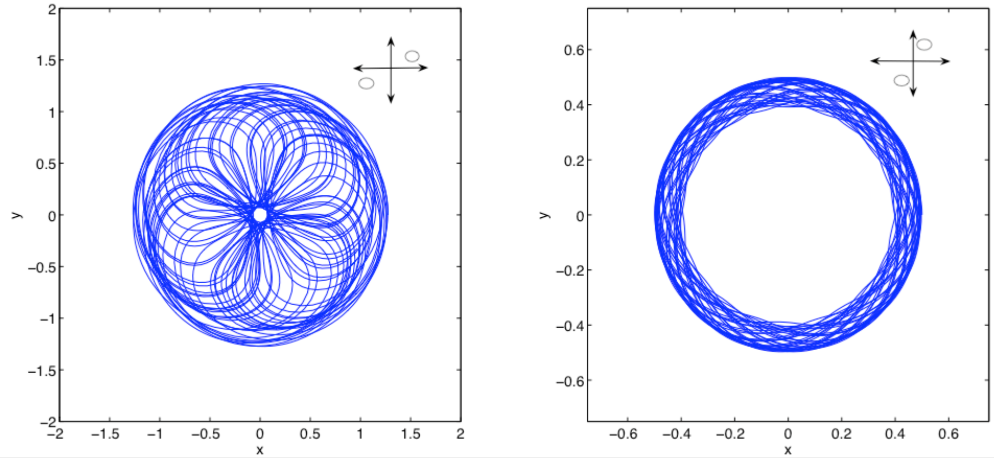
(McWilliams et al 1999)

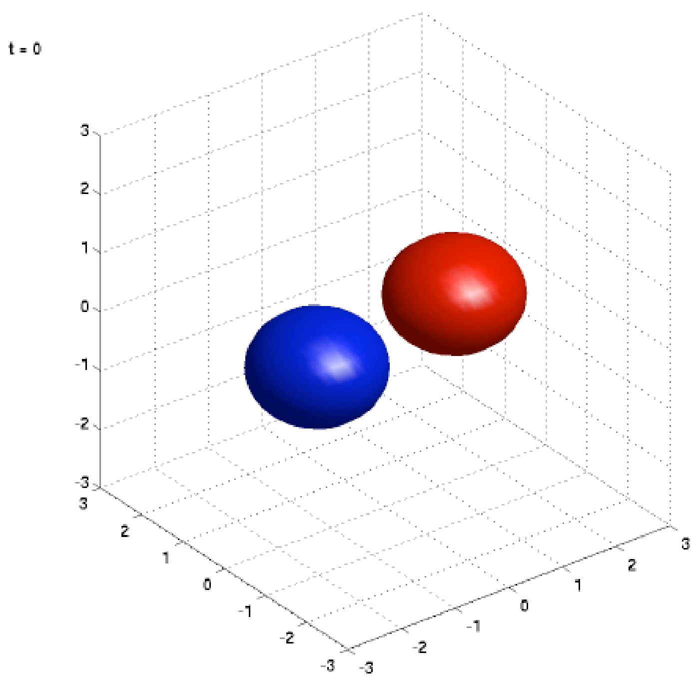
initial conditions

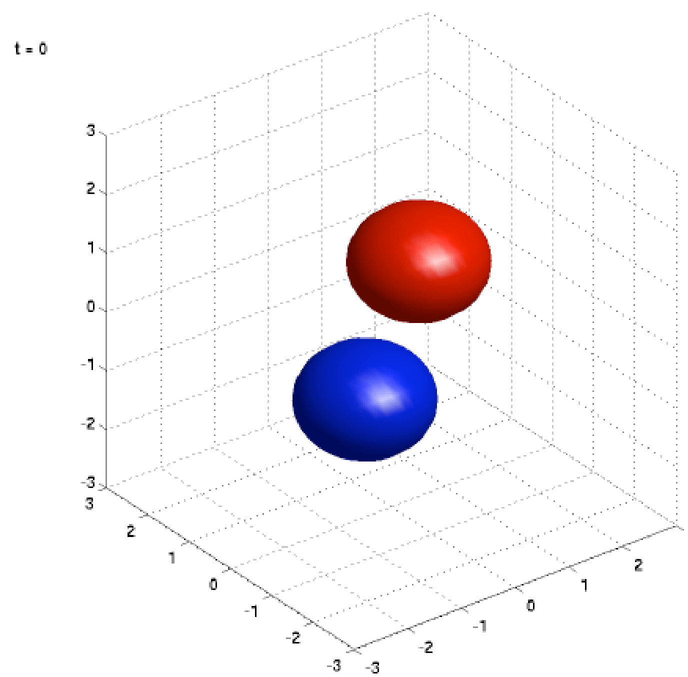
- 2 identical vortices
- aspect ratio = 0.8
- volume $4\pi/3$, $r_h = 1.08$



