Spectrally condensed turbulence in two dimensions

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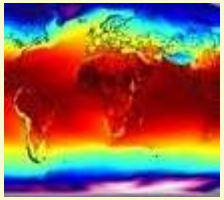


TOY2008: Summer school: Geophysical Turbulence, Boulder, Colorado

Motivation

Turbulence often coexists with coherent flow

2D turbulence is capable of generating such flows

spectral condensation, crystallization

Turbulence-condensate interplay – dynamical steady-state

energy transfer from turbulence to flows

effects of shear flows on turbulence

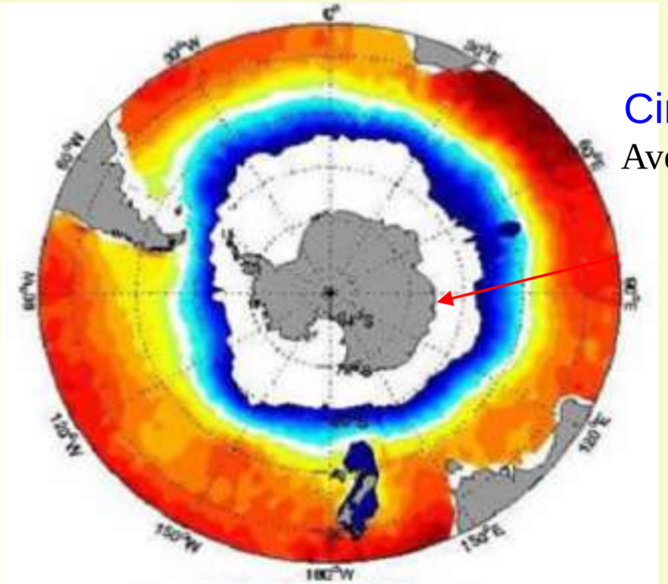
Practical applications

atmospheric and oceanic processes,

magnetically confined plasma, etc.

2D turbulence and spectral condensation

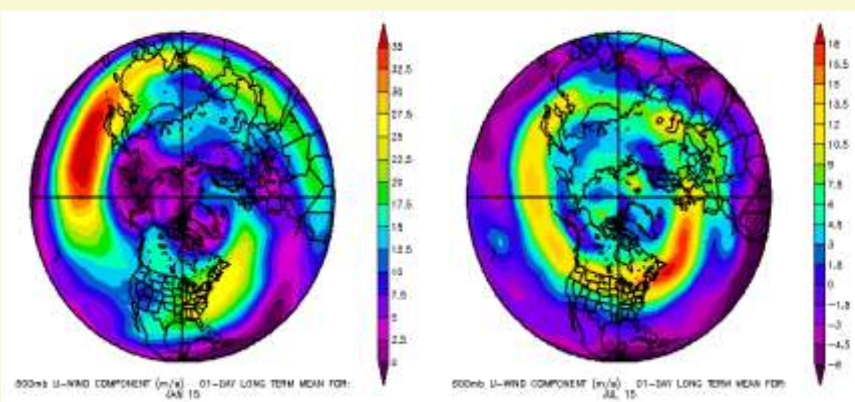
Large coherent flows coexist with turbulence



Antarctic
Circumpolar Current
Average volume transport
$\sim 1.5 \quad 10^{8}\ \mathrm{m^3 s^{-1}}$

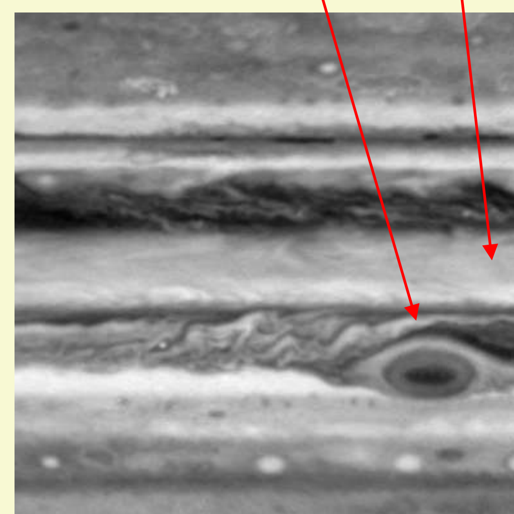
Jupiter's
zonal flows

GRS



Earth: atmospheric zonal winds

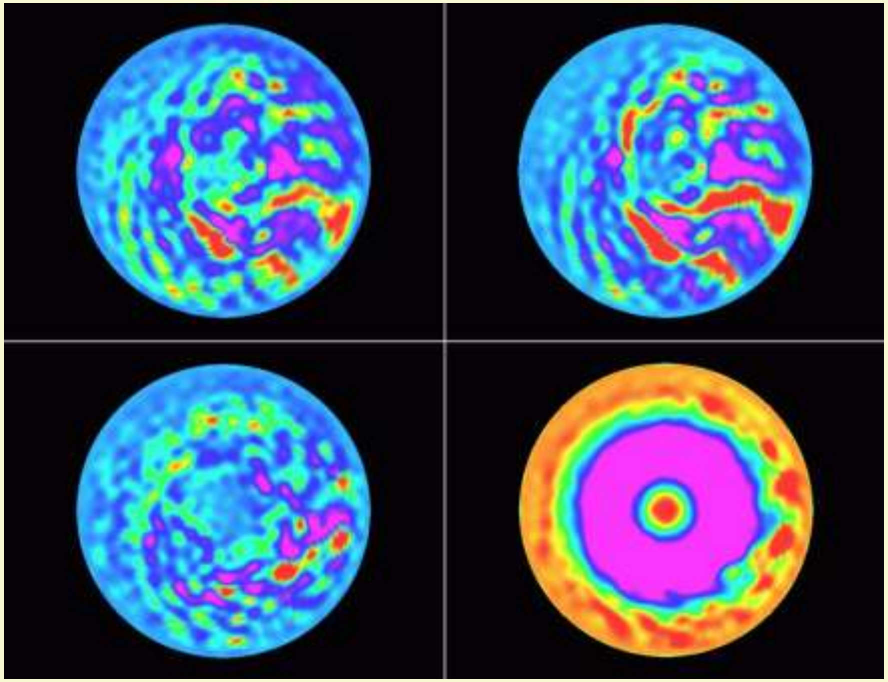
Planetary atmospheres are dominated by turbulent structures (cyclones, zonal winds, *etc*)



