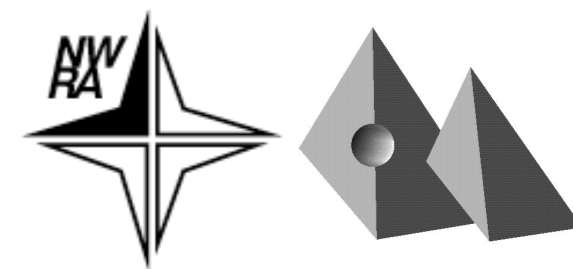


Gravity Wave Breaking in the Atmosphere

Joe Werne, Dave Fritts, Tom Lund, Ling Wang, Kam Wan
NWRA/CoRA, Boulder, CO

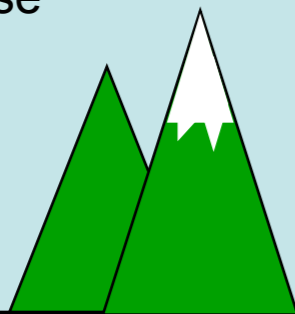
Mark Berliner
Ohio State University, Columbus, OH



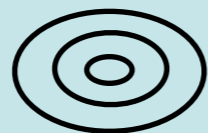
Sources of Stratospheric Turbulence

stratopause

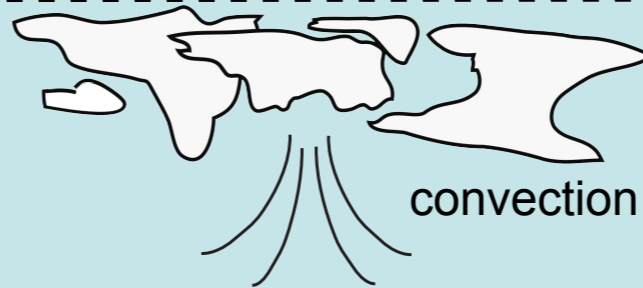
tropopause



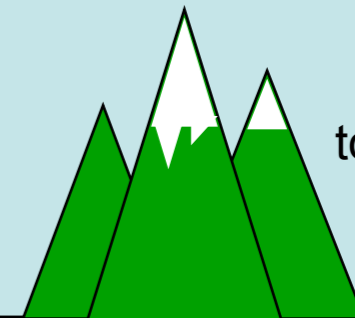
summer



jet instability



convection



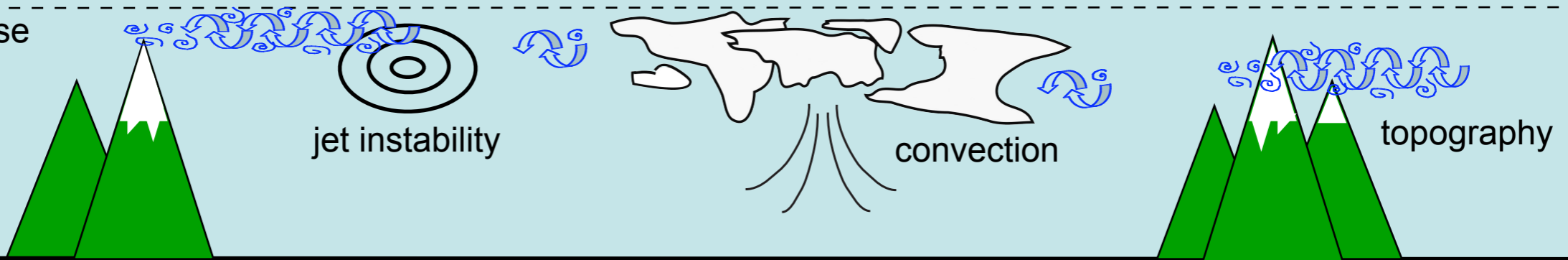
topography

winter

Sources of Stratospheric Turbulence

stratopause

tropopause



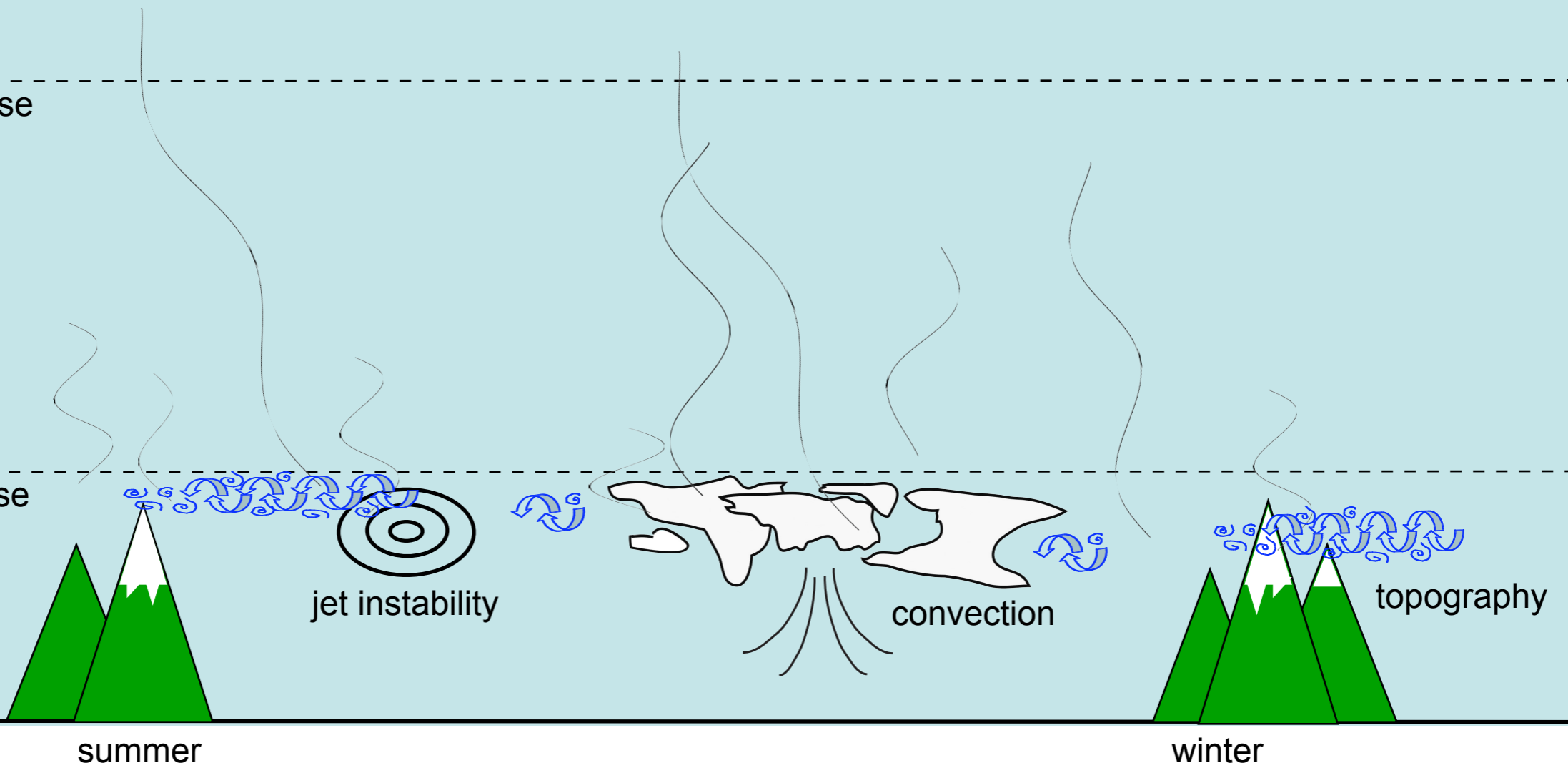
summer

winter

Sources of Stratospheric Turbulence

stratopause

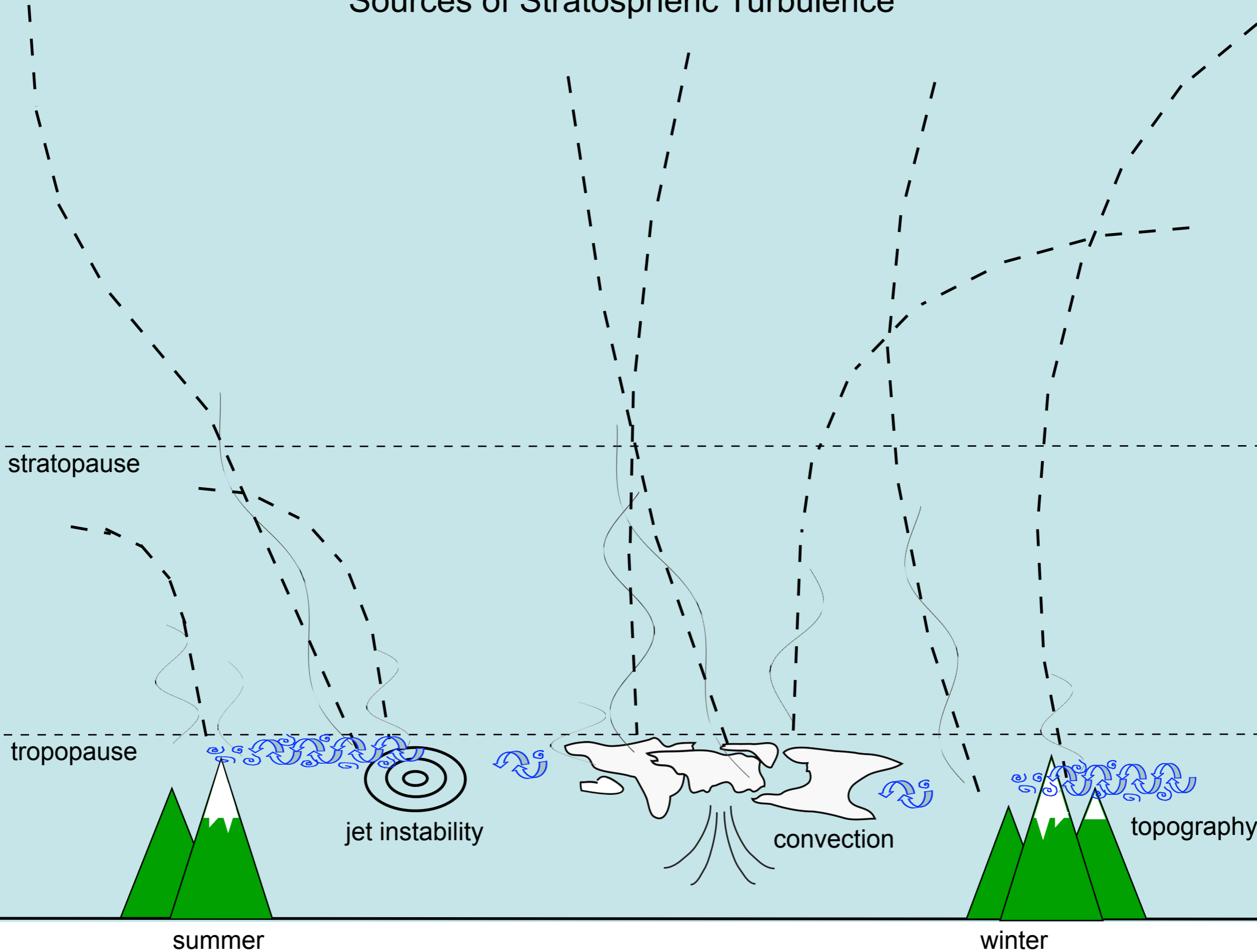
tropopause



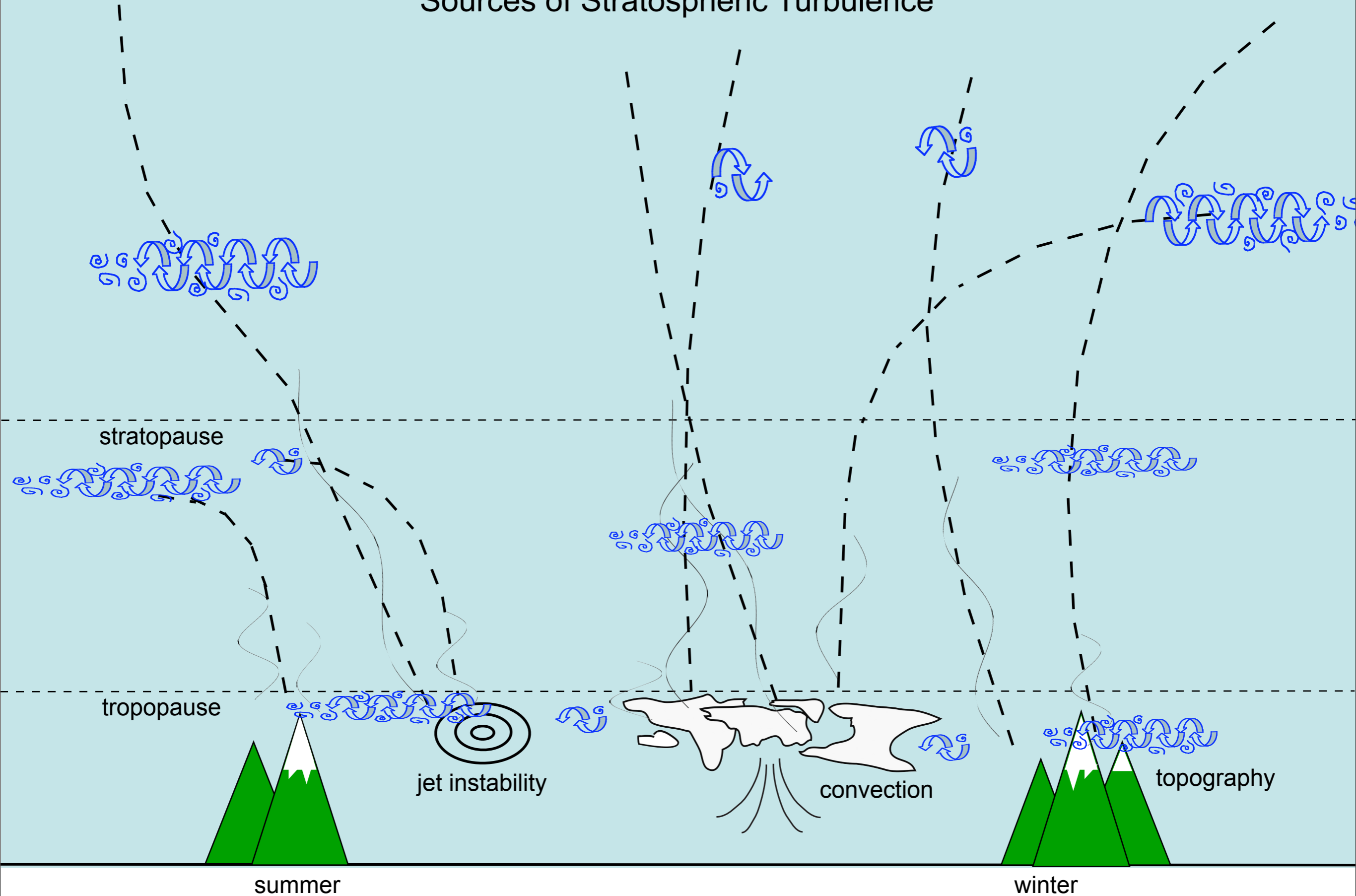
summer

winter

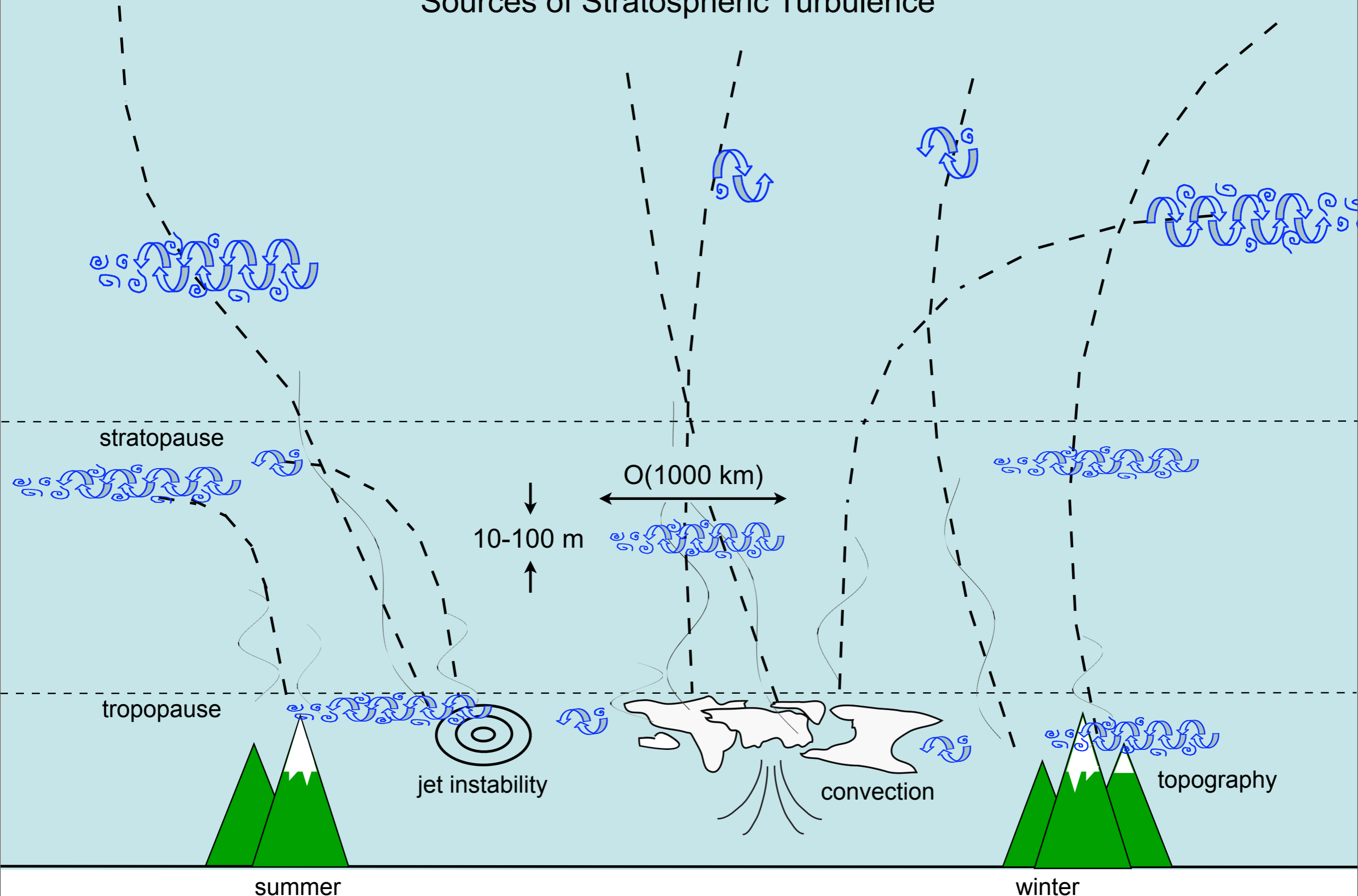
Sources of Stratospheric Turbulence



Sources of Stratospheric Turbulence



Sources of Stratospheric Turbulence



summer

winter

Sources of Stratospheric Turbulence

Good News:

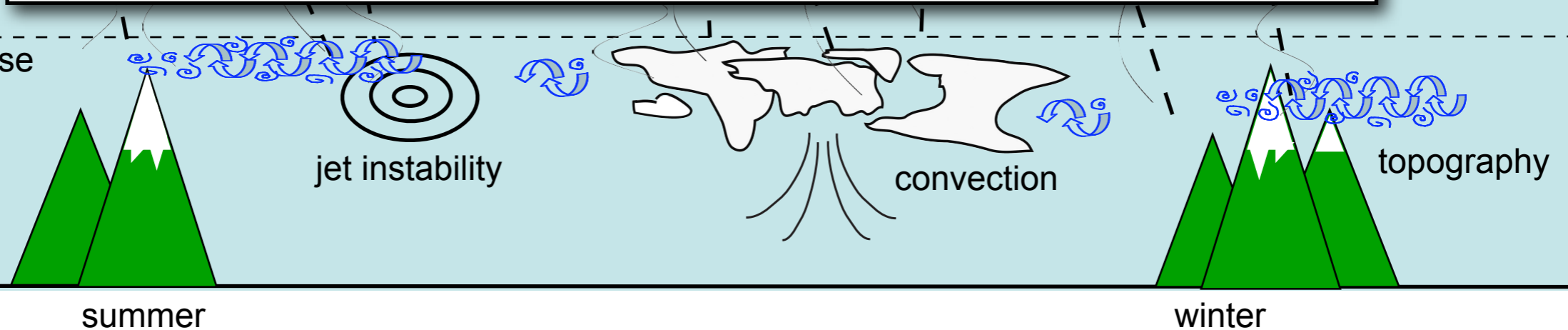
We know the processes that are responsible for stratospheric turbulence: wind shear and gravity-wave breaking.

