Reduced models of large-scale ocean circulation

R. M. Samelson
College of Oceanic and Atmospheric Sciences
Oregon State University

rsamelson@coas.oregonstate.edu

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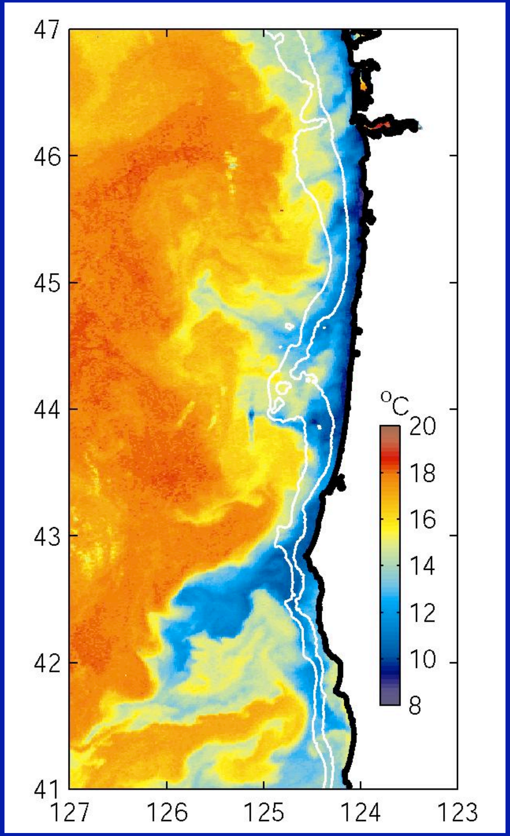
The Coastal Ocean

Sea-surface Temperature



24 Aug 2003

Off Oregon and California, the coastal ocean includes shelf (depths less than 200 m), slope, adjacent ocean interior (depths greater than 3000 m)

Oregon coastal domain:

- several hundred km offshore
- 41ºN-47ºN alongshore

Equations (Physical Principles)

- Mass
- Momentum
- Salt
- Thermodynamic energy
- Equation of state

$$\boxed{\frac{DF}{Dt} = \frac{\partial F}{\partial t} + \mathbf{u} \cdot \nabla F}$$

$$\begin{aligned}
\frac{D\rho}{Dt} + \rho \nabla \cdot \mathbf{u} &= 0, \\
\rho \left(\frac{D\mathbf{u}}{Dt} + 2\Omega \times \mathbf{u} \right) &= -\nabla p - \rho \nabla \Phi - \nabla \cdot \mathbf{d} \\
\frac{DS}{Dt} &= k_S \nabla^2 S, \\
\rho \left[\frac{D\hat{e}}{Dt} + p \frac{D}{Dt} \left(\frac{1}{\rho} \right) \right] &= k_T \nabla^2 T + \chi + \rho Q, \\
\rho &= \mathcal{R}(p, T, S).
\end{aligned}$$

Scales of ocean fluid (continuum) motion

Motions of interest:

at least 10 km = 10^4 m (horizontal)

at least 10 m (vertical)

Smallest scale: of order 1 mm = 10^{-3} m

Ratio large/small:

at least 10^7 (horizontal)

at least 10^4 (vertical)

Degrees of freedom: at least $10^7 \times 10^7 \times 10^4 = 10^{18}$

DNS is well beyond current computing capacity

=> This is still a physics (& mathematics) problem!

Ocean Circulation:
Mathematics or Applied Mathematics?

…Applied Mathematics???

There is no such thing. (Mathematics is not paint.)

Theorem: The object of study either has a mathematical structure, or it does not.

Proof: Examination.

Corollary (Remark):

Any such structure either is interesting, or is not.

Proof: *De gustibus non est disputandum.*

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$$
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\rho &= \mathcal{R}(p, T, S).
\end{aligned}
$$

Scales of motion

Motions of particular interest have typical scales…

10^4 m or greater (horizontal)
10 m or greater (vertical)
1 m s^{-1} or less (horizontal velocity)
10^{-3} m s^{-1} or less (vertical velocity)
10^4 s or more (time)
15 °C or less (temperature)
35 parts per 1000 or less (salinity)

=> Use *a-priori* estimates of individual terms
to simplify equations

Equations (Physical Principles)

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$$
\begin{aligned}
\frac{D\rho}{Dt} + \rho\nabla\cdot\mathbf{u} &= 0, \\
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\rho\left[\frac{D\hat{e}}{Dt} + p\frac{D}{Dt}\left(\frac{1}{\rho}\right)\right] &= k_T\nabla^2 T + \chi + \rho Q, \\
\rho &= \mathcal{R}(p, T, S).
\end{aligned}
$$

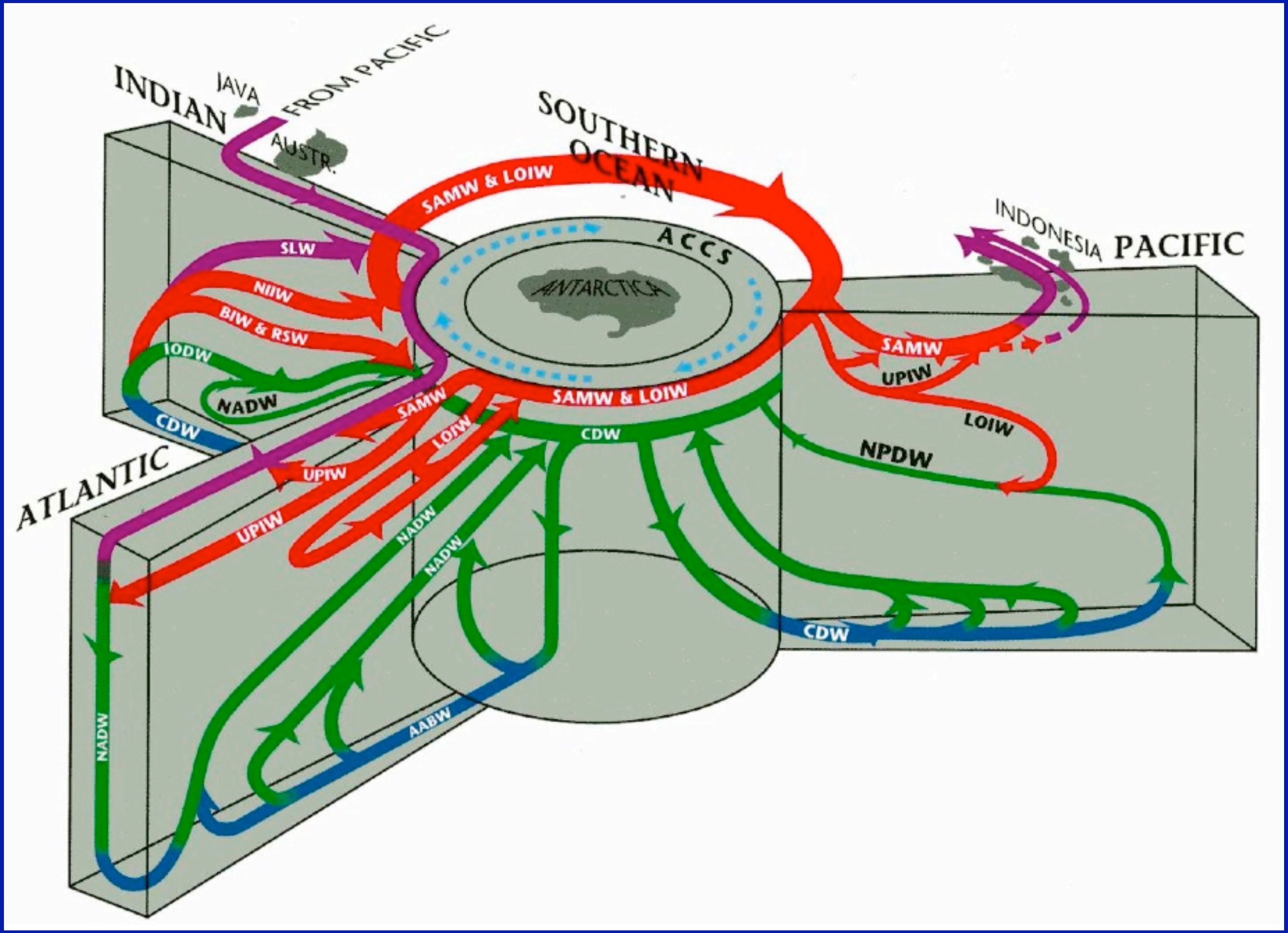
Hydrostatic Boussinesq Primitive Equations for Ocean Circulation Modeling

- Mass
- Momentum
- Salt
- Thermodynamic energy
- Equation of state

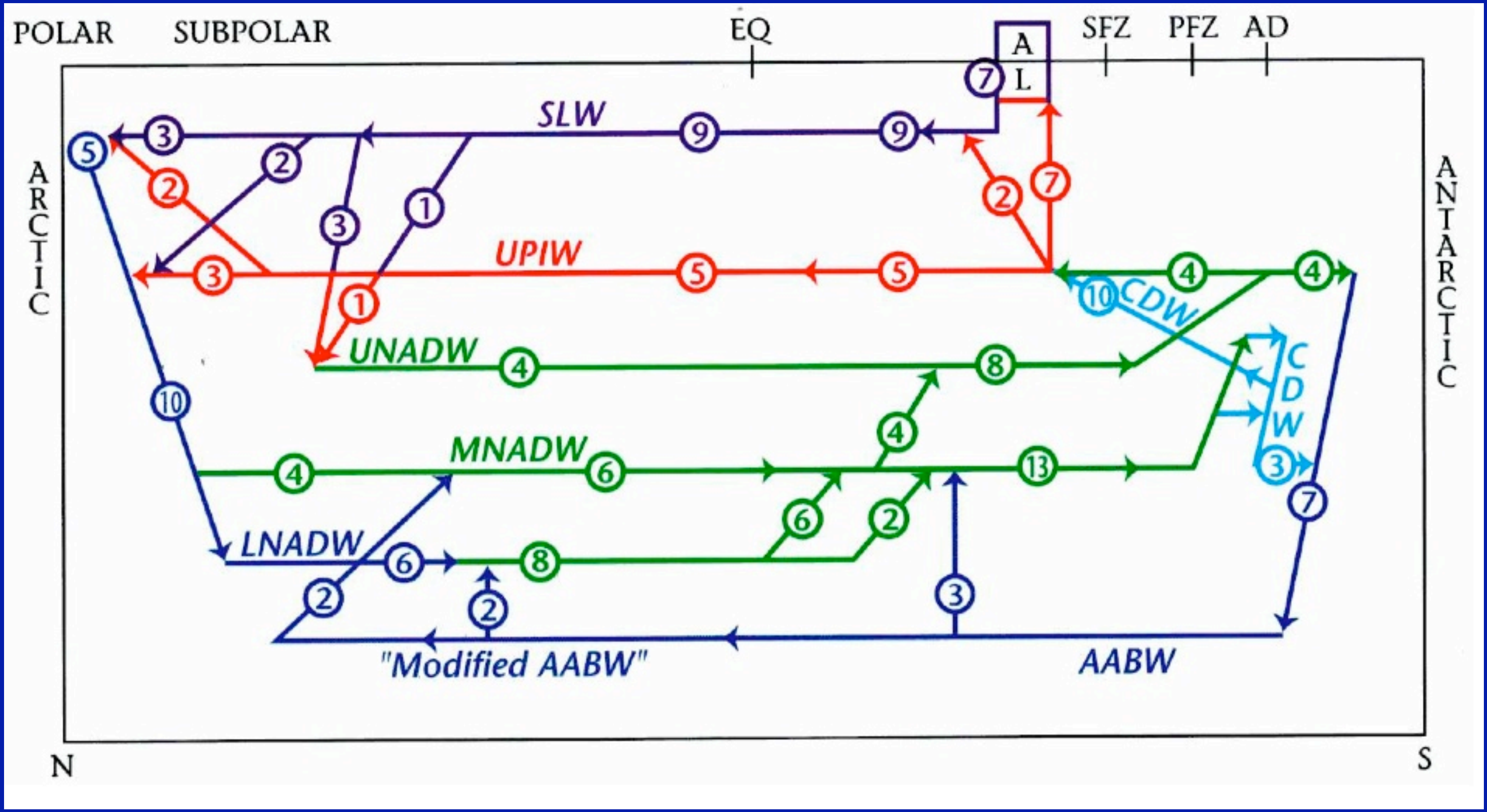
$$\begin{aligned}
\nabla \cdot \mathbf{u} &= 0 \\
\frac{D\mathbf{u}_H}{Dt} + 2\Omega \sin\phi\, \mathbf{k} \times \mathbf{u}_H &= -\frac{1}{\rho_0}\nabla_2 p, \quad \frac{\partial p}{\partial z} = -g\rho \\
\frac{DS}{Dt} &= 0 \\
\frac{DT}{Dt} &= 0 \\
\rho &= \mathcal{R}(p, T, S)
\end{aligned}$$

…plus models of turbulent diffusion.

Schmitz (1996): Thermohaline circulation



Schmitz (1996): Atlantic overturning



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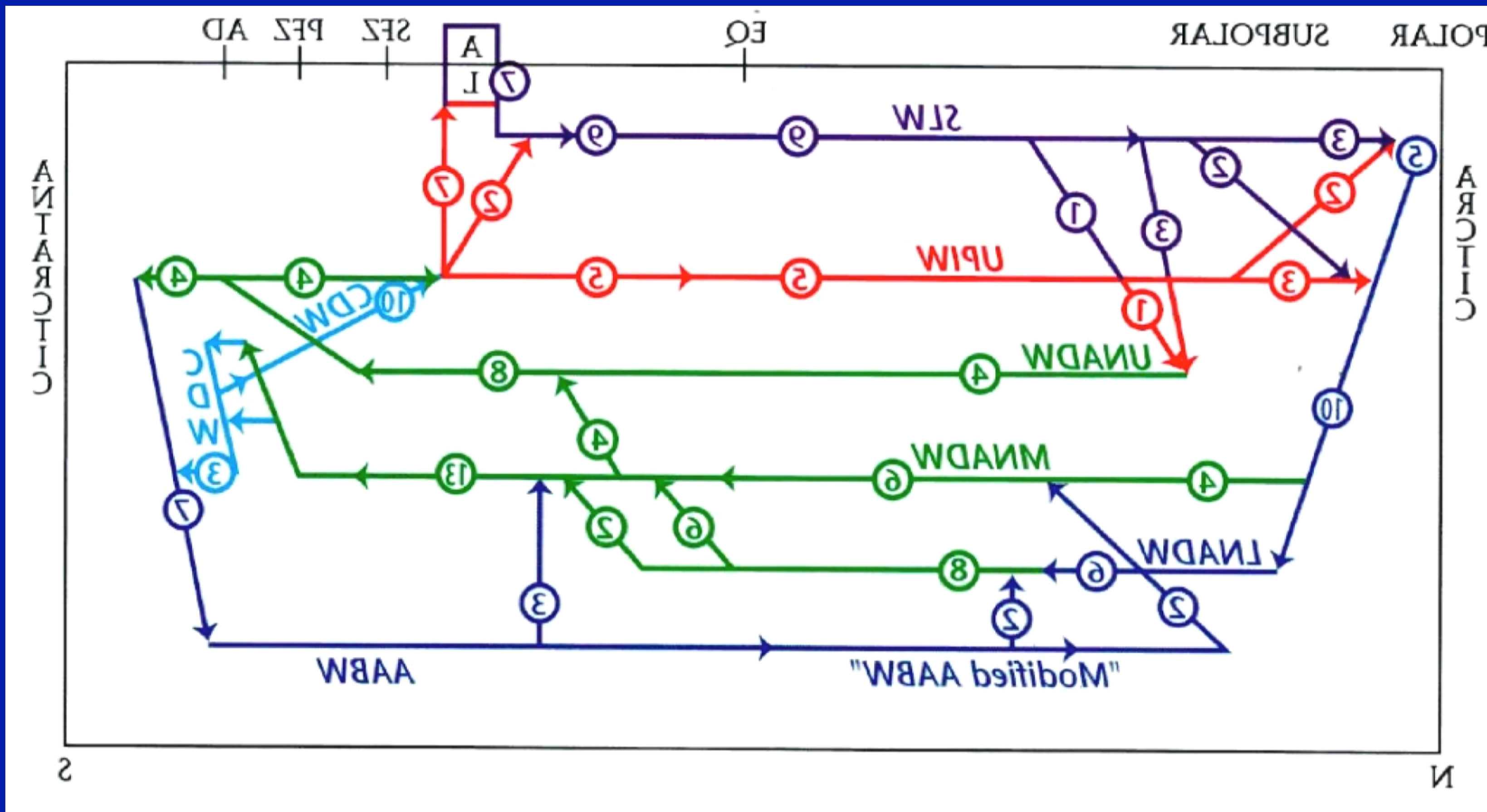
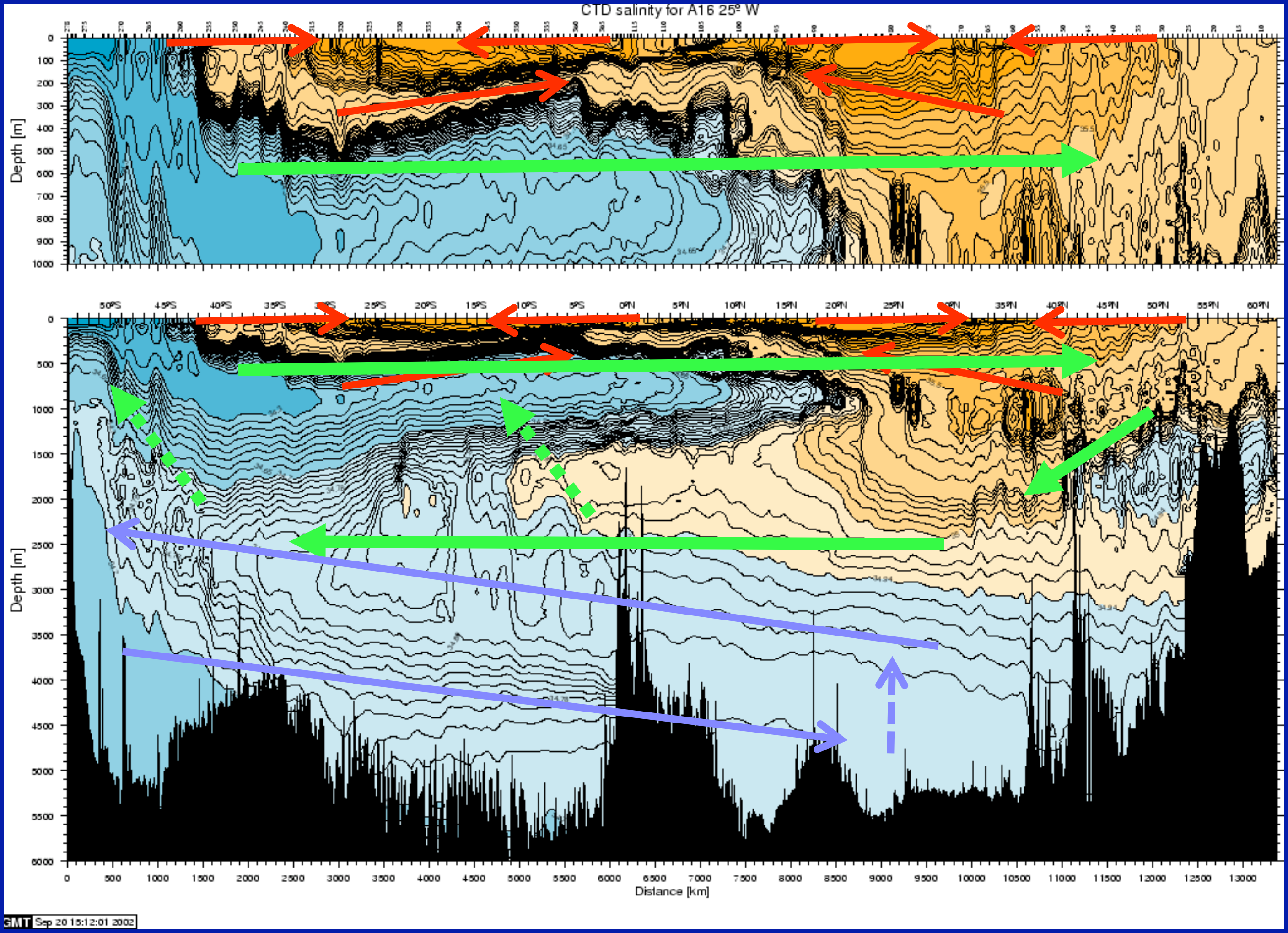


