How Republican is the Ocean?
Some Challenges of non-Conservative Ocean Dynamics in Applied Mathematics

William K. Dewar, FSU

collaborators:
R.J. Bingham, R. Iverson, L. St. Laurent, D. Nowacek, P. Wiebe, P. Berloff, A. Hogg, JC McWilliams, MJ Molemaker
Consider ‘Mixing’.

Munk and Wunsch, 1998
Mixing energy (2.4TW) supplied by external sources.

St Laurent and Simmons (2006)
How much energy are we talking about anyway?

How large of a column of the ocean would a typical kitchen mixer mix?

\[
\frac{2.4 \times 10^{12} W}{3.5 \times 10^{14} m^2} = \frac{.007 W}{m^2} \quad \frac{200 W m^2}{.007 W} : (200 m)^2
\]
Point: The Ocean is ‘extremely’ conservative in its properties, but the ‘weak’ non-conservative effects are essential to its dynamics.

Key problem: How to model? Enormous scale disparity, probably requiring parameterization.

Objective: Discuss two examples and outline open questions.
Numerical problems associated with capturing this weak level of mixing have prompted novel model constructions. Ex: MICOM – isopycnals
HYCOM, GOLD – Hybrid MICOM
ROMS – Terrain following

Because of the EOS, what is a good isentropic surface? Can we numerically close ocean energy budgets?
Physeter Macrocephalus (aka sperm whale)
Architeuthis dux (aka giant squid)

Our Hypothesis:
Swimmers by kinetic activities mix the ocean
Marine Biosphere impacts ocean mixing as effectively as the winds and tides.
Diel Migrators

Wiebe, 1979
Recent Efforts – work in progress

Tongue of the Ocean

Principles: Nowacek St. Laurent
This is a poorly studied problem in turbulence
There are at least three length scales in this problem
1. individual
2. inter-cloud separation
3. cloud scale (actually two of these)

*Catton, Webster and Yen (OS, 2008), in tank experiments, conclude krill aggregations define the effective length scale of their mixing.*

Can the cloud mixing ‘efficiency’ be computed?
Can we apply to schools of fish?
What about direct fluid transport ala Dabiri/Katija?
But, beware: its not all zooplankton!
Q: How many giant squid are there?

A: 1 Billion

\[10^9 \text{ A. dux} / 3.5 \times 10^8 \text{km}^2 = 3 \text{ per square km}\]
Run of the Mill TMR is easily 1W/kg
Swimming inefficiency of 10%
~30W/sqkm

Can we develop a swimming theory for cephalopods like that for thunniforms?
Mesonychoteuthis
Part II: Balanced Energetics

Turn on a global ocean model and what do you see?

http://www7320.nrlssc.navy.mil/GLBhycom1-12/navo/globalsss_nowcast_anim30d.gif
Many definitions exist, but all have diagnostic momentum equations.

\[ u_t + uu_x + vu_y + wu_z + fv = -p_x \]
\[ v_t + uv_x + vv_y + wv_z - fu = -p_y \]

The simplest example is geostrophy:

\[ fv = p_x \]
\[ fu = -p_y \]

Tend to large scales and subinertial frequencies.
More formally, balanced flows have a ‘potential vorticity’ that is diagnostically linked to the dynamical fields:

$$\nabla^2 p = q$$

A consequence: difficult for these flows to dissipate. Energetics budgets in models?
FIGURE 8. Comparison of the full (top), balanced (middle) and unbalanced (bottom) components of the displacement field (in a $y = 0$ cross section) at 20 QG time units for $c = 10$ and for the effective Rossby numbers indicated. The contour intervals for the full and balanced fields are $\Delta = 0.008$. The unbalanced contour intervals are $1/50$th of the balanced contour intervals.
Comparable pv question:

$$\frac{\partial}{\partial t} q = -\nabla g F; \ F = uq - Xx \nabla \rho - \omega H$$
What about external effects, eg topography
Temperature at western wall
A Theory of Wall Interaction

\begin{align*}
    u_t + uu_x + vu_y - fv &= -M_x \\
    v_t + uv_x + vv_y + fu &= -M_y \\
    M &= p + \rho g z \\
    q_t + uq_x + vq_y &= 0; \quad q = (f + v_x - u_y) / z \rho
\end{align*}

EOMs in density coordinates
At the wall, normal flow vanishes

\[
\begin{align*}
y_t + u y_x + v y_y - f v &= -M_x \\
v_t + u v_x + v v_y + f u &= -M_y
\end{align*}
\]

\[
M_\rho = g z
\]

\[
M = p + \rho g z
\]

\[
q_t + u q_x + v q_y = 0; \quad q = (f + v_x - y_y) / z_\rho
\]

At the wall, normal flow vanishes
\[ f(v_g + v') = M_{gx} + M'_x \]
\[ v'_t + \left( \frac{v'^2}{2} \right)_y + (v_g v')_y + M'_y = -v_{gt} - v_g v_y - M_{gy} \]

**Exact pv solution**

\[ \frac{(f - v_x)}{z_{\rho}} = q(x, y, \rho, t) = q(0, y_0, \rho, 0) = \frac{fg}{M_{\rho\rho}} \]

\[ M'_{xx} = \frac{f^2}{M_{\rho\rho}} M'_{\rho\rho} \]

The only assumption: hydrostatics!
Solutions:

Total Velocity, Northward Moving Anticyclone

Day 0

Day 4

Total Velocity, Southward Moving Cyclone

Day 0

Day 4

MITgcm
When linearized, above set has yielded much useful information about quasi-1d cases:

\[
\begin{align*}
\nu_t' + \left( \frac{\nu'^2}{2} \right)_y + (\nu_g \nu')_y + M'_y &= -v_{gt} - v_g \nu_{gy} - M_{gy} \\
M'_{xx} &= \frac{f^2}{M_{\rho\rho}} M'_{\rho\rho}
\end{align*}
\]

and the interesting nonlinear eigenvalue problem

\[
M'_{\rho\rho} + \lambda^2 \frac{1}{\sqrt{1 - \frac{2M'}{c^2}}} = \lambda^2 \\
M'_{\rho} = g z'(x, y, \rho, t) = 0 \text{ at } \rho = \rho_b, \rho_s
\]
\[ v'_t + \left( \frac{v'^2}{2} \right)_y + (v_g v'_y)_y + M'_y = -v_{gt} - v_g v_{gy} - M_{gy} \]

\[ M'_{xx} = \frac{f^2}{M_{\rho\rho}} M'_{\rho\rho} \]

Also shows need for non-hydrostatic parameterization
Generalization to ‘realistic’ topography?
There is no theory for this case.
Connections to models

An ultra-fine embedded solution for Monterey Bay
Summary: The ocean is extremely conservative, but:

- non-conservative processes cannot be ignored for climate modeling purposes
  - set water mass distributions and the energy levels of the balanced flows
- are extremely subtle to capture correctly
- certainly involve processes that are poorly understood AND parameterized in suspect forms in all current climate models

Two examples:
Mixing by clouds of smallish migrants and large unusually shaped organisms
Topography - Candidate equation gives hopes for parameterization of pv fluxes
- Generalization to more complex topographies and turbulent settings?