Bad News:

1. Current forecast models cannot resolve them.
2. Resulting turbulence is challenging to model (layered, non-stationary, anisotropic, inhomogeneous, fossil events precondition future events, gravity waves provide non-local unresolved momentum transfer).

stratopause

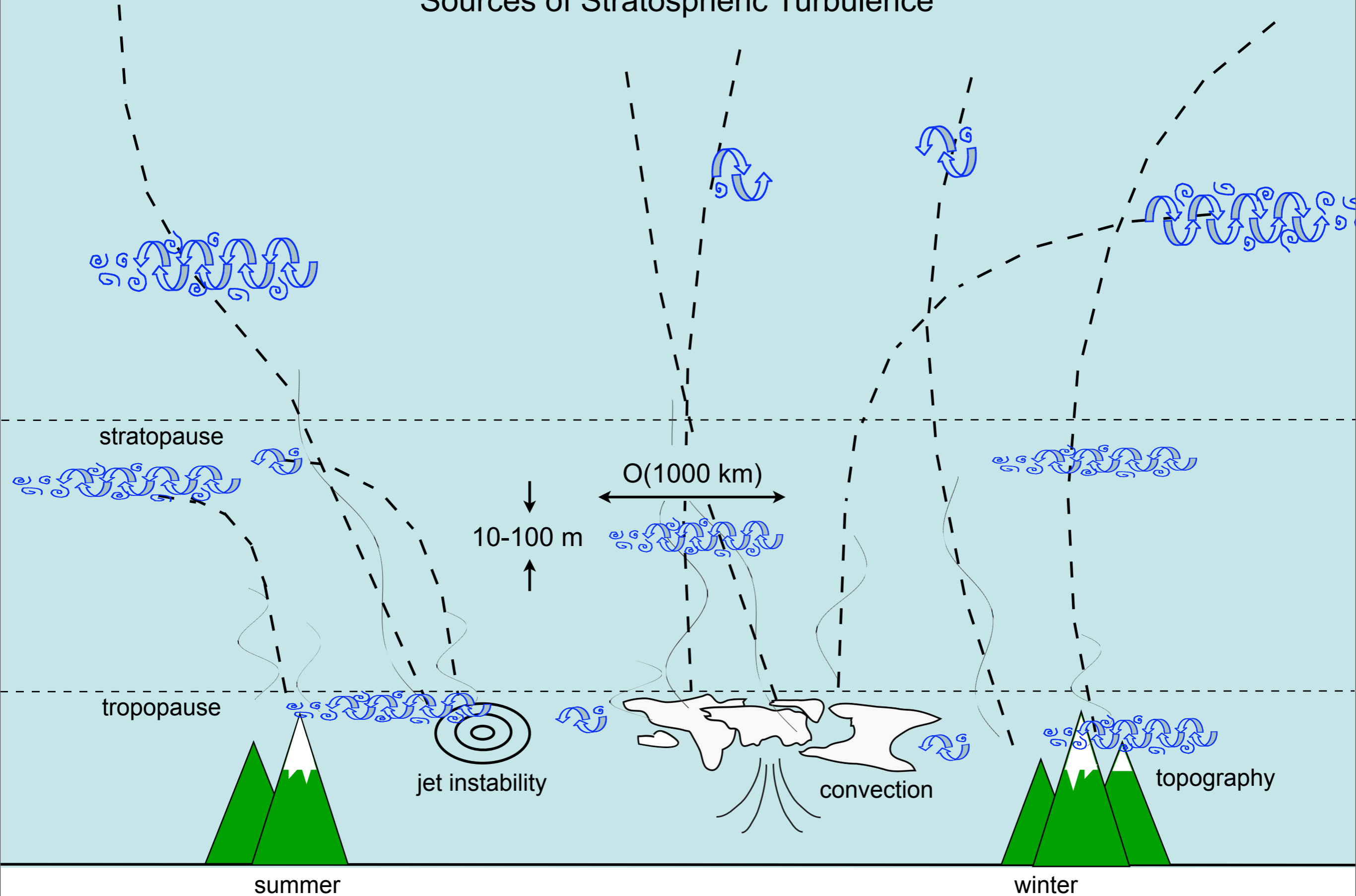
tropopause



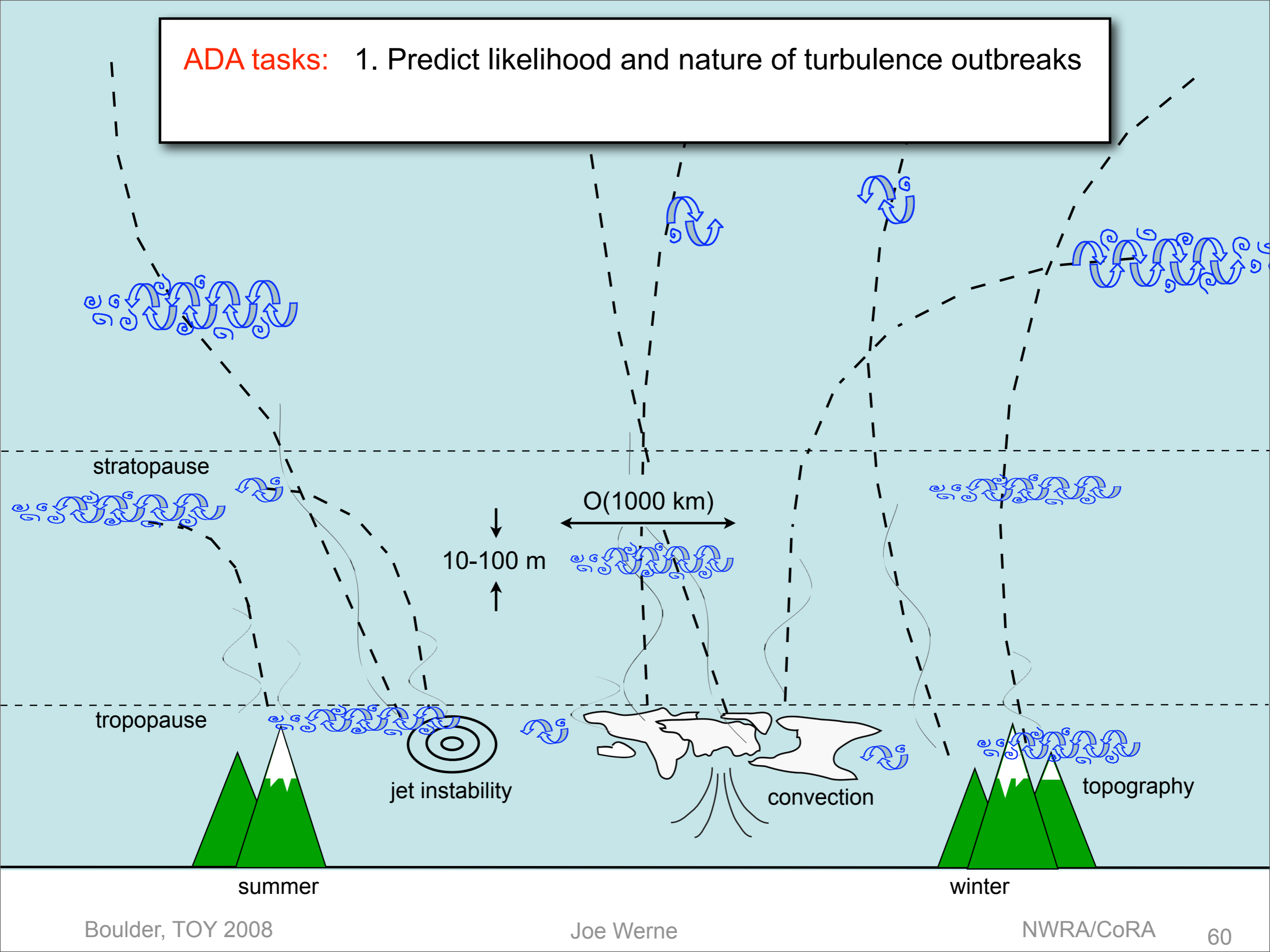
summer

winter

Sources of Stratospheric Turbulence



ADA tasks: 1. Predict likelihood and nature of turbulence outbreaks

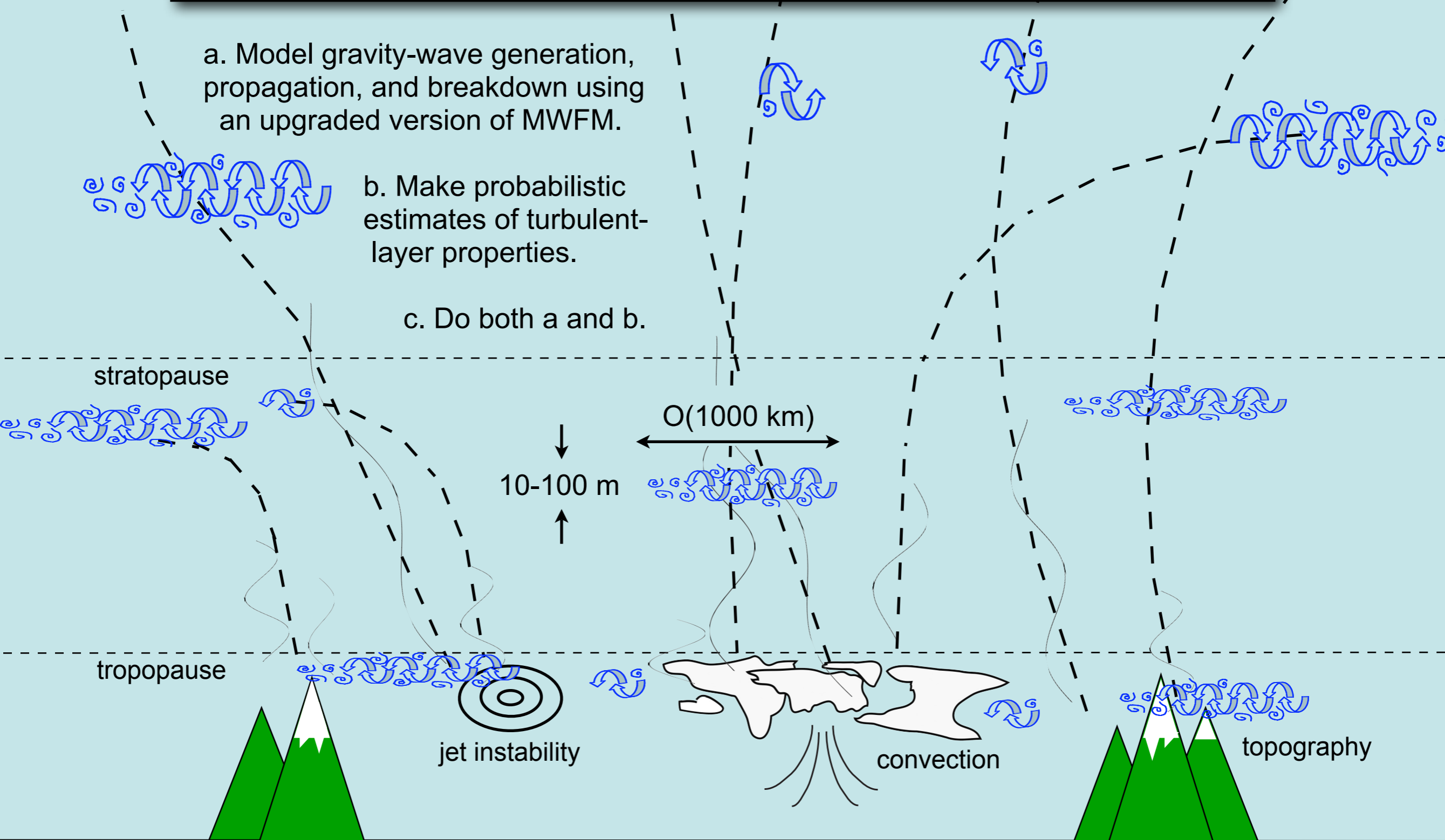


ADA tasks: 1. Predict likelihood and nature of turbulence outbreaks

a. Model gravity-wave generation, propagation, and breakdown using an upgraded version of MWFM.

b. Make probabilistic estimates of turbulent-layer properties.

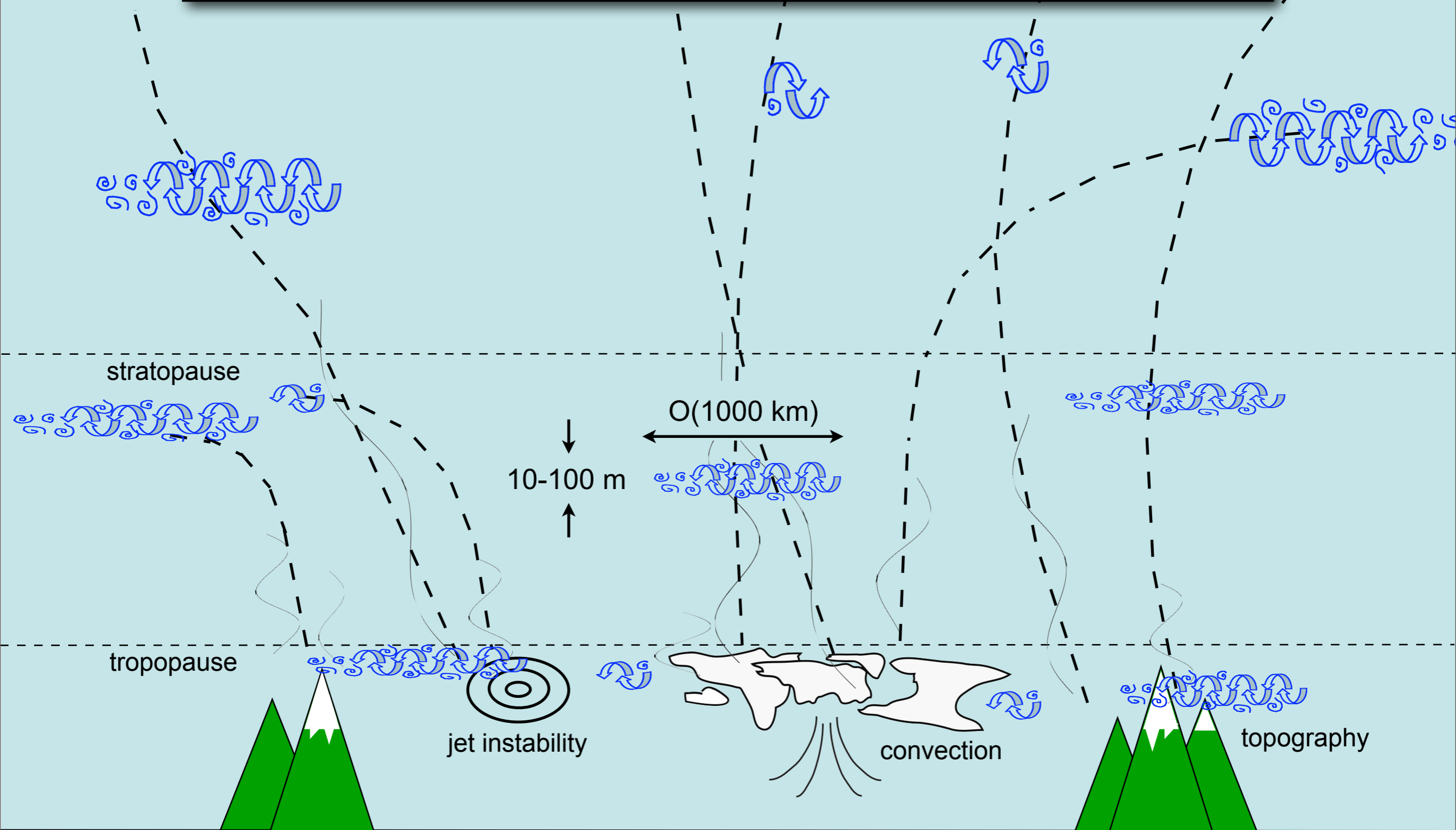
c. Do both a and b.



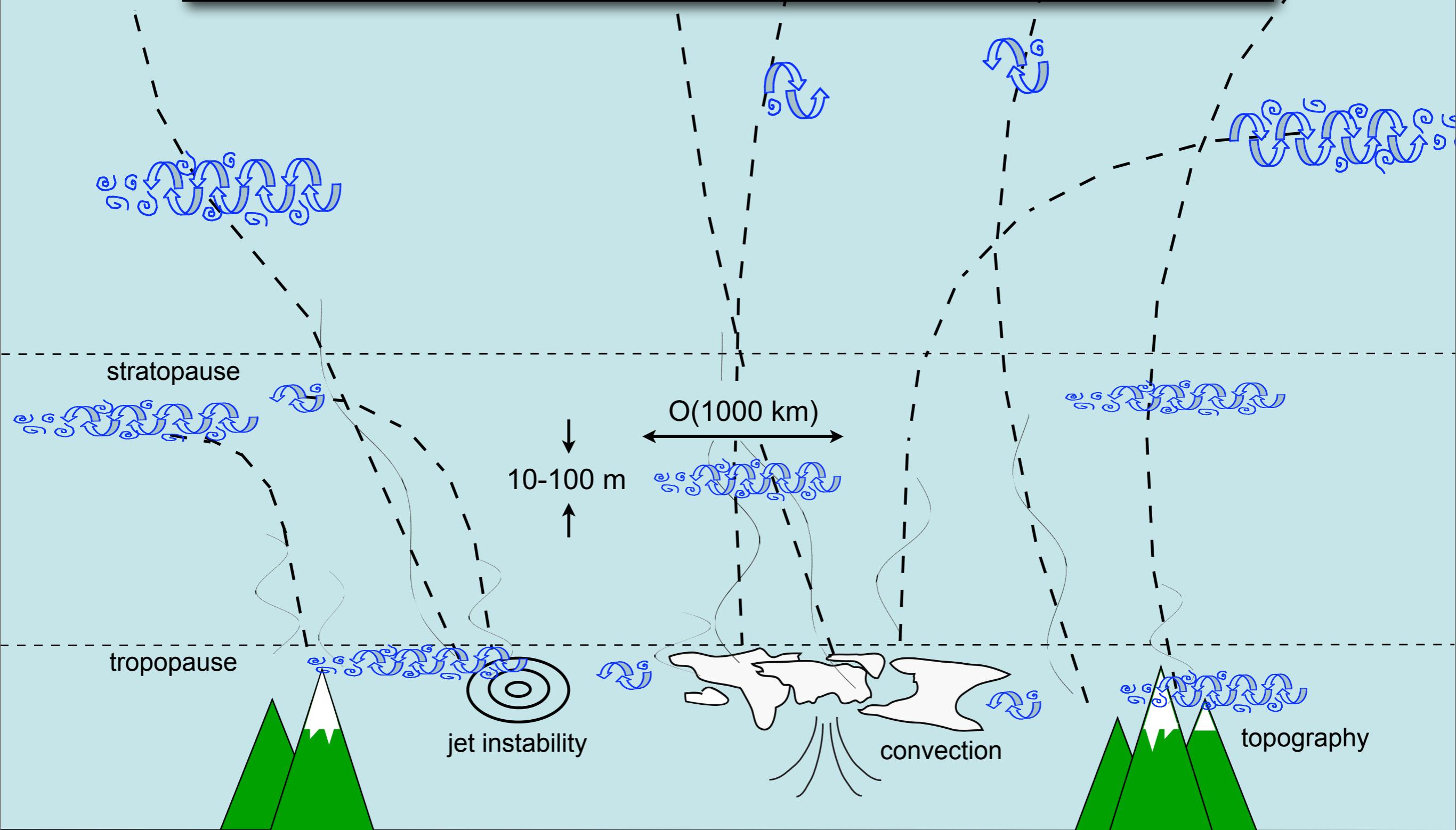
summer

winter

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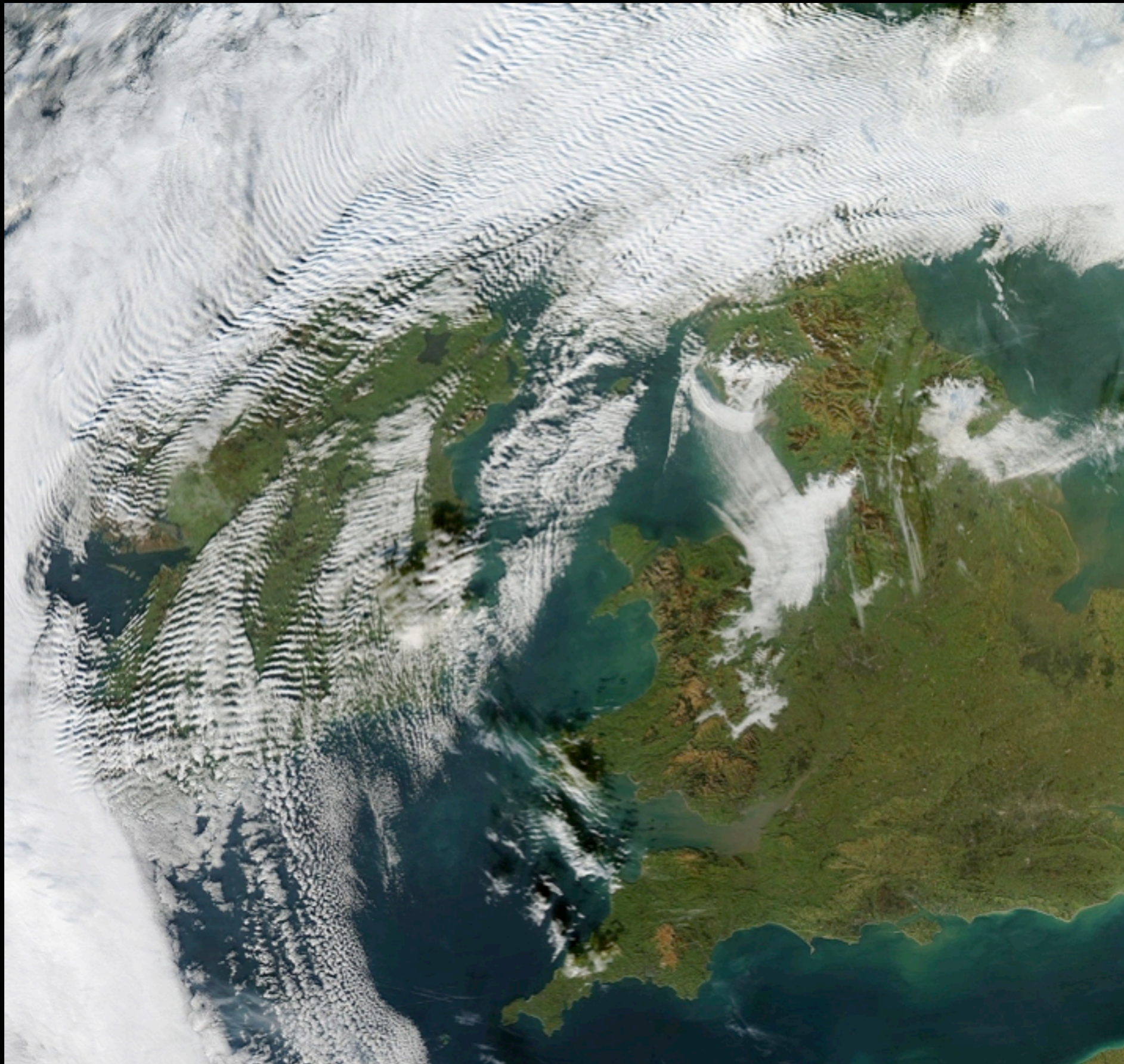


ADA tasks: 1. Predict likelihood and nature of turbulence outbreaks
2. Estimate impact on resolved-scale dynamics

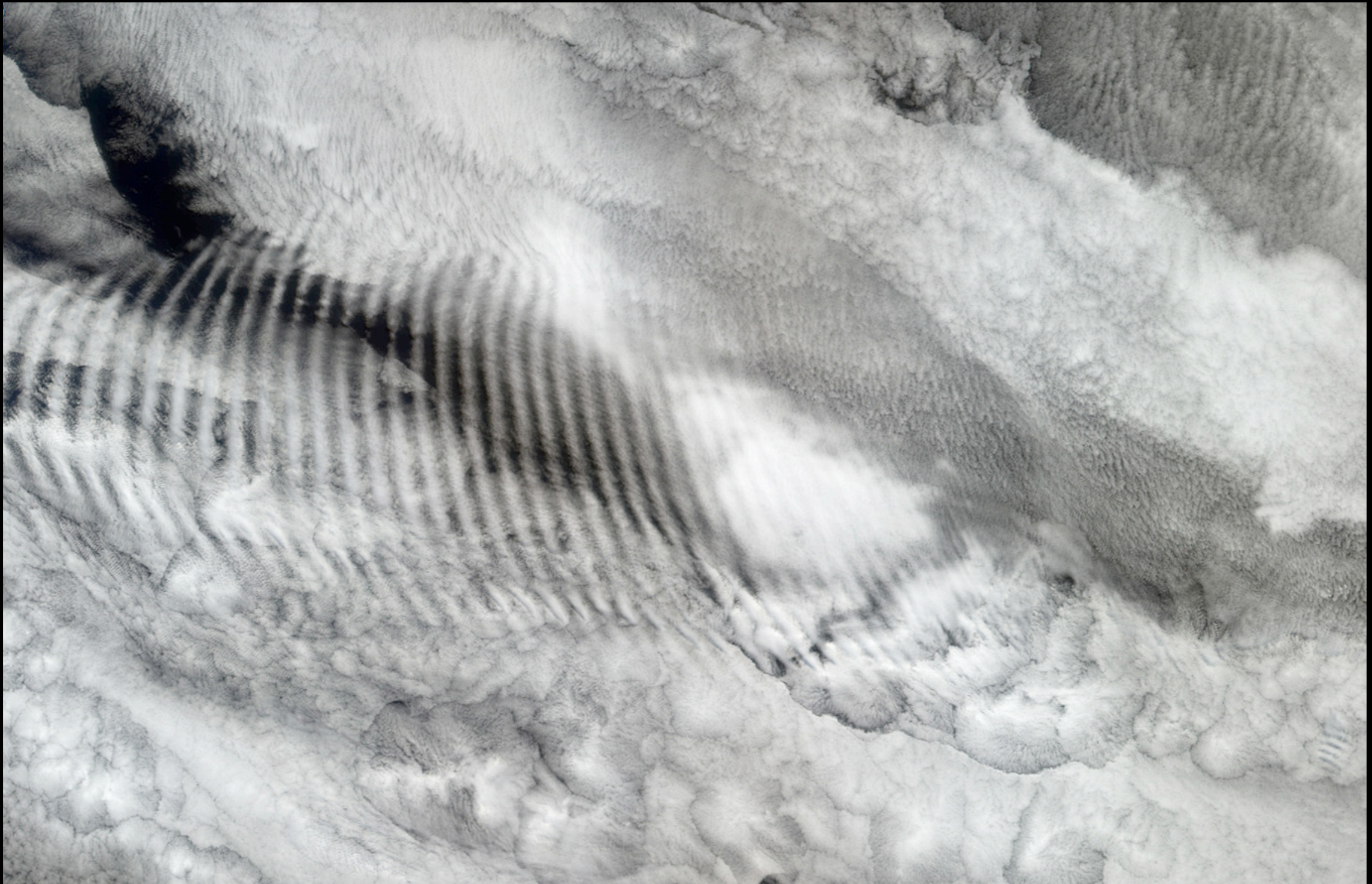


summer

winter



Wave clouds over Ireland & Scotland, 2003 (Aqua MODIS, NASA/GSFC)