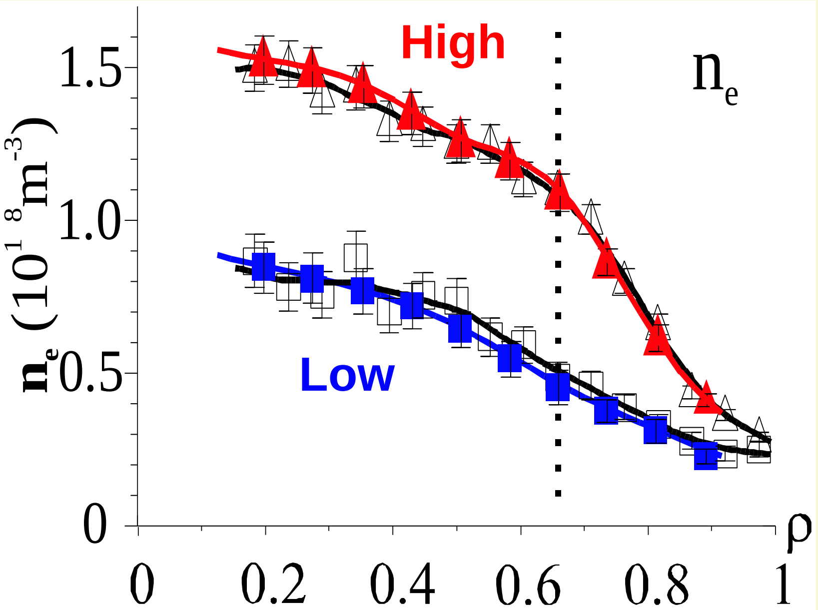
Cassini spacecraft

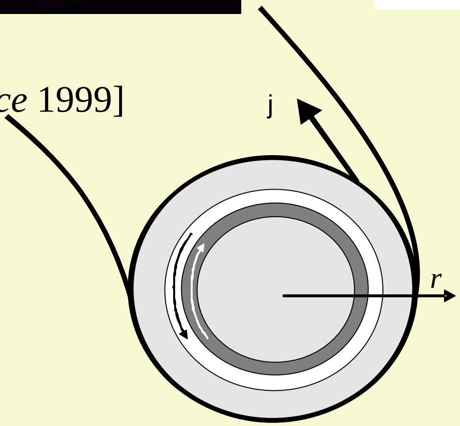
Courtesy NASA

Turbulence-driven structures in fusion plasma



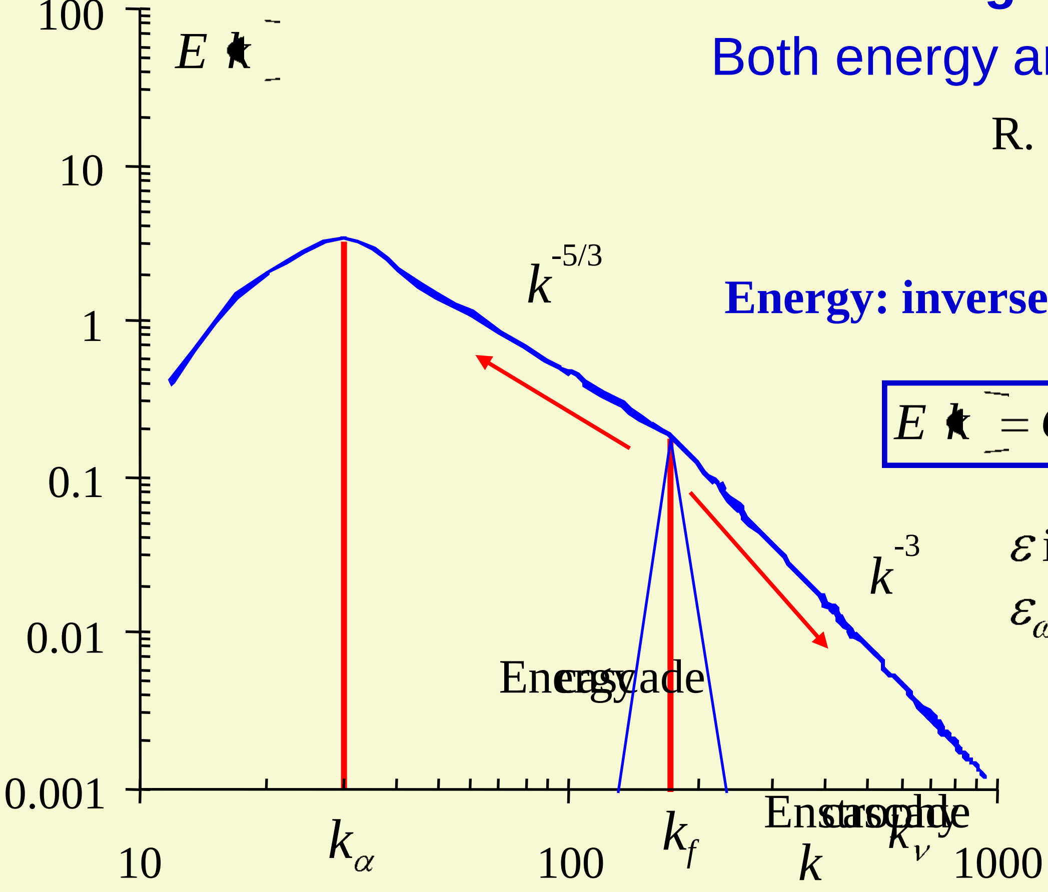
Zonal flows in plasma
Numerical simulations [Z. Lin, *Science* 1999]





In magnetically confined plasma, turbulence-driven anisotropic flows develop, which inhibit radial transport of particles and energy

2D turbulence



Idealized homogeneous isotropic 2D turbulence

Both energy and enstrophy are conserved

R. Kraichnan (1967)

Dual cascade:

Energy: inverse cascade, Enstrophy: forward cascade

$$\boxed{E(k) = C_k \varepsilon^{2/3} k^{-5/3}}$$

$$\boxed{E(k) = C_\omega \varepsilon_\omega^{2/3} k^{-3}}$$

ε is the energy dissipation rate

ε_ω is the enstrophy dissipation rate

$$\boxed{\Omega = \frac{1}{2}\int_V |\boldsymbol{\omega}|^2 dV}$$

enstrophy

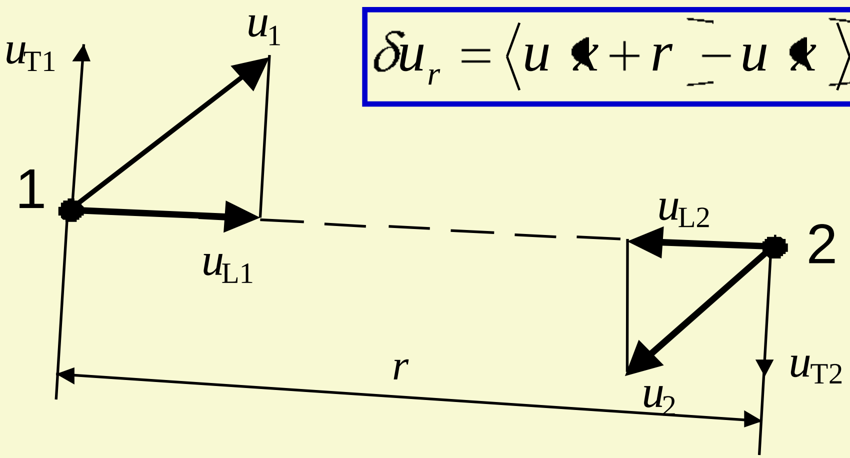
Opposite to 3D, energy flows from smaller to larger scales

Basis for self-organization

Structure functions and Kolmogorov law

Label an ‘eddy’ by a velocity increment δu_l across a distance *r:*

$$\delta u_r = \left\langle u(x+r) - u(x) \right\rangle$$



Statistical moments of this increment are called *structure functions* of the nth order:

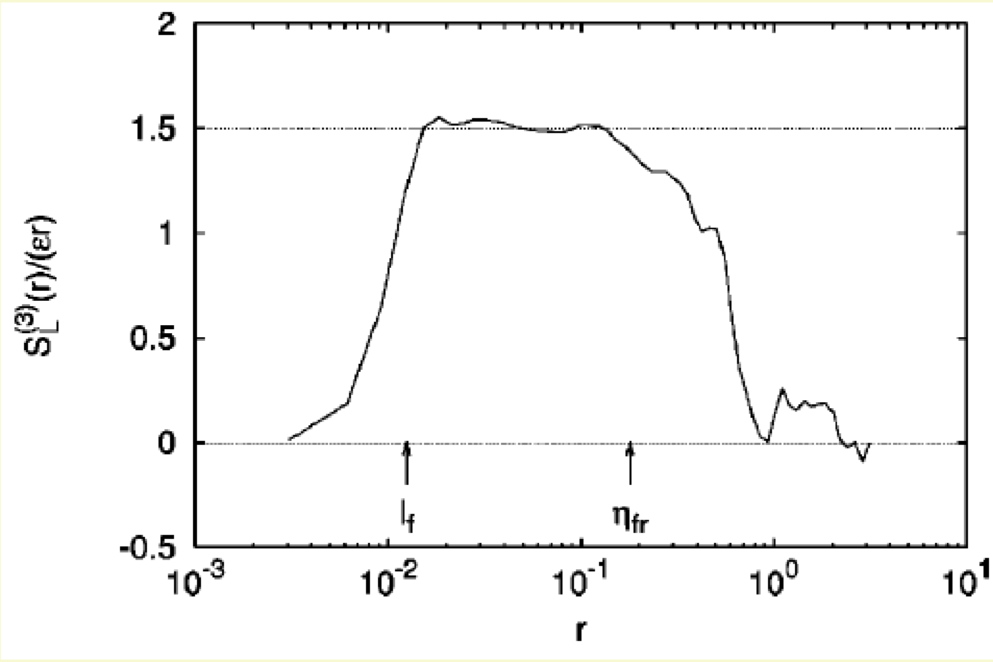
$$S_n(r) = \left\langle (\delta u_r)^n \right\rangle = \left\langle [u(x+r) - u(x)]^n \right\rangle$$

Kolmogorov law

relates the third-order longitudinal structure function of turbulence to the mean energy dissipation per unit mass ε

in 2D (e.g. [Lindborg 1999]):

$$S_{3L}(r) = \left\langle \delta V_L^3(r) \right\rangle = \frac{3}{2}\varepsilon r$$



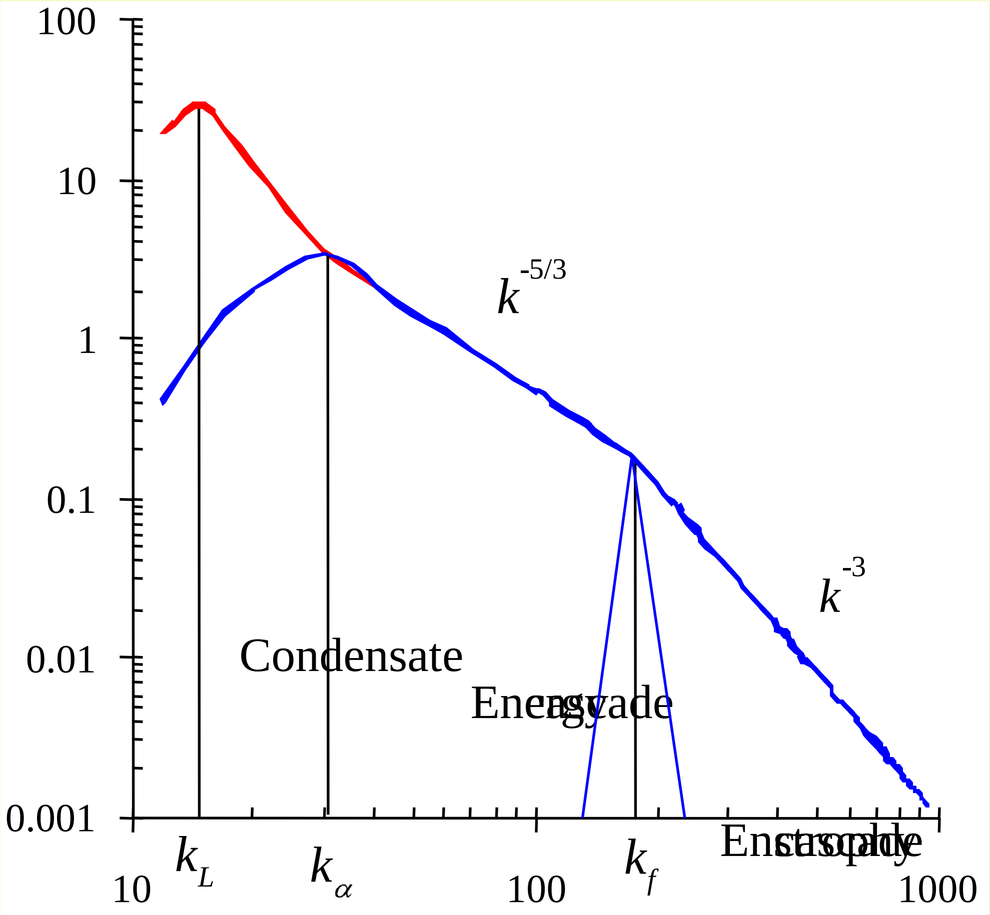
[G. Boffetta, A. Celani, M. Vergassola, 2000]