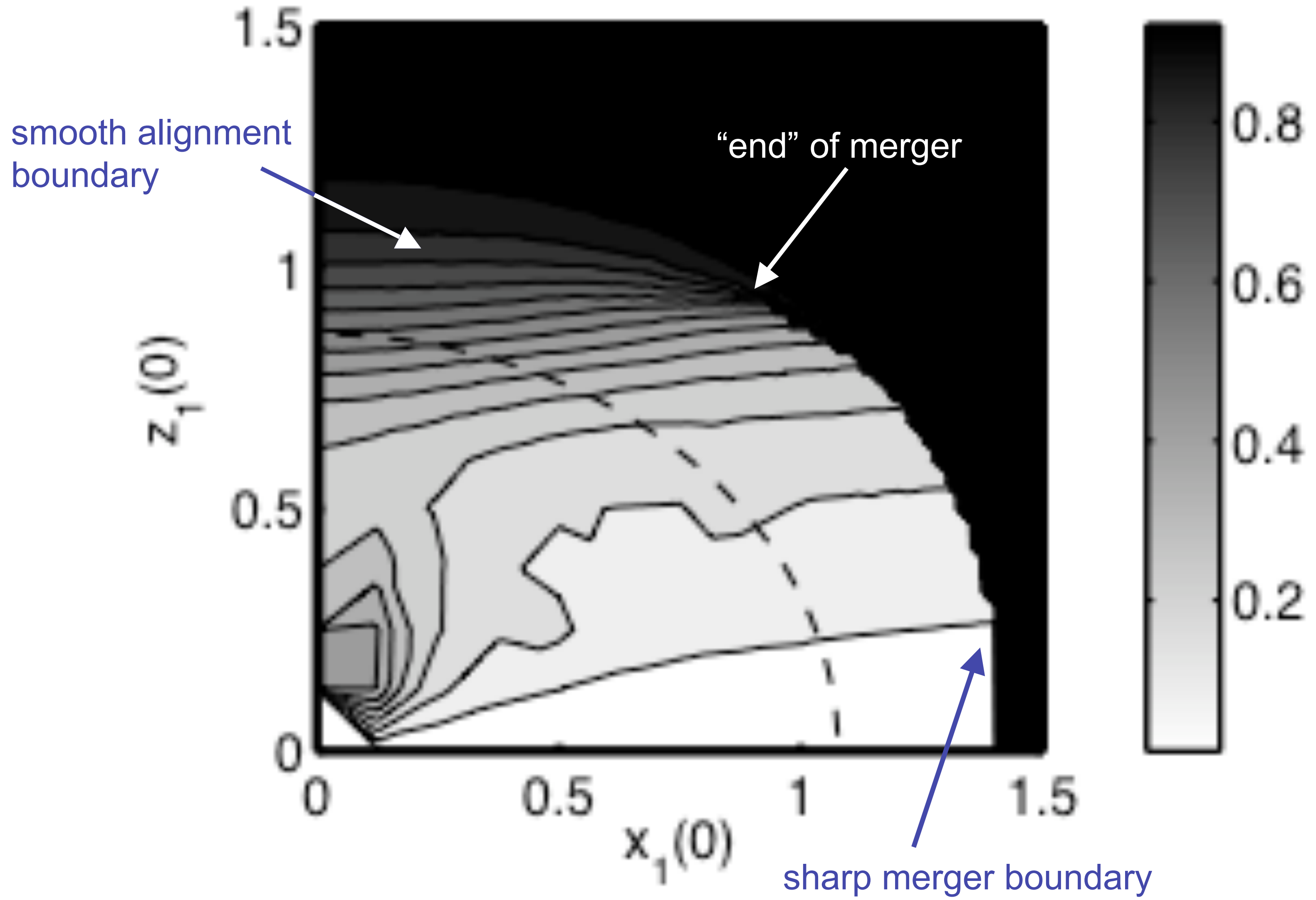
trajectory projected onto (x,y) plane





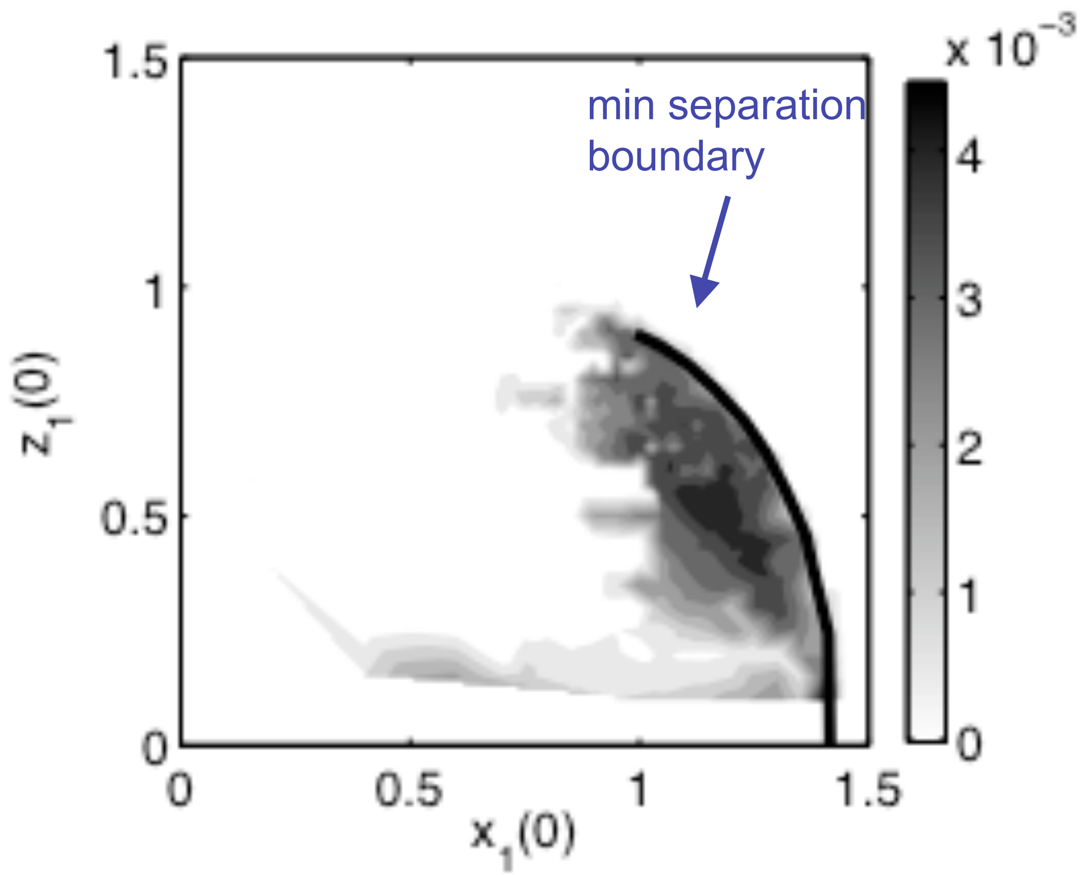


- minimum horizontal separation



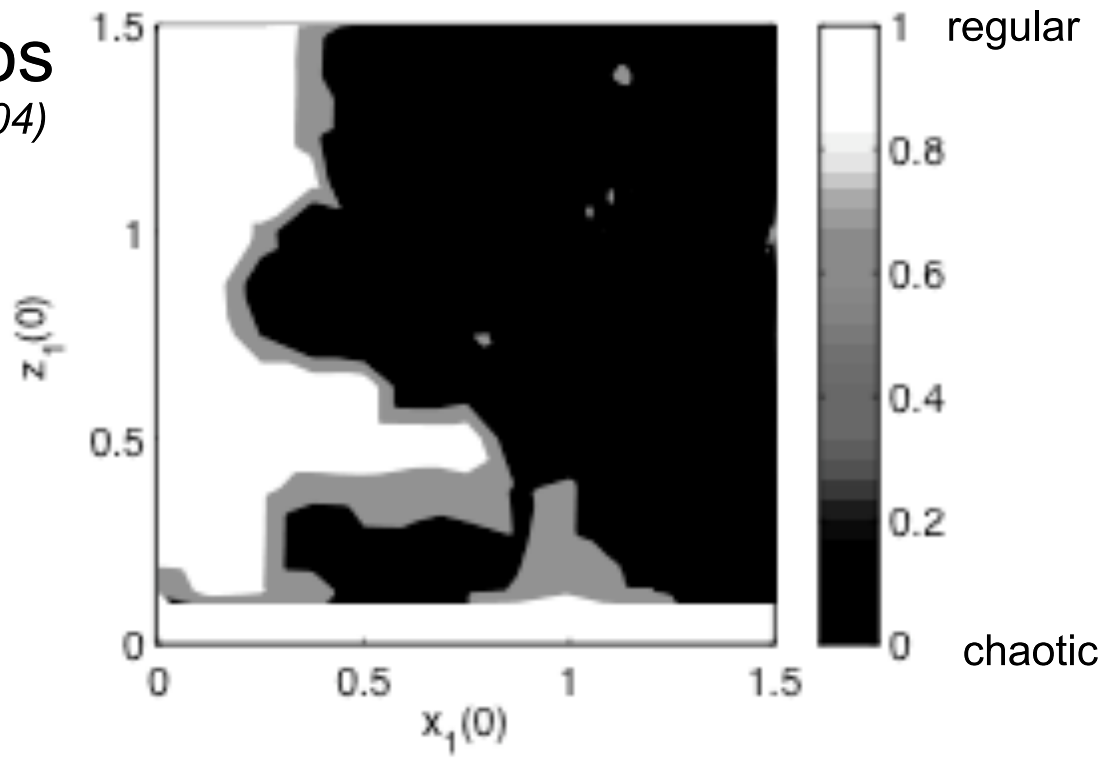
Lyapunov exponents

- in 2d, merger occurs at onset of chaos
- large Lyapunov exponents align with boundary in min separation



Gottwald-Melbourne 0-1 Test

- New test for chaos
  (Gottwald and Melbourne, 2004)



- no correspondence with min separation or large Lyapunov exponent
- strong and weak chaos in high-D phase space