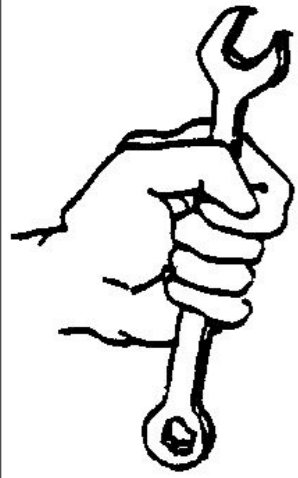
Convection induced waves over the Indian Ocean, 2003 (MISR, NASA/GSFC/LaRC/JPL)



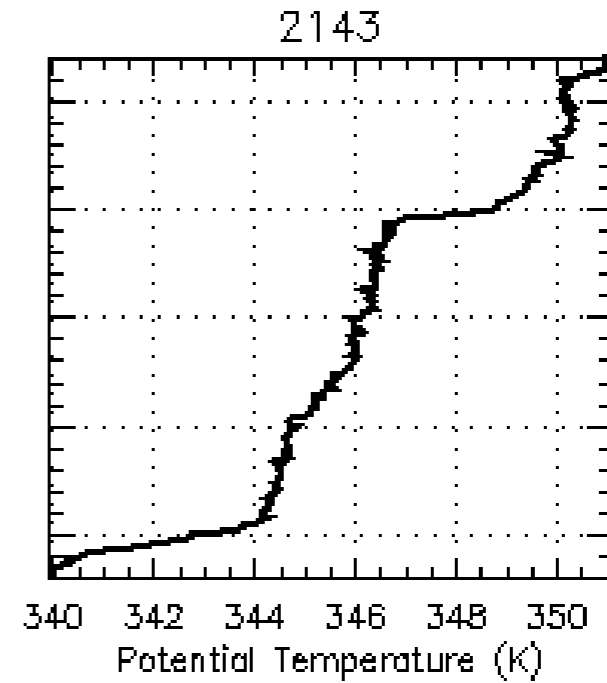
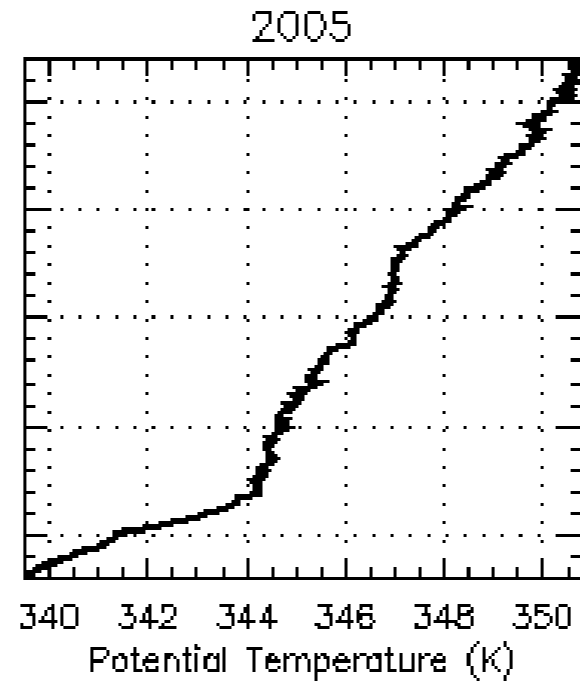
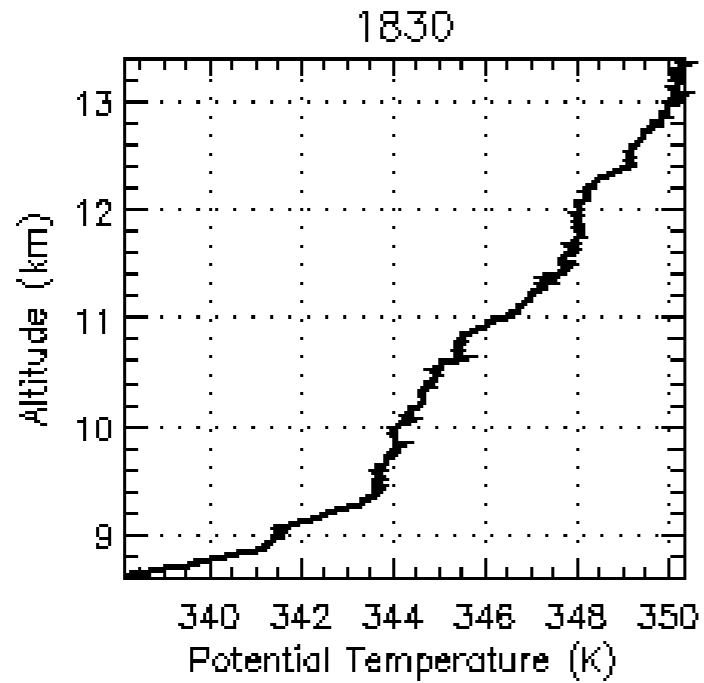
Noctilucent Clouds, Kustavi, Finland, 1989 (photo by Pekka Parviainen)

3km Deep Mixing Layer Triggered by a Gravity Wave

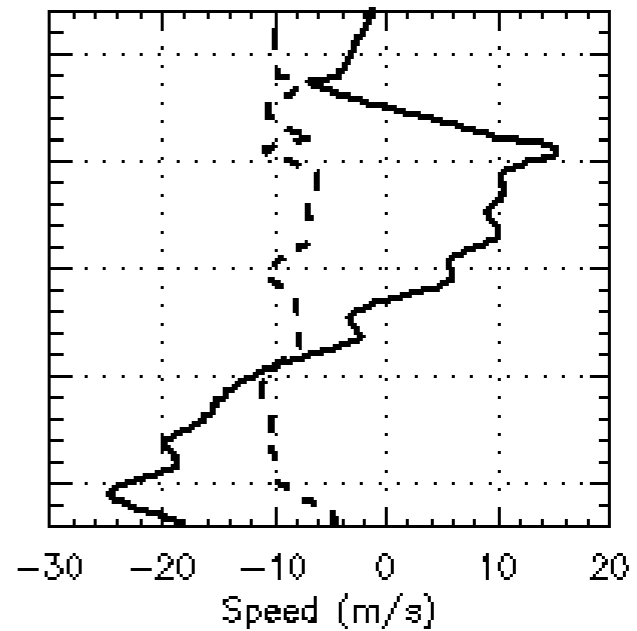
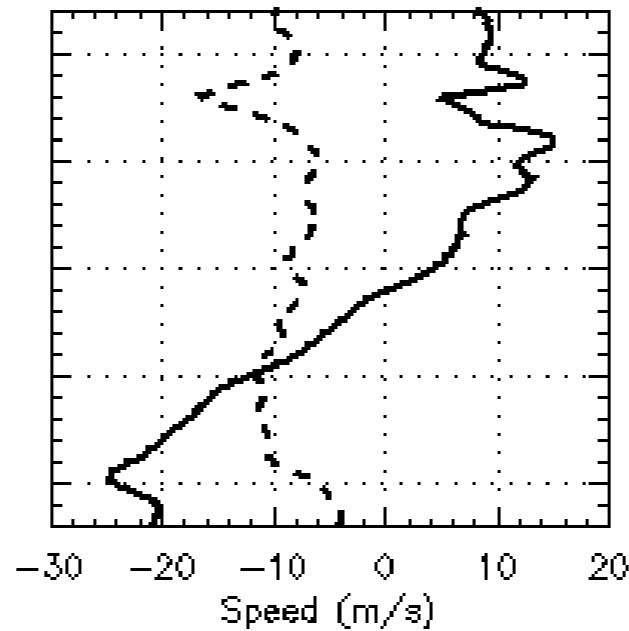
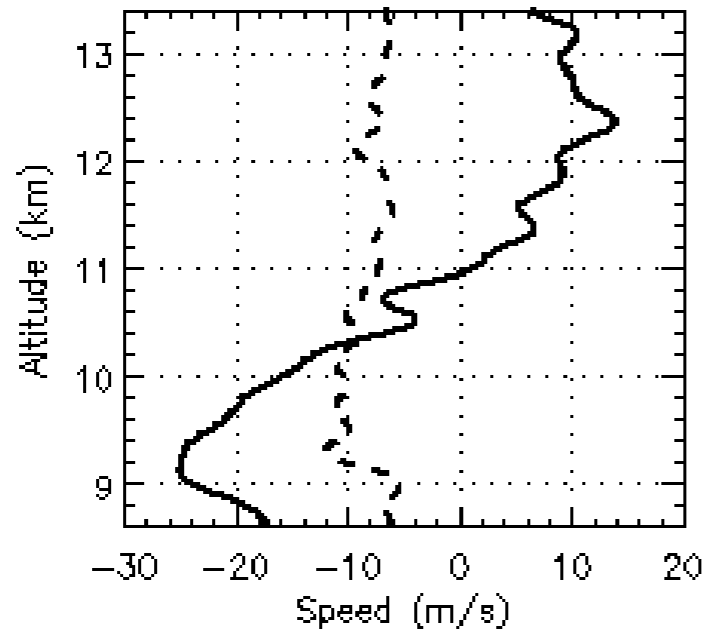
Kelley, Chen, Beland, Woodman, Chau & Werne, GRL (2005)



T

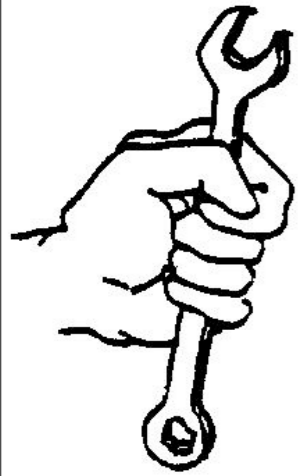


U

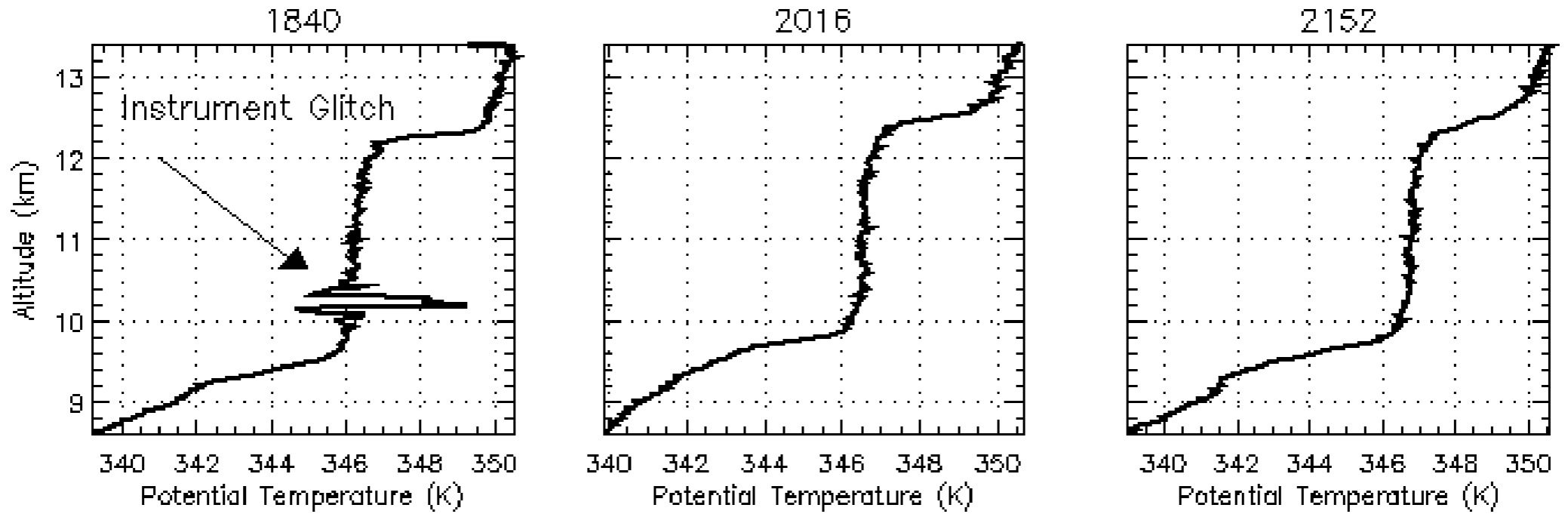


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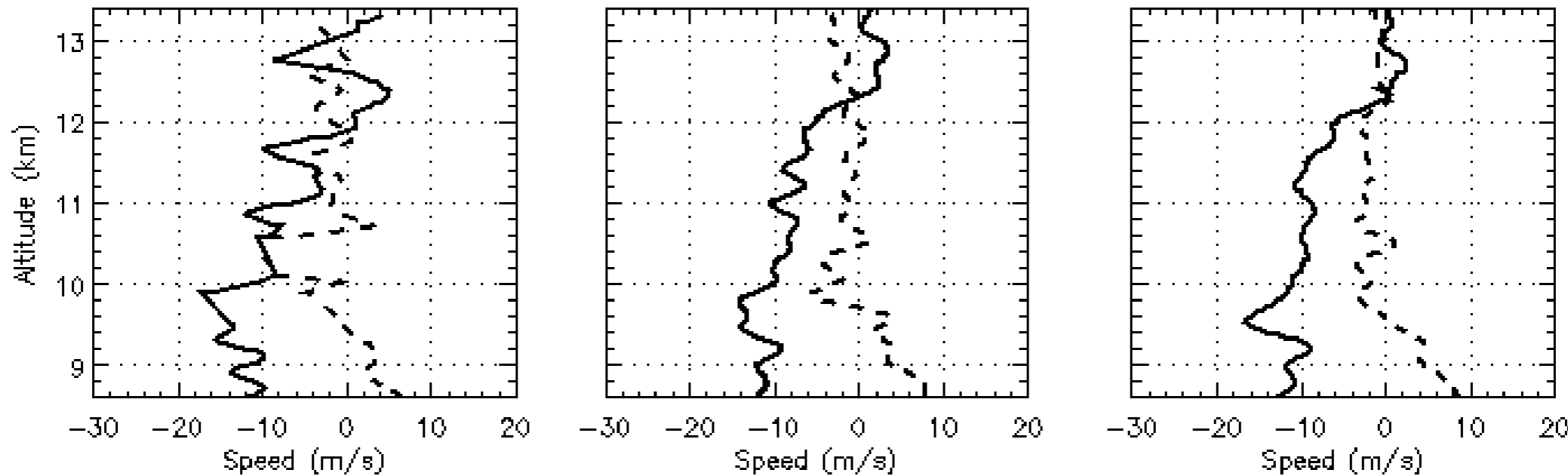
Kelley, Chen, Beland, Woodman, Chau & Werne, GRL (2005)



T



U



Unresolved Gravity Waves: Are They Important?

Examples mentioned previously all involve gravity waves:

- DC-8 cargo plane with missing engine and 12' of wing.
- U2 incidents involving aborted missions, loss of aircraft and death of a pilot.
- Woman killed on United flight over the Pacific Ocean when the aircraft dropped 1000 feet.

Gravity waves also play important roles in atmospheric dynamics:

- The mesopause (~90km altitude) is colder in the summer than in the winter as a result of meridional circulations forced by overturning gravity waves.
- Gravity waves also play an important role in the quasi-biennial oscillation (QBO), a quasi-periodic oscillation of the zonal wind with a period that varies from 22 to 34 months.



The New York Times

December 29, 1997

Jet Hits Turbulence; 110 Hurt and a Woman Dies

A United Airlines jumbo jetliner with 393 people aboard hit severe air turbulence over the Pacific Ocean on Sunday night, killing one Japanese woman and injuring 110 other passengers.

Passengers and serving carts were flung to the ceiling as the plane dived 1,000 feet when it flew into the turbulence at 33,000 feet, officials said.

The plane, flight 826 bound for Honolulu

What is an internal gravity wave?

equations of motions for a compressible, rotating atmosphere

$$\frac{du}{dt} - fv + \frac{1}{\rho} \frac{\partial p}{\partial x} = X, \quad (1)$$

$$\frac{dv}{dt} + fu + \frac{1}{\rho} \frac{\partial p}{\partial y} = Y, \quad (2)$$

$$\frac{dw}{dt} + \frac{1}{\rho} \frac{\partial p}{\partial z} + g = 0, \quad (3)$$

$$\frac{1}{\rho} \frac{d\rho}{dt} + \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0, \quad (4)$$

$$\frac{d\theta}{dt} = Q, \quad (5)$$

mean state: $\bar{\rho} = \rho_0 e^{-(z-z_0)/H} \quad \bar{u}(z) \quad \bar{v}(z)$

What is an internal gravity wave?

solving linear perturbation equations in the WKB approximation gives:

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$$\left(u', v', w', \frac{\theta'}{\bar{\theta}}, \frac{p'}{\bar{p}}, \frac{\rho'}{\bar{\rho}} \right) = (\tilde{u}, \tilde{v}, \tilde{w}, \tilde{\theta}, \tilde{p}, \tilde{\rho}) \exp [i(kx + ly + mz - \omega t) + z/(2H)]$$

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wave amplitude
grows with z

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1 Gravity Waves

1.1 Dispersion Relation

$$\hat{\omega}^2 = \frac{N^2(k^2 + \ell^2) + f^2(m^2 + \frac{1}{4H^2})}{k^2 + \ell^2 + m^2 + \frac{1}{4H^2}} \quad \text{or} \quad m^2 = \frac{(k^2 + \ell^2)(N^2 - \hat{\omega}^2)}{\hat{\omega}^2 - f^2} - \frac{1}{4H^2} \quad (1)$$

1.2 Polarization Relations

$$\frac{\tilde{p}}{\hat{\omega}^2 - f^2} = \frac{\tilde{u}}{\hat{\omega}k + if\ell} = \frac{\tilde{v}}{\hat{\omega}\ell - ifk} = \frac{\tilde{w}(m + \frac{i}{2H})}{\hat{\omega}(k^2 + \ell^2)} = \frac{\tilde{\theta} gi (m + \frac{i}{2H})}{N^2(k^2 + \ell^2)} \quad (2)$$

where $\hat{\omega} = \omega - k\bar{u} - l\bar{v}$ is the wave's intrinsic frequency.

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- note:**
1. evanescence (internal reflection) when $m^2 < 0$
 2. critical levels (wave/mean-flow interaction)

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solving linear perturbation equations in the WKB approximation gives:

group velocity:

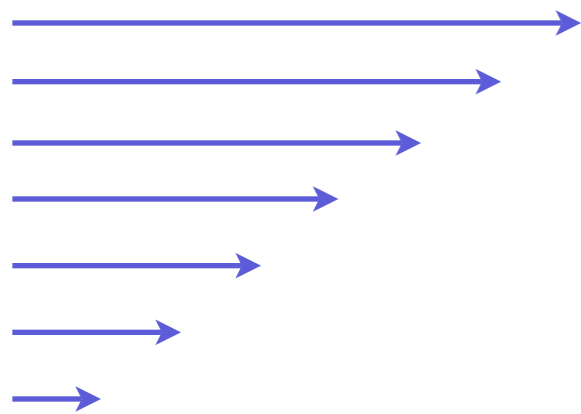
$$\begin{aligned}(c_{gx}, c_{gy}, c_{gz}) &= \left(\frac{\partial \omega}{\partial k}, \frac{\partial \omega}{\partial l}, \frac{\partial \omega}{\partial m} \right) \\ &= (\bar{u}, \bar{v}, 0) + \frac{[k(N^2 - \hat{\omega}^2), l(N^2 - \hat{\omega}^2), -m(\hat{\omega}^2 - f^2)]}{\hat{\omega} (k^2 + l^2 + m^2 + \frac{1}{4H^2})}\end{aligned}$$

phase velocity:

$$(c_x, c_y, c_z) = \frac{\omega}{k^2 + l^2 + m^2} (k, l, m)$$

Wave / mean-flow interaction

wave amplitude grows with z , eventually leading to nonlinear effects and turbulence, depositing momentum into the mean flow, decelerating the upper level winds.



$c_g, m, \hat{\omega} \uparrow$

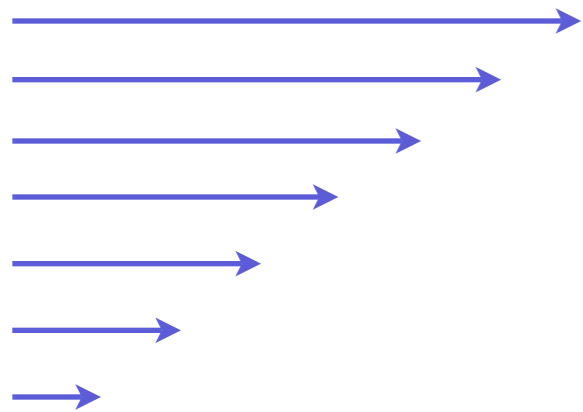
critical-level absorption

$c_g, m, \hat{\omega} \downarrow$

wave amplitude grows with $\hat{\omega}^{-1}$ near critical level and breaking occurs, with wave depositing momentum into background, accelerating the low-level winds.

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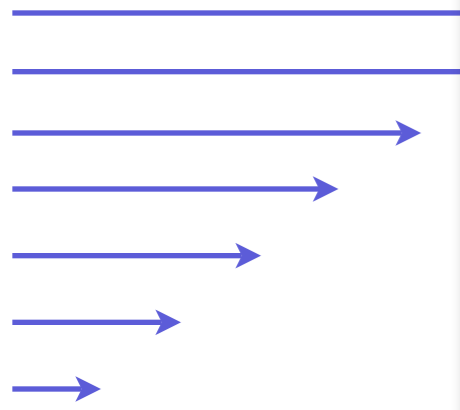
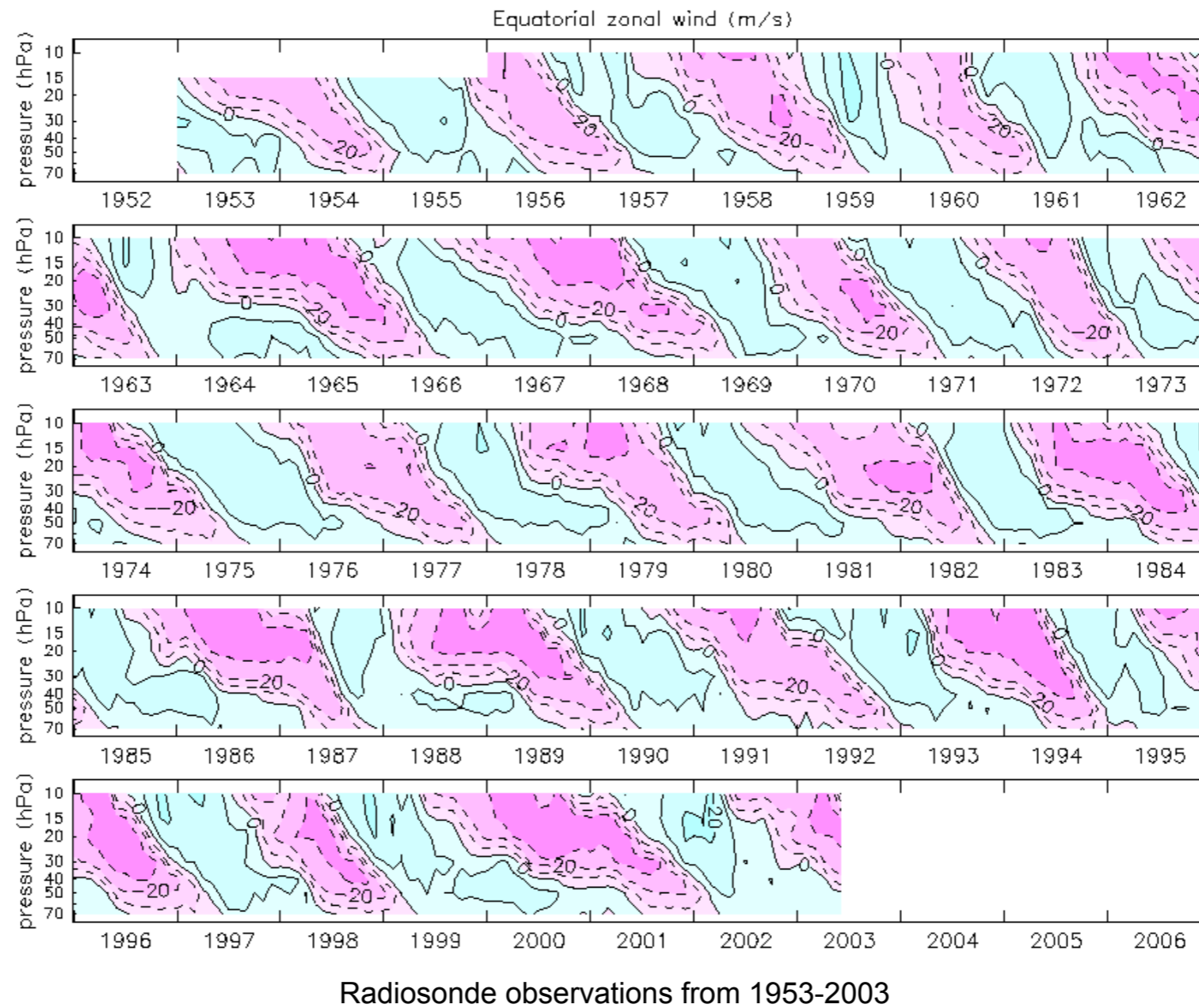
$c_g, m, \hat{\omega} \downarrow$

wave amplitude grows with $\hat{\omega}^{-1}$ near critical level and breaking occurs, with wave depositing momentum into background, accelerating the low-level winds.