The mid-depth meridional overturning: NADW and the warm return flow

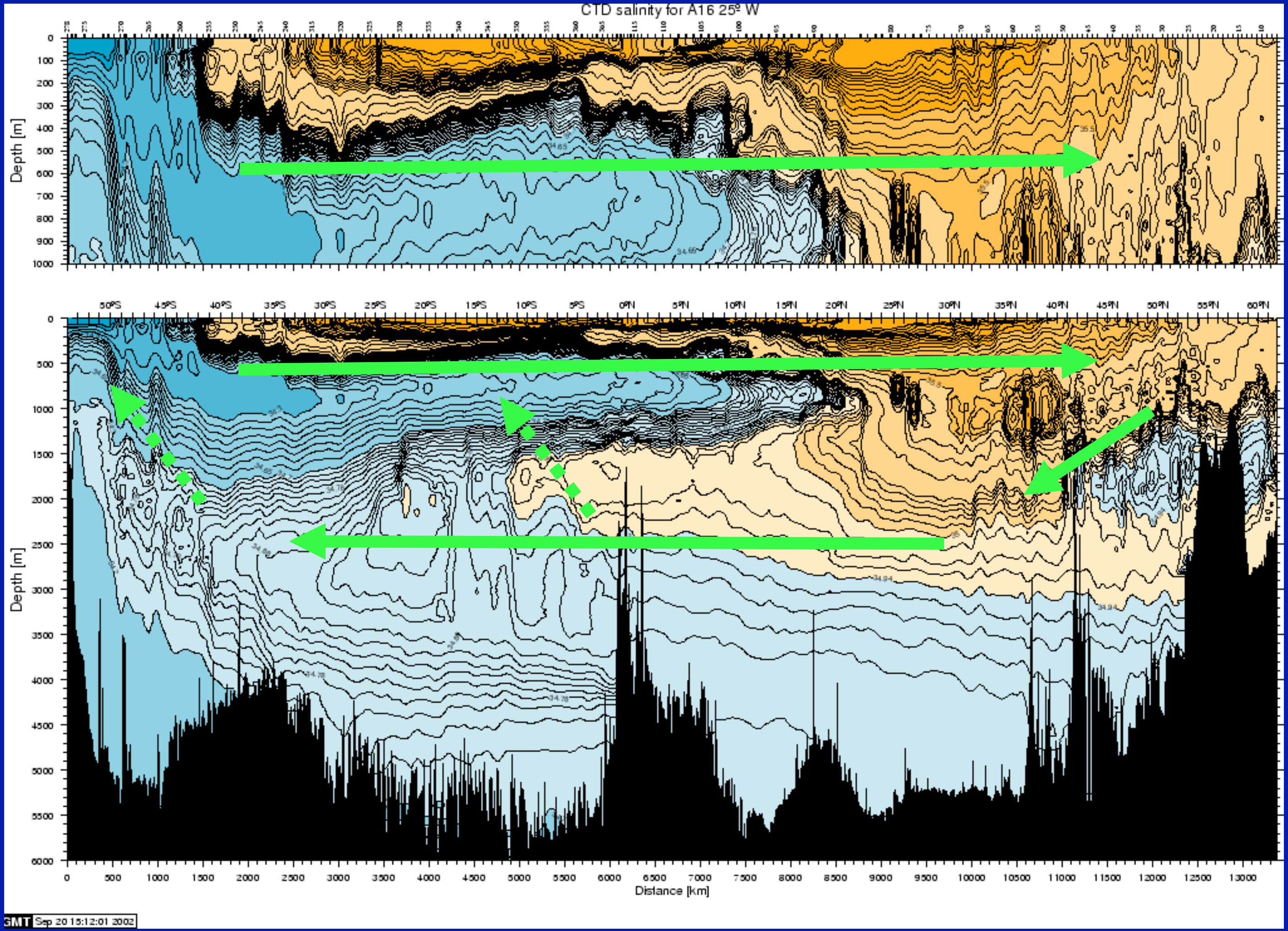


55 S

WOCE A16 Salinity - 25 W

60 N

The mid-depth meridional overturning: NADW and the warm return flow



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Planetary Geostrophic (PG) equations

Start with:

'Rotating compressible Euler plus thermodynamic and salt equations and empirical EOS.'

Then nondimensionalize for planetary-scale motion and approximate:

1. Hydrostatic approximation (vertical momentum equation)
2. Geostrophic approximation (horizontal momentum equation)
3. Boussinesq approximation (mass and thermodynamic equations; momentum equations)
4. Linearized equation of state (often)
5. Geometric simplification (distortion, for convenience: β-plane)

Classical PG equations:

$$\begin{aligned} -fv &= -p_x \\ fu &= -p_y \\ 0 &= -p_z + T \\ u_x + v_y + w_z &= 0 \\ T_t + uT_x + vT_y + wT_z &= 0 \end{aligned}$$

where (β-plane)

$$f = f_0 + \beta(y - y_0).$$

NB: T may represent temperature, potential temperature, or a combined temperature-salinity ('buoyancy') variable. Salinity S has been dropped or subsumed into the T variable. The S conservation equation, nonlinear equation of state, and independent boundary conditions can be reinstated, but PV conservation breaks down and analysis becomes more difficult.

Sverdrup vorticity relation:

$$\beta v = \beta f^{-1} p_x = f w_z$$

Geostrophic Sverdrup transport balance:

$$\beta V = \beta \int v\, dz = f(w_{top} - w_{bottom}) \quad \{= f w_E = f[(\tau^y/f)_x - (\tau^x/f)_y)] \quad \text{if} \quad w_{bottom} = 0\}$$

Potential vorticity (PV) conservation:

$$Q = f T_z \quad \Rightarrow \quad Q_t + u Q_x + v Q_y + w Q_z = 0$$

Bernoulli conservation:

$$B = p - zT \quad \Rightarrow \quad B_t + u B_x + v B_y + w B_z = B_t$$

Welander M-equation:

$$M_z = p, \quad M_x = \beta^{-1} f^2 w \quad \Rightarrow \quad M_{zzt} + f^{-1}(M_{xz} M_{zzy} - M_{yz} M_{zzx}) + \beta f^{-2} M_x M_{zzz} = 0$$

Classical PG equations for layers $j,\ j=M,\ldots,N$:

$$\begin{aligned}
-fv_j &= -p_{jx} \\
fu_j &= -p_{jy} \\
0 &= -p_{jz}+T_j \\
h_{jt}+(u_jh_j)_x+(v_jh_j)_y &= 0 \quad (j>M) \\
h_{Mt}+(u_Mh_M)_x+(v_Mh_M)_y &= -w_E
\end{aligned}$$

Here j increases downward from the uppermost (M) to the deepest moving (N) layer, and h_j is the layer-j thickness. The layer-j potential vorticity and Bernoulli functions are:

$$Q_j=\frac{f}{h_j}, \quad B_j=p_j.$$

Since by assumption layer $N+1$ is stagnant ($\nabla_2 p_{N+1}=0$),

$$p_j=\sum_{k=j}^{N}\gamma_k H_k, \quad \text{where} \quad H_j=\sum_{k=M}^{j}h_k \quad \text{and} \quad \gamma_j=T_j-T_{j+1}.$$

NB: The vertical coordinate z has been replaced by the index j, the T_j are constants, and all variables $(u_j,v_j,p_j,h_j,Q_j,B_j,H_j)$ are functions of (x,y) and t only.

Diffusive driving and energetics:

Consider the steady PG equations with horizontal friction, diffusive heat flux, and convective adjustment in basin with vertical sidewalls and flat bottom.

$$
\begin{aligned}
-fv &= -p_x + \mathscr{F}^x \\
fu &= -p_y + \mathscr{F}^y \\
0 &= -p_z + T \\
u_x + v_y + w_z &= 0 \\
uT_x + vT_y + wT_z &= \kappa_v T_{zz} + \kappa_h \Delta_2 T + \mathscr{C}
\end{aligned}
$$

No flow or heat flux through solid boundaries, no flow through surface:

$$
\begin{aligned}
&(u,v)\cdot\mathbf{n} = \kappa_h \nabla_2 T \cdot \mathbf{n} = 0 \quad \text{on sidewalls} \\
&w = \kappa_v T_z = 0 \quad \text{at} \quad z = -D \qquad \text{and} \qquad w = 0 \quad \text{at} \quad z = 0
\end{aligned}
$$

Mechanical energy equation:

$$
\int wT\,dxdydz = \mathscr{D} = -\int (u\mathscr{F}^x + v\mathscr{F}^y)\,dxdydz > 0
$$

since

$$
\int (u,v,w)\cdot\nabla p\,dxdydz = \int p(u,v,w)\cdot\mathbf{n}\,dS = 0
$$

Thermodynamic equation:

$$
\begin{aligned}
\int_{-D}^{z}\int_{(x,y)}(u,v,w)\cdot\nabla T\,dxdydz &= \int \nabla\cdot[(u,v,w)T]\,dxdydz \\
&= \int_{-D}^{z}\oint T\,(u,v)\cdot\mathbf{n}\,dl dz + \left[\int_{(x,y)} wT\,dxdy\right]_{-D}^{z} \\
&= \int_{(x,y)} wT\,dxdy|_{z} \\
&= \int_{-D}^{z}\int_{(x,y)} \nabla\cdot(\kappa_h\nabla_2 T,\kappa_v T_z)\,dxdydz + \int_{-D}^{z}\int_{(x,y)}\mathscr{C}\,dxdydz \\
&= \int_{-D}^{z}\int_{(x,y)} \nabla\cdot(\kappa_h\nabla_2 T,\kappa_v T_z)\,dxdydz + \int\mathscr{C}\,dxdydz \\
&= \int_{-D}^{z}\int_{(x,y)} \nabla_2\cdot\kappa_h\nabla_2 T\,dxdydz + \int_{(x,y)}\int_{-D}^{z}\kappa_v T_{zz}\,dzdxdy + \int\mathscr{C}\,dxdydz \\
&= \int_{-D}^{z}\oint(\kappa_h\nabla_2 T\cdot\mathbf{n})\,dl dz + \left[\int_{(x,y)}\kappa_v T_z\,dxdy\right]_{-D}^{z} + \int\mathscr{C}\,dxdydz \\
&= \int_{(x,y)}\kappa_v T_z\,dxdy|_{z} + \int_{-D}^{z}\int_{(x,y)}\mathscr{C}\,dxdydz
\end{aligned}
$$

So:

$$
\begin{aligned}
\int wT\,dxdydz = \mathscr{D} &= \int_{-D}^{0}\left[\int_{(x,y)}\kappa_v T_{z'}\,dxdy|_{z'} + \int\mathscr{C}\,dxdydz'\right]dz \\
&= \int_{(x,y)}\kappa_v[T(z=0)-T(z=-D)]\,dxdy + \int_{-D}^{0}\left[\int_{-D}^{z'}\int_{(x,y)}\mathscr{C}\,dxdydz'\right]dz
\end{aligned}
$$

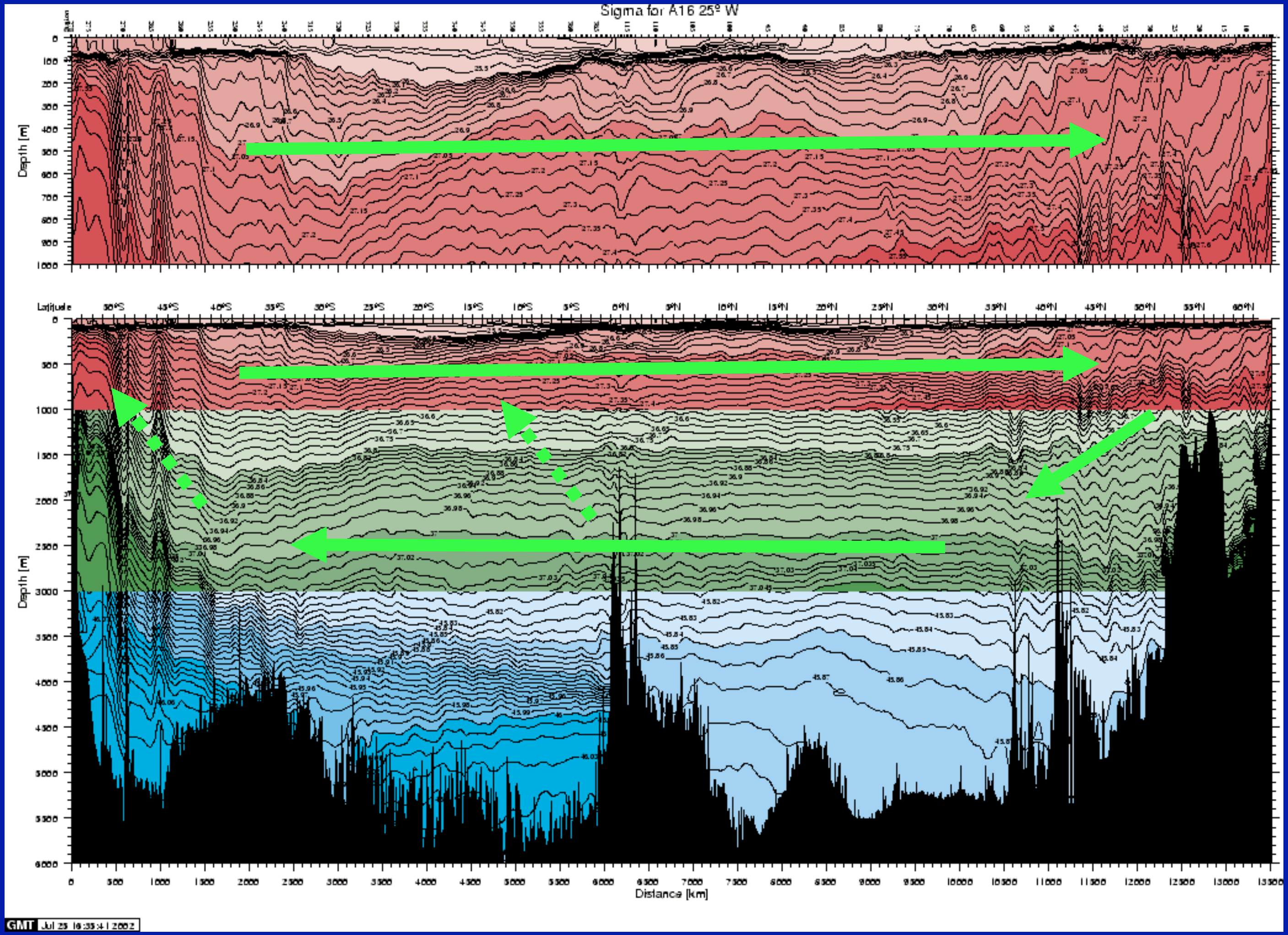
The convective adjustment $\mathscr{C}$ reduces mechanical (potential) energy, so it is a form of dissipation $\left(\int_{-D}^{z}\int_{(x,y)} \mathscr{C}\, dxdydz < 0\right)$, as is $\mathscr{D}$. Thus, the circulation is driven **mechanically**, by the turbulent diffusion acting on the difference between top and bottom temperatures (densities):

$$\int_{(x,y)} \kappa_v [T(z=0) - T(z=-D)]\, dxdy = \text{ Dissipation}$$

It follows that surface fluxes do not drive the circulation, and that this type of circulation is most accurately described as '**diffusively-driven**.' (Thus, to first order the ocean is not a heat engine.)