Compare with 3d simulations

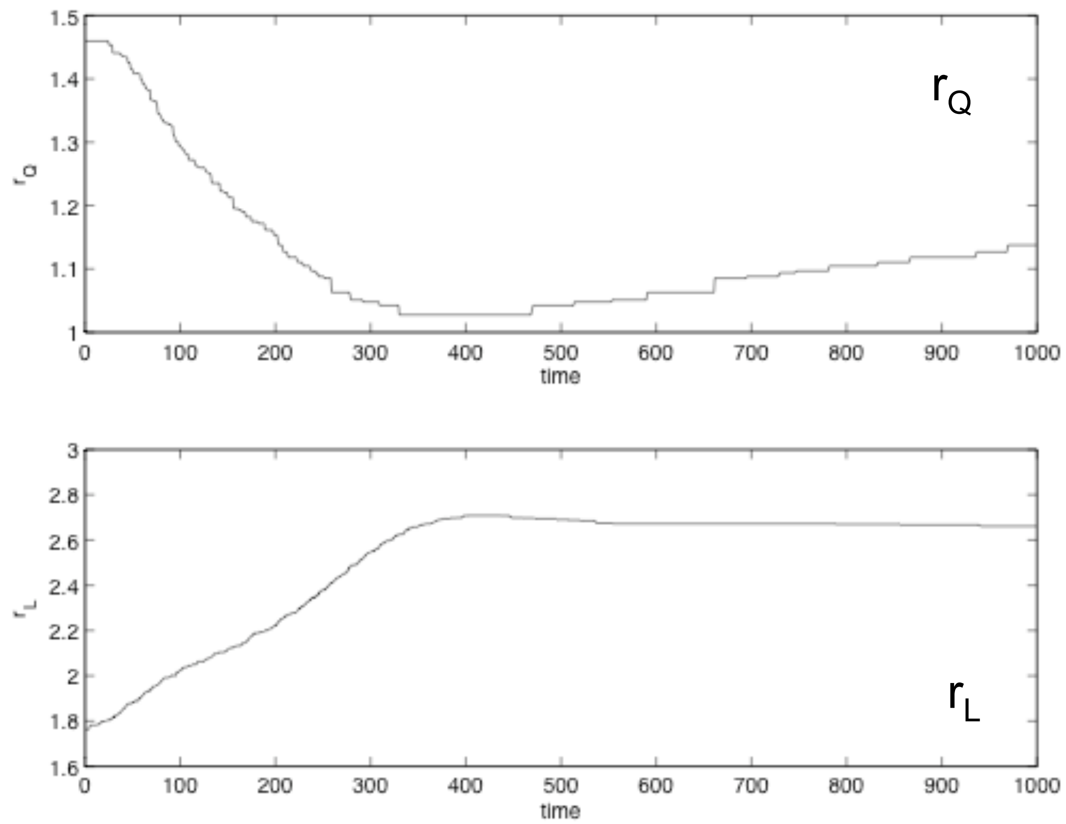
- 3d QG fluid equations with Newtonian dissipation
- tanh profiles for vortices
- measure merger/alignment by median radii
  - inviscid evolution is a rearrangement of properties
  - consider circulation inside cylinder C_r with radius r