The net effect is to act to reverse the mean shear.

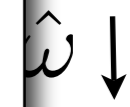
Wave / mean-flow interaction

This process operates in the QBO - an oscillation of the mean zonal wind in the equatorial lower stratosphere with a period from 22 to 34 months.



The net effect is to reverse the direction of the mean zonal wind.

absorption



with $\hat{\omega}^{-1}$ making the wind, winds.

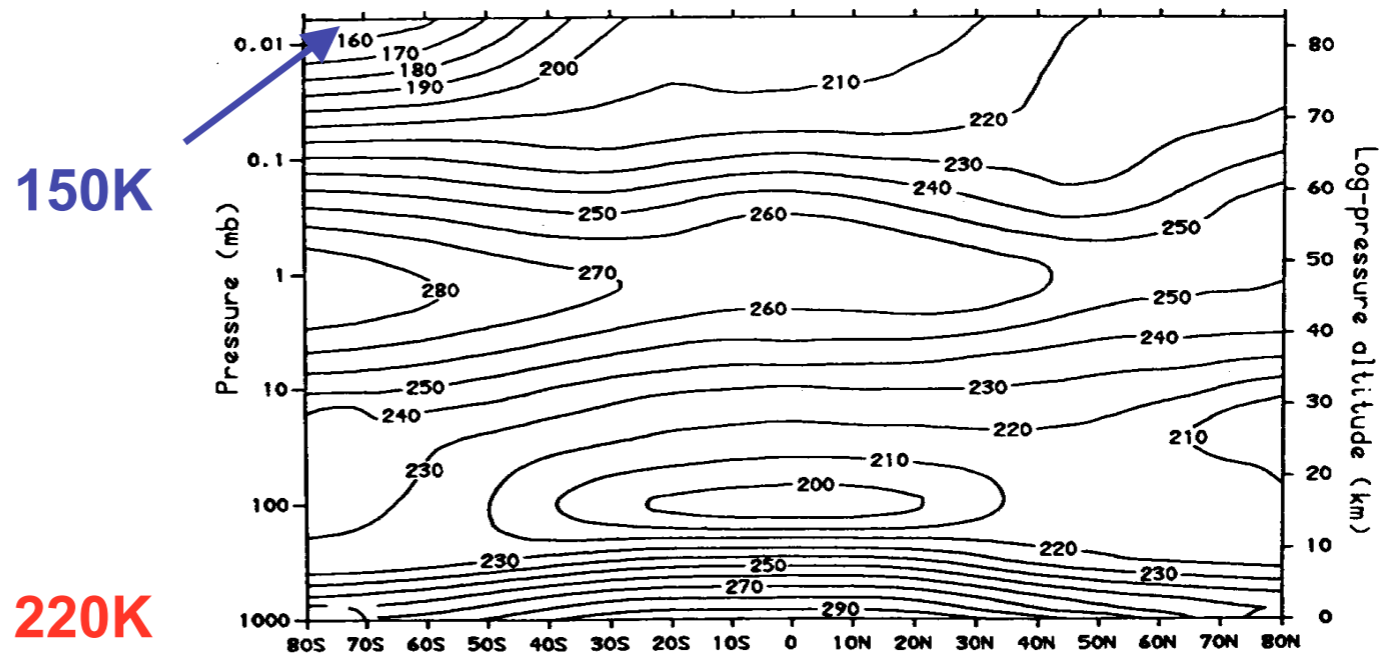
Wave / mean-flow interaction

Wave / mean-flow interactions are also important in the mesopause region (~90km).

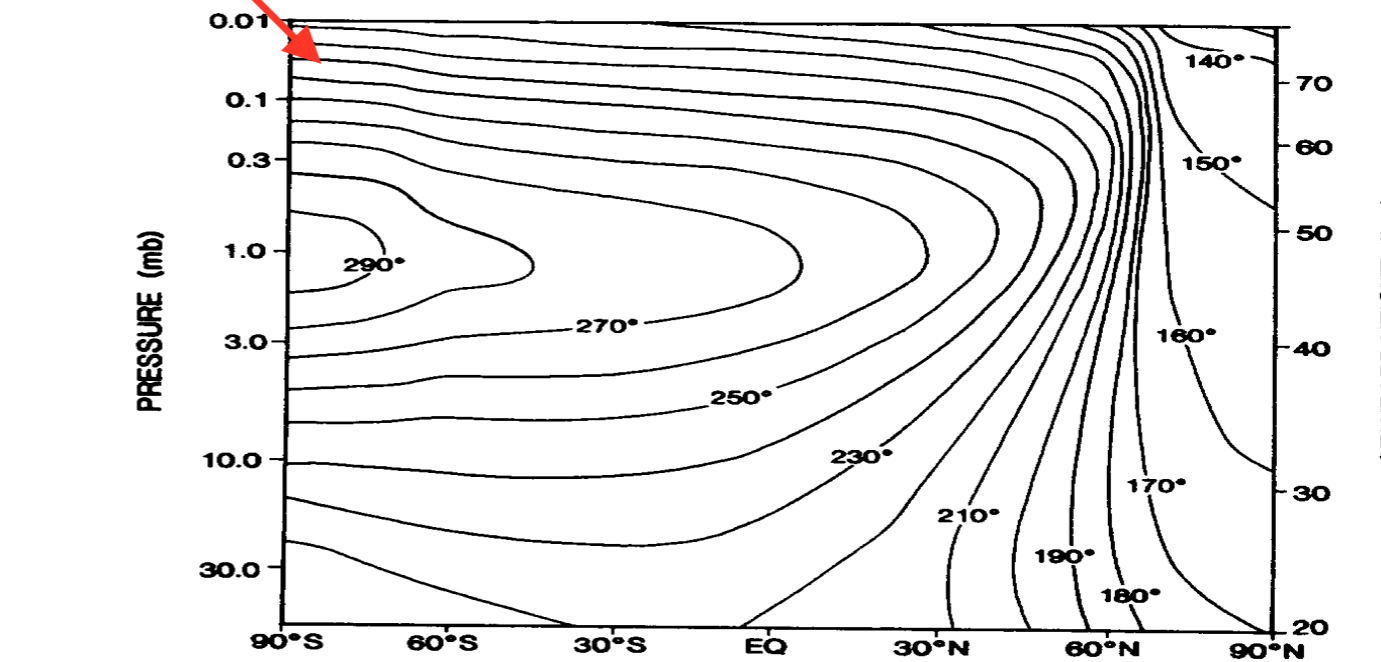
The summer mesopause is 70K colder than the winter mesopause, and also 70K colder than radiative equilibrium temperatures.

Adiabatic cooling and residual circulations driven by gravity waves are responsible.

Observed Temperature (January)



"Radiative" Temperature (January)



from Fels (1987)

Modeling unresolved gravity-wave effects

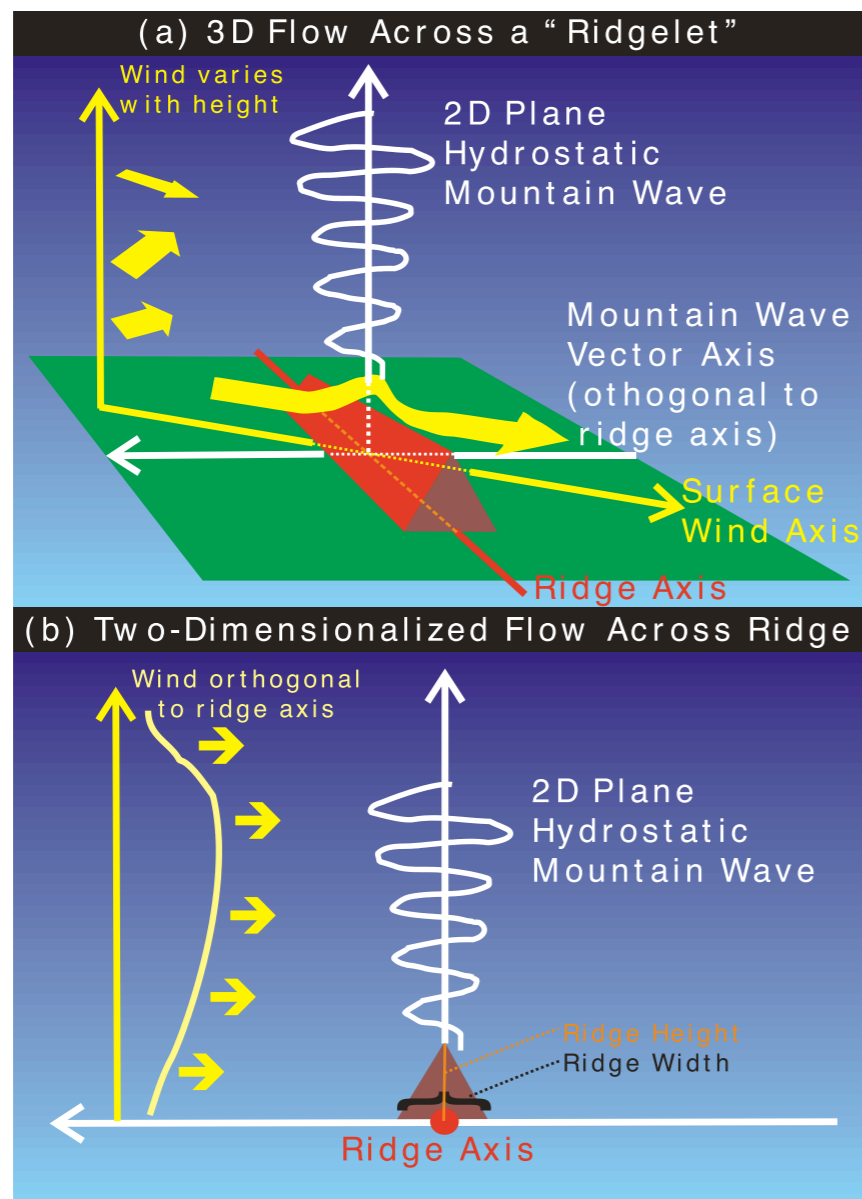
Waves contribute organized, coherent motions that can have important dynamical implications.

During field experiments, meso-scale simulations of stratified dynamics often hint at mountain wave initiation, but subsequent evolution is severely damped.

In practice, an astute operator will forecast wave dynamics and turbulence to caution aircraft making field measurements, but not because WRF (MM5) predicted it, but rather because waves were briefly glimpsed before unphysically damped by the forecast model.

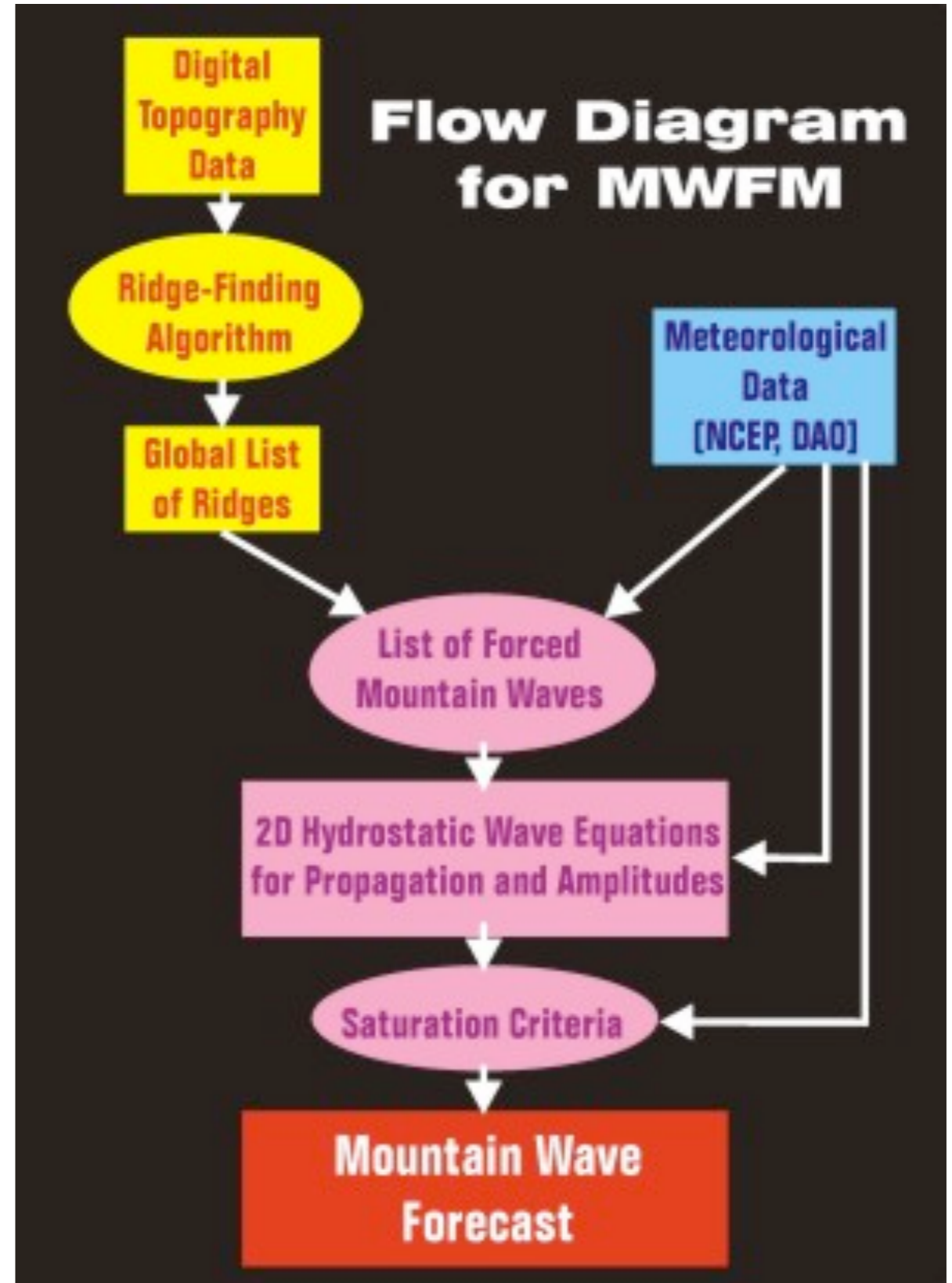
How to include waves (or, how to put them back) ...

Linear, 2D MWFM-2 response

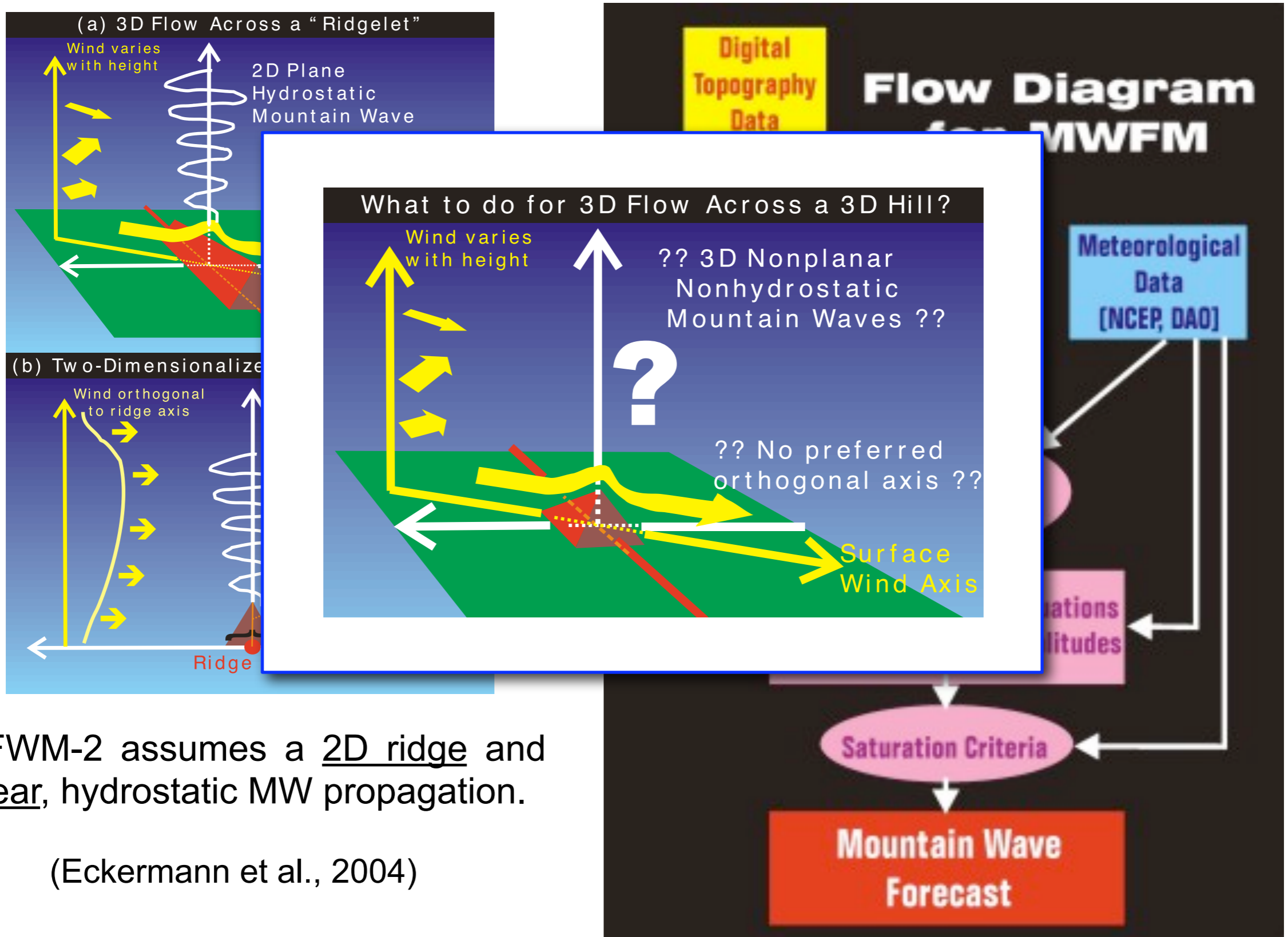


MFWM-2 assumes a 2D ridge and linear, hydrostatic MW propagation.