NB: Surface fluxes are necessary to maintain the difference between surface and bottom temperatures, without which the turbulent diffusion has no effect.

The mid-depth meridional overturning: NADW and the warm return flow



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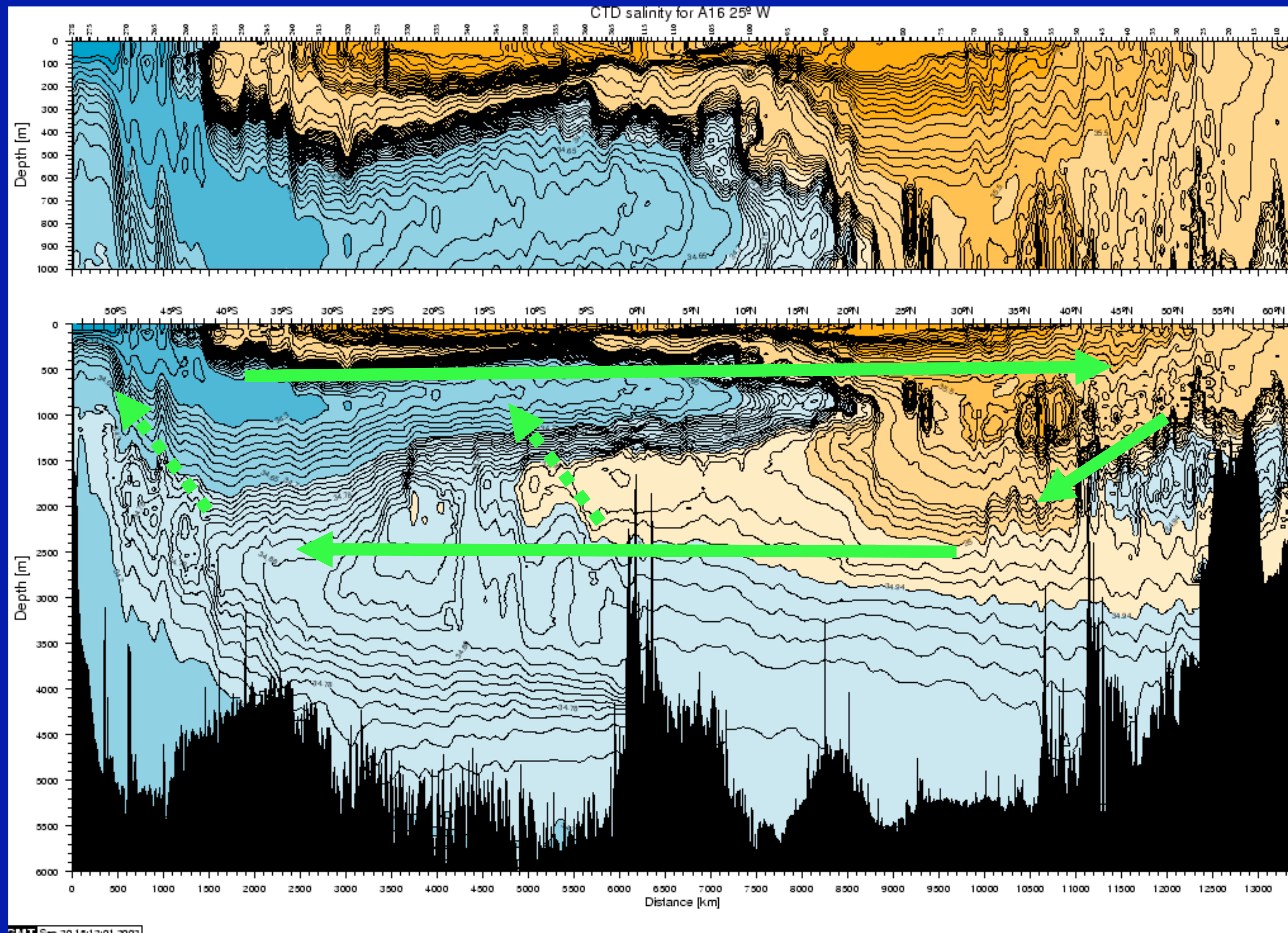
WOCE A16 Density - 25 W

60 N

The mid-depth meridional overturning: NADW and the warm return flow



55 S

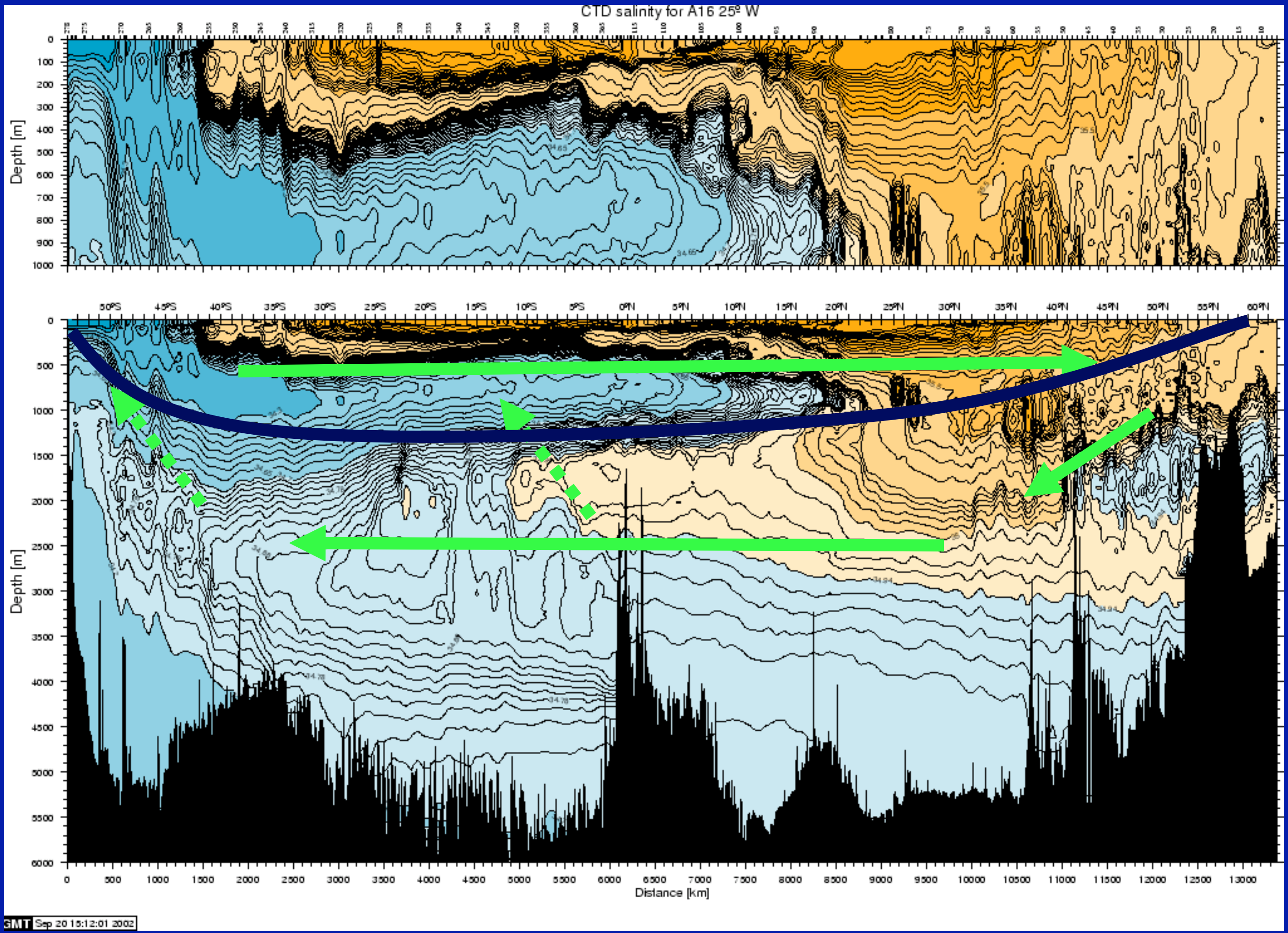
WOCE A16 Salinity - 25 W

60 N

The Model

- Reduced-gravity, one-layer, shallow-water, beta-plane, geostrophic + wind stress + linear (Stommel) friction
- Rectangular basin with circumpolar channel
- Analytical, zonally symmetric ACC (Samelson, 1999, 2004)
- Ekman transport across ACC into warm layer
- Eddy flux of warm water across ACC (parameterized)
- Surface heating and cooling, diapycnal flux across base of warm-water layer (parameterized)
- Steady state: total flux of water into warm layer must vanish

The mid-depth meridional overturning: NADW and the warm return flow



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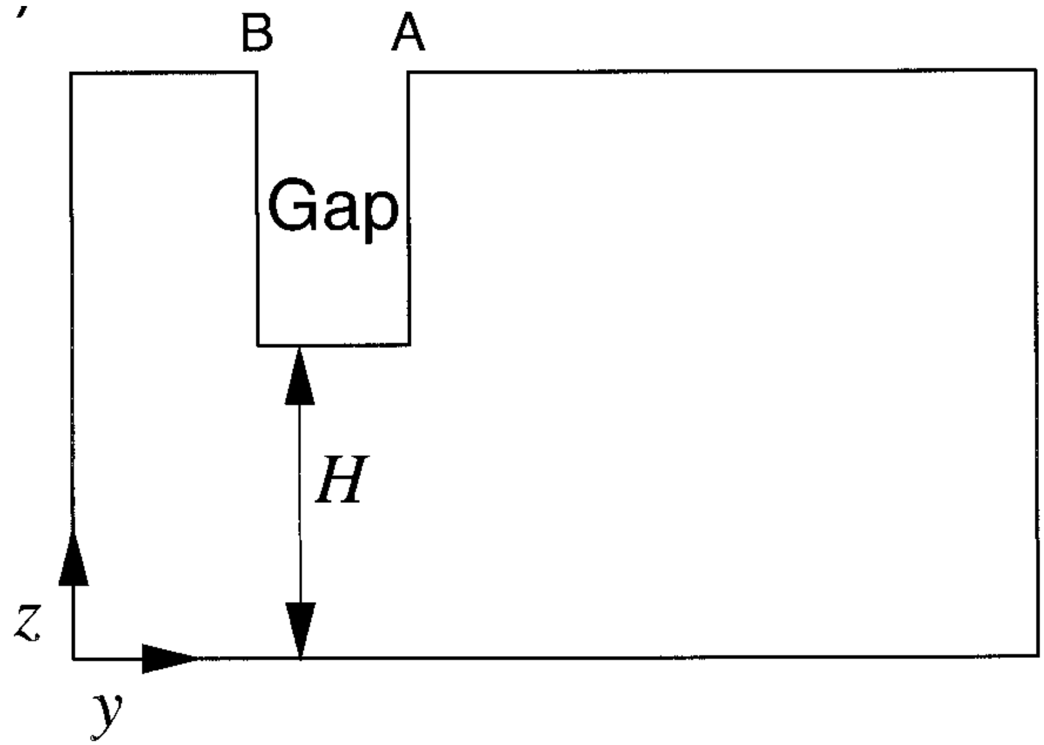
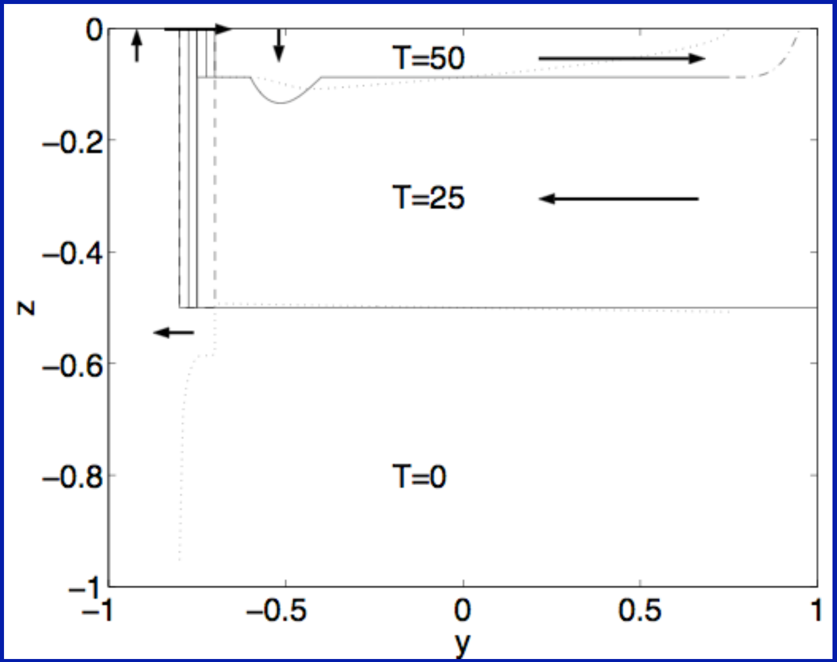

B
A
Gap
H
z
y

Meridional overturning circulation: The Antarctic Circumpolar Current

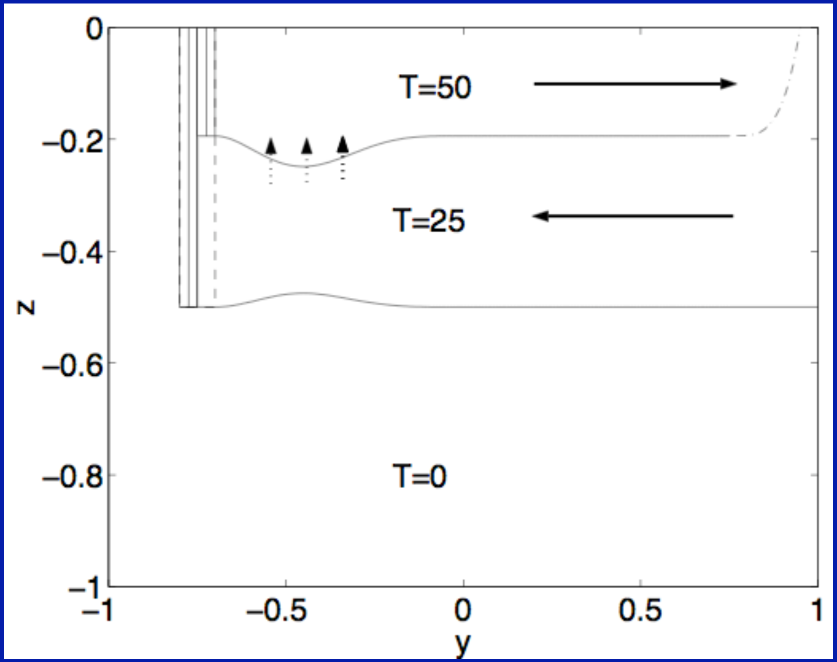
The geostrophic constraint ($\oint v\,dx = \oint f^{-1} p_x\,dx = f^{-1} \oint p_x\,dx = 0$) prevents meridional geostrophic flow across the Southern Ocean to a depth of 1500-2000 m. The northward Ekman transport must therefore be returned southward by geostrophic flow beneath the still depth. In the simplest (PG) models, this forces the development of a warm mid-depth ocean at mid-latitudes, and a thermal current that can be identified as an analogue of the Antarctic Circumpolar Current.