$$Q(r) = \int_{C_r} q dV$$

  - similar for angular momentum

$$L(r) = \int_{C_r} \sqrt{x^2 + y^2} q dV$$

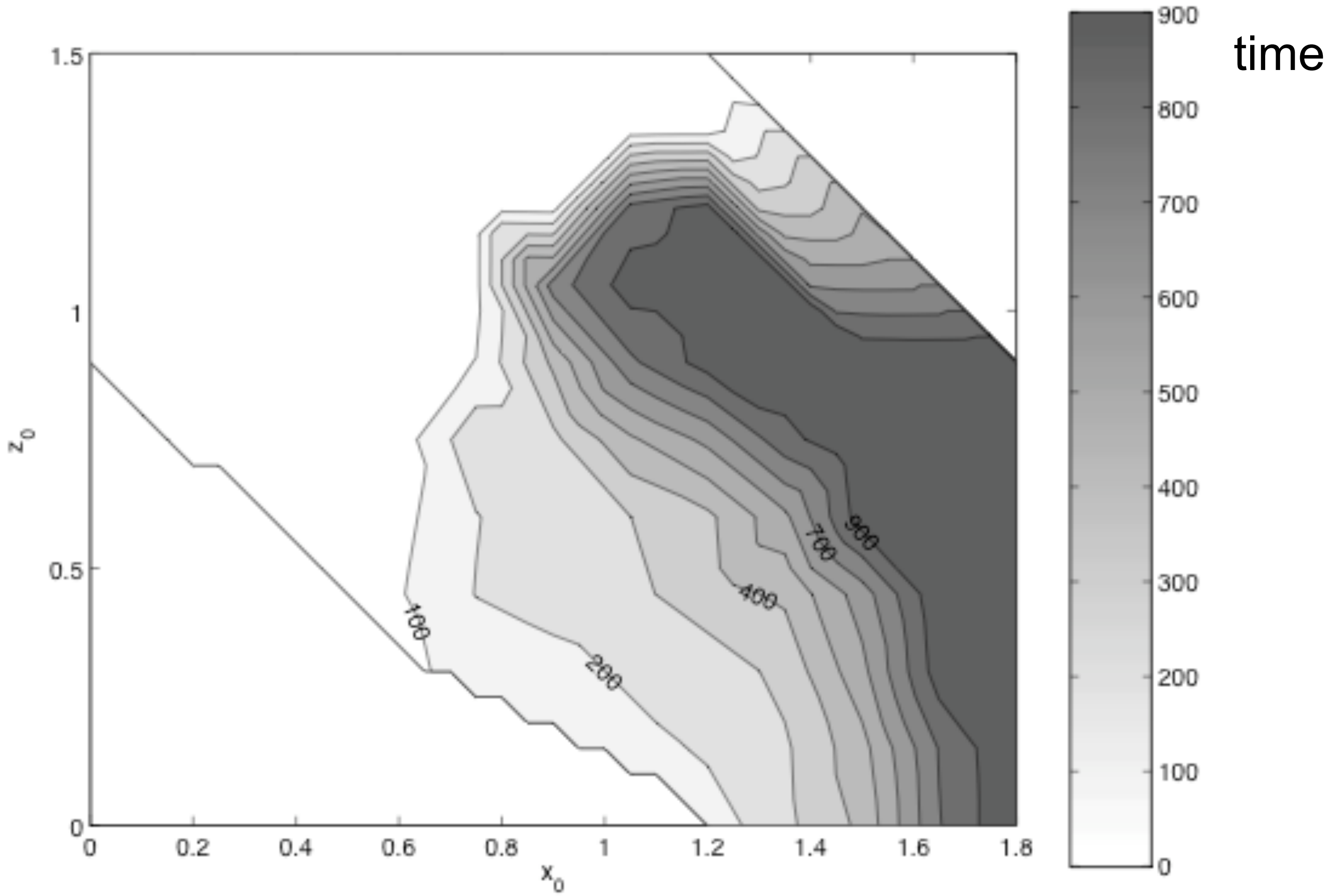
- median radii
  - r_Q: $Q(r_Q) = 1/2$
  - r_L: $L(r_L) = 1/2$
- merger/alignment:
  - r_Q shrinks
  - r_L grows