(Eckermann et al., 2004)



Linear, 2D MWFM-2 response



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(Eckermann et al., 2004)

Linear, 2D MWFM-2 response

(Linear waves computed by ray tracing)

$$\frac{dx_i}{dt} = V_i + \frac{\partial \omega_{Ir}}{\partial k_i} = V_i + c_{g_i}$$
$$\frac{dk_i}{dt} = -k_j \frac{\partial V_j}{\partial x_i} - \frac{\partial \omega_{Ir}}{\partial x_i},$$

$$m^2 = \frac{k_H^2 N^2}{\omega_{Ir}} - k_H^2 - \frac{1}{4H^2}$$

$$\vec{k} = (k, l, m) \quad k_H^2 = k^2 + l^2 \quad \omega_{Ir} = \omega - kU - lV$$

Convective plume parameterization:

Vertical Body Force Model

$$\frac{\partial u}{\partial t} + \frac{1}{\bar{\rho}} \frac{\partial p'}{\partial x} - f v = F_x(\mathbf{x}) \mathcal{F}(t),$$

$$\frac{\partial v}{\partial t} + \frac{1}{\bar{\rho}} \frac{\partial p'}{\partial y} + f u = F_y(\mathbf{x}) \mathcal{F}(t),$$

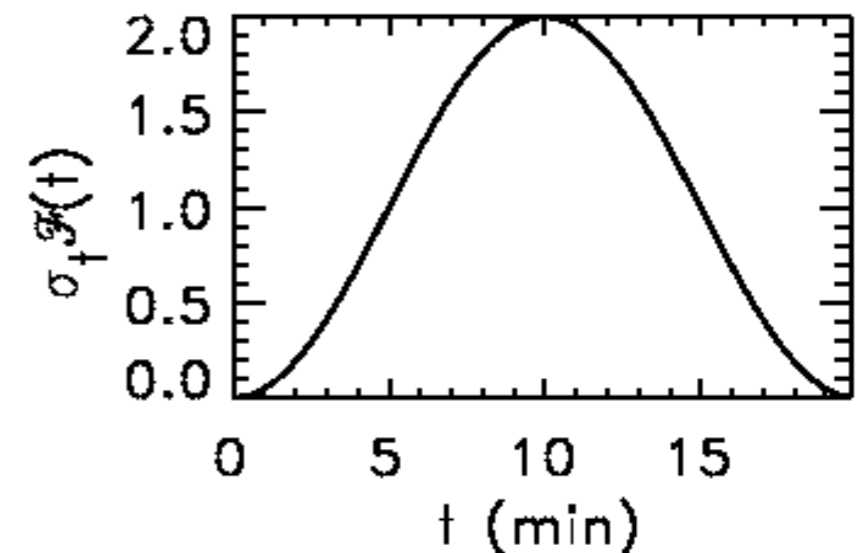
$$\frac{\partial w}{\partial t} + \frac{1}{\bar{\rho}} \frac{\partial p'}{\partial z} - \frac{g}{\bar{\theta}} \theta' = F_z(\mathbf{x}) \mathcal{F}(t),$$

$$\frac{\partial \theta'}{\partial t} + \frac{\bar{\theta} N^2}{g} w = \frac{\bar{\theta}}{g} J(\mathbf{x}) \mathcal{F}(t),$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0,$$

- Exact solutions to the linearized f-plane fluid, non-dissipative equations using Laplace transforms

GW solutions are in Fourier space at time t



Vertical Body Force:

$$F_z(\mathbf{x}) = w_0 \exp \left(- \left[\frac{(x - x_0)^2}{2\sigma_x^2} + \frac{(y - y_0)^2}{2\sigma_y^2} + \frac{(z - z_0)^2}{2\sigma_z^2} \right] \right)$$

(Vadas and Fritts, 2001)

$$u(x, y, z, t) = \frac{1}{(2\pi)^3} \int \int \int e^{-ikx-ily-imz} \tilde{u}(k, l, m, t) dk dl dm$$

The spectral solution after the forcings/heatings finish is

$$\begin{aligned} \tilde{u}_F(t) &= \frac{-ilN^2\tilde{F}_\zeta - mlf\tilde{J}}{\mathbf{k}^2\omega^2} - \frac{Dm\hat{a}^2}{k_H^2} \left\{ \left(k\omega A_F - \frac{lfB_F}{\omega} \right) \mathcal{S} - (lfA_F + kB_F)\mathcal{C} \right\}, \\ \tilde{v}_F(t) &= \frac{ikN^2\tilde{F}_\zeta + mkf\tilde{J}}{\mathbf{k}^2\omega^2} - \frac{Dm\hat{a}^2}{k_H^2} \left\{ (kfA_F - lB_F)\mathcal{C} + \left(\frac{kfB_F}{\omega} + l\omega A_F \right) \mathcal{S} \right\}, \\ \tilde{w}_F(t) &= \left\{ D \hat{a}^2 (A_F\omega\mathcal{S} - B_F\mathcal{C}) \right\}, \\ \tilde{\Theta}_F(t) &= \frac{mf(iN^2\tilde{F}_\zeta + mf\tilde{J})}{\mathbf{k}^2\omega^2} + DN^2\hat{a}^2 \left\{ A_F\mathcal{C} + \frac{B_F\mathcal{S}}{\omega} \right\}, \\ \tilde{P}_F(t) &= \frac{if(iN^2\tilde{F}_\zeta + mf\tilde{J})}{\mathbf{k}^2\omega^2} + \frac{iD\hat{a}^2(N^2 - \omega^2)}{m} \left\{ A_F\mathcal{C} + \frac{B_F\mathcal{S}}{\omega} \right\}, \end{aligned}$$

Now take Laplace transform of equations, solve linear equations, then take inverse Laplace transform. Solutions are a function of k,l,m,t.

where

$$D = \frac{1}{\sigma_t\omega^2(\hat{a}^2 - \omega^2)}$$

$$P = p'/\bar{p},$$

$$\Theta = g\theta'/\bar{\Theta},$$

$$\mathcal{S} = \sin \omega t + \sin \omega(\sigma_t - t),$$

$$\mathcal{C} = \cos \omega t - \cos \omega(\sigma_t - t),$$

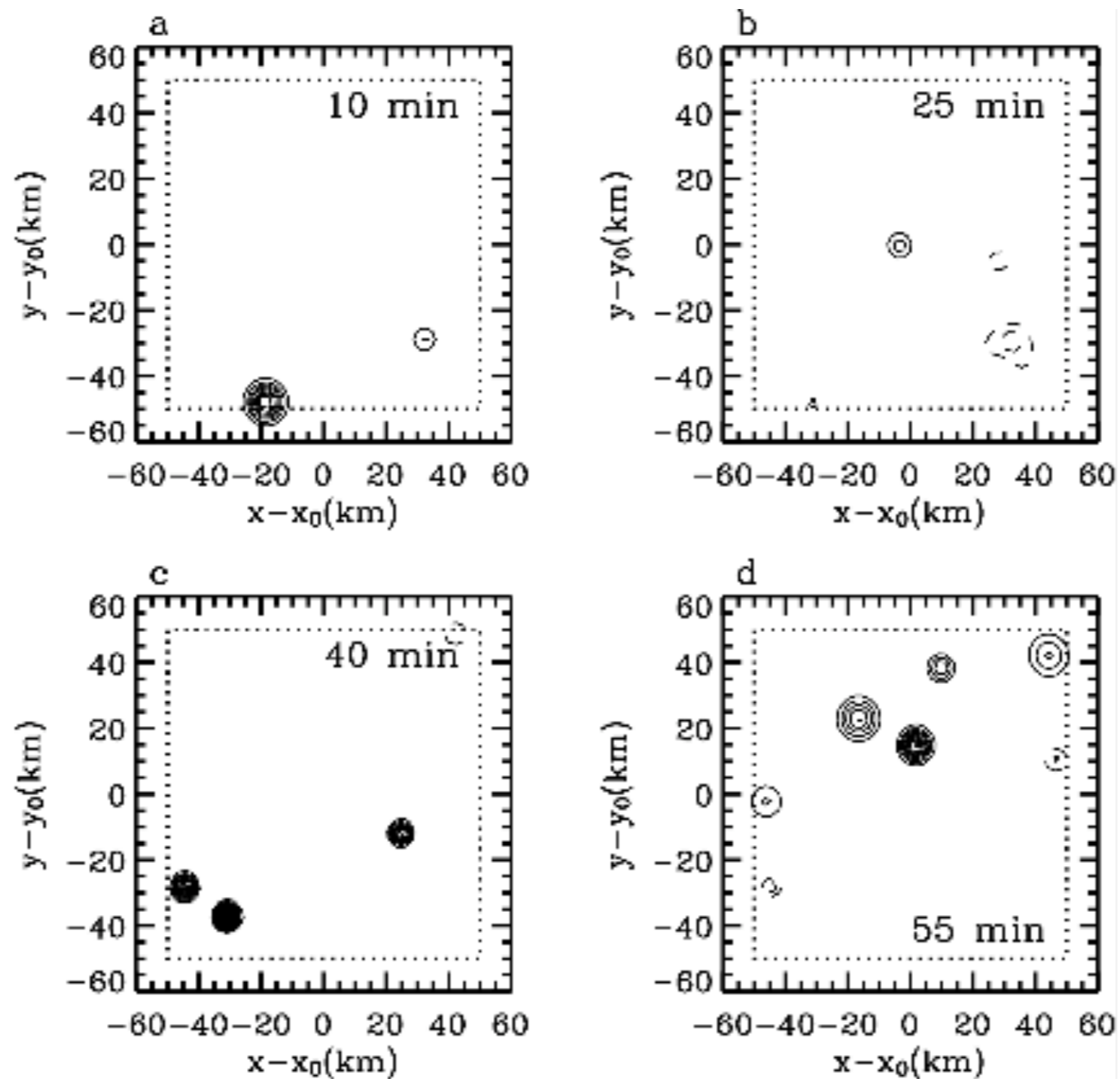
$$F_\zeta = \partial F_y/\partial x - \partial F_x/\partial y,$$

$$F_\delta = \partial F_x/\partial x + \partial F_y/\partial y + \partial F_z/\partial z,$$

$$A_F = \bar{F}_z - \frac{im\bar{F}_\delta}{\mathbf{k}^2},$$

$$B_F = \frac{k_H^2\bar{J}}{\mathbf{k}^2} - \frac{imf\bar{F}_\zeta}{\mathbf{k}^2}$$

Modeled GWs from mesoscale convective complexes (MCCs) modeled as vertical body forces

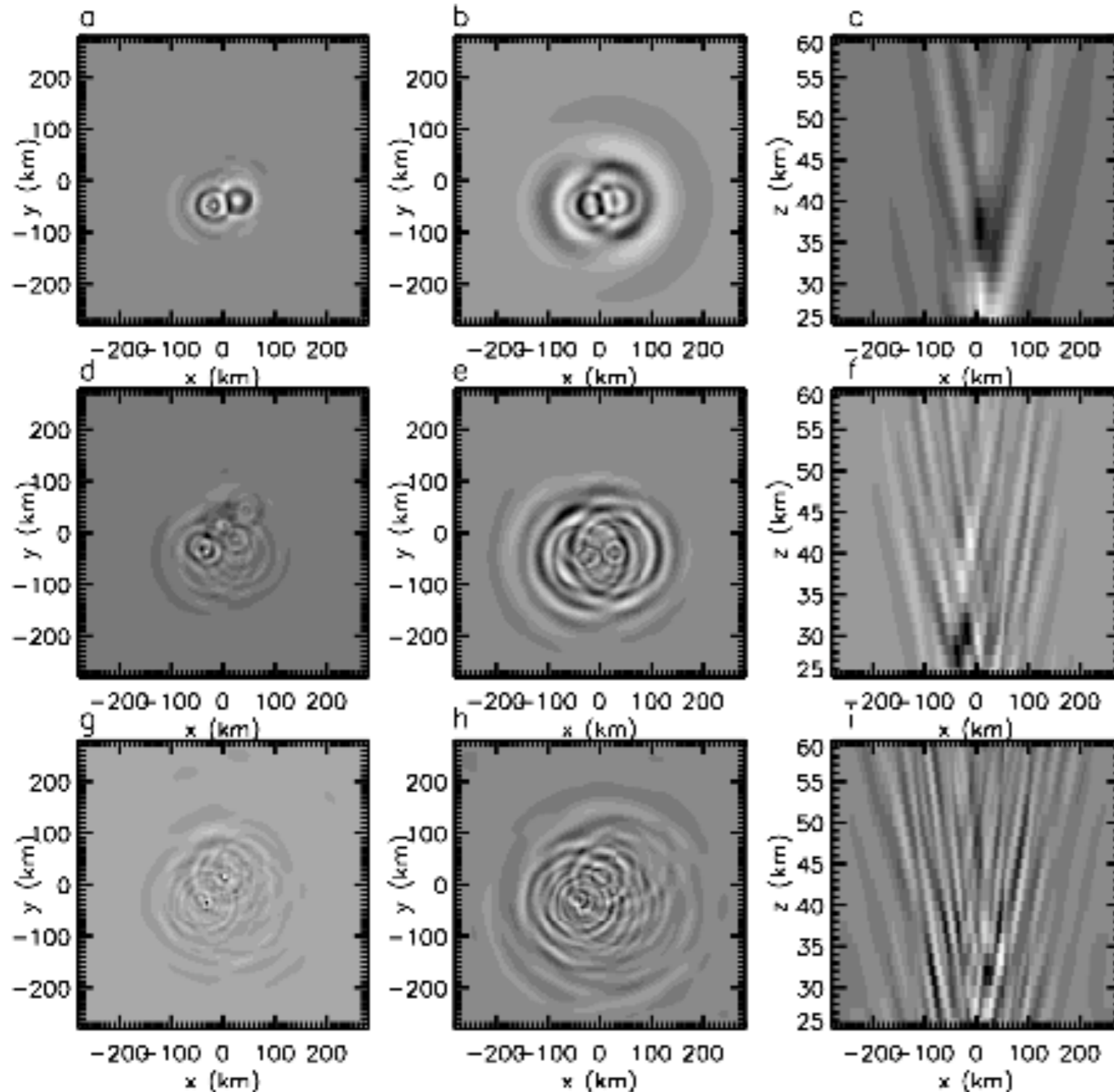


Response from modeled body forces:

Vertical velocities of GWs 30, 60, and 90 min after convective initiation

z=25 km

Z=50 km



- Deep, tropical convective systems efficiently excite GWs
- GWs radiate as concentric rings upwards and away from convective cells in 3D nonlinear numerical models (Piani et al, 2000; Lane et al, 2001; Horinouchi et al, 2002)

X (km)

X (km)

X (km)

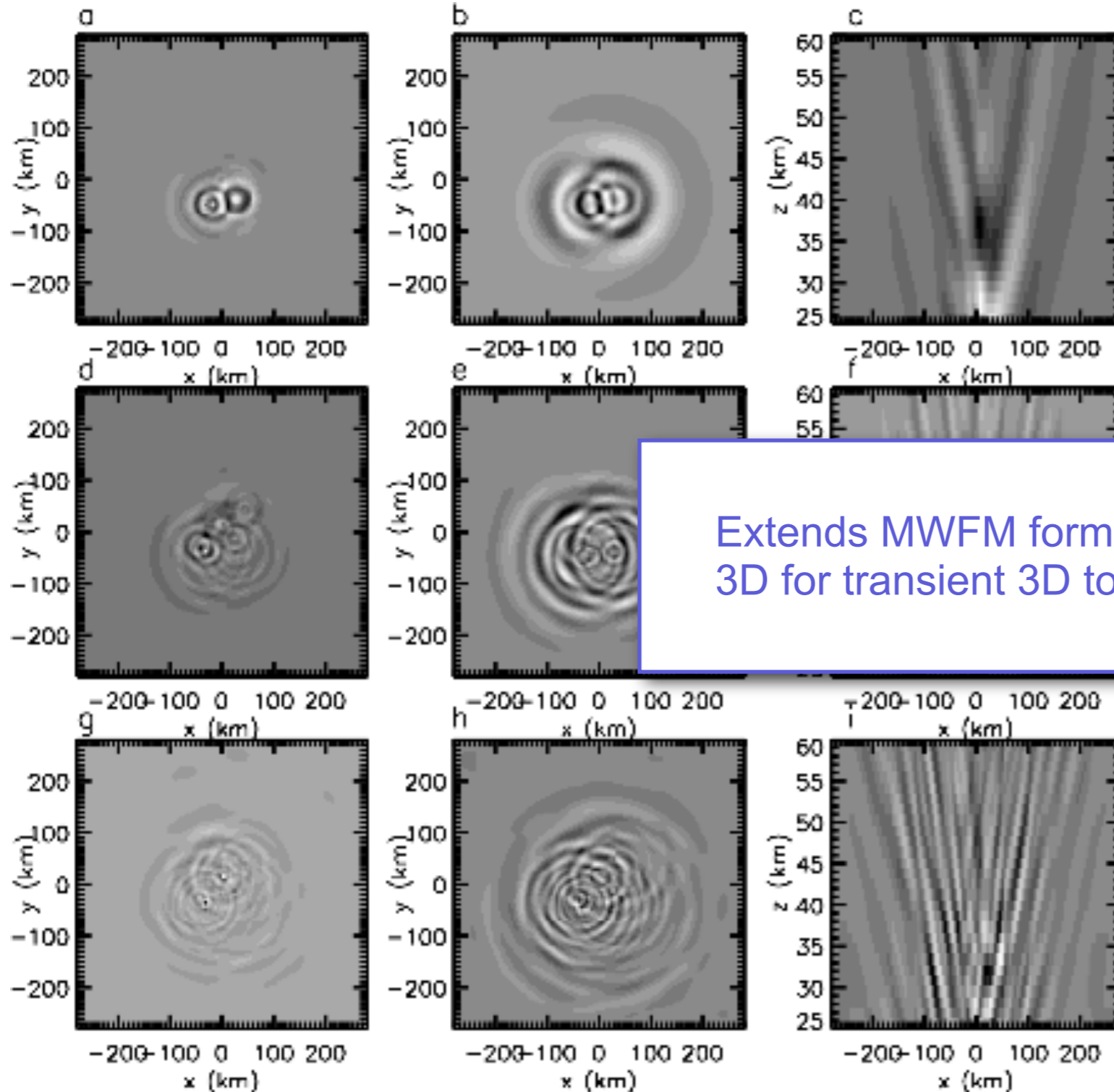
(Vadas and Fritts, 2004)

Response from modeled body forces:

Vertical velocities of GWs 30, 60, and 90 min after convective initiation

z=25 km

Z=50 km



- Deep, tropical convective systems efficiently excite GWs
- GWs radiate as concentric rings upwards and away from convective cells in 3D nonlinear numerical models (Piani et al, 2000; Lane et al, 2001; Horinouchi et al, 2002)

Extends MWFM formalism to 3D for transient 3D topography

X (km)

X (km)

X (km)

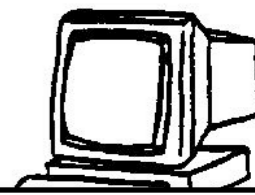
(Vadas and Fritts, 2004)

Bayesian SGS Modeling

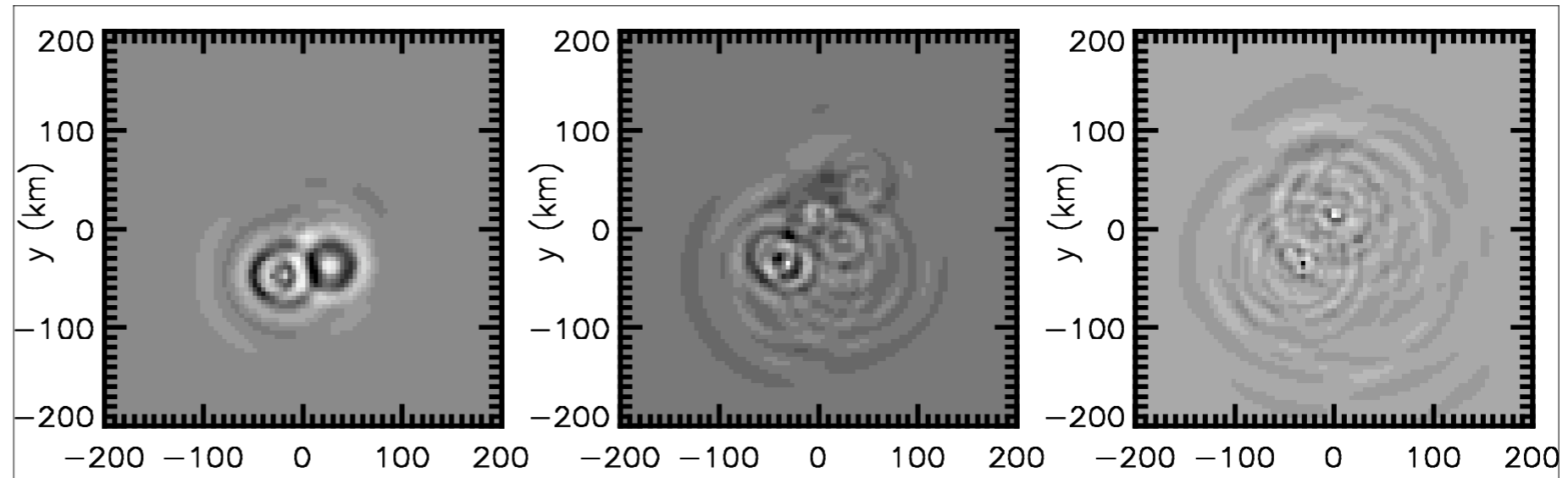
$$\int_{\Delta}^{\infty} [A | F, Y] [Y | F] [F] dY$$

Estimate with ensemble runs
or knowledge of F uncertainty

resolved by NWP or secondary model

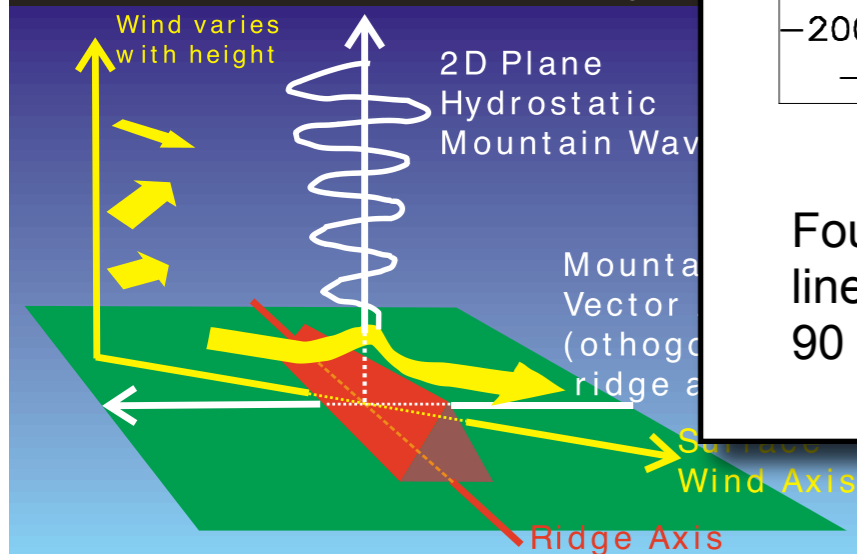


Fourier-Laplace GW propagation

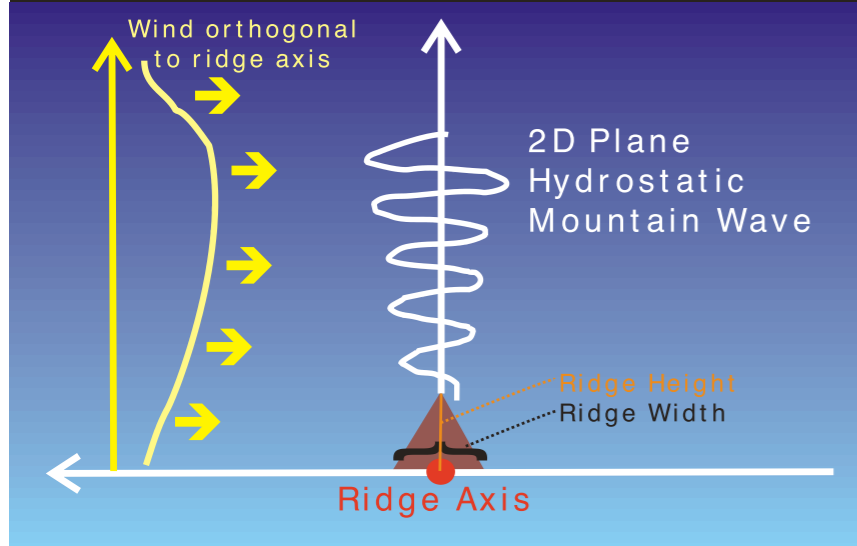


Fourier-Laplace method permits fast, accurate 3D linear wave propagation for transient linear sources. Vertical velocity at 22 km above specified convection cells 30, 60, and 90 minutes after excitation. Vadas & Fritts (2004).