Spectral condensation of 2D turbulence

The maximum of the energy spectrum lies in the low-*k* range, at k_α, in the absence of the energy dissipation at large scales k_α can not be constant in time since it accumulates spectral energy

$$k_\alpha = f(\varepsilon, t)$$

System size < dissipation scale



Dissipation at large scales (bottom damping) α stabilizes the maximum of the spectrum at the scale

$$k_\alpha \approx \left(\alpha^3/\varepsilon\right)^{1/2}$$

Kraichnan, 1967:
predicted condensate

At low dissipation in a bounded system, at $k_\alpha << k_L$ spectral energy accumulates in a box-size coherent structure

Spectral condensation of turbulence in thin layers

Experiments: Sommeria (1986), Paret &Tabeling (1998), Shats et al (2005, 2007)

Time evolution to condensed state
[Shats *et al* (2005)]

Numerical simulations of 2D turbulence:

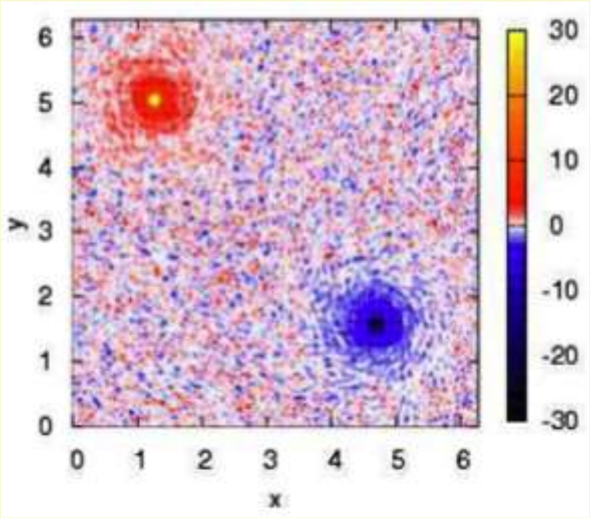
Hossain (1983), Smith &Yakhot (1993)…

van Heijst, Clercx, Molenaar (2004-2006),

Chertkov et al. (2007)

Periodic boundary condition – dipole

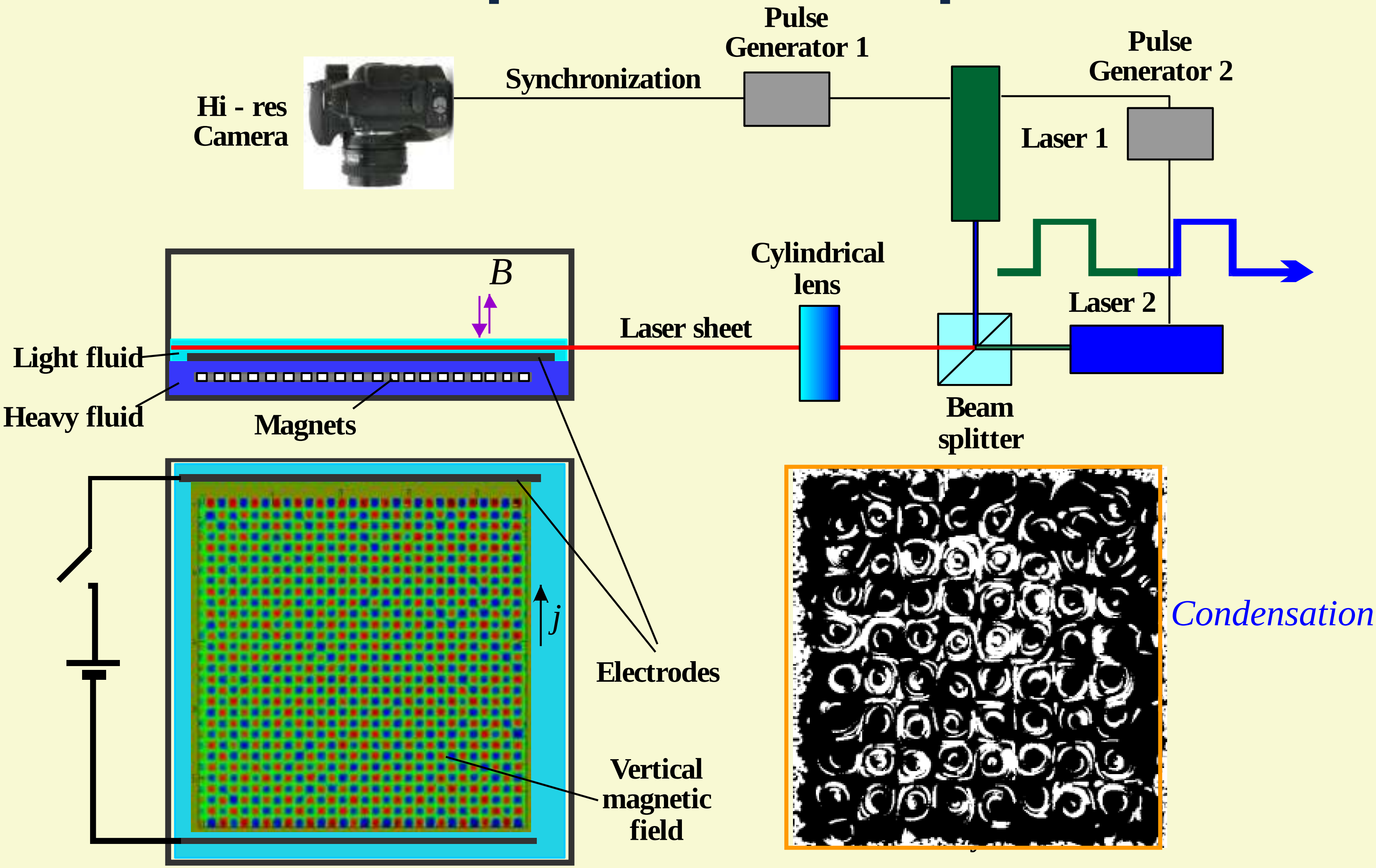
No-slip boundary – single vortex



Vorticity of the condensate

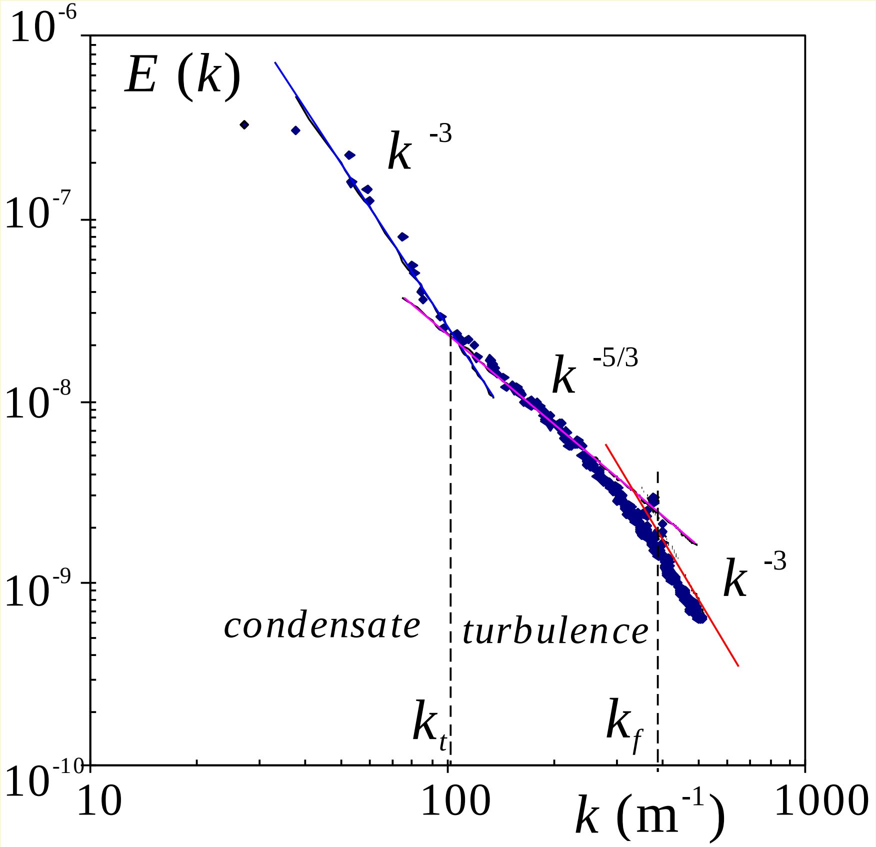
[M. Chertkov et al. (2007)]