NB: See Stommel (1957), Gill (1968), and Gill and Bryan (1971)

A conceptual model: the role of the ACC



Wind-driven overturning



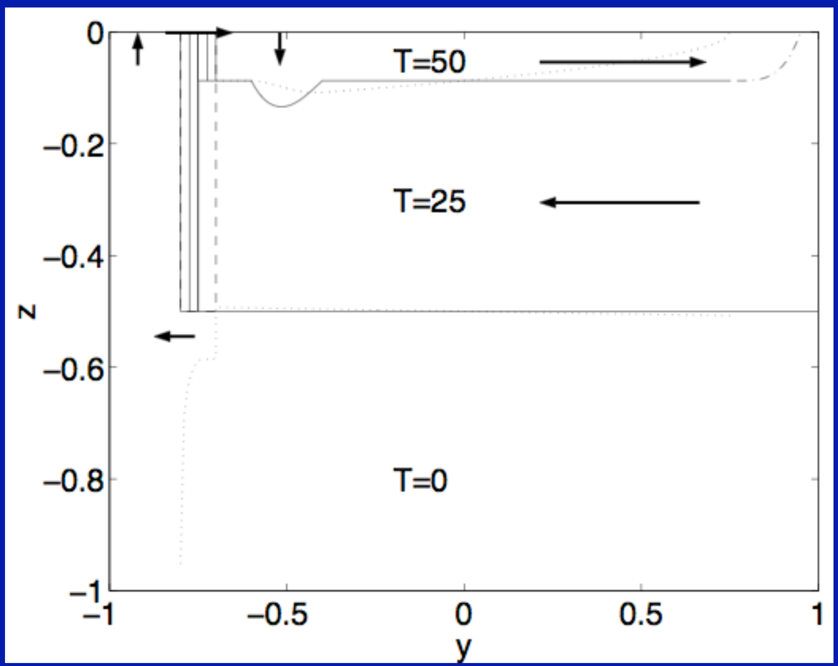
Diffusively-driven overturning

Gill, 1968; Gill and Bryan, 1971; Toggweiler and Samuels, 1995; Samelson, 1999….

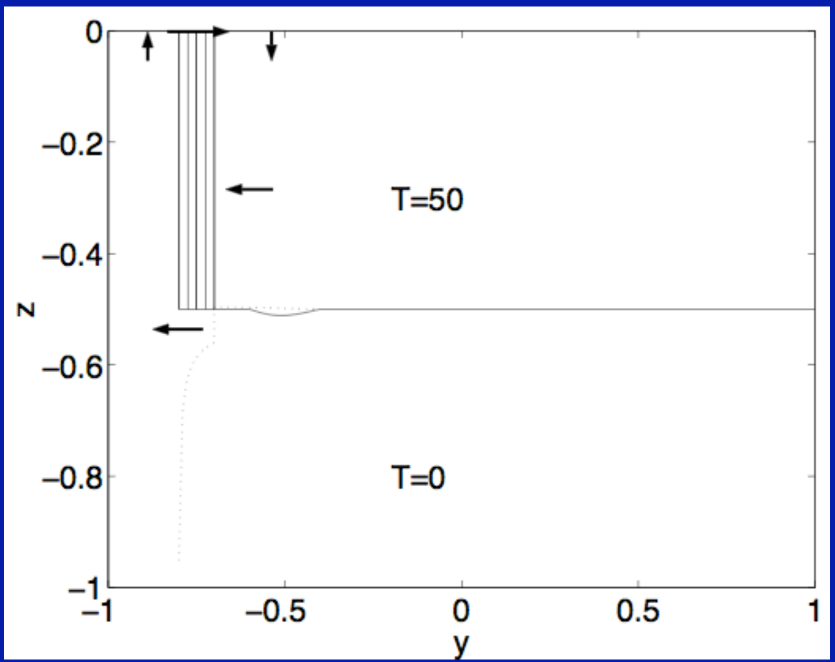
The simple ‘PG + gap’ models suggest:

- The ACC is a thermal, not wind-driven, current, but would not exist without the wind.
- The mid-latitude mid-depth stratification is strong because of the presence of the gap.
- The mid-depth overturning can be driven by the wind, or by turbulent diffusion.
- The overturning mechanism can be understood as a ‘pump and valve’ system, in which the Southern Ocean winds or turbulent diffusion are the pump, and the Northern Hemisphere cooling is the valve.
- The wind-driven and diffusively-driven overturning systems cannot be easily distinguished from the thermal (density) structure.

A conceptual model: the pump and valve



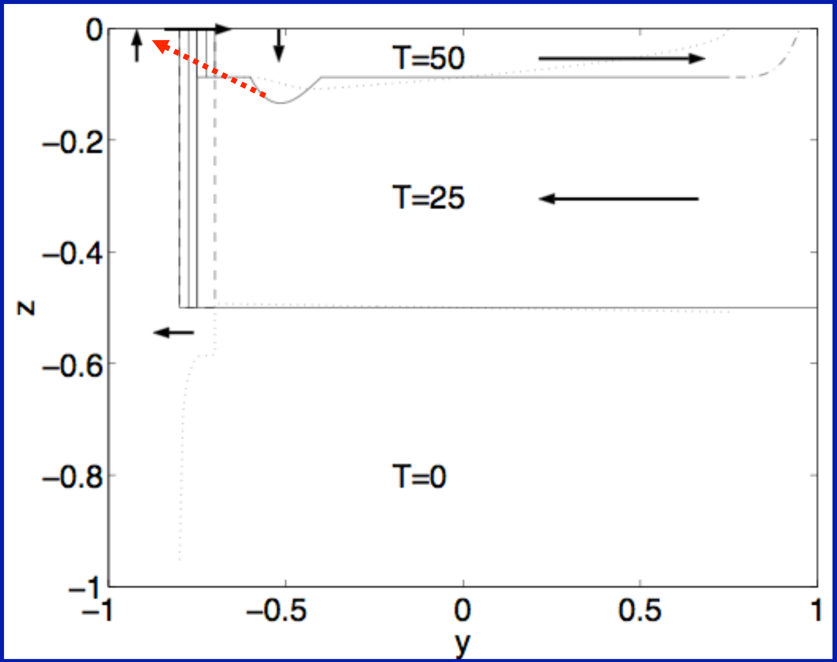
NH cooling (valve) on



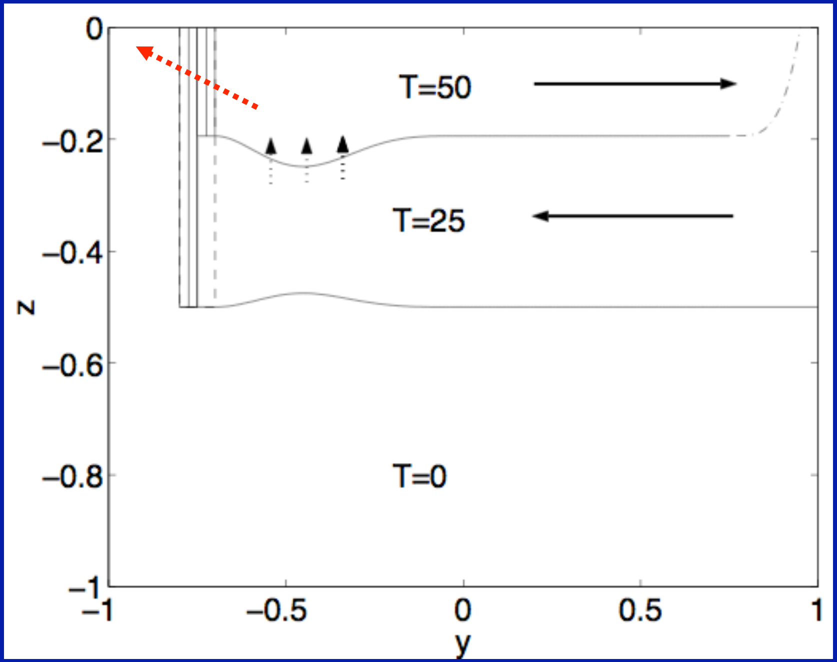
NH cooling (valve) off

Broecker, 1987; Stocker and Wright, 1991; Samelson, 2004….

A conceptual model: the role of eddy fluxes?



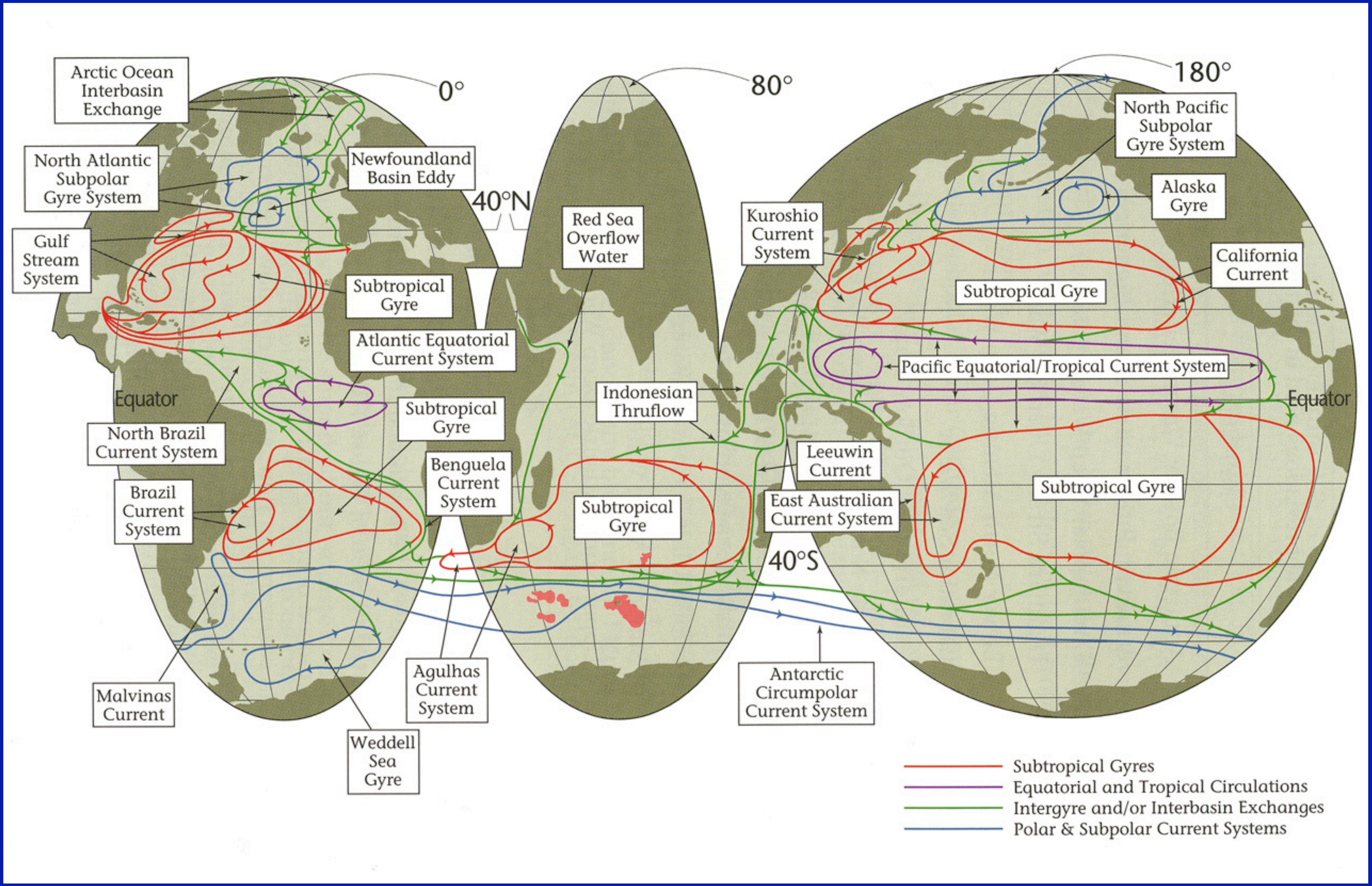
Wind-driven overturning



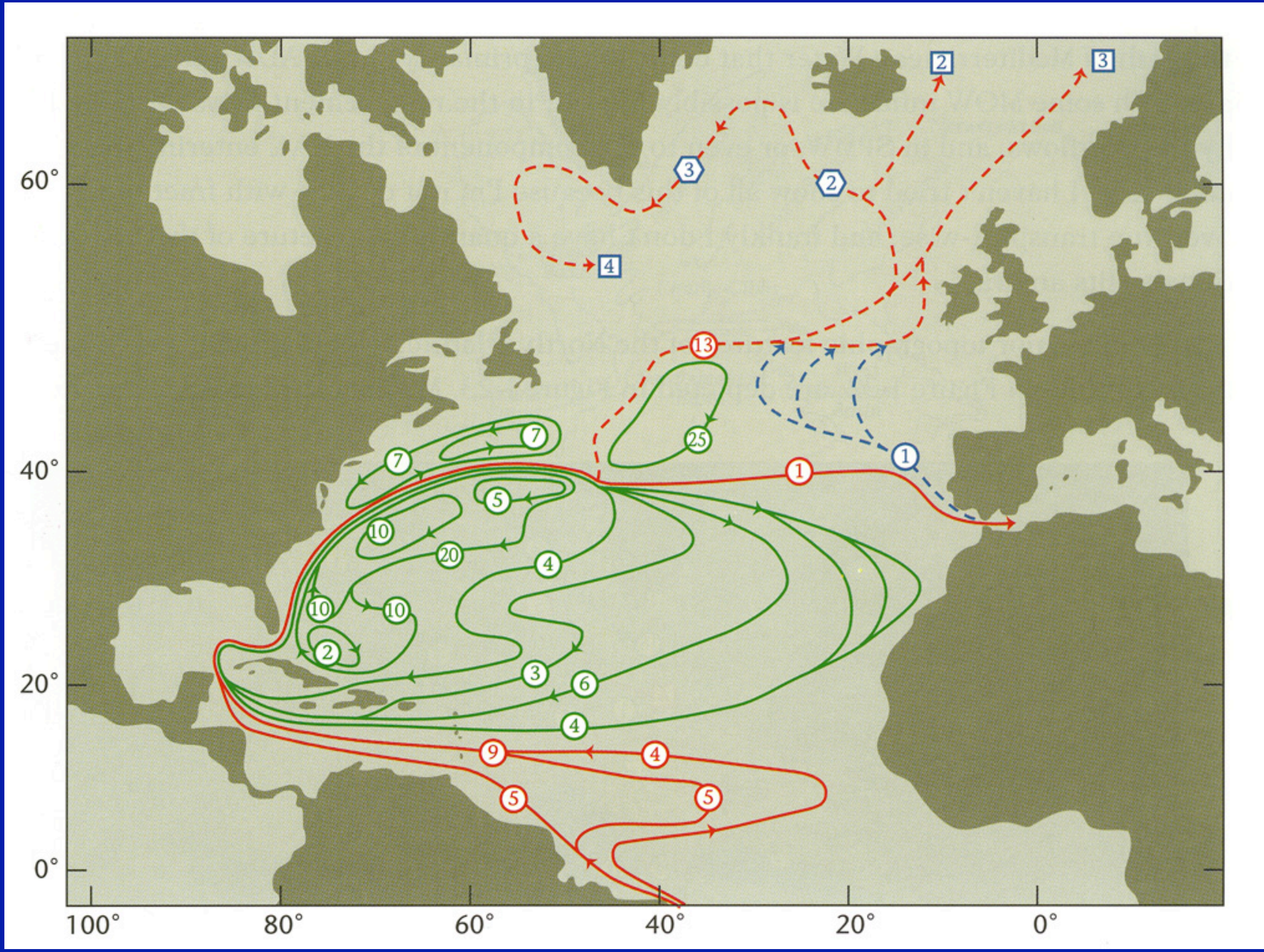
Diffusively-driven overturning

Gnanadesikan, 1999; MacCready and Rhines, 2001; Hallberg and Gnanadesikan, 2001;….