(a) 3D Flow Across a "Ridgele



(b) Two-Dimensionalized Flow Across Ridge



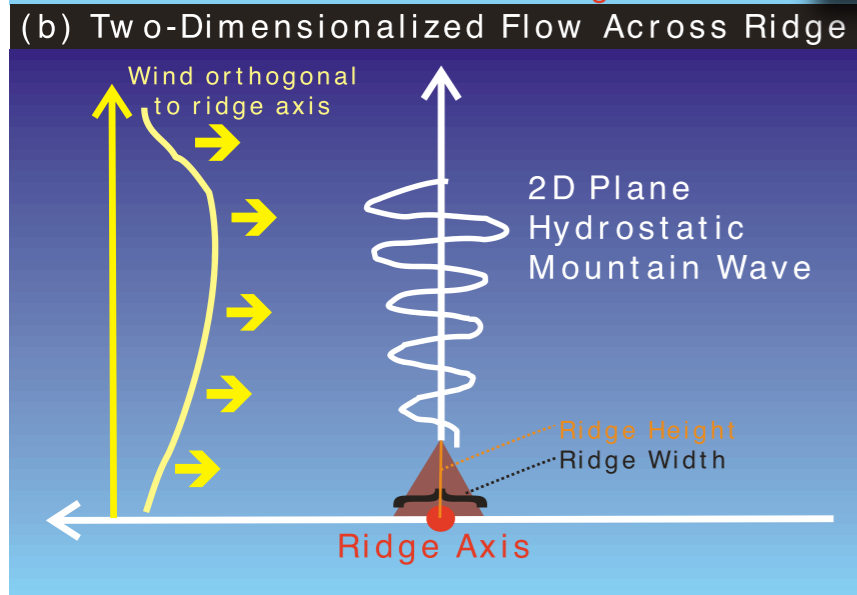
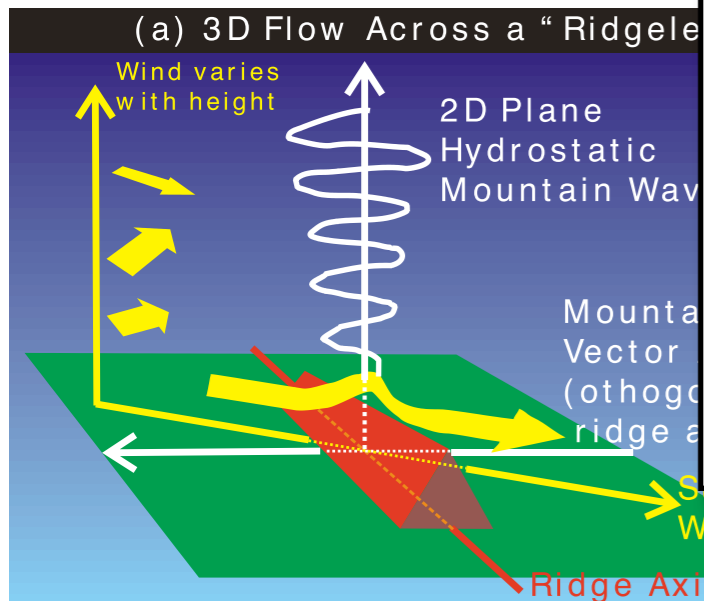
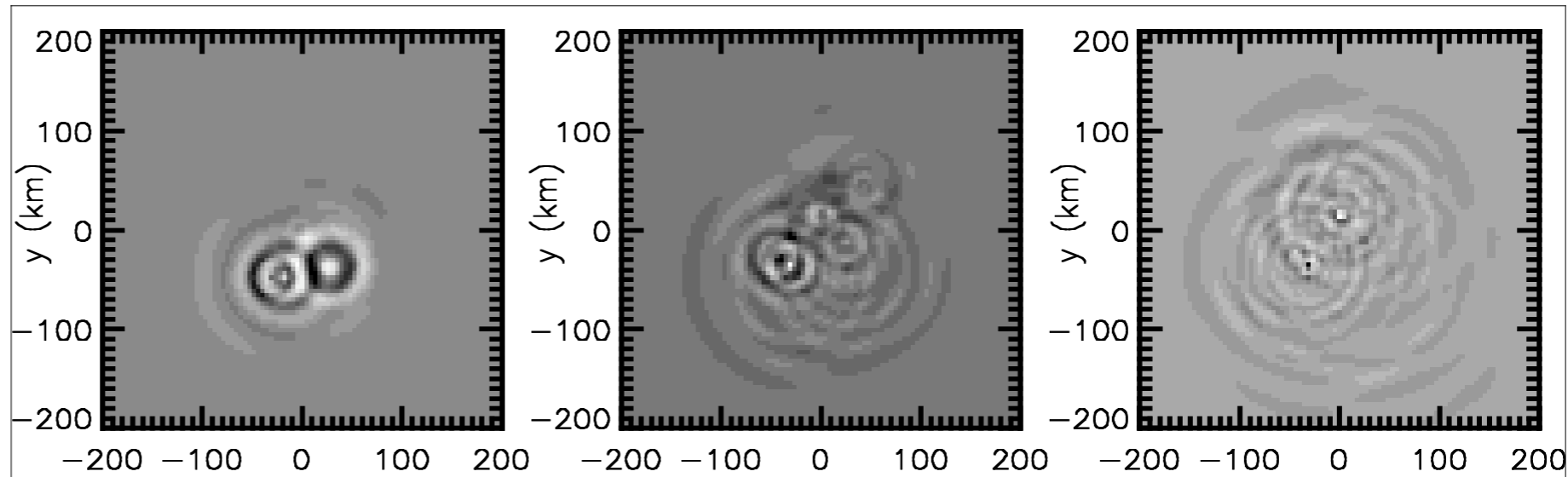
$$\int_{\Delta}^{\infty} [A | F, Y] ([Y | F] [F]) dY$$

resolved by NWP or secondary model

Bayesian SGS Modeling



Fourier-Laplace GW propagation



Prescribed wave-saturation conditions permit prediction of turbulent-patch locations

for transient 30, 60, and

$$\int_{\Delta} |A| F, Y (|Y| F) |F| dY$$

resolved by NWP or secondary model

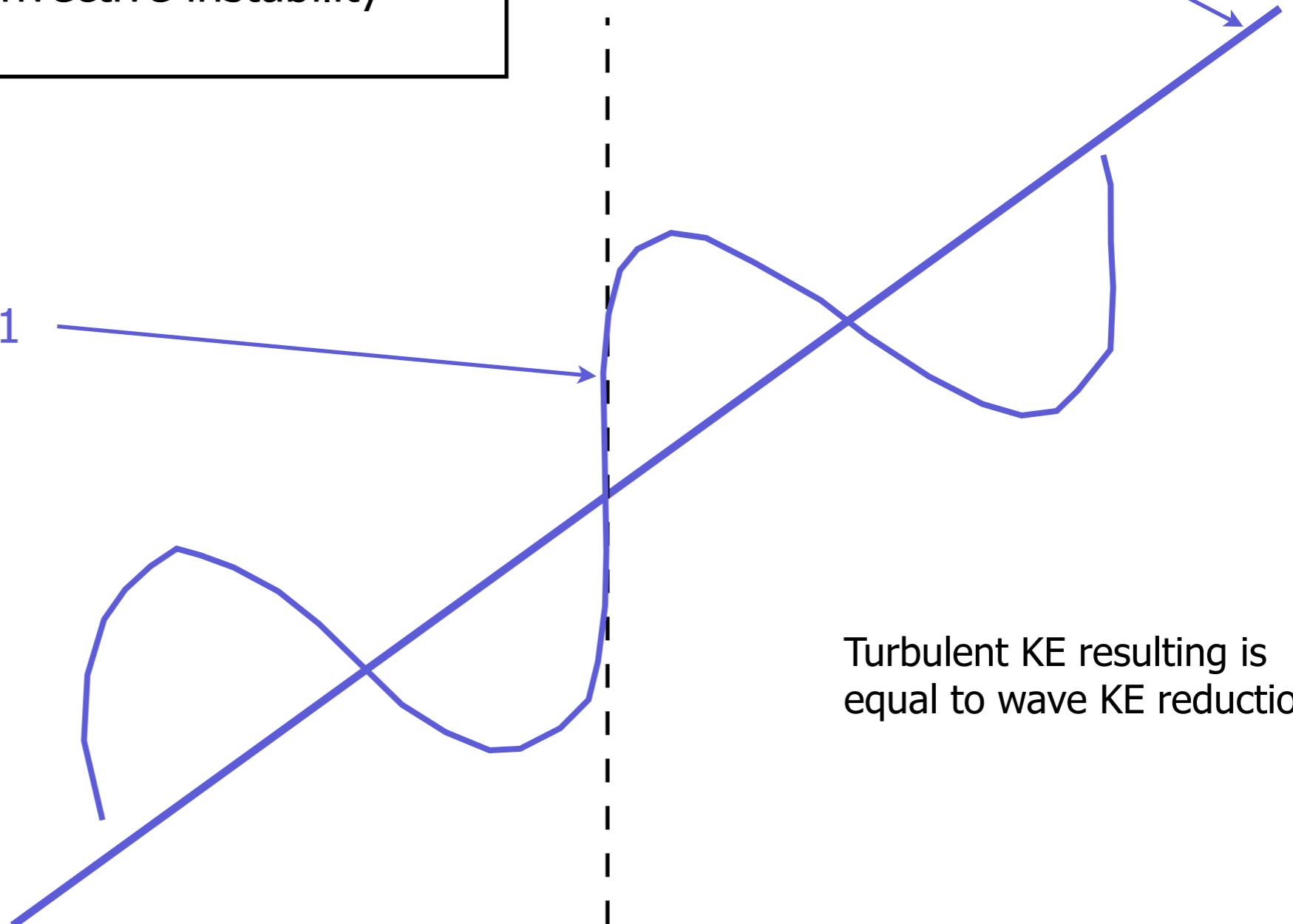
Modeling unresolved gravity-wave effects

Wave saturation model:

if wave grows above critical amplitude, reduce amplitude to marginal convective instability

Critical wave with $A=1$

$$\frac{\partial \bar{T}}{\partial z}$$



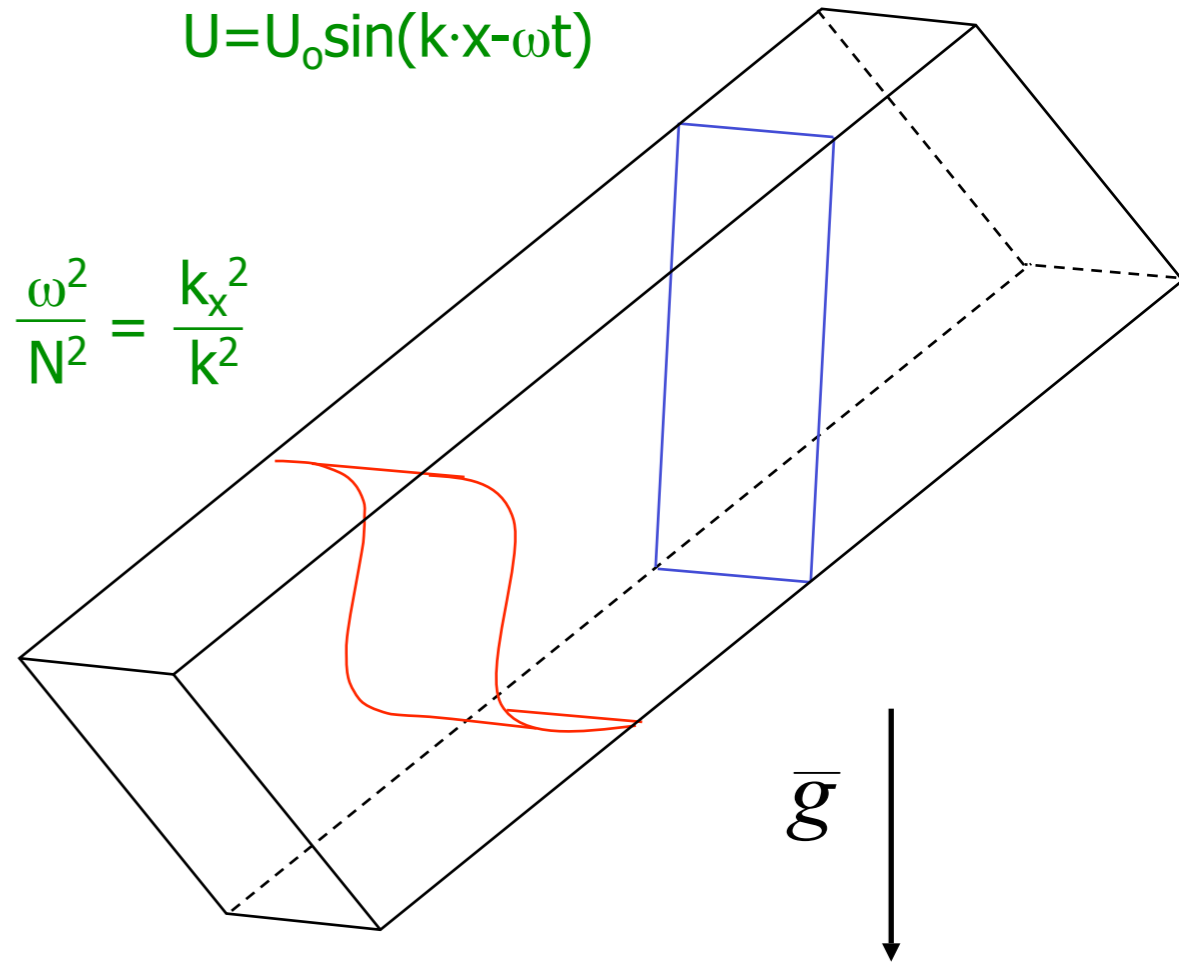
Turbulent KE resulting is equal to wave KE reduction.

Gravity-Wave Breaking Simulation

$$\theta = \theta_0 \cos(\mathbf{k} \cdot \mathbf{x} - \omega t)$$

$$U = U_0 \sin(\mathbf{k} \cdot \mathbf{x} - \omega t)$$

$$\frac{\omega^2}{N^2} = \frac{k_x^2}{k^2}$$



$$Ri = \frac{N^2 h^2}{U_0^2} \quad Re = \frac{U_0 h}{\nu} \quad Pe = \frac{U_0 h}{\kappa}$$

$$\partial_t \mathbf{u} + \boldsymbol{\omega} \times \mathbf{u} = Re^{-1} \nabla^2 \mathbf{u} - \nabla P + Ri \theta \hat{\mathbf{z}}$$

$$\partial_t \theta + \mathbf{u} \cdot \nabla \theta = Pe^{-1} \nabla^2 \theta$$

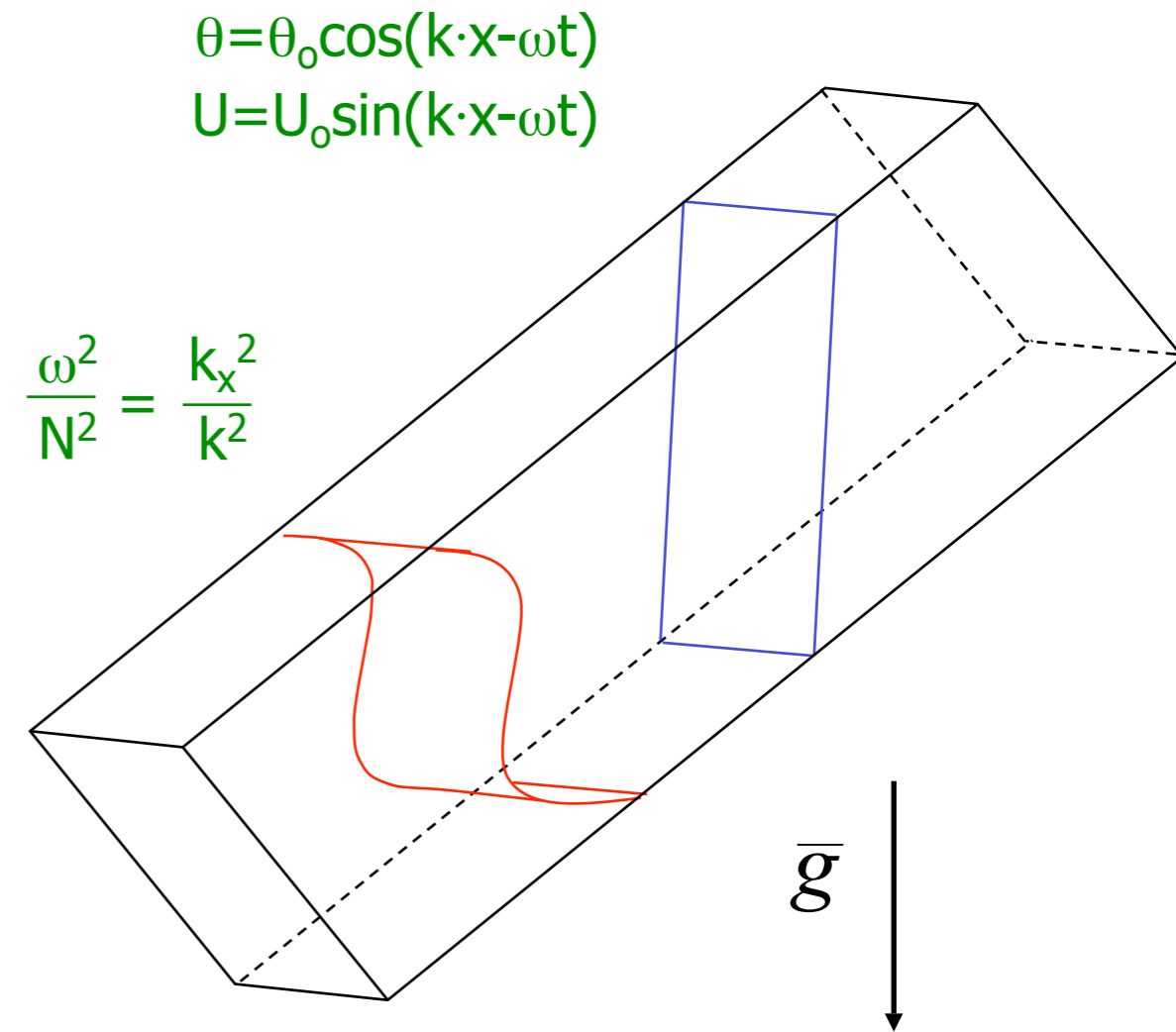
$$\nabla \cdot \mathbf{u} = 0$$

Gravity-Wave Breaking Simulation

gravity-wave asymptotic linear stability

$\theta=30$	$k_x=0.9$	$k_y=0.9$
$\theta=45$	$k_x=3.8$	$k_y=0.0$
$\theta=60$	$k_x=4.0$	$k_y=0.0$
$\theta=72$	$k_x=0.0$	$k_y=0.9$
$\theta=80$	$k_x=3.9$	$k_y=4.0$
$\theta=90$	$k_x=0.0$	$k_y=4.0$

Lombard & Riley, Phys. Fluids, 1996



$$Ri = \frac{N^2 h^2}{U_0^2} \quad Re = \frac{U_0 h}{\nu} \quad Pe = \frac{U_0 h}{\kappa}$$

$$\partial_t \mathbf{u} + \boldsymbol{\omega} \times \mathbf{u} = Re^{-1} \nabla^2 \mathbf{u} - \nabla P + Ri \theta \hat{\mathbf{z}}$$

$$\partial_t \theta + \mathbf{u} \cdot \nabla \theta = Pe^{-1} \nabla^2 \theta$$

$$\nabla \cdot \mathbf{u} = 0$$

Gravity-Wave Breaking Simulation

gravity-wave asymptotic linear stability

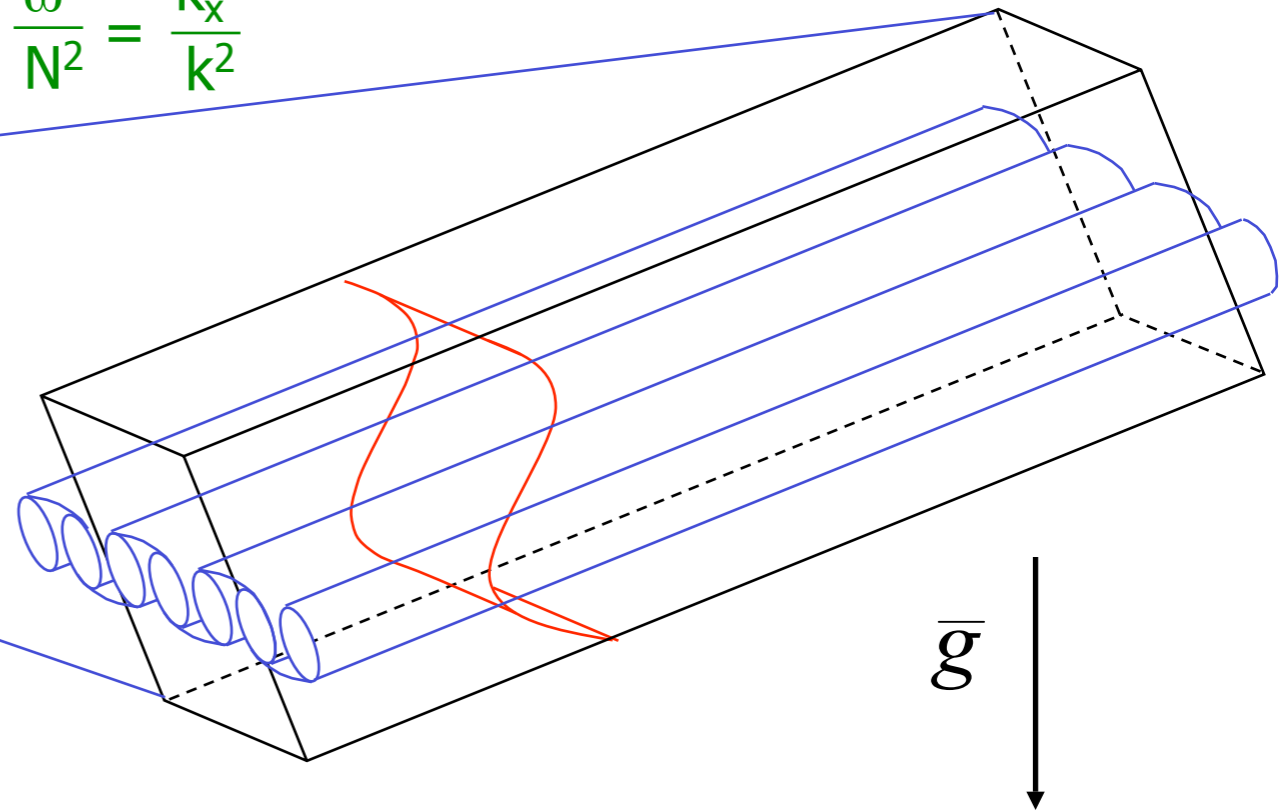
$\theta=30$	$k_x=0.9$	$k_y=0.9$
$\theta=45$	$k_x=3.8$	$k_y=0.0$
$\theta=60$	$k_x=4.0$	$k_y=0.0$
$\theta=72$	$k_x=0.0$	$k_y=0.9$
$\theta=80$	$k_x=3.9$	$k_y=4.0$
$\theta=90$	$k_x=0.0$	$k_y=4.0$

Lombard & Riley, Phys. Fluids, 1996

$$\theta = \theta_0 \cos(\mathbf{k} \cdot \mathbf{x} - \omega t)$$

$$U = U_0 \sin(\mathbf{k} \cdot \mathbf{x} - \omega t)$$

$$\frac{\omega^2}{N^2} = \frac{k_x^2}{k^2}$$



$$Ri = \frac{N^2 h^2}{U_0^2} \quad Re = \frac{U_0 h}{\nu} \quad Pe = \frac{U_0 h}{\kappa}$$

$$\partial_t \mathbf{u} + \boldsymbol{\omega} \times \mathbf{u} = Re^{-1} \nabla^2 \mathbf{u} - \nabla P + Ri \theta \hat{\mathbf{z}}$$

$$\partial_t \theta + \mathbf{u} \cdot \nabla \theta = Pe^{-1} \nabla^2 \theta$$

$$\nabla \cdot \mathbf{u} = 0$$

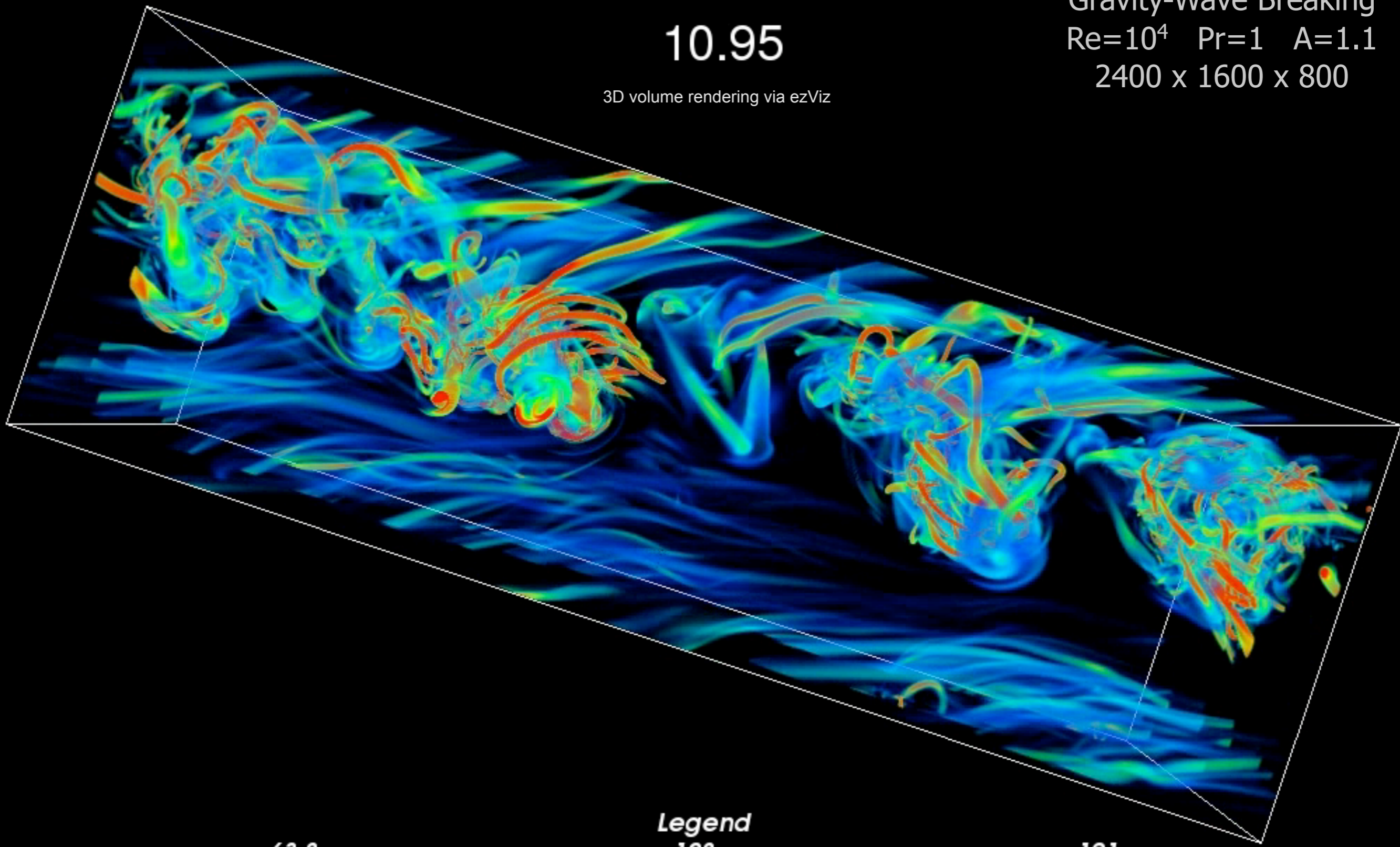
Gravity-Wave Breaking
Re=10⁴ Pr=1 A=1.1
2400 x 1600 x 800

3D volume rendering via ezViz

Gravity-Wave Breaking
Re=10⁴ Pr=1 A=1.1
2400 x 1600 x 800

10.95

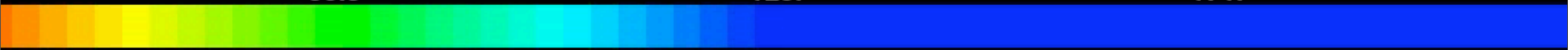
3D volume rendering via ezViz



63.8

Legend
128.