Experimental setup



- ❑ Bottom layer: isolator Fluorinert FC-77 (resist. = 2×10^{15} Ohm cm; SG = 1.78)
- ❑ Top layer: electrolyte NaCl solution (SG = 1.04)

Condensed turbulence spectrum is robust



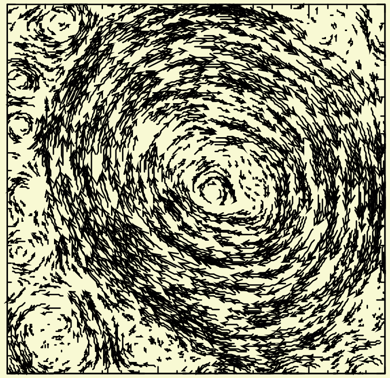
$k^{-(3\text{-}4)}$ at large scales

$k^{-5/3}$ at meso-scales

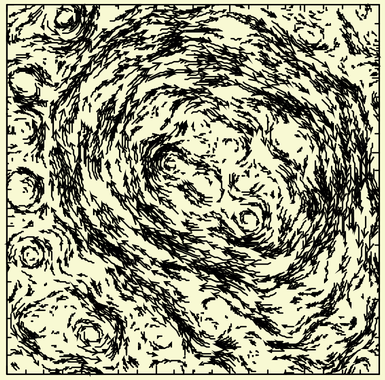
$k^{-(3\text{-}4)}$ at small scales

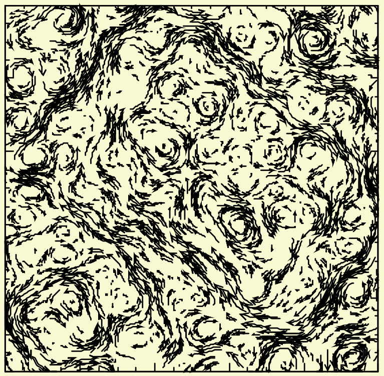
-Bottom drag,
-boundary size,
-forcing
affect condensate strength and topology

Time-averaged $L = 0.1$ m



strong



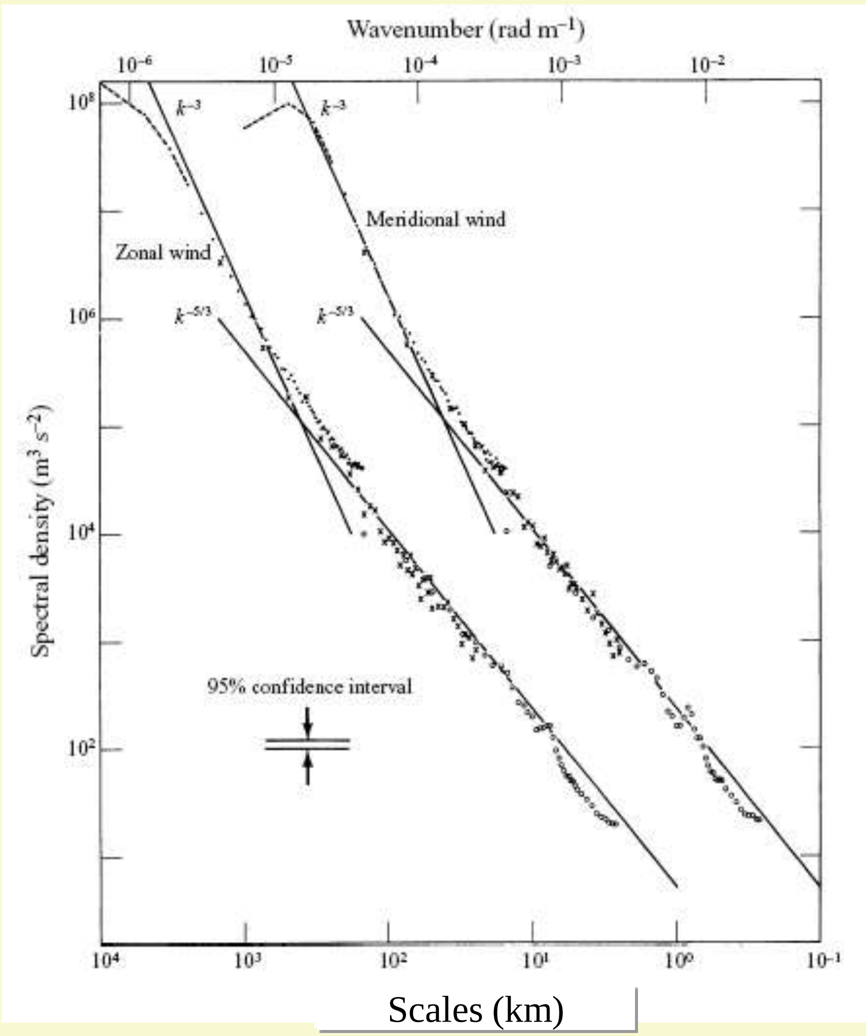


weak

Nastrom-Gage spectrum of atmospheric winds

Atmospheric spectrum

[Nastrom, Gage, Jasperson, Nature (1984)]



k^{-3} and $k^{-5/3}$ ranges are present but in the reversed order compared to the Kraichnan theory

$$E(k) = C_k \varepsilon^{2/3} k^{-5/3} \quad at\ k < k_f$$

$$E(k) = C_\omega \varepsilon_\omega^{2/3} k^{-3} \quad at\ k > k_f$$

What is the origin of

k^{-3} and $k^{-5/3}$ ranges in atmosphere?

Meso-scale $k^{-5/3}$ range can be due to

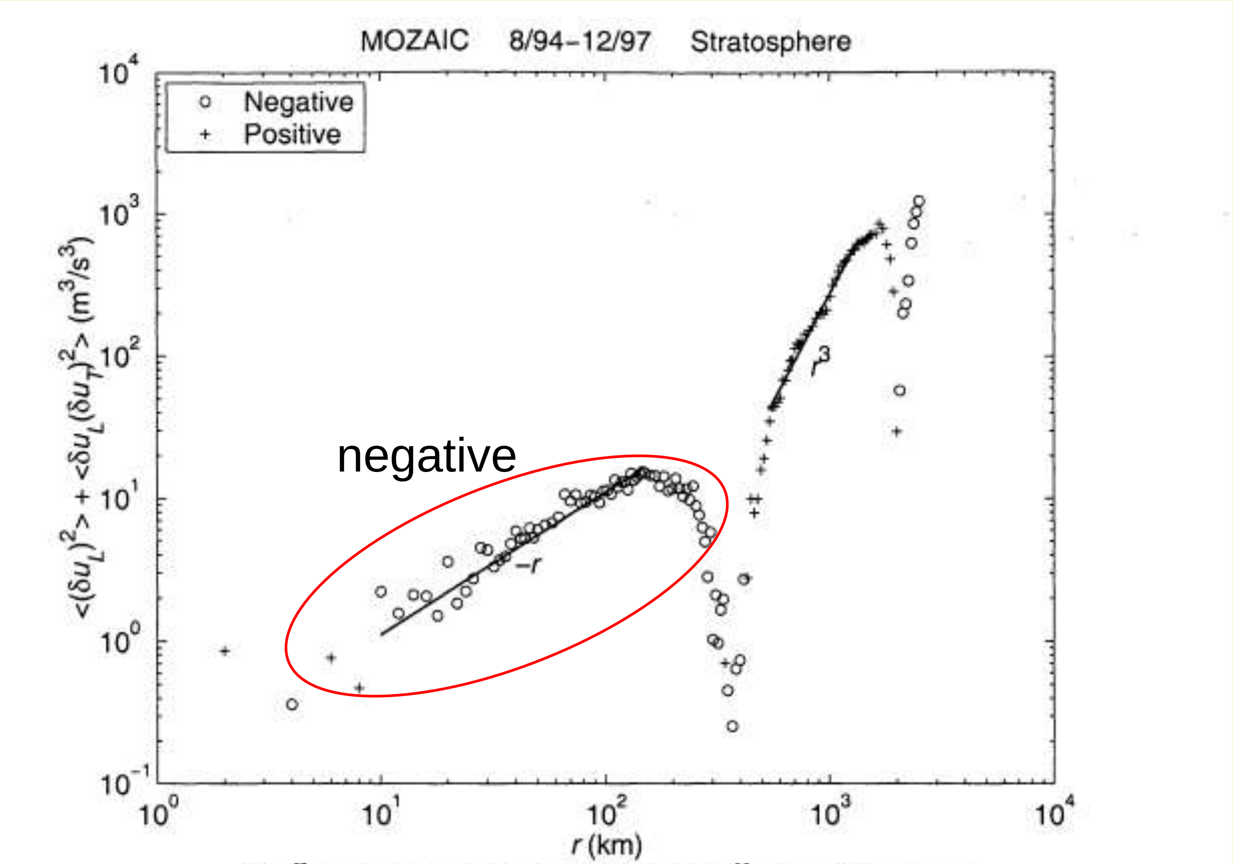
- *3D (downscale) direct energy cascade,*
- *2D inverse (upscale) cascade*