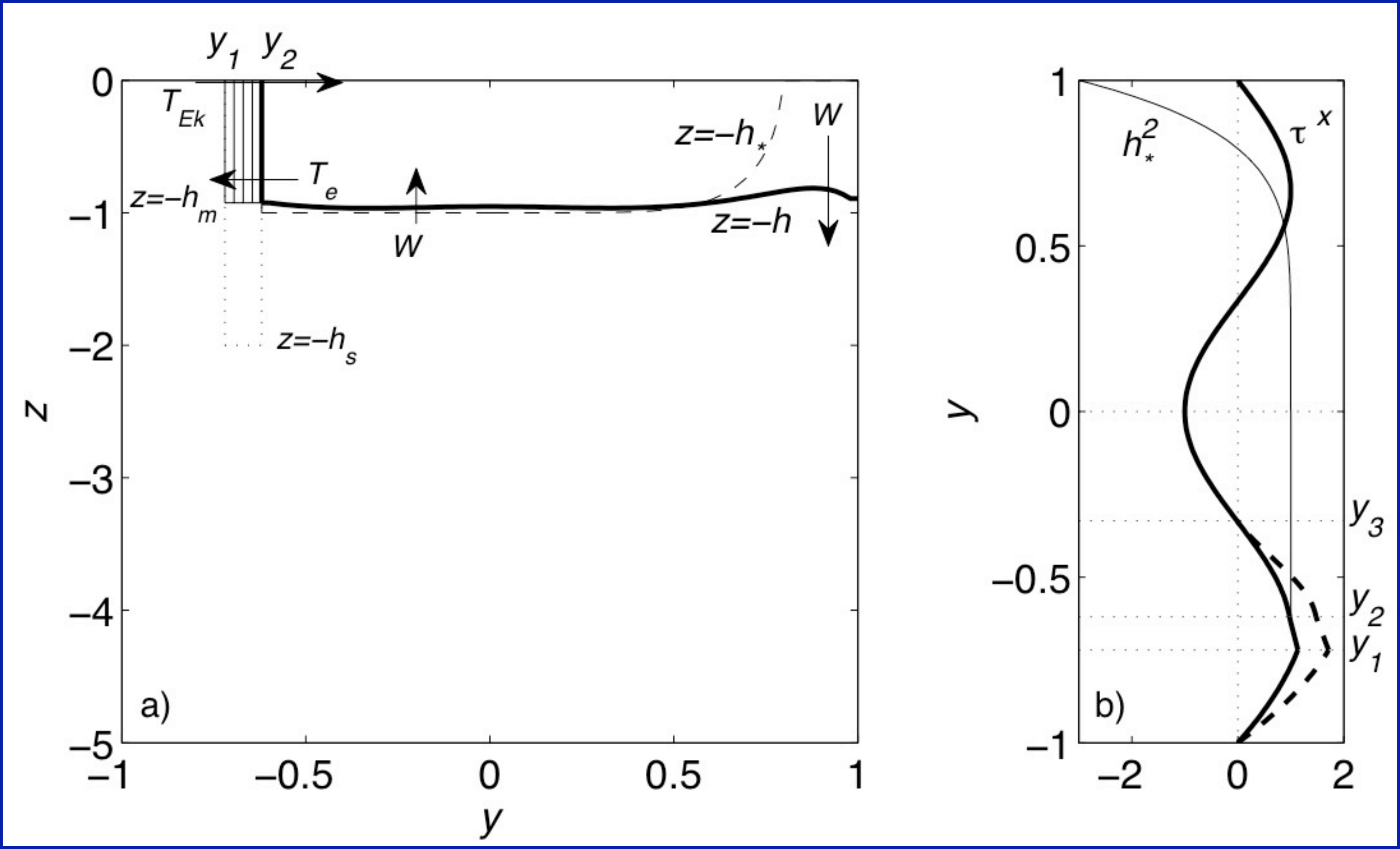
Schmitz (1996): Thermohaline circulation



Schmitz (1996): North Atlantic warm-water circulation



The Model - schematic



The Equations

$$
\begin{aligned}
-fvh &= -\gamma h h_x - rhu + \tau^x(x,y), \\
fuh &= -\gamma h h_y - rhv + \tau^y(x,y).
\end{aligned}
$$

$$(hu)_x + (hv)_y = W(x,y)$$

$$hu = -\Psi_y - \Phi_x, \quad hv = \Psi_x - \Phi_y$$

$$
\begin{aligned}
\Phi_{xx} + \Phi_{yy} &= -W \\
(r\Psi_x)_x + (r\Psi_y)_y + \beta\Psi_x &= \tau^y_x - \tau^x_y + J(r,\Phi) + \beta\Phi_y - fW
\end{aligned}
$$

The Equations, cont'd

$$W(x,y) = -\alpha_w[h^2(x,y) - h_*^2(x,y)]$$

$$u(y,z) = -\frac{1}{f}\frac{\gamma}{y_2 - y_1}(z + h_m), \quad y_1 < y < y_2, \quad -h_m < z < 0$$

$$h_m^2 = \frac{1}{x_E - x_W}\int_{x_W}^{x_E} h^2(x, y_2)\, dx$$

$$T_e = V_e(x_E - x_W), \quad V_e = -\alpha_e h_m^2$$

The Equations, cont'd

$$T_{Ek} + T_e + T_{w2} = 0$$

$$T_{w2} = T_w(y_2), \quad T_w(y) = \int_y^{y_N} \int_{x_W}^{x_E} W \, dx \, dy = \int_{x_W}^{x_E} \Phi_y(x, y) \, dx$$

$$\int_{x_W}^{x_E} \left(\alpha_w \int_{y_2}^{y_N} h^2(x, y) \, dy + \alpha_e \, h^2(x, y_2) \right) dx = \alpha_w \int_{y_2}^{y_N} \int_{x_W}^{x_E} h_*^2 \, dx \, dy + T_{Ek}$$

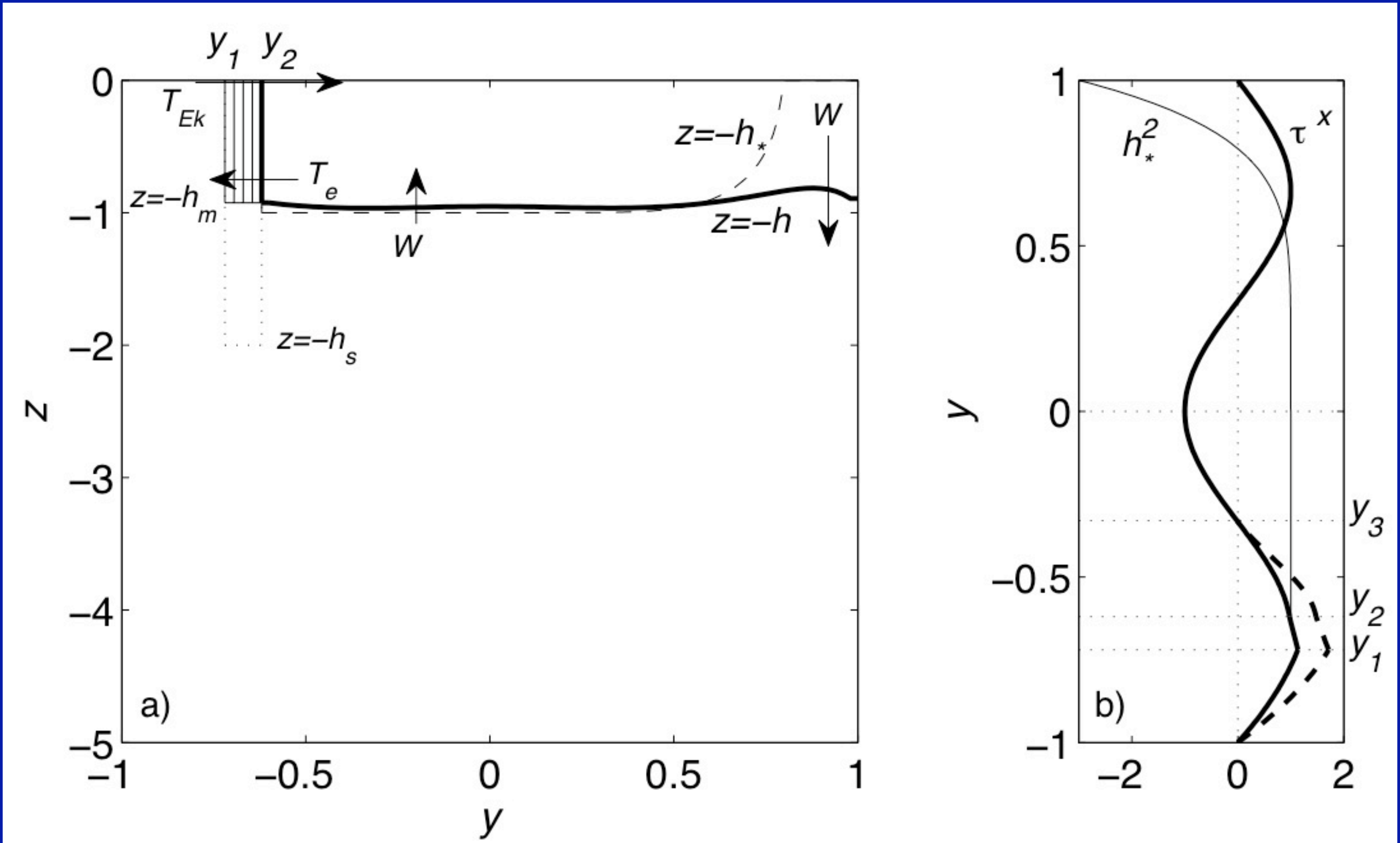
$$\begin{aligned} \frac{\partial}{\partial x}(h^2) &= \frac{2}{\gamma}(\tau^x + r\Phi_x - f\Phi_y + f\Psi_x + r\Psi_y) \\ \frac{\partial}{\partial y}(h^2) &= \frac{2}{\gamma}(\tau^y + r\Phi_y + f\Phi_x + f\Psi_y - r\Psi_x) \end{aligned}$$

The Forcing

$$h_*^2(y) = \begin{cases} h_0^2 - \delta h_N^2 y^6, & 0 < y < y_N = 1 \\ h_0^2, & y_2 < y < 0, \\ 0, & y_S < y < y_2. \end{cases}$$

$$\tau^x = \begin{cases} \tau_0 \cos\frac{3\pi y}{2}, & y_3 = -\frac{1}{3} < y < y_N = 1 \\ \tau_0 \cos\frac{3\pi y}{2} + \tau_1 \frac{1}{2}\left(1 - \cos\frac{\pi(y-y_3)}{y_2-y_3}\right), & y_2 < y < y_3 = -\frac{1}{3} \end{cases}$$

The Model - schematic



Scales and Parameters

$$\gamma = \frac{\tilde{\gamma} H}{f_0 U L} = \frac{g \Delta \rho H}{\rho_0 f_0 U L} = \frac{g \Delta \rho H^2}{\tau_* L} = 20, \qquad f = \frac{\tilde{f}}{\beta_0 L} = \beta y, \quad \beta = \frac{\tilde{\beta}}{\beta_0} = 1,$$

$$\tau_0 = \frac{\tilde{\tau}_0}{\tau_* / \rho_0} : -2 \le \tau_0 \le 0, \qquad \tau_1 = \frac{\tilde{\tau}_1}{\tau_* / \rho_0} : -0.5 \le \tau_1 \le 0.5,$$

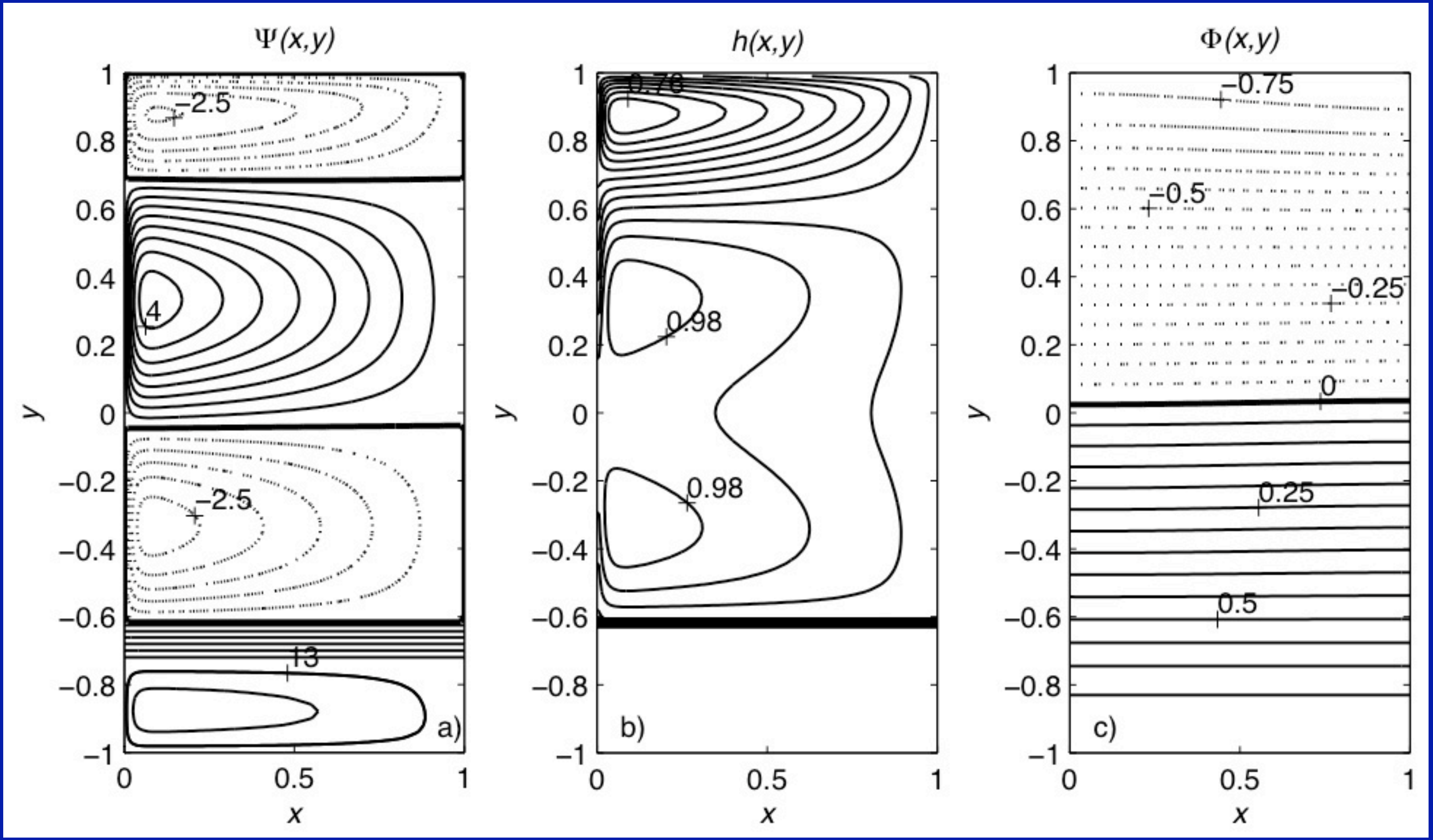
$$h_0^2 = \frac{\tilde{h}_0^2}{H^2} = 1, \qquad \delta h_N^2 = \frac{\delta \tilde{h}_N^2}{H^2} : 0 \le \delta h_N^2 \le 6,$$

$$(\alpha_e, \alpha_w) = (\tilde{\alpha}_e, \tilde{\alpha}_w H) T \; : \; 0 \le (\alpha_e, \alpha_w) \le 5, \qquad r = \frac{\tilde{r}}{f_0} = 0.02.$$