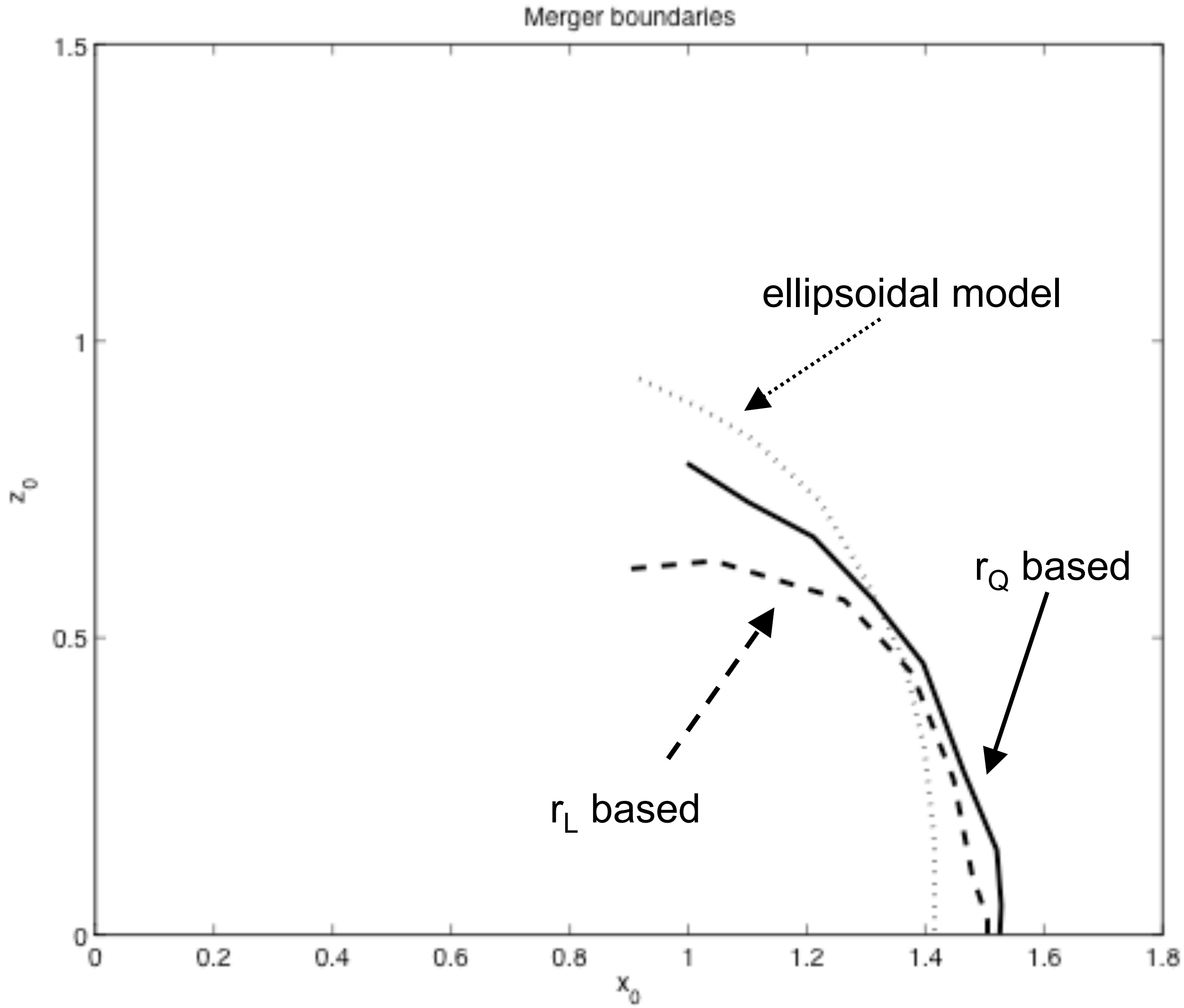
time

time to reach min r_Q

- dissipation eventually causes merger
- diagnose merger by time to reach min r_Q

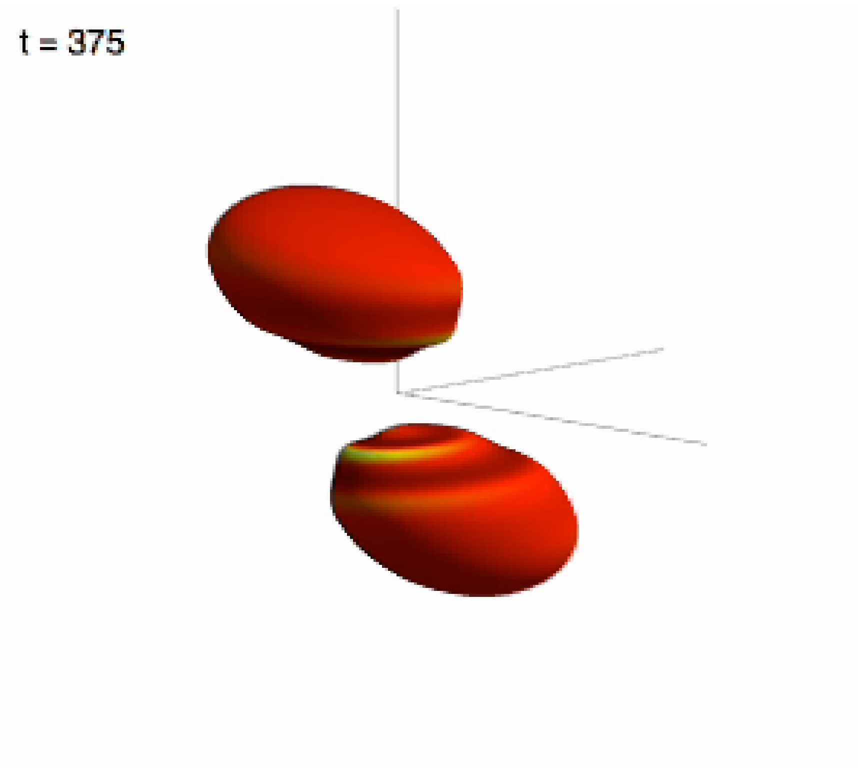


merger boundary



alignment

- no dramatic alignment
- interesting wave phenomena

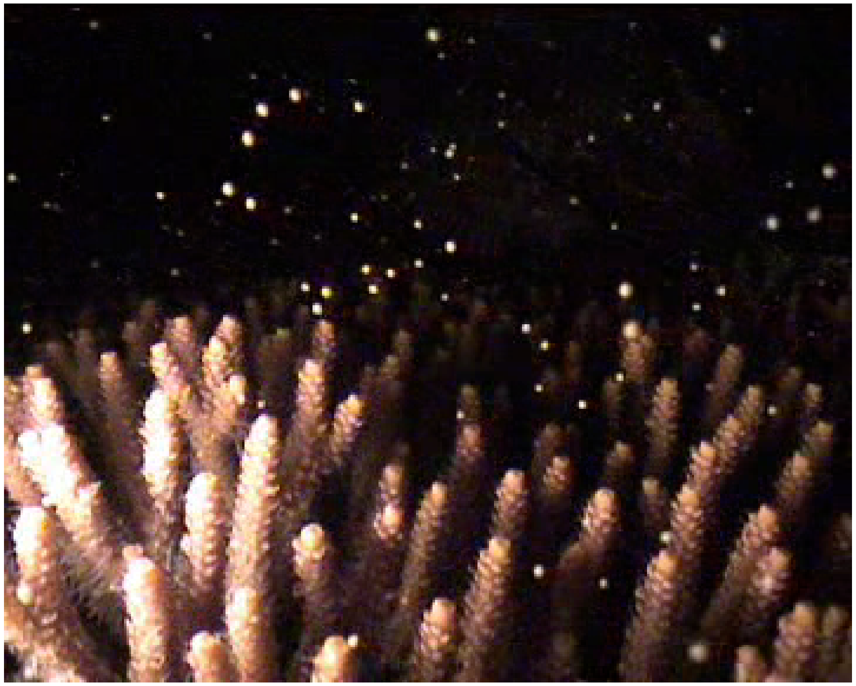


- similar to vortex Rossby waves proposed for hurricanes *(e.g. Reasor and Montgomery, 2001)*
  - suggests alignment is a subtle multi-event adjustment process