Large-scale k^{-3} range can be due to

- *direct enstrophy cascade (large-scale forcing)*
- *spectral condensation*

Kinetic energy spectrum alone cannot resolve the question of the sources

Energy flux in atmospheric turbulence



Circles and crosses indicate negative and positive values

Third-order velocity moment gives the energy flux direction

$$S_3(r) = \frac{3}{2}\varepsilon r$$

Negative S_3 at scales up to 500 km interpreted as evidence against inverse energy cascade in the mesoscale range

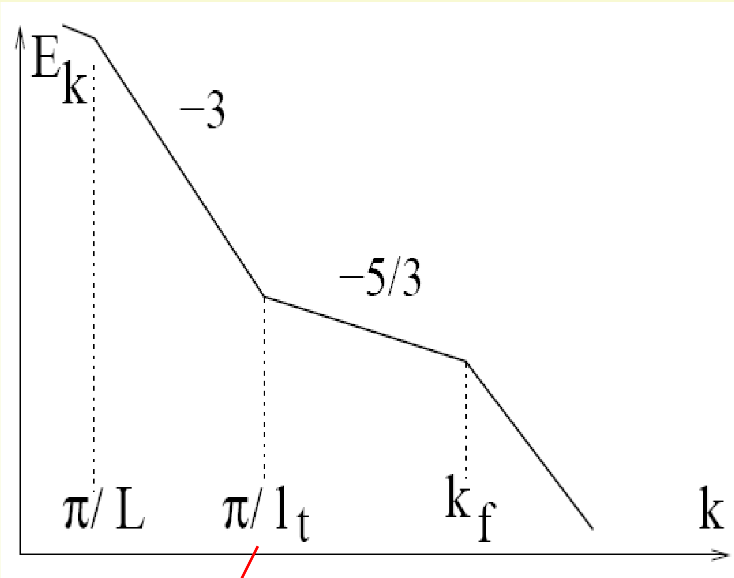
[Cho, Lindborg, J. Geophys. Res. (2001)]

Need to understand spectral flux in the presence of large coherent flow, which may affect higher moments

Condensate – coherent flow – self-generated by turbulence

In the lab can control strength and spectral extent of condensate (?)

Model of the spectrum



$$k_t = \frac{\pi}{l_t} \approx \pi L^{-3/2} \left(C\alpha/2\right)^{-3/4} \varepsilon^{1/4}$$

In the inverse cascade, the turnover time of the eddy of scale l is $t_l = l\big/\sqrt{S_2} \approx l^{2/3} C^{-1/2} \varepsilon^{-1/3}$

1. Assume that the condensate (vortex) appears when the system size L is such that $t_L\ \alpha < 1$
2. Characterize the condensate amplitude by its mean velocity V.

This velocity can be estimated from the energy balance, $\alpha V^2 \cong 2\varepsilon$ which gives $V \cong \sqrt{2\varepsilon/\alpha}$

3. We estimate that the condensate related velocity fluctuation on the scale l as Vl/L. Then we expect the knee of the spectrum to be at the scale l_t defined by

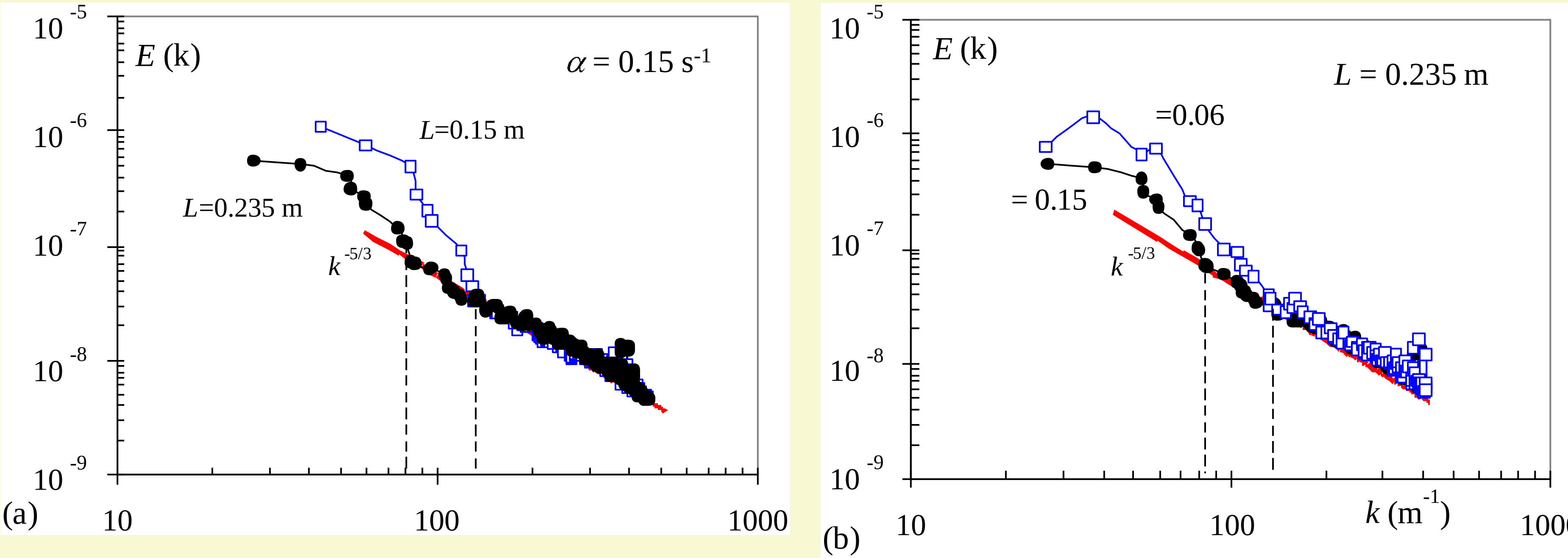
$$Vl_t/L \cong C^{1/2} \left(\varepsilon l_t\right)^{1/3}$$

This gives

$$l_t \approx L^{3/2} \left(C\alpha/2\right)^{3/4} \varepsilon^{-1/4}$$

Knee of the spectrum shifts with α and *L*

$$k_t = \frac{\pi}{l_t} \approx \pi L^{-3/2} \left(C\alpha / 2 \right)^{-3/4} \varepsilon^{1/4}$$



k_t increases with the decrease in the boundary size L

k_t increases with the decrease in the damping rate α

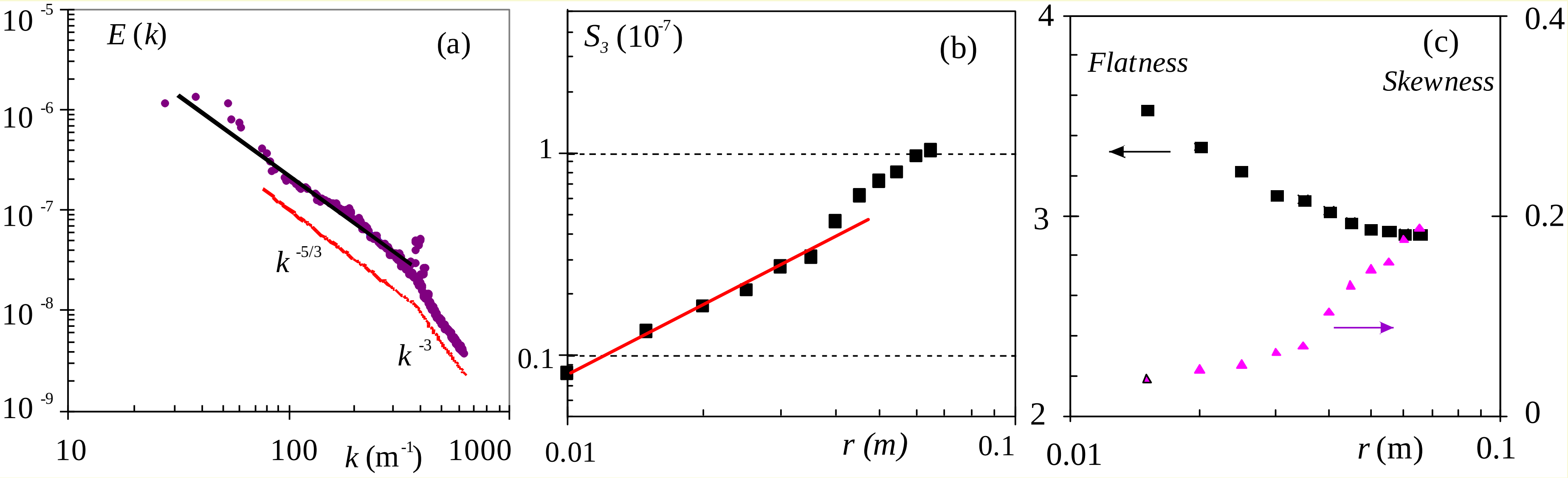
In the lab we can control strength, spectral extent of the condensate