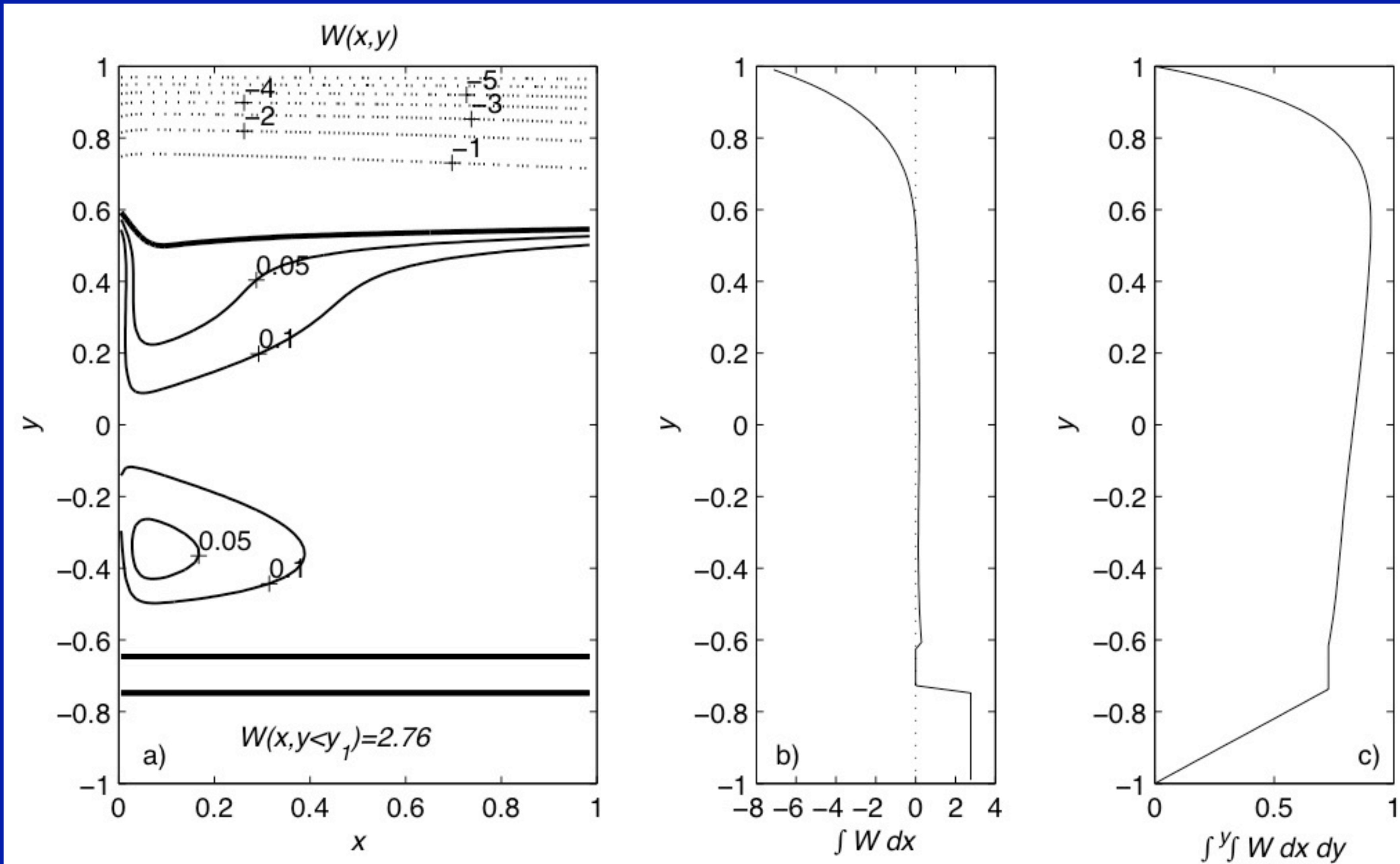
$$\begin{aligned} g &= 10 \text{ m s}^{-2} \\ \rho_0 &= 1025 \text{ kg m}^{-3} \\ \Delta\rho/\rho_0 &= 10^{-3} \\ H &= 1000 \text{ m} \\ L &= 5 \times 10^6 \text{ m} \\ f_0 &= 10^{-4} \text{ s}^{-1} \end{aligned}$$

$$\begin{aligned} \beta_0 &= 2 \times 10^{-11} \text{ m}^{-1} \text{ s}^{-1} \\ \tau_* &= 0.1 \text{ N m}^{-2}. \\ U &= \frac{\tau_0}{\rho_0 f_0 H} = 10^{-3} \text{ m s}^{-1} \\ T &= L/U = 5 \times 10^9 \text{ s} = 160 \text{ yr} \\ UHL &= 5 \times 10^6 \text{ m}^3 \text{ s}^{-1} = 5 \text{ Sv} \end{aligned}$$

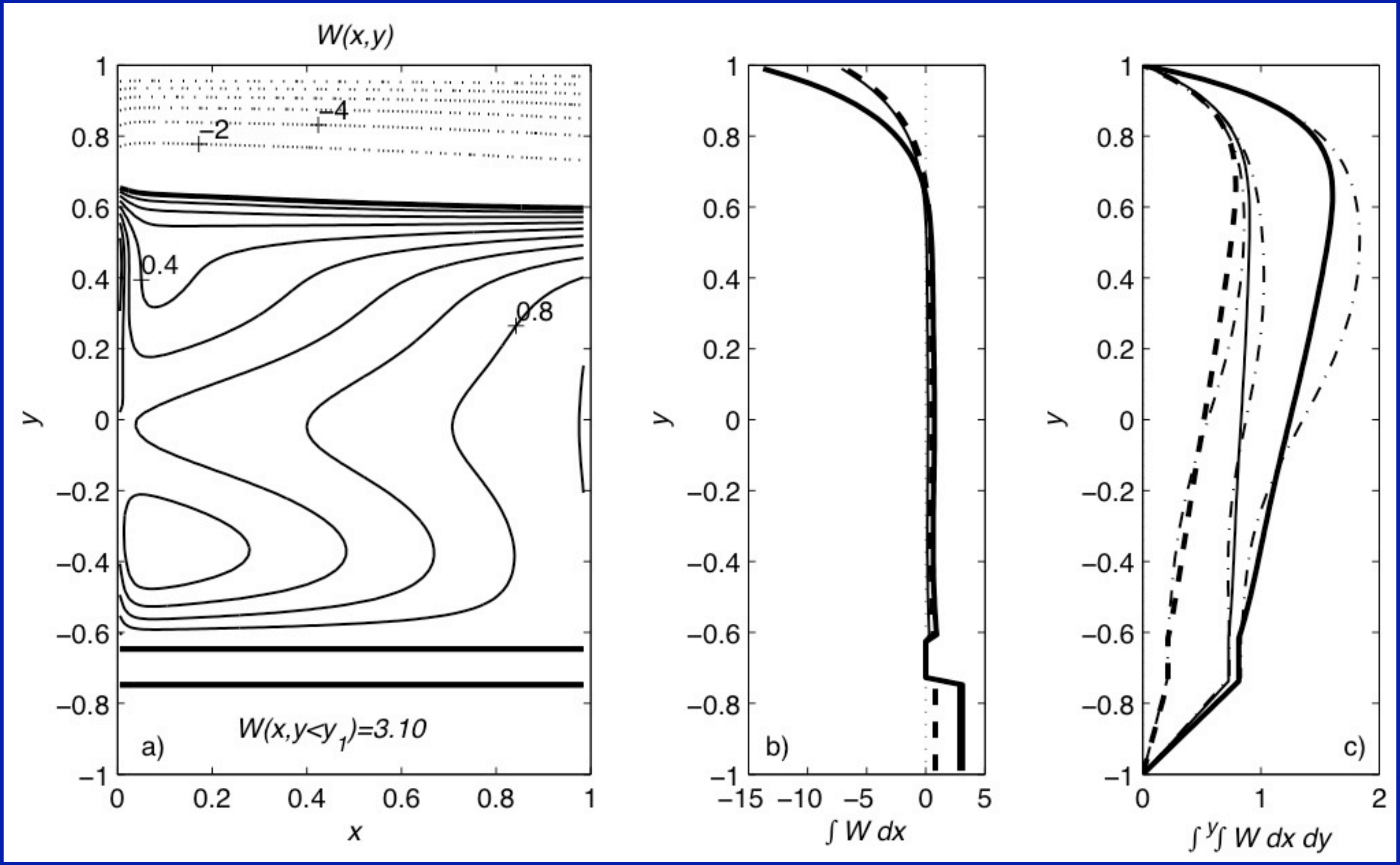
The Solution



The Solution, cont'd



The Solution, cont'd



Analytical solution
(weak friction and diabatic forcing)

$$\boxed{\begin{aligned} h_E^2 &\approx \frac{V_{Ek} - \alpha_w (y_N - y_2)\left[\bar{D}_0^2(y_2) - \bar{h}_*^2(y_2)\right]}{\alpha_e + \alpha_w (y_N - y_2)} \\ V_e &\approx -\alpha_e h_E^2 \\ V(y_2) &\approx V_{Ek} - \alpha_e h_E^2 \end{aligned}}$$

$$\boxed{\begin{aligned} \lambda = \frac{2f^2}{\beta\gamma}\alpha_w \\ \lambda(x_E - x_W) << 1 \end{aligned}}$$

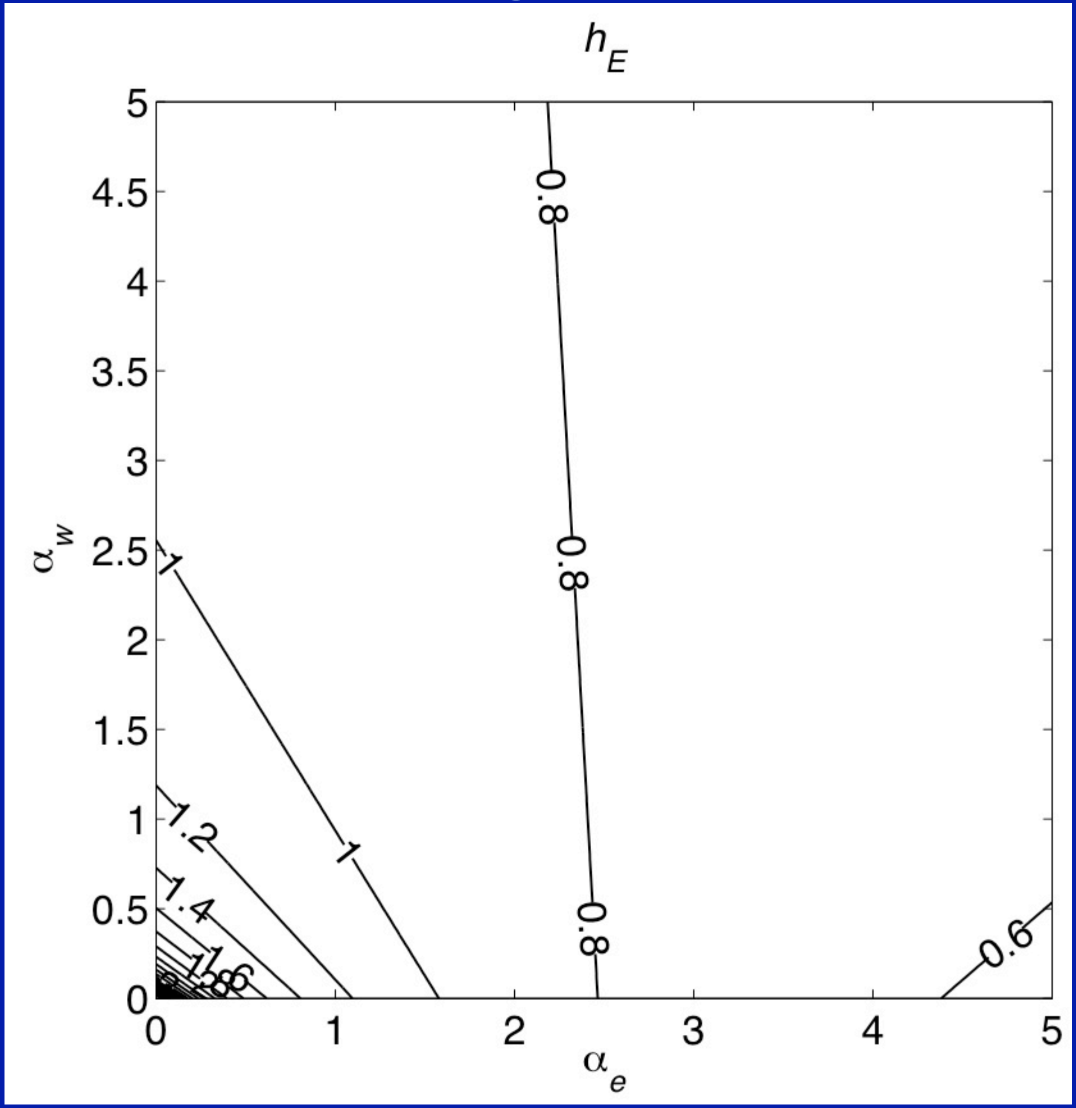
$$h^2(x,y) \approx h_E^2 + \frac{2f^2}{\beta\gamma}(x_E - x)\frac{\partial}{\partial y}\left(\frac{\tau^x}{f}\right)$$

$$\bar{h}_*^2(y) = \frac{1}{y_N - y}\int_y^{y_N} h_*^2(y')\,dy'$$

$$\bar{D}_0^2(y) = \frac{x_E - x_W}{y_N - y}\int_y^{y_N} \frac{f^2}{\beta\gamma}\frac{\partial}{\partial y}\left(\frac{\tau^x}{f}\right)\,dy$$

h_E vs. diabatic forcing and eddy coefficients

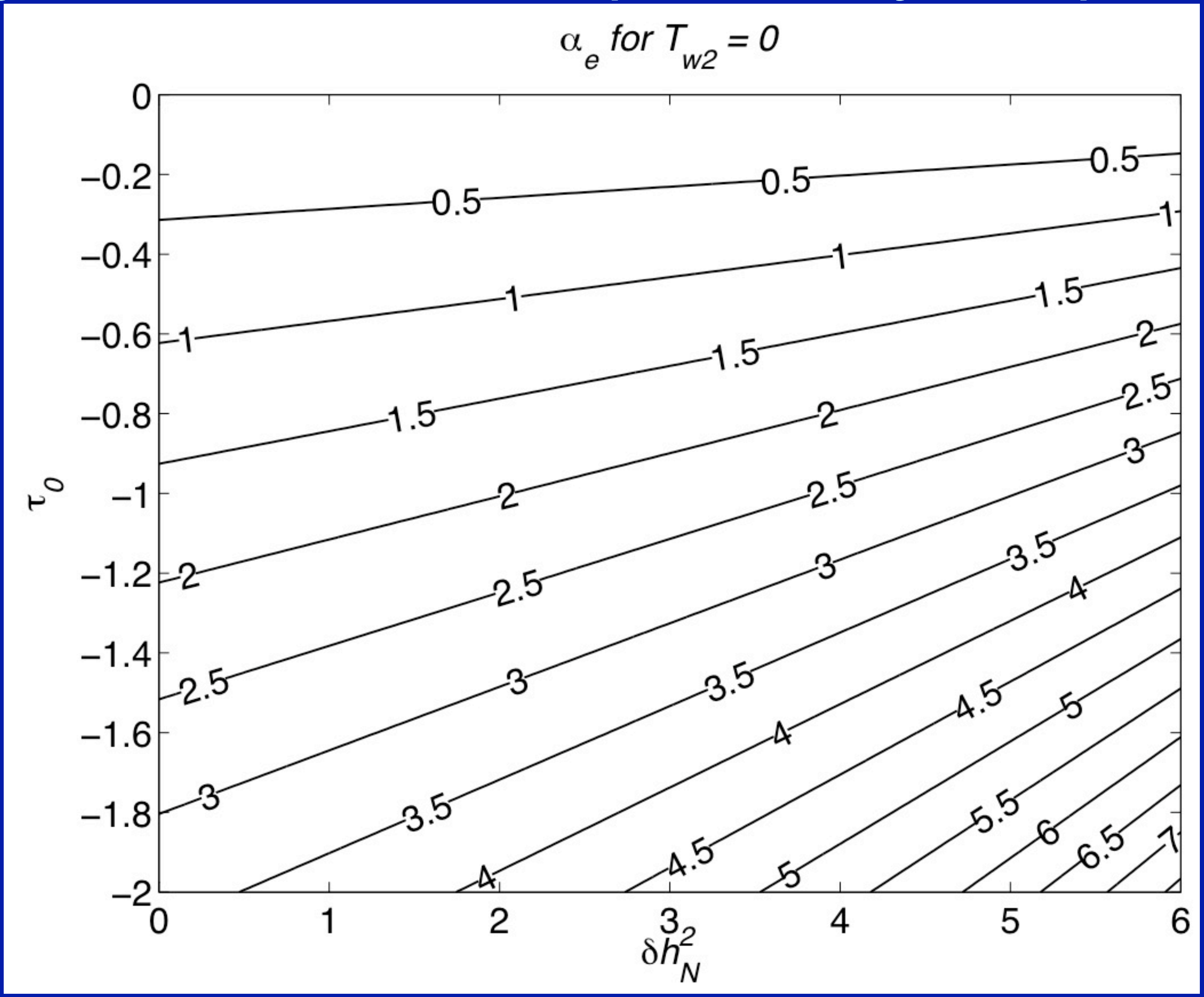
Diabatic forcing coefficient



Eddy coefficient

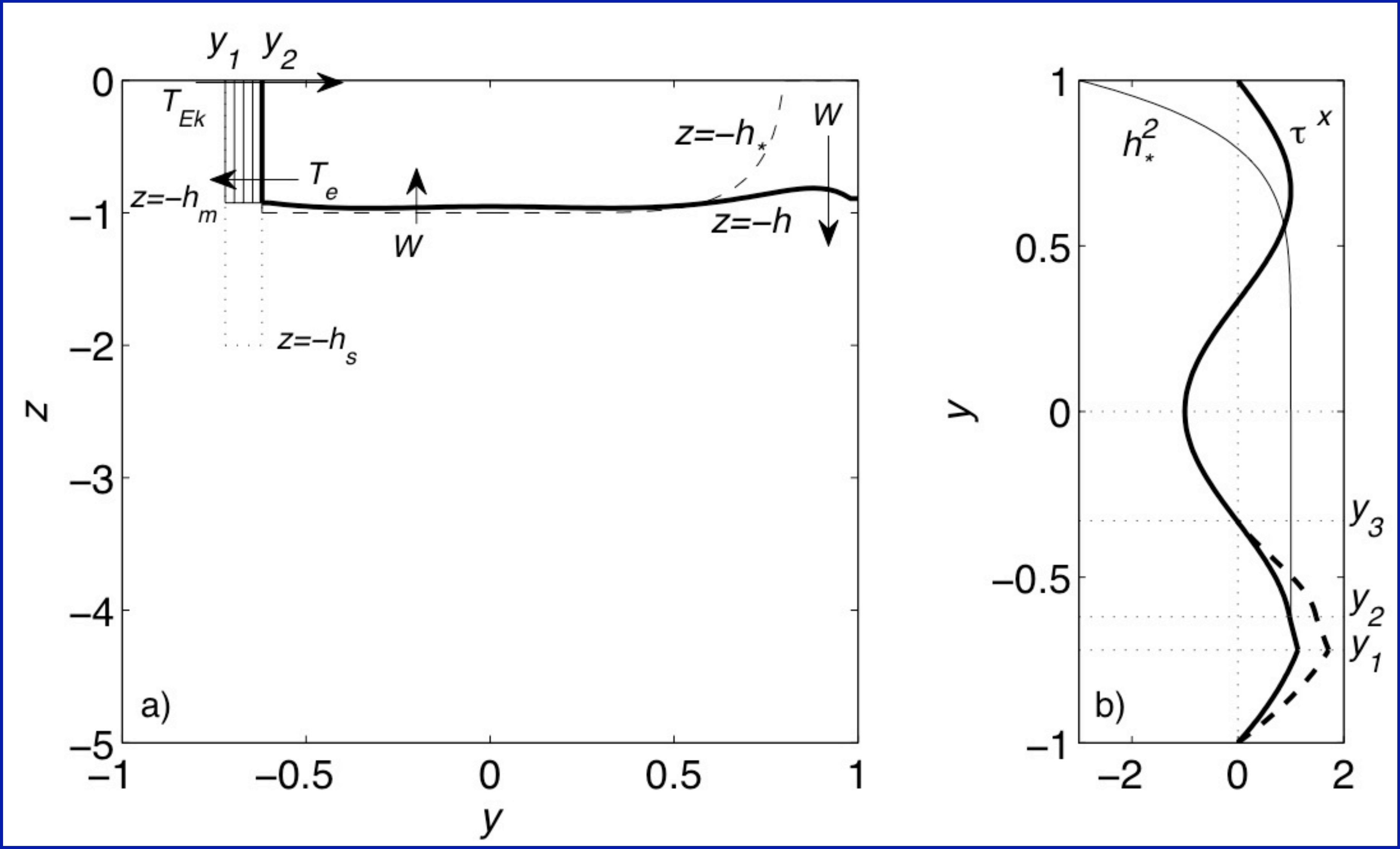
Eddy coefficient for complete eddy compensation