4. Reactions in vortices

(Crimaldi, et al 2006, 2008)

- Motivating problem: coral fertilization
  - 2008 annual mass spawning off Palau
- broadcast spawning:
  - sperm and egg released into flow
  - two reacting scalars separated by third scalar
- fertilization rates:
  - field measurements:
    - 5% - 90%
  - eddy diffusion:
    - 0.01% - 1%



reefvid.org

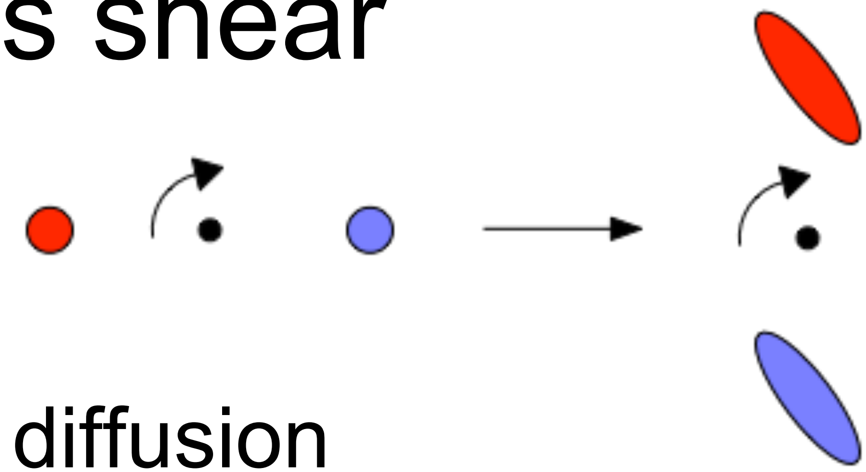
chemical reactions

- two scalars $C_A(x,y,t)$, $C_B(x,y,t)$ in 2d
- local reaction rate = kC_AC_B
- total reaction rate given by overlap

$$\theta(t) = \iint dx\,dy\,C_A(x,y,t)\;C_B(x,y,t)$$

- work in low-concentration limit
  - Damkohler number Da to 0
- C given by advection-diffusion equation

vortex produces shear

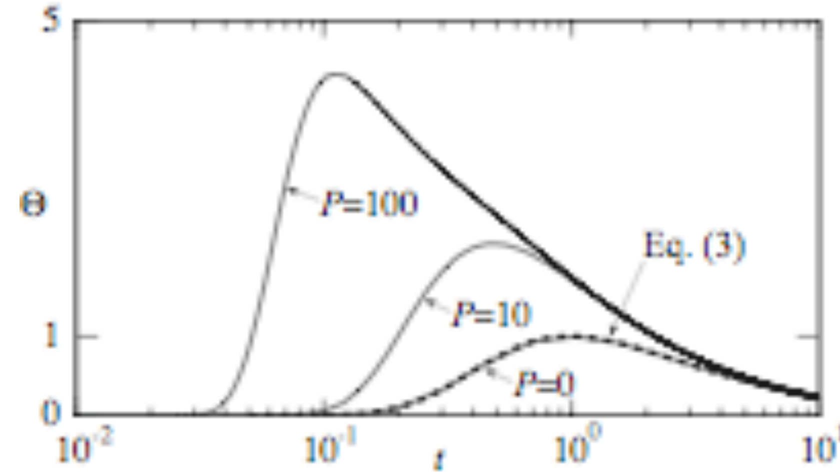


- shear enhanced diffusion
  - normal diffusion: $\Delta x^2 \sim t$
  - shear enhanced: $\Delta x^2 \sim t^3$
- problem governed by Peclet number P

  Γ = vortex circulation, D = diffusion

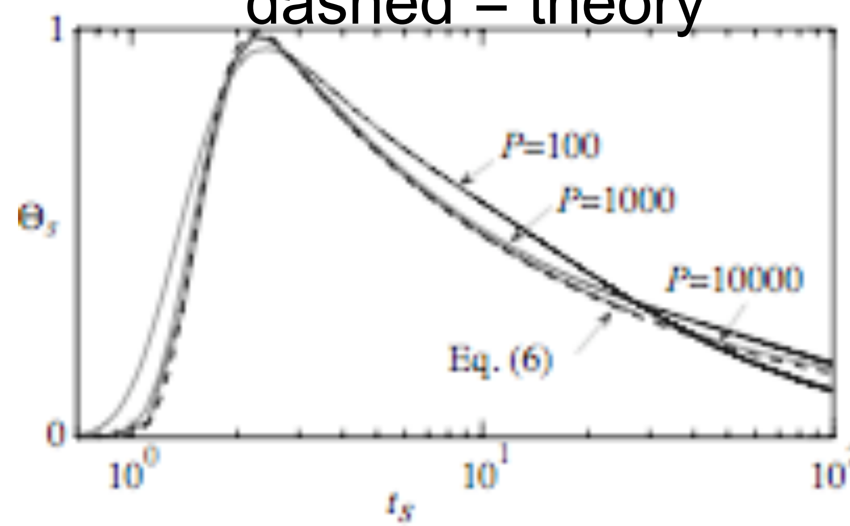
$$P = \frac{t_{diffusion}}{t_{advection}} \sim \frac{\Gamma}{D}$$