Case of weak condensate

$$E(k) = C_k \varepsilon^{2/3} k^{-5/3} \qquad S_3(r) = \left(\langle \delta V_L^3 \rangle + \langle \delta V_L \delta V_T^2 \rangle\right)/2 = \varepsilon r$$

$$Sk = S_3 / (S_2)^{3/2}$$

$$F = S_4 / S_2^2$$



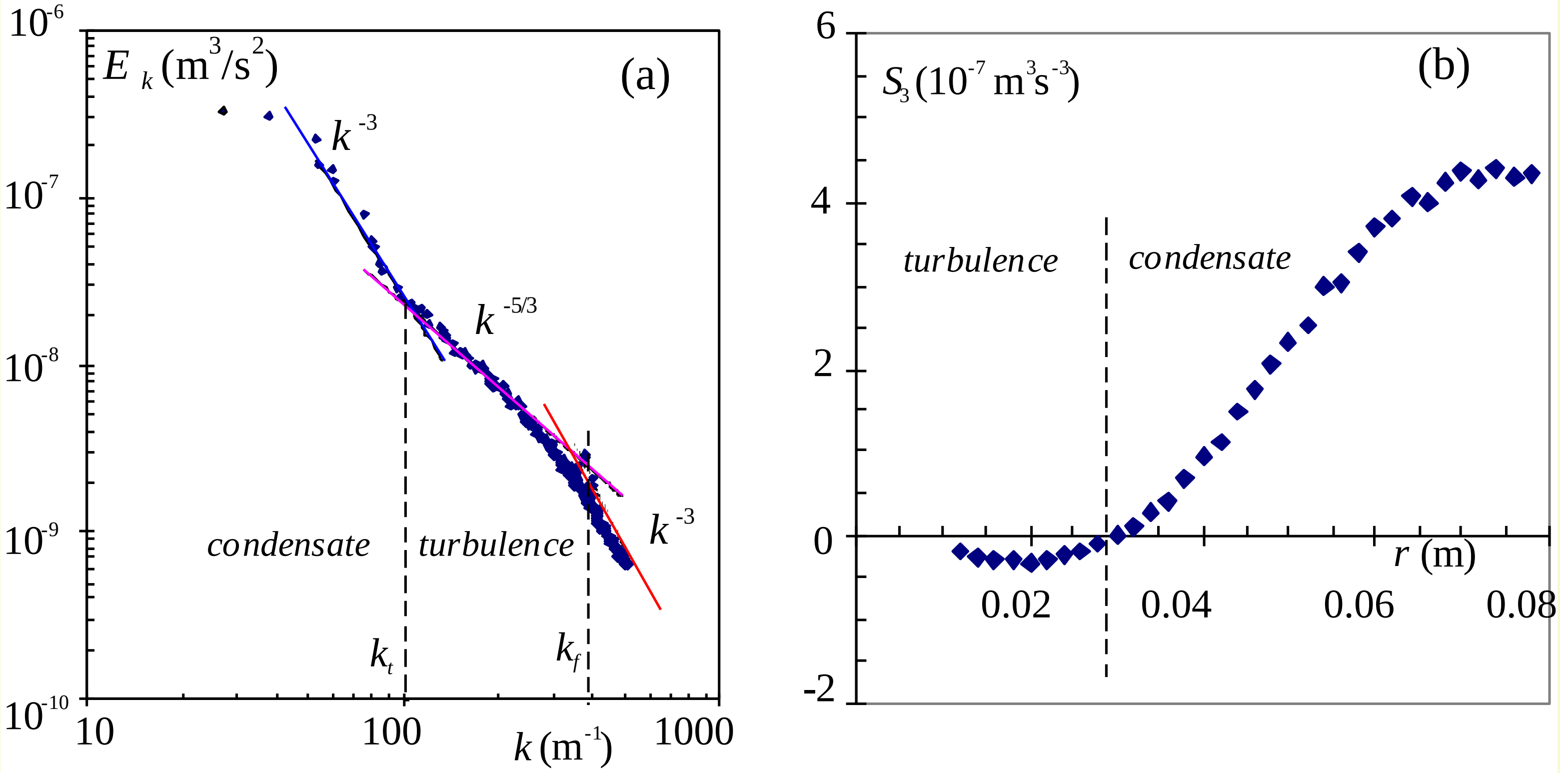
Weak condensate case shows small differences with isotropic 2D turbulence

~ $k^{-5/3}$ spectrum in the energy range

Kolmogorov law – linear S_3 (r) dependence; Kolmogorov constant C ≈ 5.6

Skewness and flatness are close to their Gaussian values (*Sk* = 0, *F* = 3)

Case of stronger condensate



Mean shear flow (condensate) $\delta\bar{V}$

changes all velocity moments:

$$\delta V = \delta\bar{V} + \delta\tilde{V}$$

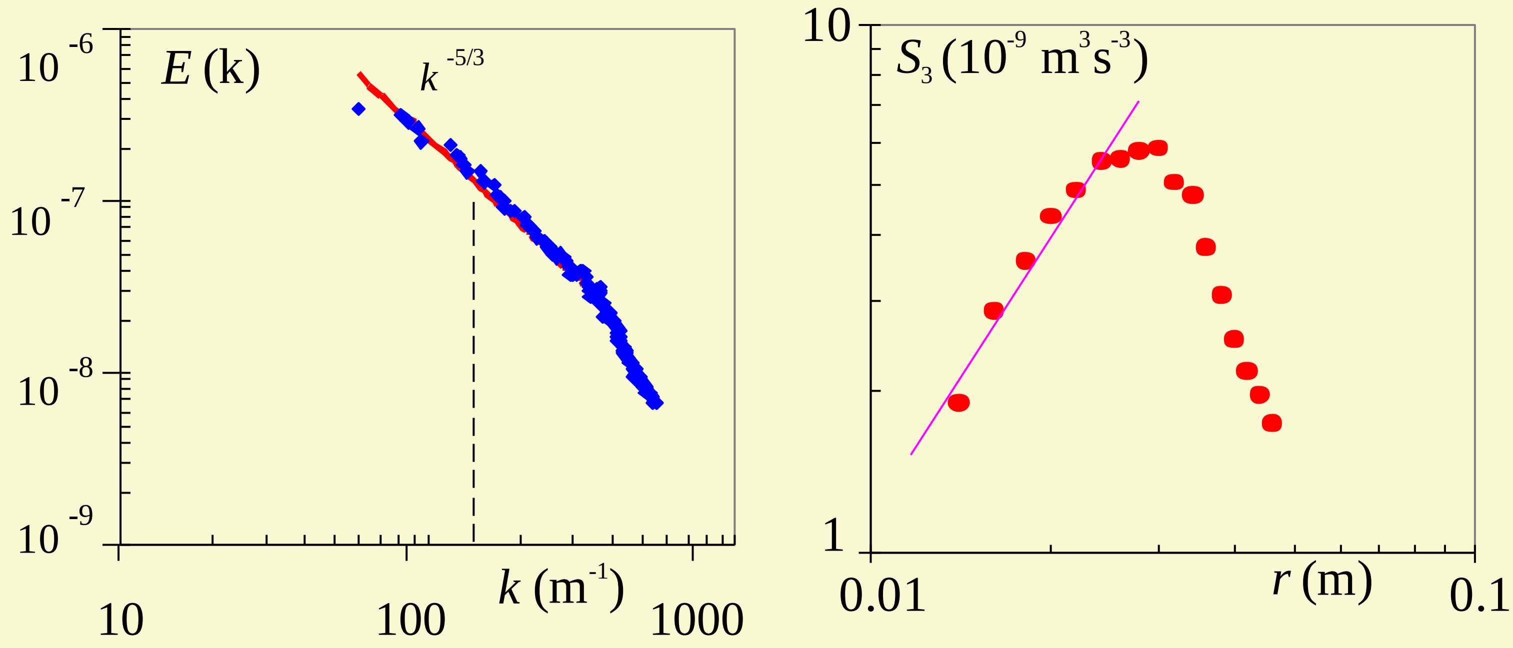
$$\left\langle \delta V^2 \right\rangle = \left\langle \delta\bar{V}^2 + 2\delta\bar{V}\delta\tilde{V} + \delta\tilde{V}^2 \right\rangle$$

$$\left\langle \delta V^3 \right\rangle = \left\langle \delta\bar{V}^3 - 3\delta\bar{V}^2\delta\tilde{V} + 3\delta\bar{V}\delta\tilde{V}^2 - \delta\tilde{V}^3 \right\rangle$$

Mean subtraction recovers isotropic turbulence

1.Compute time-average velocity field (*N*=400): $\overline{V}(x,y) = 1/N \sum_{n=1}^{N} V(x,y,t_n)$