191.



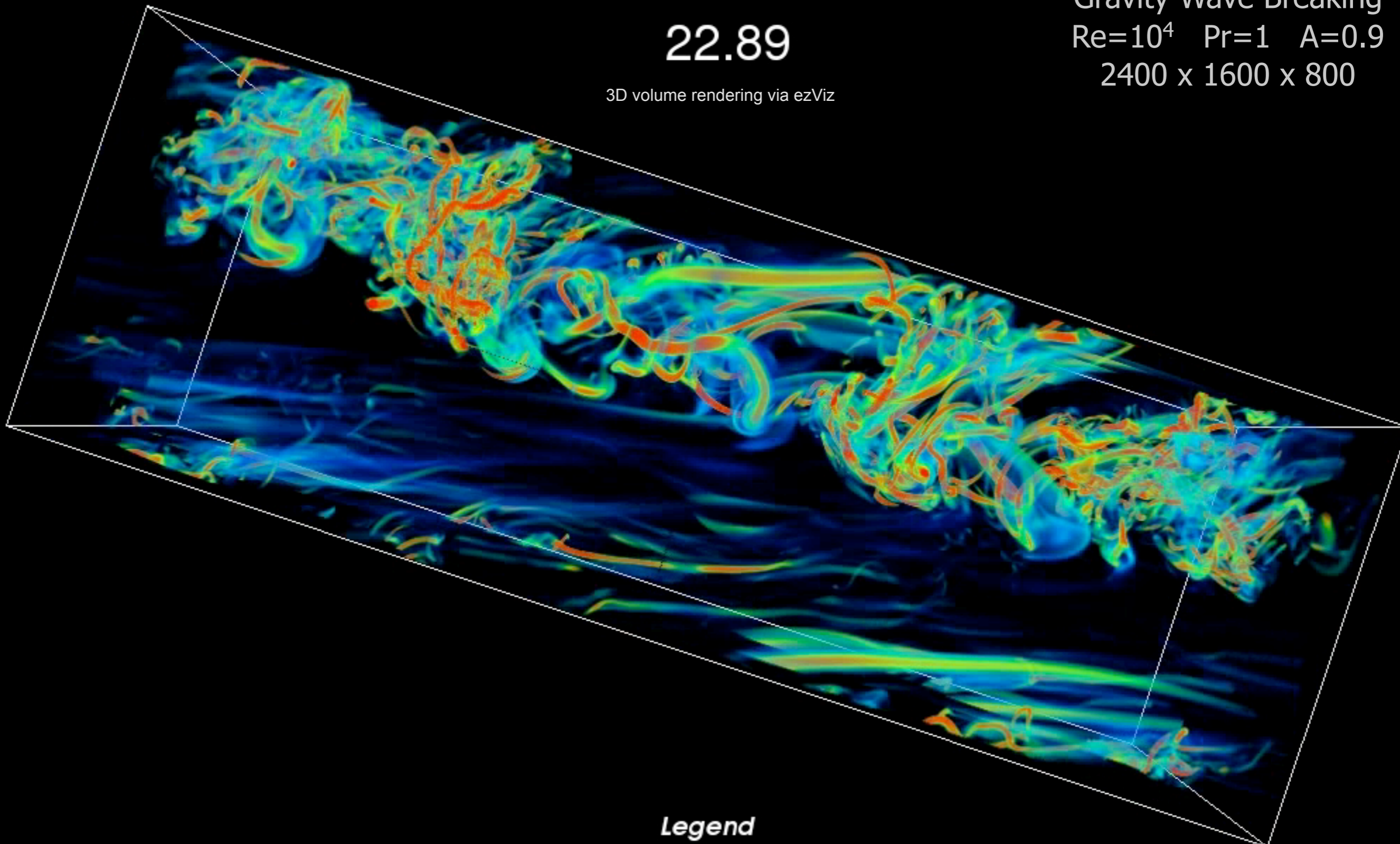
Gravity-Wave Breaking
Re=10⁴ Pr=1 A=0.9
2400 x 1600 x 800

3D volume rendering via ezViz

Gravity-Wave Breaking
Re=10⁴ Pr=1 A=0.9
2400 x 1600 x 800

22.89

3D volume rendering via ezViz



63.8

Legend
128.

191.



$A=1.1$

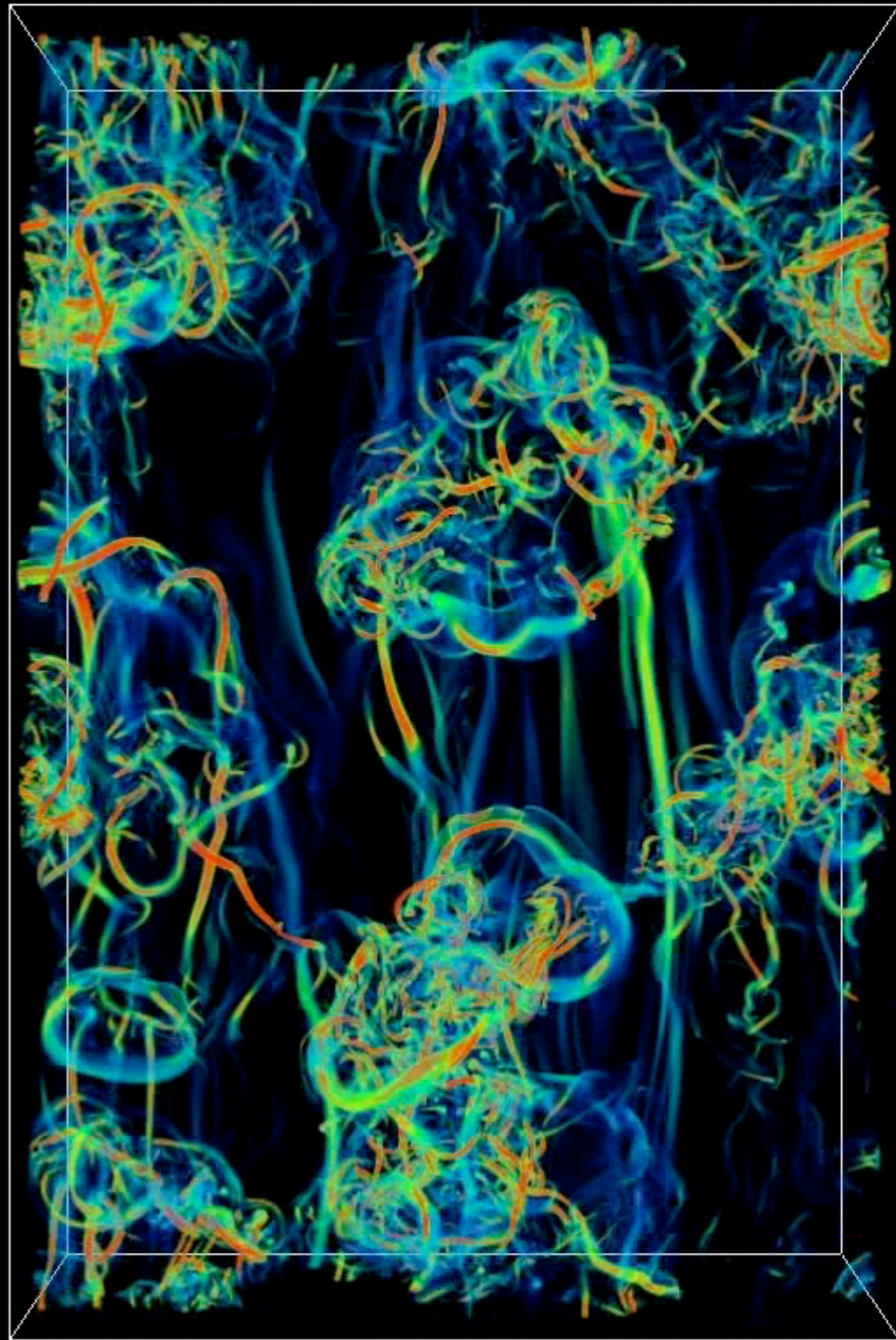
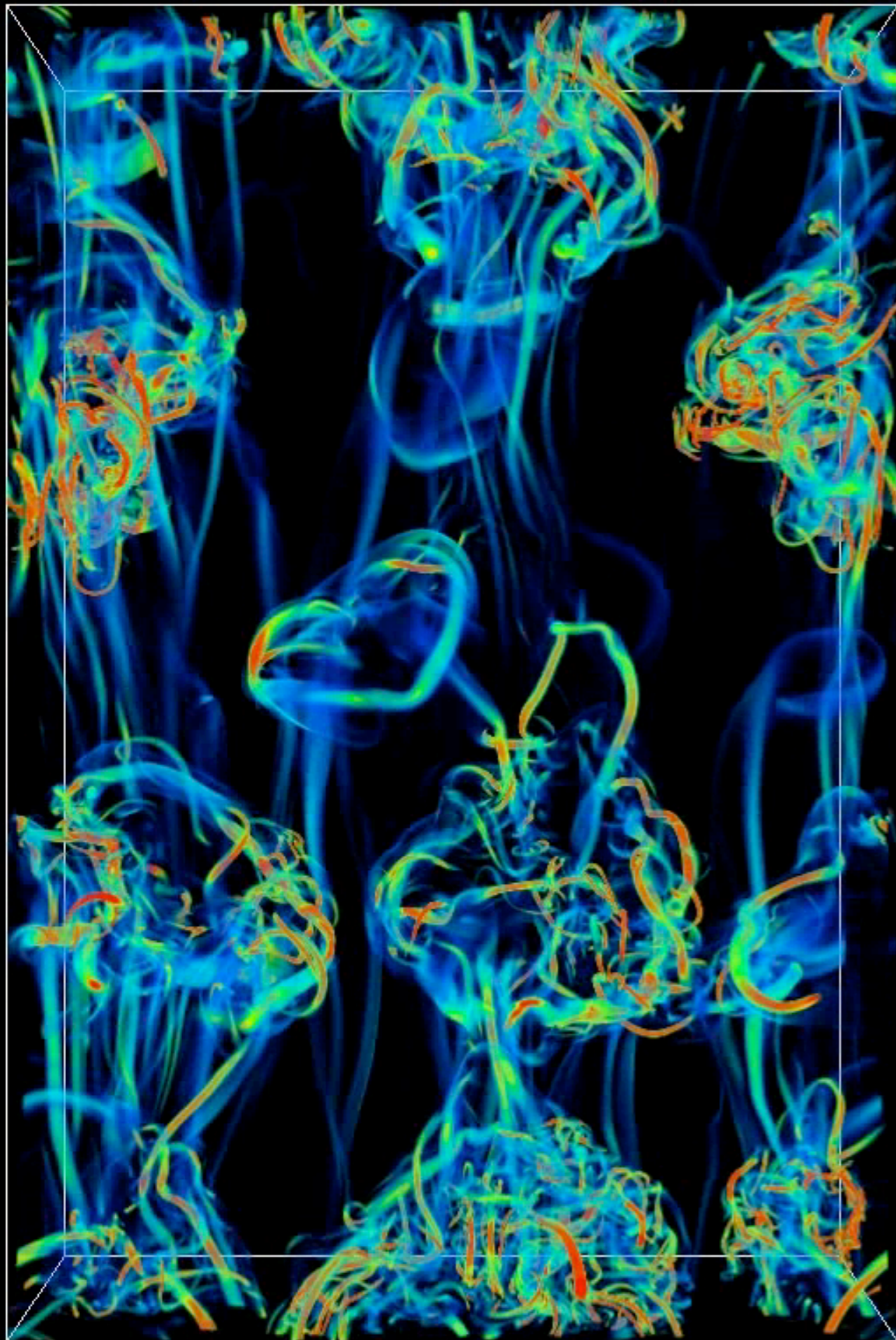
3D volume rendering via ezViz

$A=0.9$

$A=1.1$

3D volume rendering via ezViz

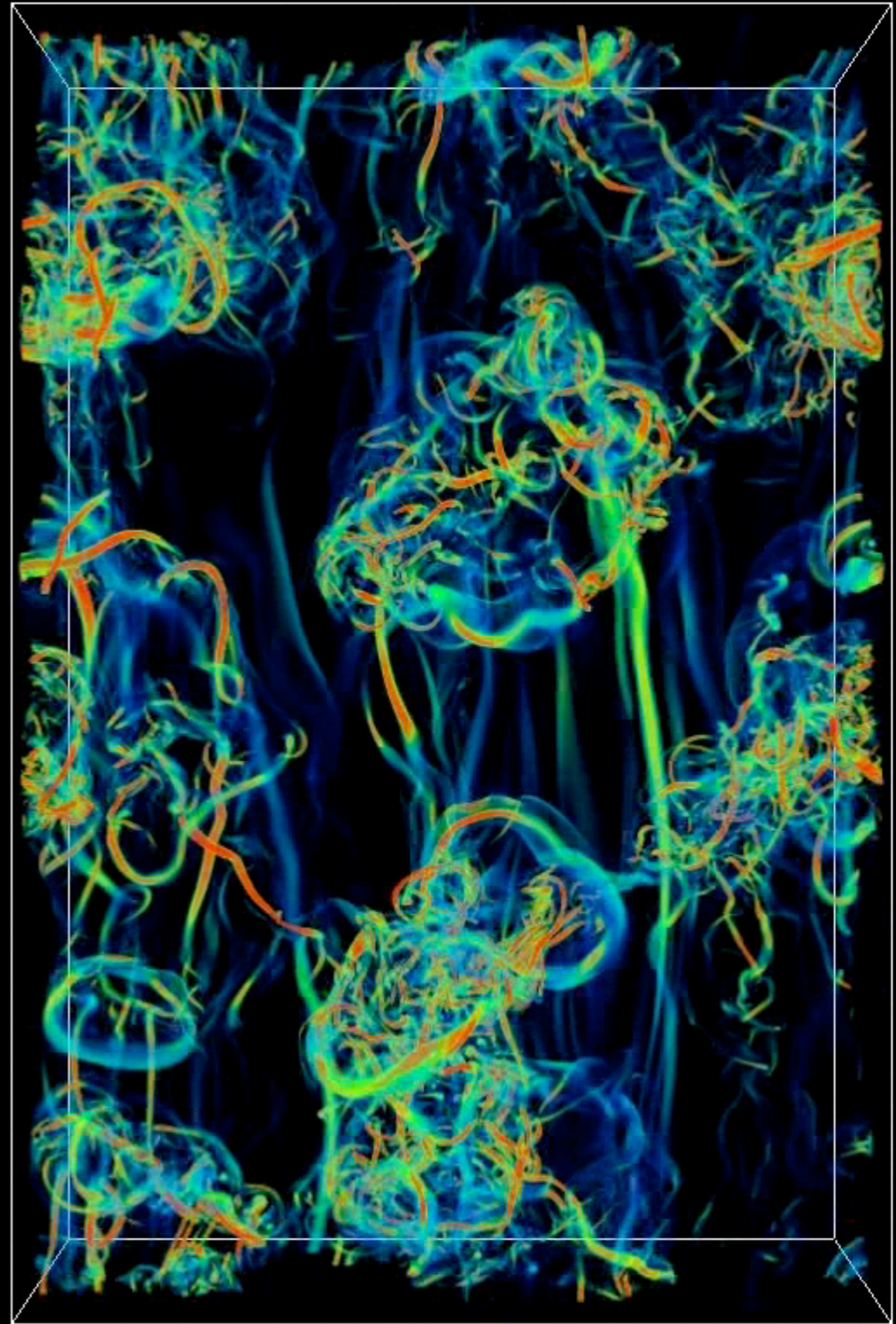
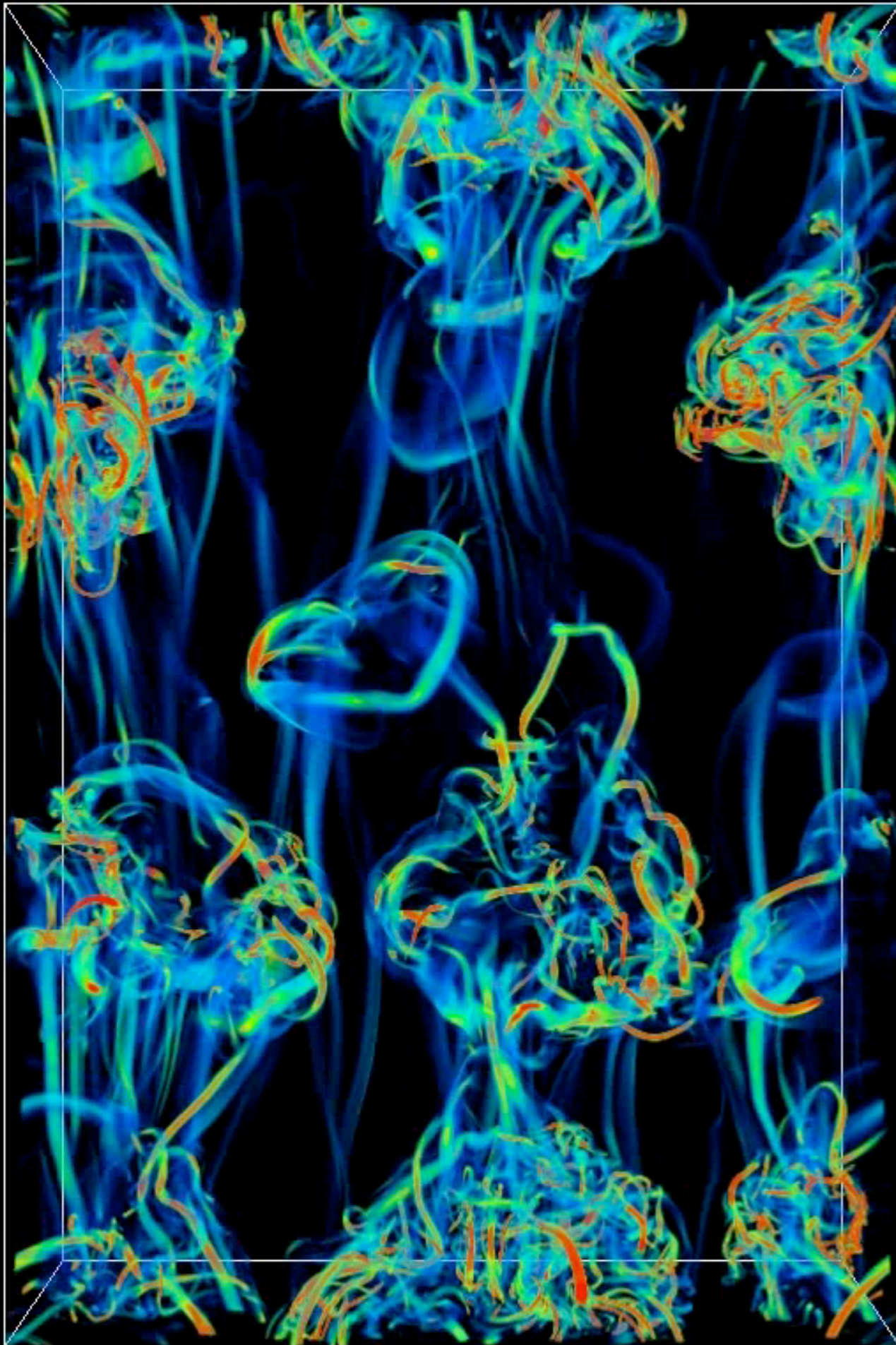
$A=0.9$



$A=1.1$

3D volume rendering via ezViz

$A=0.9$

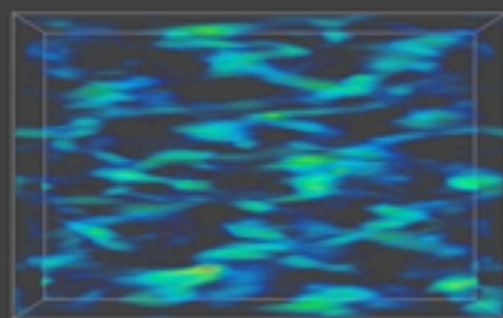
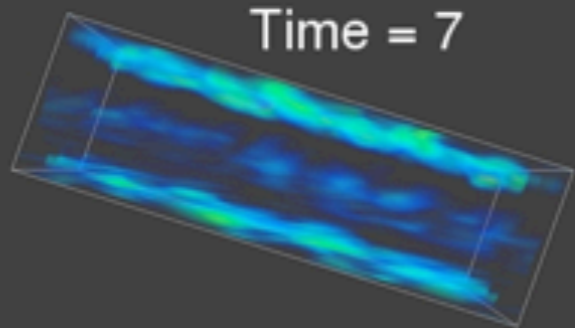


A=1.1

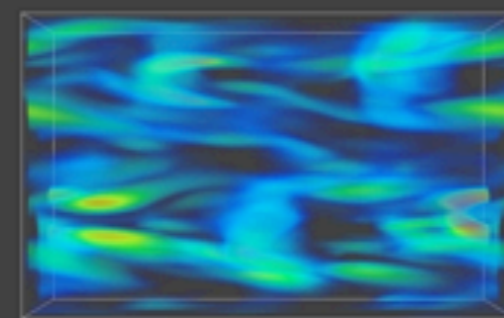
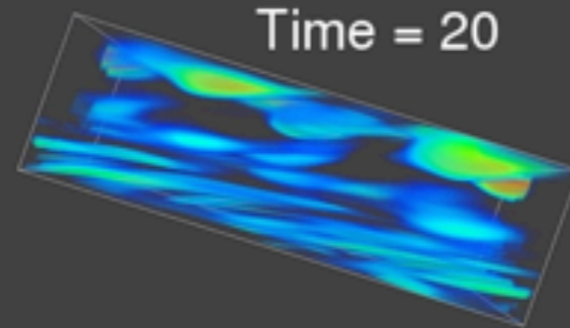
GW Breaking Simulations