Wind stress amplitude



Diabatic forcing (subpolar-tropical effective-depth difference)

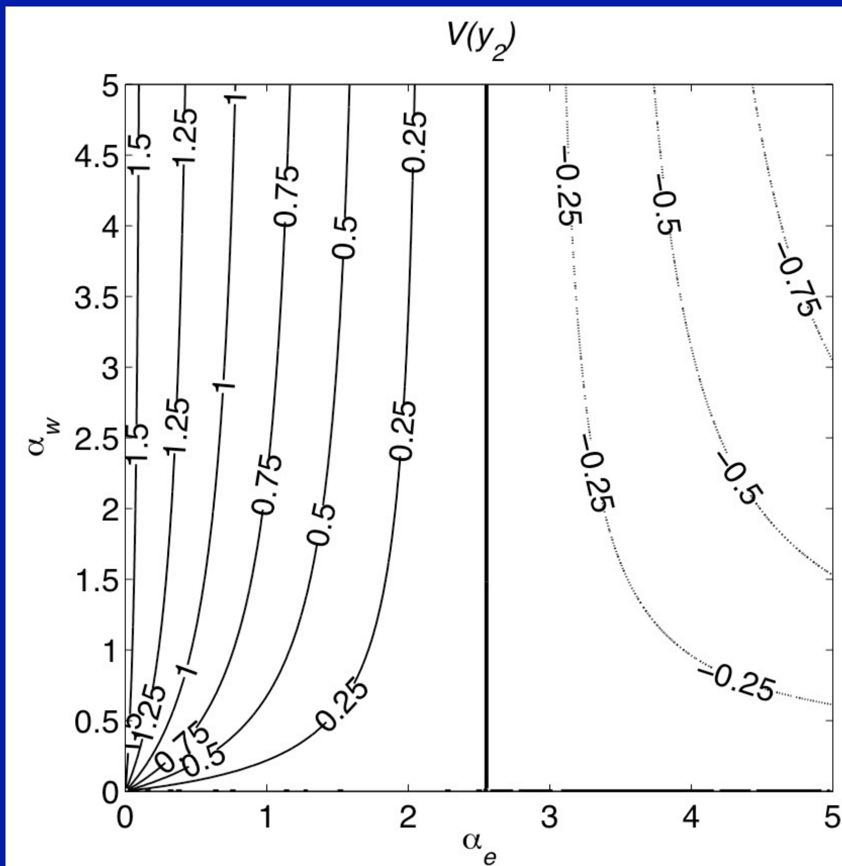
The Model - schematic



Meridional overturning transport at ACC vs. diabatic forcing and eddy coefficients

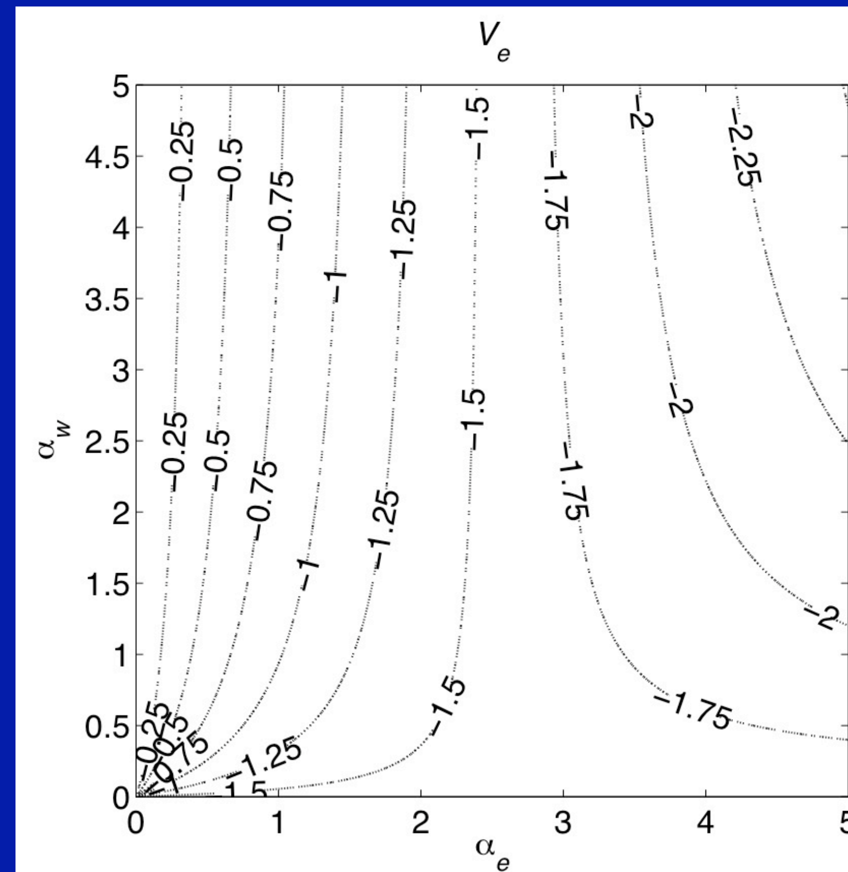
Diabatic forcing coefficient

Net transport



Eddy coefficient

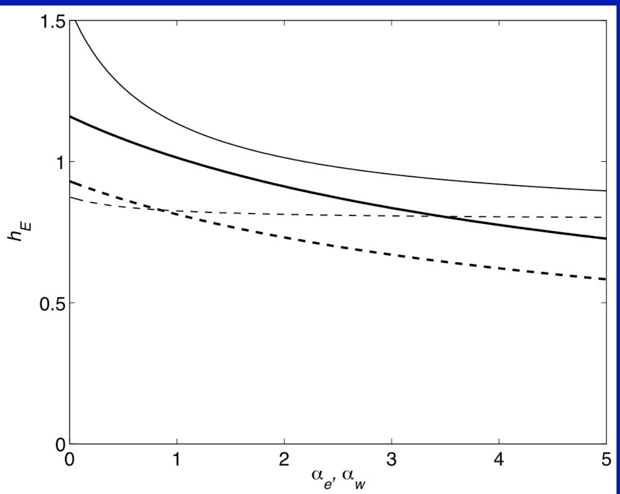
Eddy transport



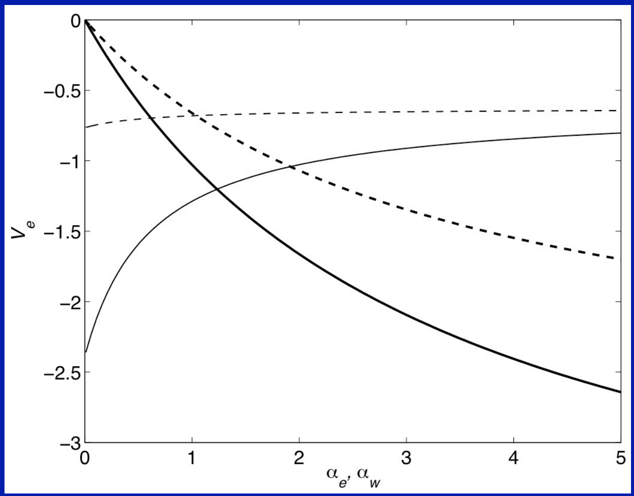
Eddy coefficient

Do southern hemisphere winds drive meridional overturning?

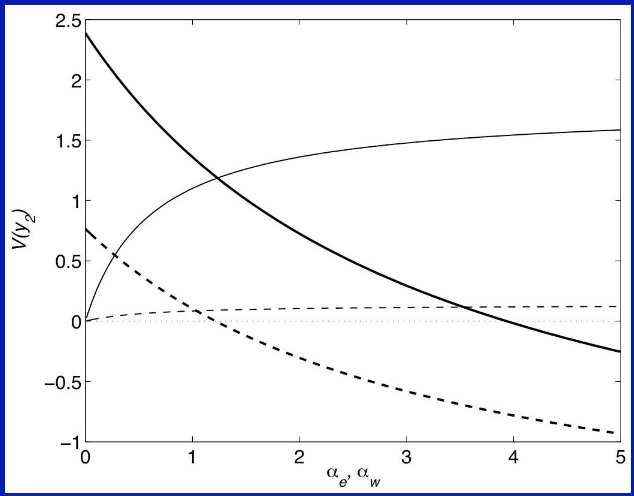
h_E



Eddy transport



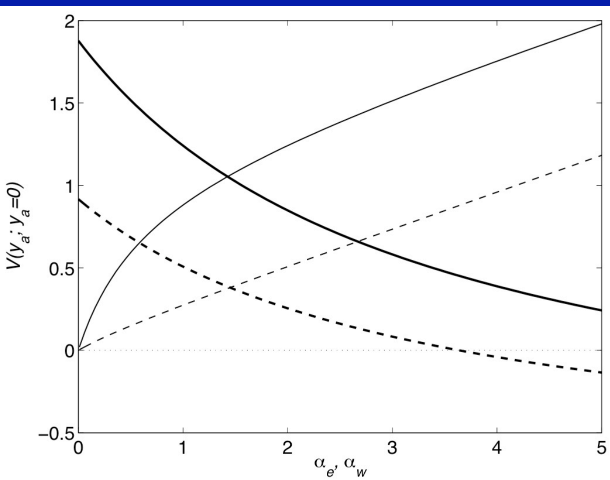
Net transport at ACC



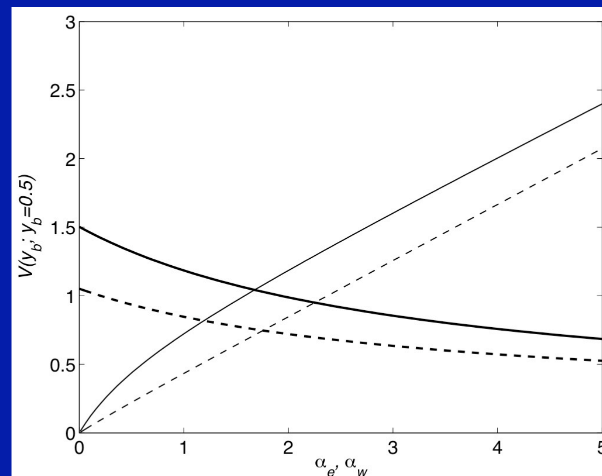
Eddy (thick lines) and diabatic forcing (thin lines) coefficients for Southern hemisphere wind stress amplitude +0.5 (solid lines) and -0.5 (dashed lines)

Do southern hemisphere winds drive meridional overturning?

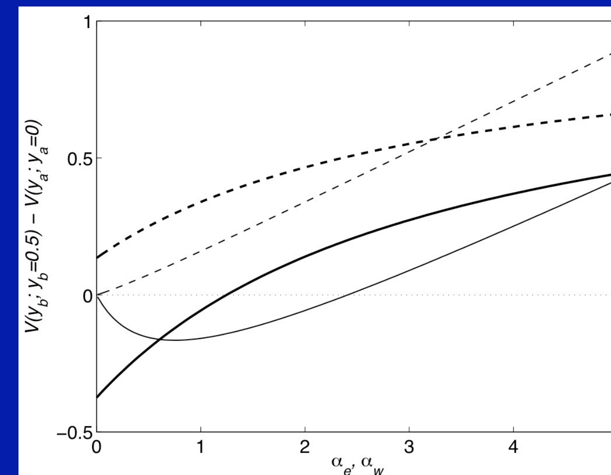
Transport at EQ



Transport at NH SPG



NH SPG - EQ difference



Eddy (thick lines) and diabatic forcing (thin lines) coefficients for Southern hemisphere wind stress amplitude +0.5 (solid lines) and -0.5 (dashed lines)

Summary

- Accessible model of warm-water branch of meridional overturning, including ACC, cross-ACC eddy fluxes, and diabatic (diapycnal) and wind-stress forcing
- Analytical solution for small friction and diabatic forcing
- Depth of warm layer controlled by three-way balance between diabatic (diapycnal) fluxes north of ACC and cross-ACC Ekman and eddy transports
- Eastern boundary depth of warm layer plays central role, controlling diabatic fluxes and cross-ACC eddy fluxes, and communicating warm-layer depth information between gyres and hemispheres; western boundary currents are passive
- Stronger southern hemisphere winds force larger eastern boundary depth, increasing compensating cross-ACC eddy fluxes and downwelling (cooling) north of ACC; meridional overturning also increases, but as part of a modified three-way balance, with spatial structure of overturning influenced by distribution of diabatic fluxes

Future directions

- Some clarification of previous ideas, but primarily a pedagogical model…should provide useful foundation for theoretical extensions

For example:

- Time-dependence
- Different (better) diabatic flux representation
- Active deep layer
- Generalization to thermohaline fluid – multiple equilibria?
- Active western boundary currents
- …?