- vortex enhances reaction rate
  - numerical simulation
  - larger overlap
  - faster reaction

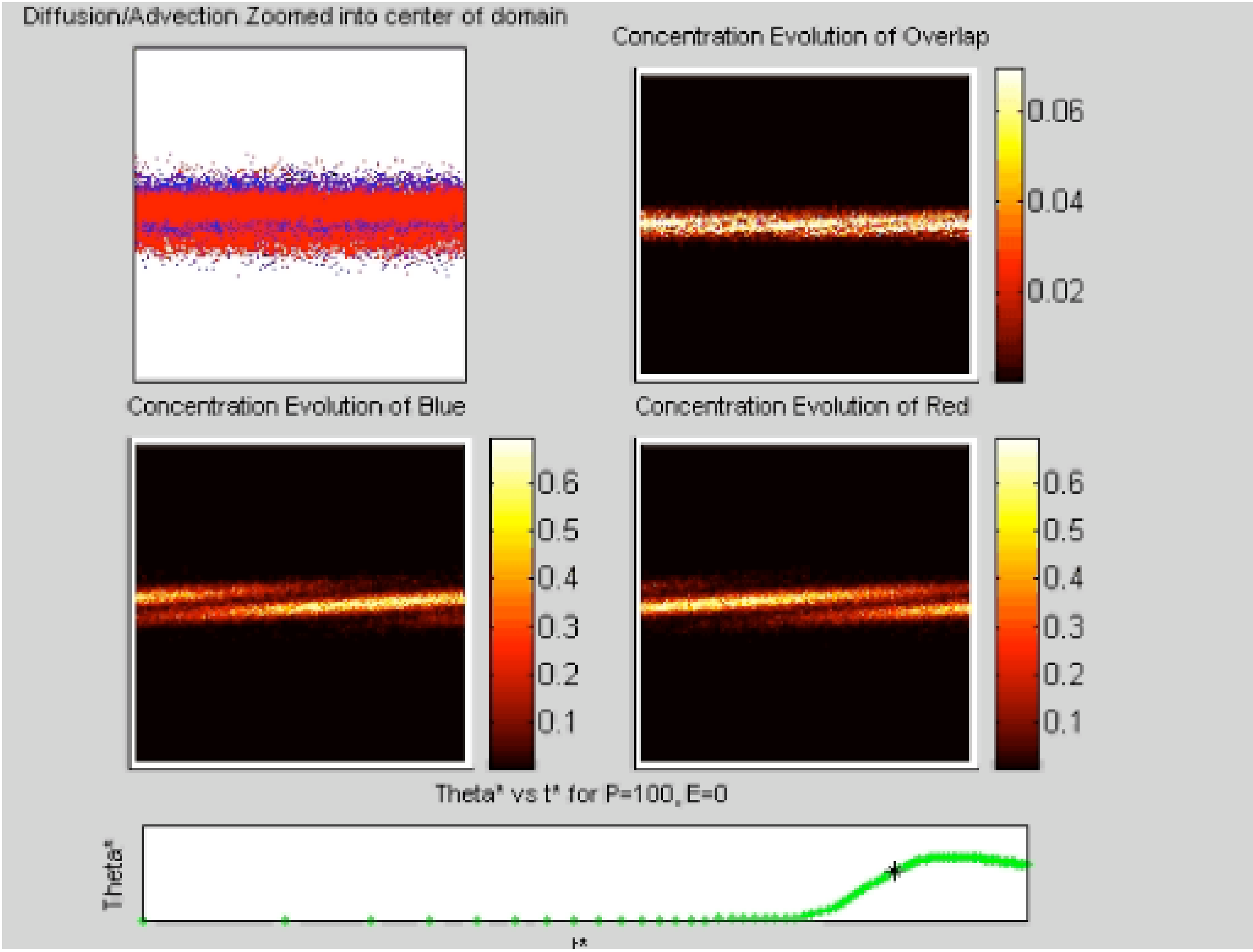


- analytic theory gives scaling
  - $\theta \sim P^{1/3}$
  - $t_{reaction} \sim P^{-2/3}$

dashed = theory



- physically: competition between
  - diffusion: reduces C
  - advection: filaments C but no reduction
- eddy diffusion always reduces C
- enhancement only weakly dependent on details of experiment
- scaling holds for Pe >> Da >> 1



Diffusion/Advection Zoomed into center of domain
Concentration Evolution of Overlap
0.06
0.04
0.02
Concentration Evolution of Blue
0.6
0.5
0.4
0.3
0.2
0.1
Concentration Evolution of Red
0.6
0.5
0.4
0.3
0.2
0.1
Theta* vs t* for P=100, E=0
Theta*
t*

Summary

- geophysical turbulence self-organizes into coherent structures
- structures dominate dynamics and transport
- ongoing progress in theory and modeling