A=0.9

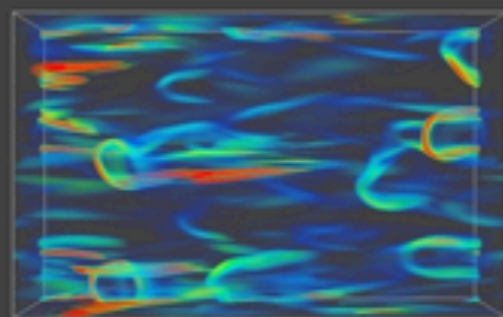
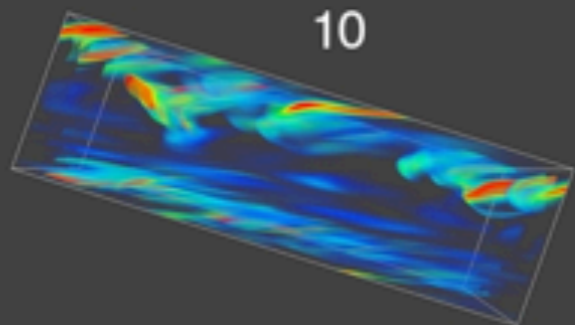
Time = 7



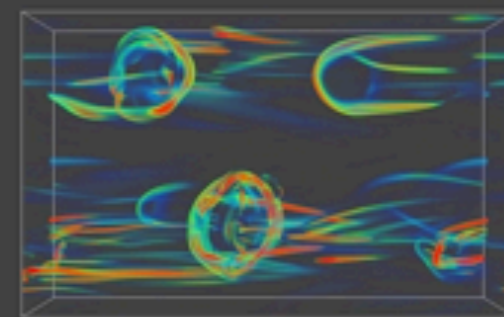
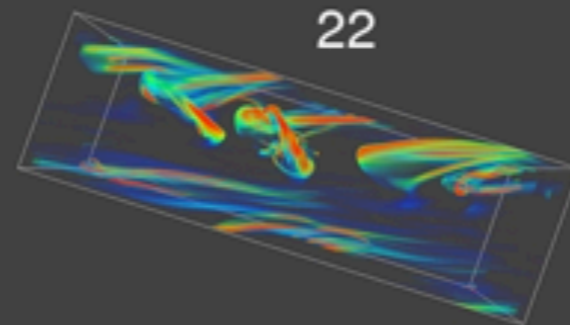
Time = 20



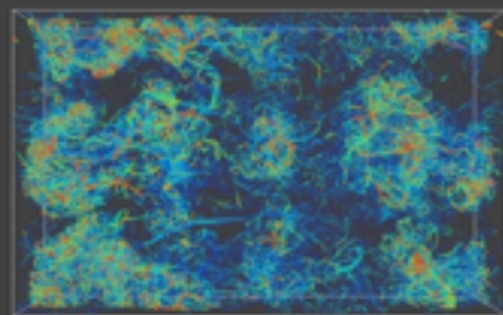
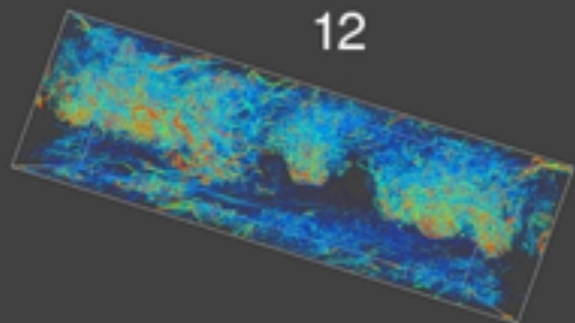
10



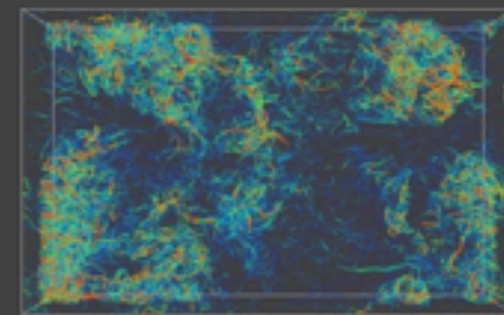
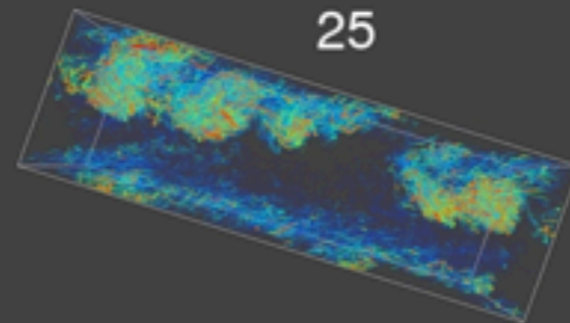
22



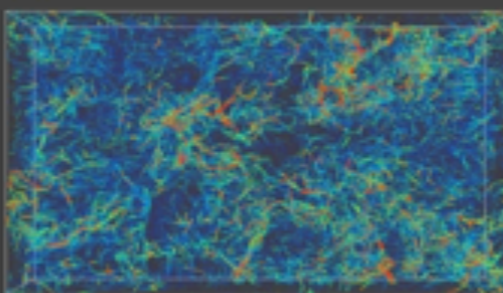
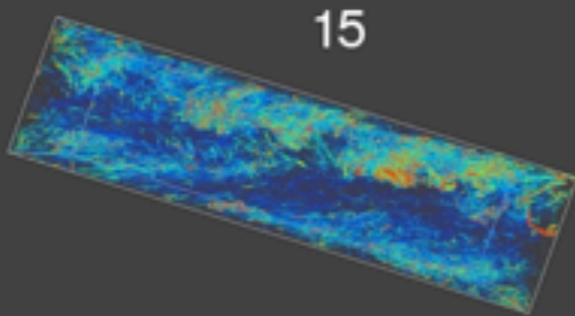
12



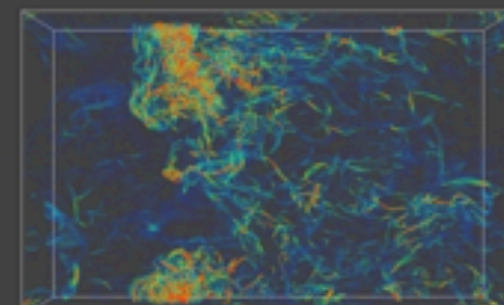
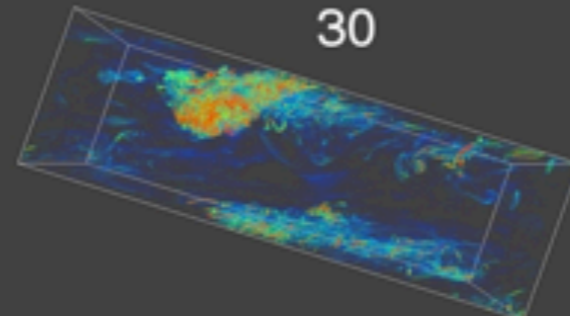
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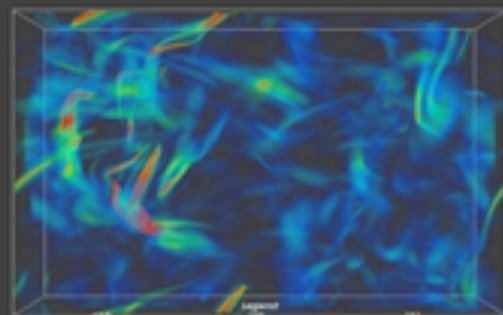
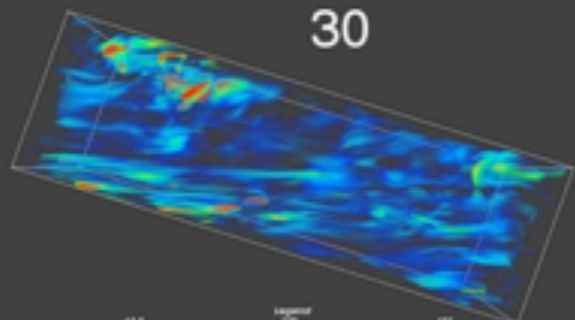
15



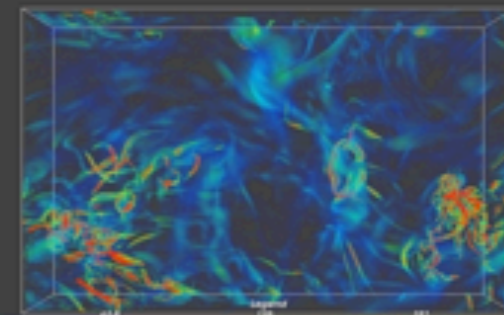
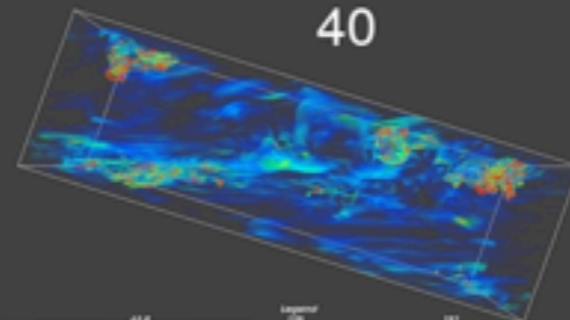
30



30



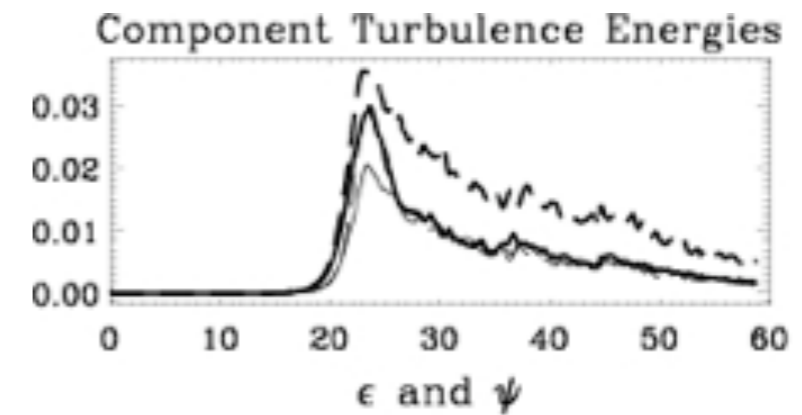
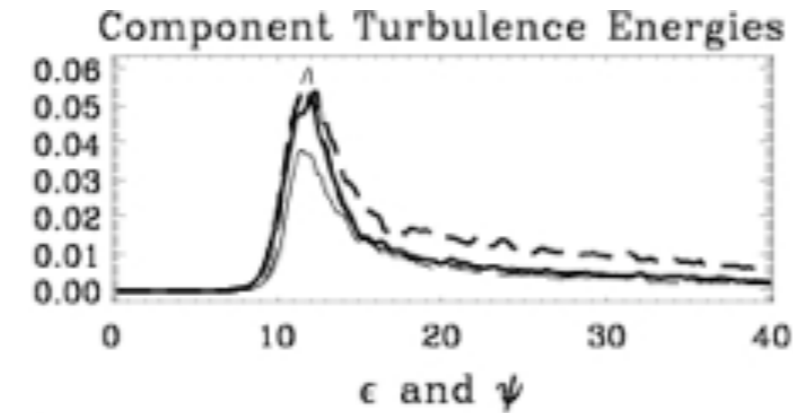
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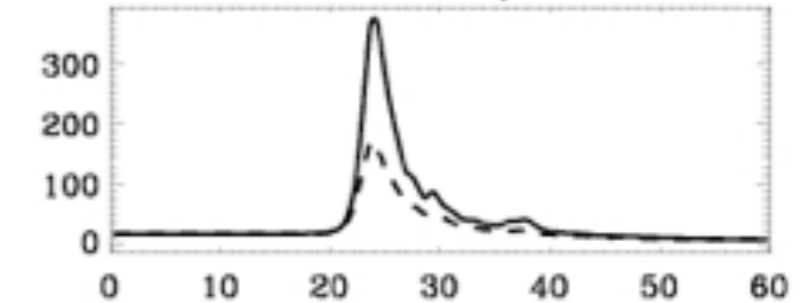
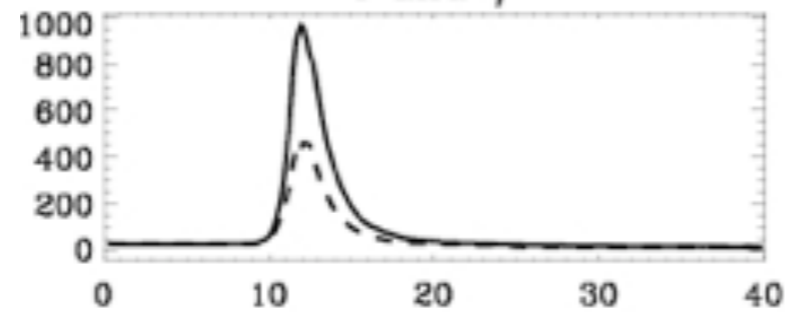
Hi-Res Wind-Shear Simulations: DNS-LES Comparisons



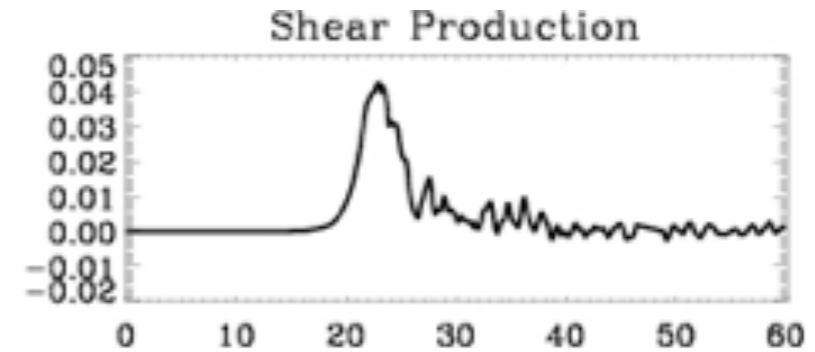
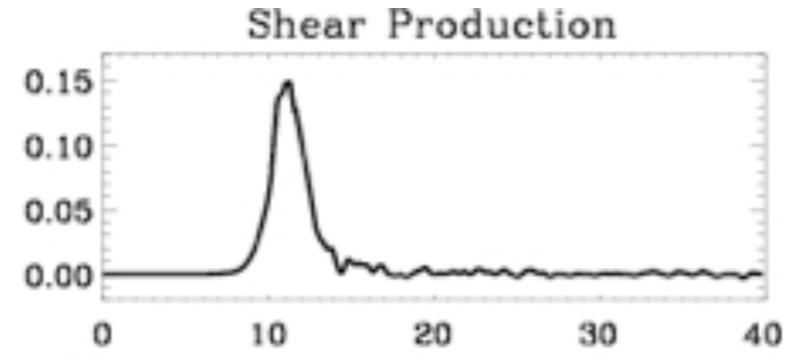
Ri $\langle \theta^2 \rangle$ $\langle u^2 \rangle$
 $\langle v^2 \rangle$ $\langle w^2 \rangle$



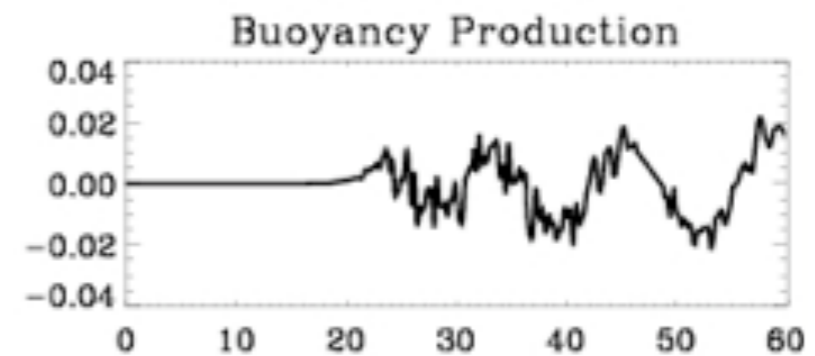
ϵ, χ



- $\langle uw \rangle \partial_z U$



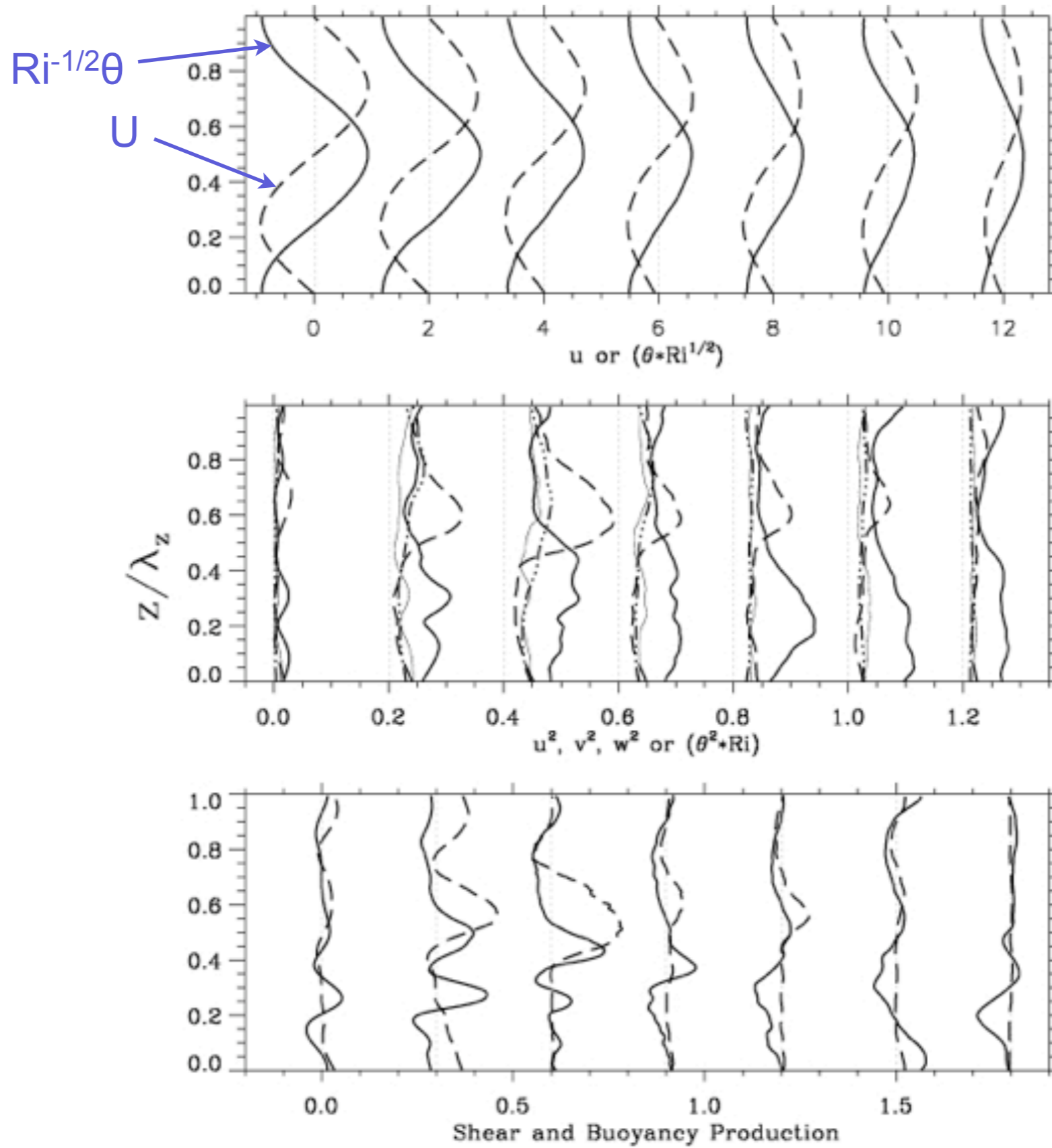
Ri $\langle \theta w \rangle$



A=1.1

A=0.9

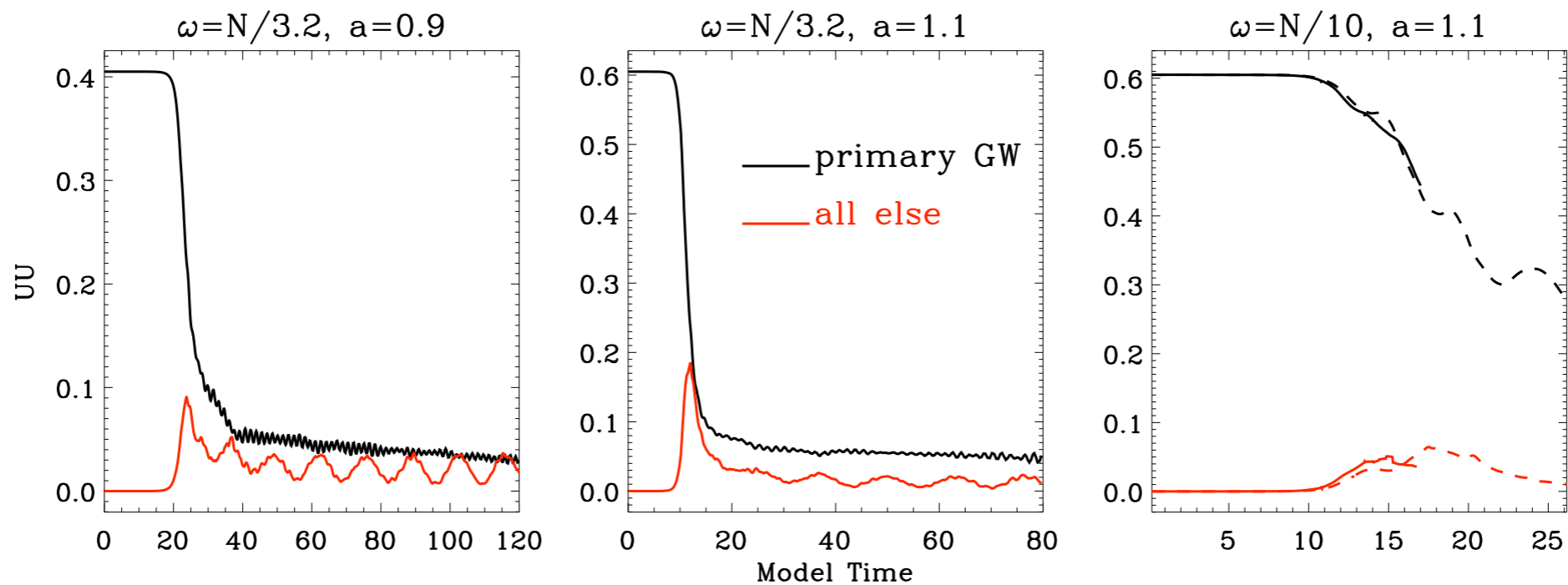
Hi-Res Wind-Shear Simulations: DNS-LES Comparisons



Hi-Res Wind-Shear Simulations: DNS-LES Comparisons



1. Amplitude drops by 70% for both $A=1.1$ AND $A=0.9$ for 72° wave ($N/3.2$).
2. Preliminary result: $A=0.9$ does not budge for 84° wave ($N/10$).



Conclusions

1. Gravity waves are important often-unresolved processes in NWP models.
2. GWs engender turbulence and couple to mean flows when overturning and breaking occurs..
3. Gravity wave breaking results after amplification in the atmosphere due to a) critical-level absorption and b) upward propagation.
4. The effects of GWs are estimated by linear wave-propagation codes operating on NWP output that does not resolve them explicitly or accurately.
5. Nonlinear effects must be modeled in wave-propagation codes, and currently convective instability is used as the criteria of choice.
5. DNS results for turbulent GW breaking indicate that convective instability under-predicts the degree of wave saturation.

Ongoing Work

1. Explore wave saturation as a function of wave parameters.