2. Subtract $\overline{V}(x,y)$ from *N*=400 instantaneous velocity fields



Recover ~ k$^{-5/3}$ spectrum in the energy range

S_3 (r) is positive – recovered inverse energy cascade

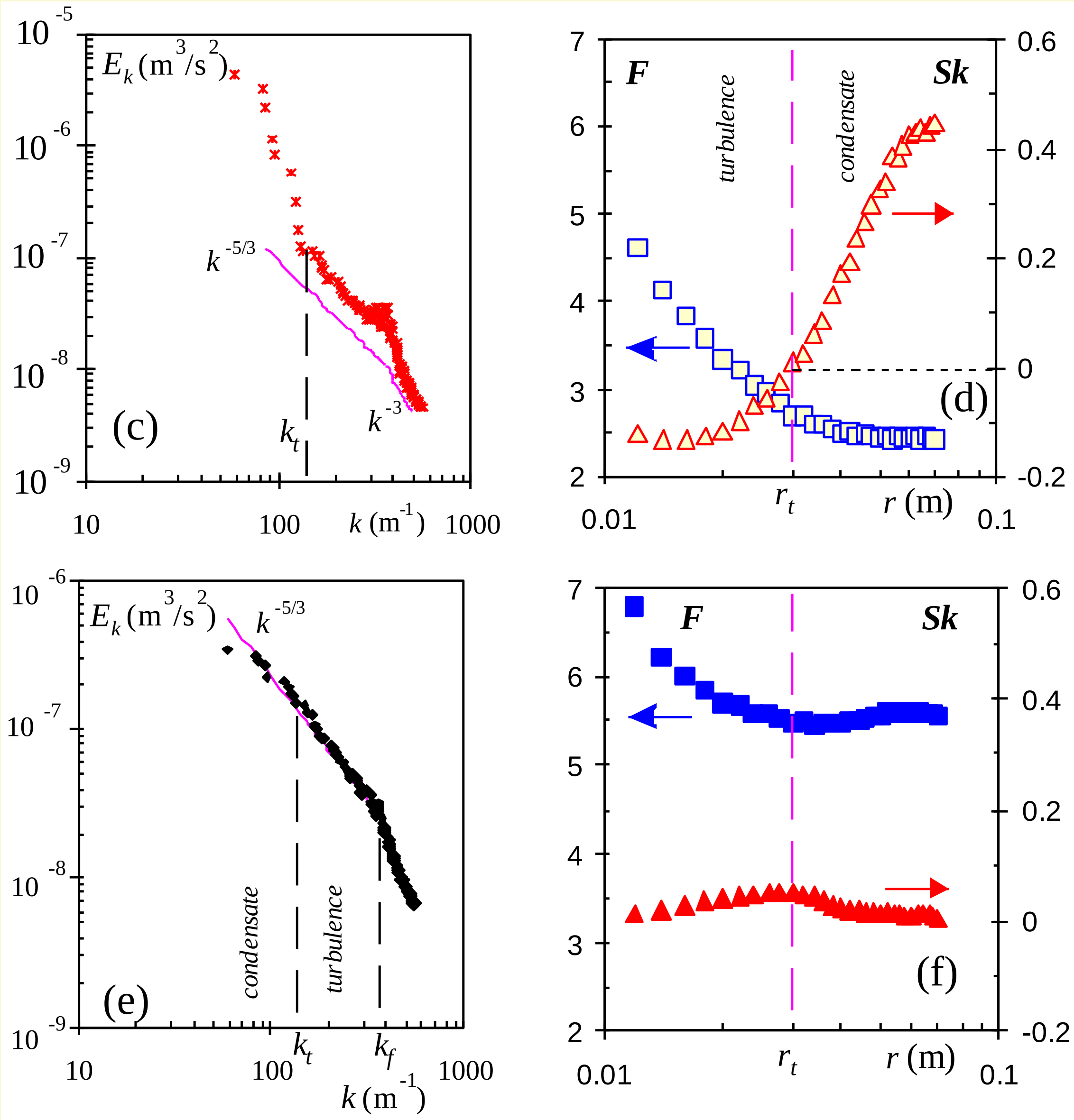
Kolmogorov law – linear S_3 (r) dependence in the “turbulence range”;

Kolmogorov constant C ≈ 7

Strong condensate: effect of mean subtraction

Before:

After:

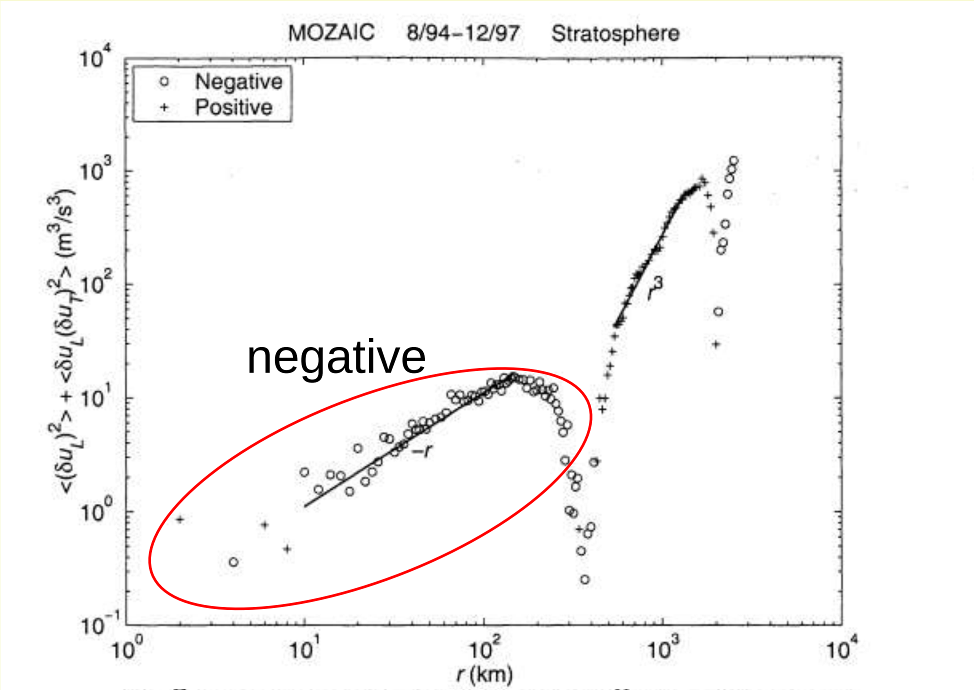


Normalized moments ~ scale-independent

Flatness is higher than in isotropic turbulence

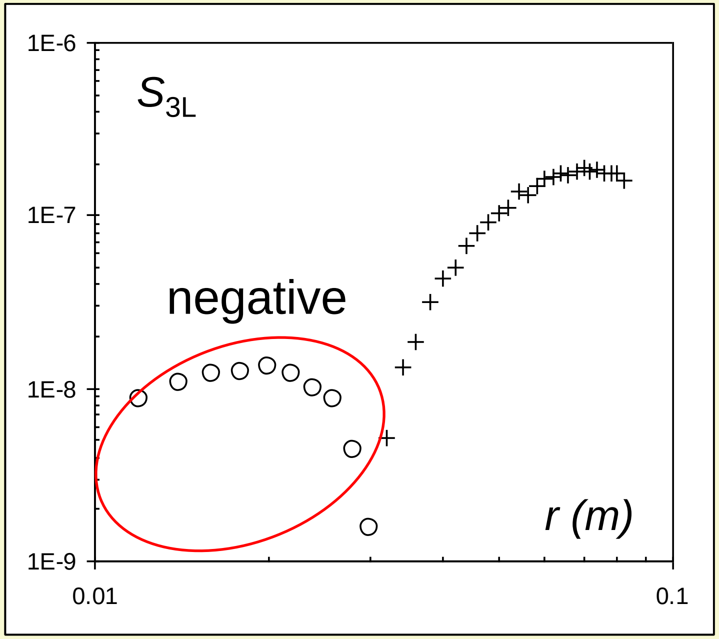
Similarity with atmospheric turbulence

[Cho, Lindborg, J. Geophys. Res. (2001)]



Circles and crosses indicate negative and positive values

Laboratory experiment
Stronger condensate, no mean subtraction



Mean shear flows present in the atmosphere affect velocity moments, similarly to laboratory experiments

Different moments affected in different ranges

Second moment

Large-scale flow is spatially smooth: $\delta V \approx sr$, $\left\langle \left(\delta V\right)^2 \right\rangle \approx s^2 r^2$

Small-scale velocity fluctuations in turbulence $\left\langle \left(\delta v\right)^2 \right\rangle \approx C \left(\varepsilon r\right)^{2/3}$

Small-scale fluctuations dominate at scales smaller than $l < l_t \approx C^{3/4} s^{-3/2} \varepsilon^{1/2}$

Third moment

Large-scale flow: $\left\langle \delta V^3 \right\rangle = \left\langle \delta V \delta v^2 \right\rangle \approx srC \left(\varepsilon r\right)^{2/3}$

Small-scale fluctuations: $\left\langle \left(\delta v\right)^3 \right\rangle \approx \varepsilon r$

Large-scale flow dominates 3rd moment in a range to much smaller scales:

$l_* \approx C^{-3/2} s^{-3/2} \varepsilon^{1/2}$, since C > 1

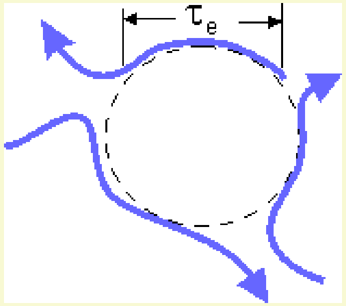
Condensate imposes different scales on different moments

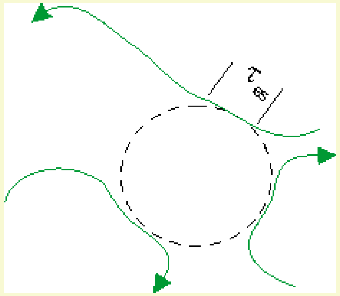
The strongest condensate can suppress turbulence

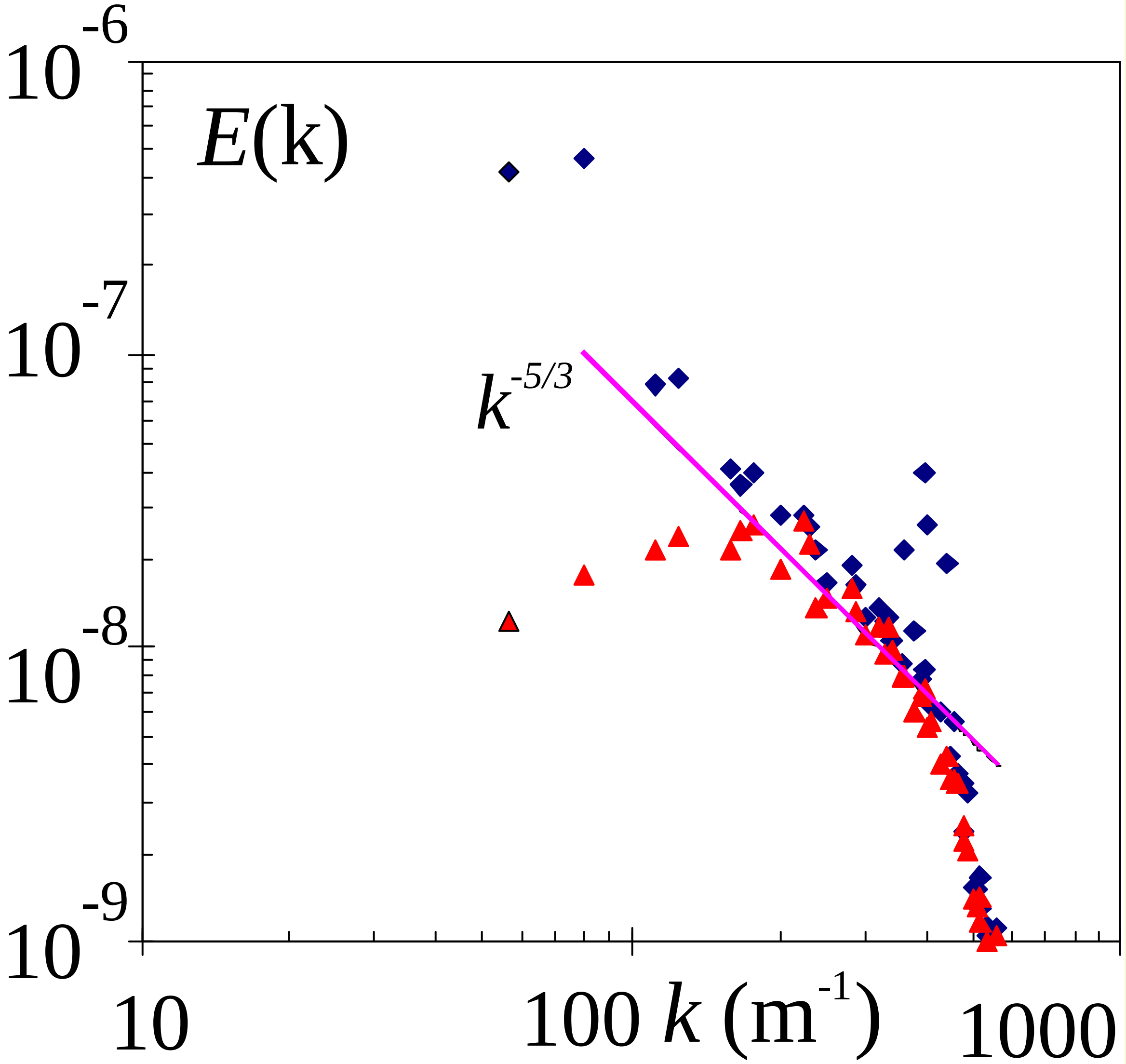
(a) Self-generated condensate

(b) Externally imposed flow

Shear

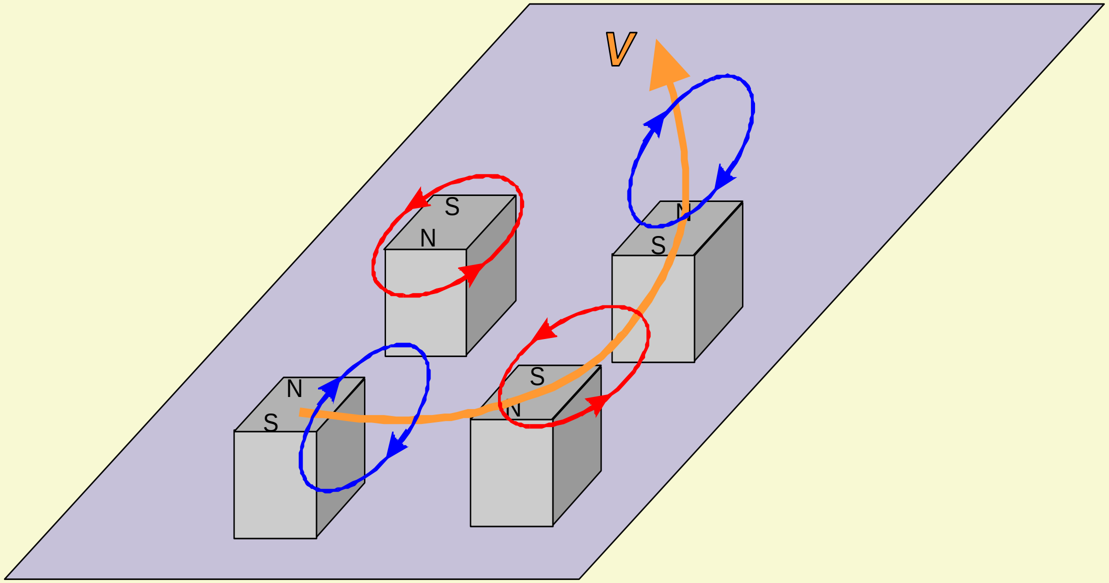


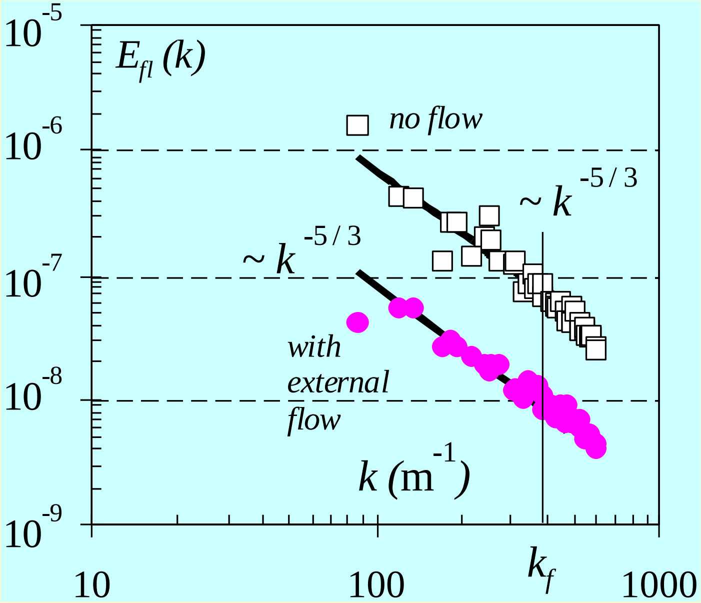


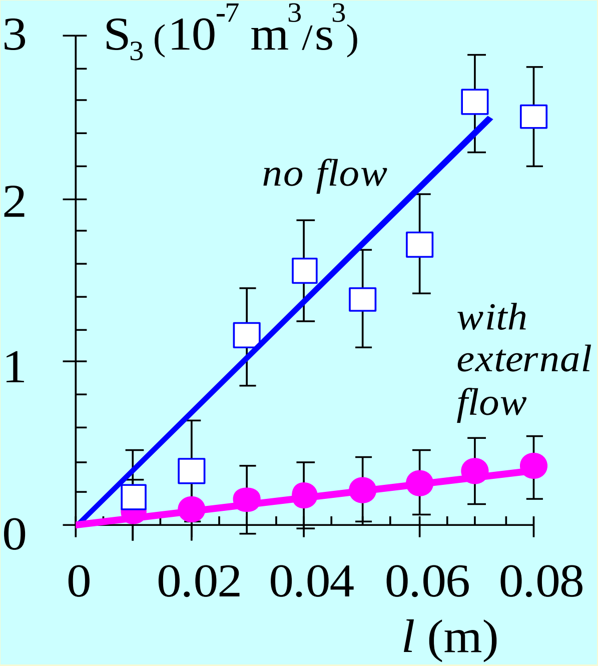


[M.G. Shats, H. Xia, H. Punzmann and G. Falkovich (2007)]

Sweeping of the forcing scale







Sweeping due to mean flow

Imposed flow affects scales down to the forcing scale

Mean flow sweeps forcing scale vortices relative to magnets

Sweeping

becomes important when

sweeping parameter

$$sw = \omega_{sw}\tau_e = \frac{V_\theta}{\sqrt{S_2}} > 1$$

Eddy life time

$$\tau_e = l / \sqrt{\left\langle \left| \delta u^2(l) \right| \right\rangle}$$

Sweeping is more efficient on small scales:

$$sw = \omega_{sw}\tau_e = \frac{V_\theta}{S_1} \propto l^{-1/3}$$

Summary

- Spectral condensation leads to the generation of mean flow coherent across the system size
- Spectrum of condensed turbulence shows 3 power laws:
  $\sim k^{-3}$ at large scales; $k^{-5/3}$ in the meso-scales, $k^{-(3\text{-}4)}$ at small scales
- Condensate modifies statistical moments of velocity fluctuations
- Different moments are affected in different ranges of scales
- Velocity moments of condensed turbulence similar to those in the atmosphere
- Coherent shear flow suppresses turbulence which generates it via shearing and sweeping