=> Better understanding of ocean's role in global climate dynamics

The Equations

$$
\begin{aligned}
-fvh &= -\gamma h h_x - rhu + \tau^x(x,y), \\
fuh &= -\gamma h h_y - rhv + \tau^y(x,y).
\end{aligned}
$$

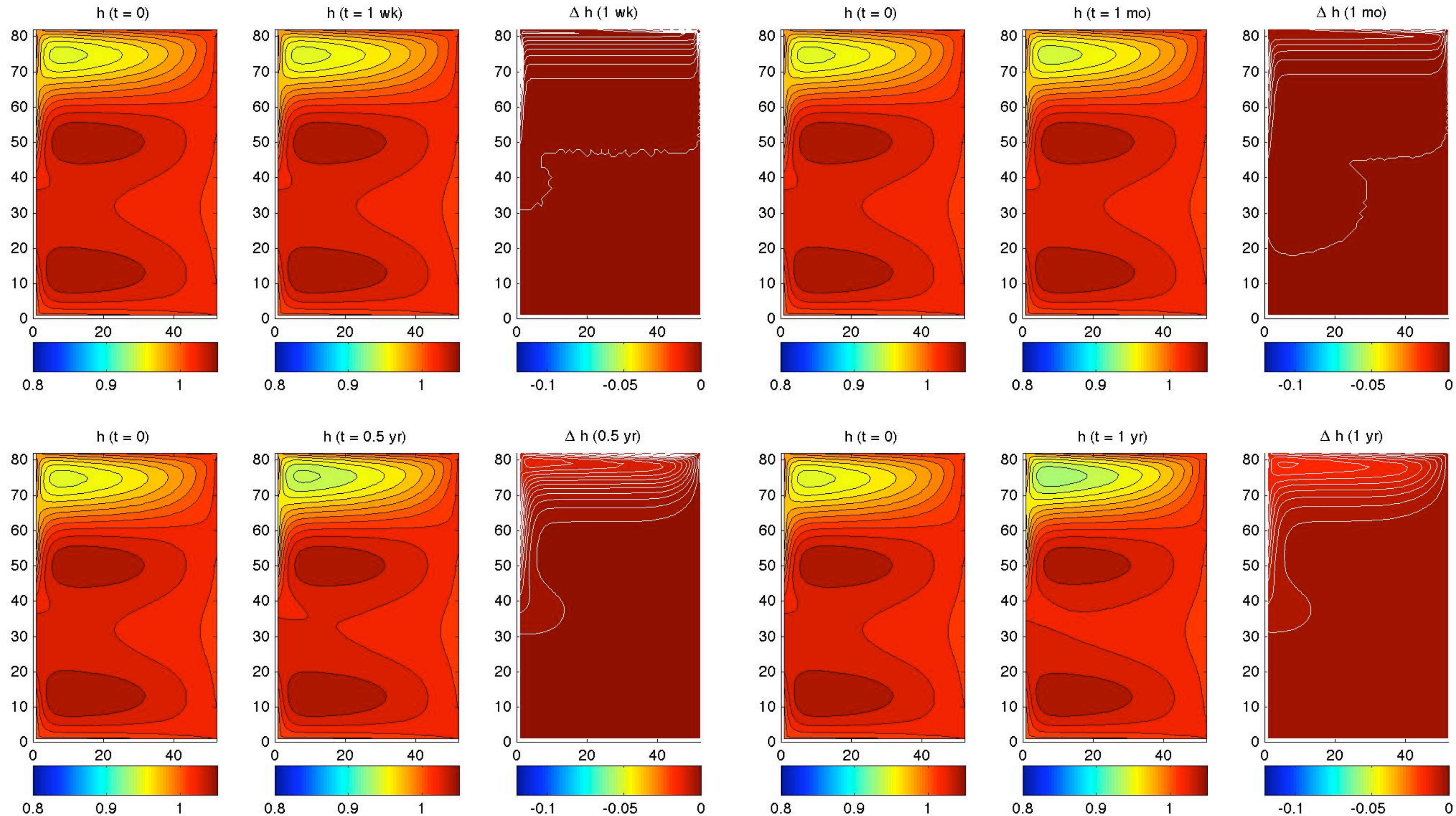
$$(hu)_x + (hv)_y = W(x,y)$$

$$hu = -\Psi_y - \Phi_x, \quad hv = \Psi_x - \Phi_y$$

$$
\begin{aligned}
\Phi_{xx} + \Phi_{yy} &= -W \\
(r\Psi_x)_x + (r\Psi_y)_y + \beta\Psi_x &= \tau_x^y - \tau_y^x + J(r,\Phi) + \beta\Phi_y - fW
\end{aligned}
$$

Time-dependence (NH cooling)

Approach: Restore h_t to mass conservation equation (PG approximation)



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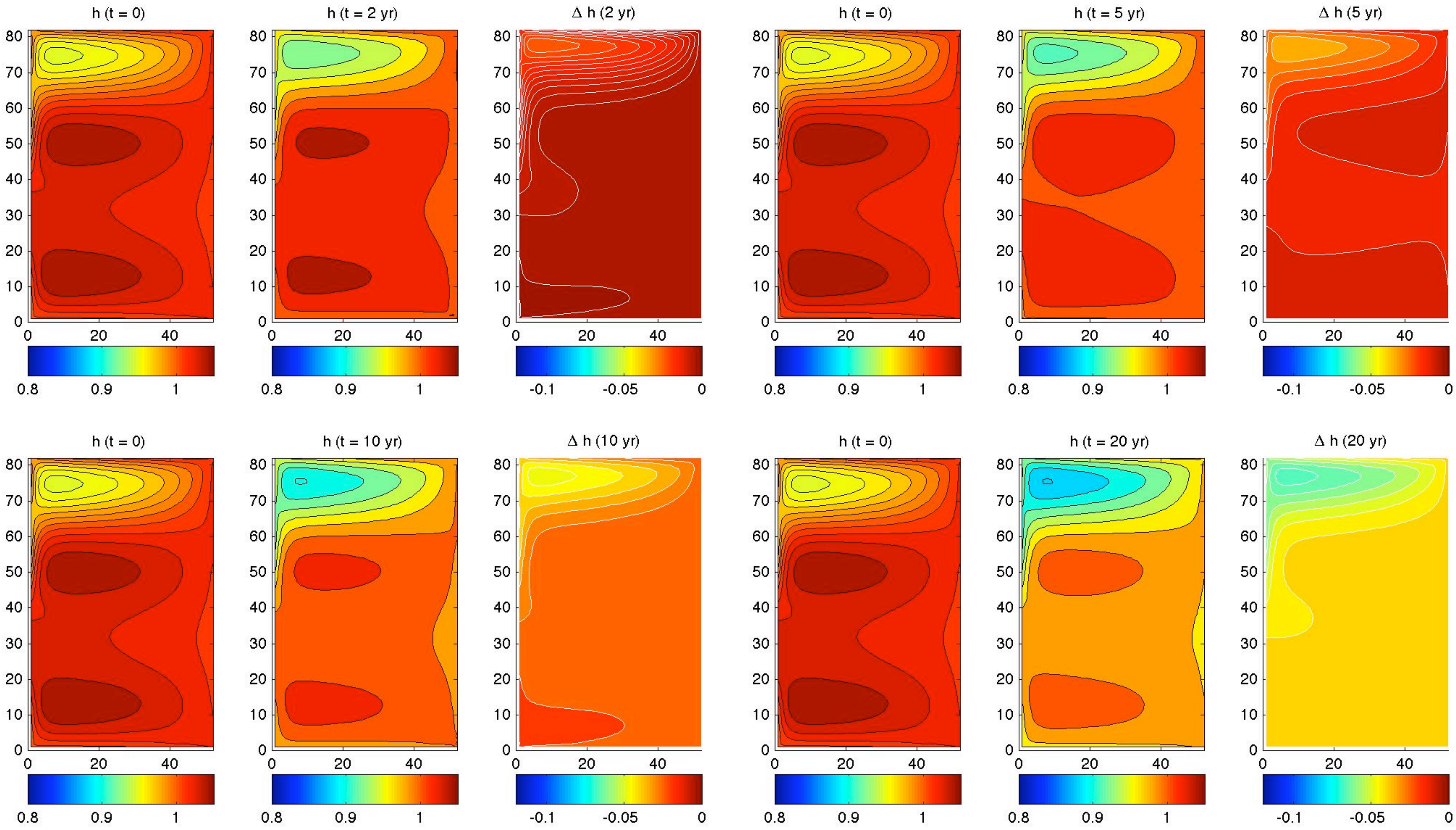

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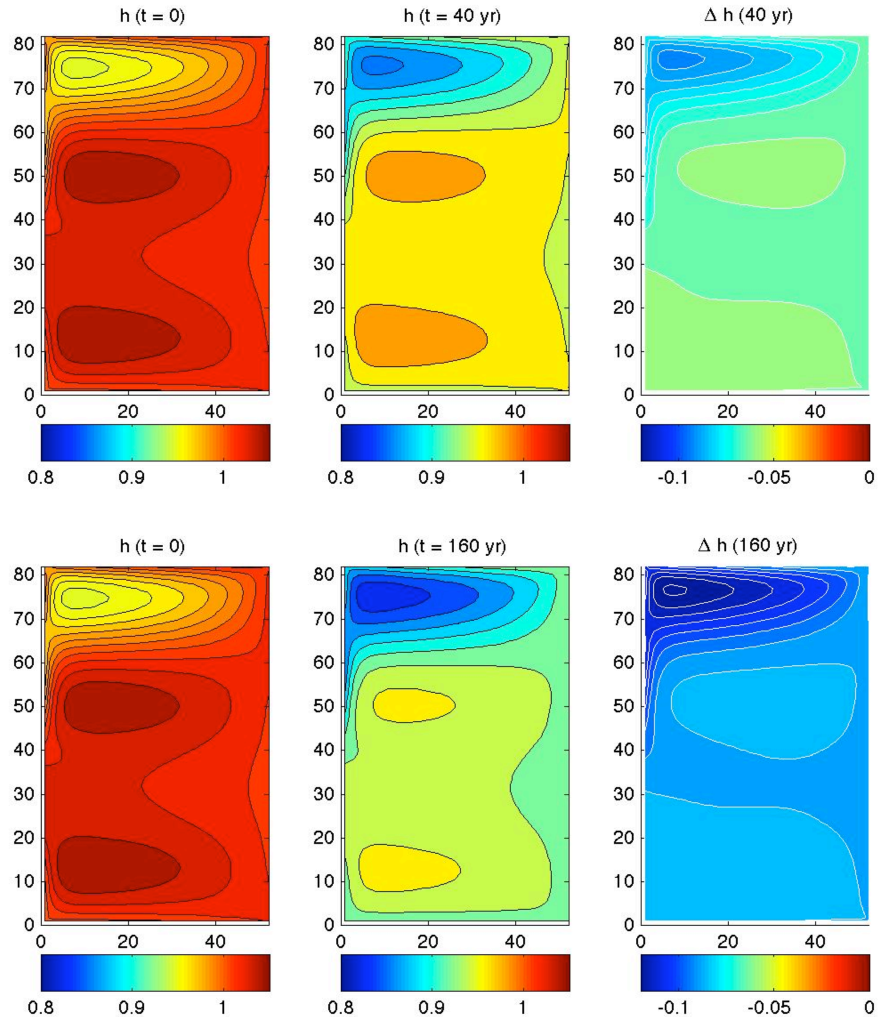

Time-dependence (SH winds)

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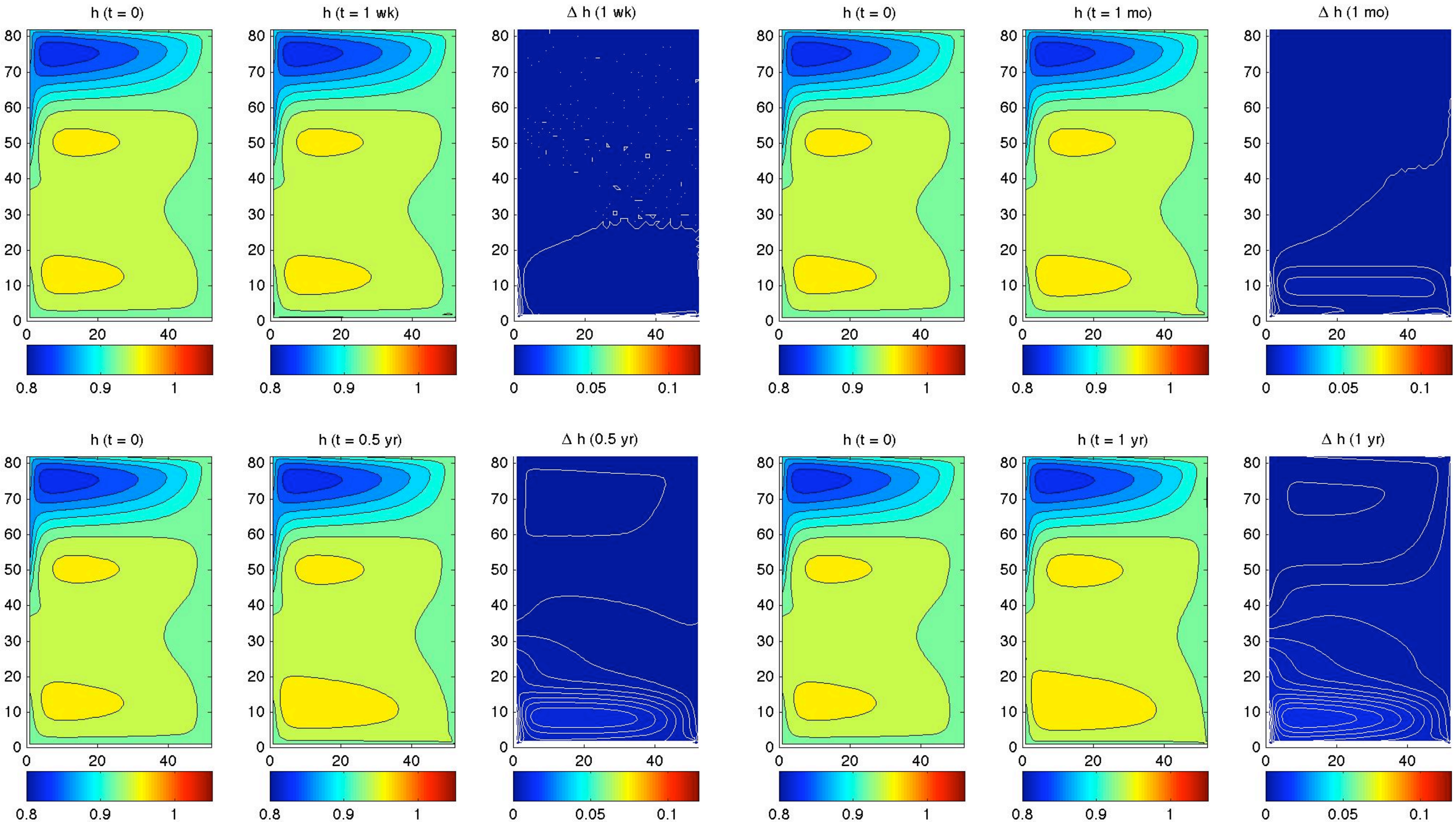

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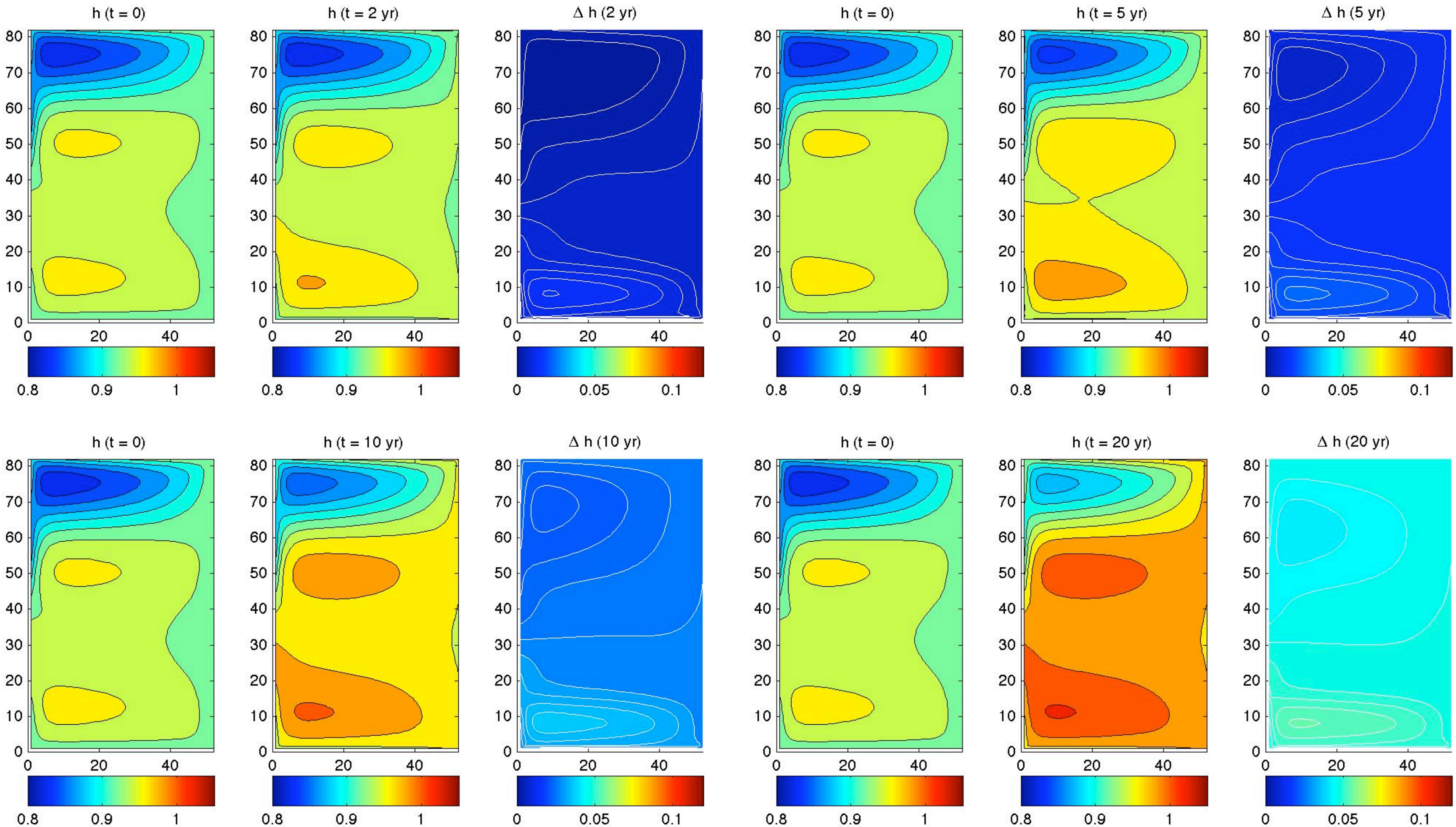

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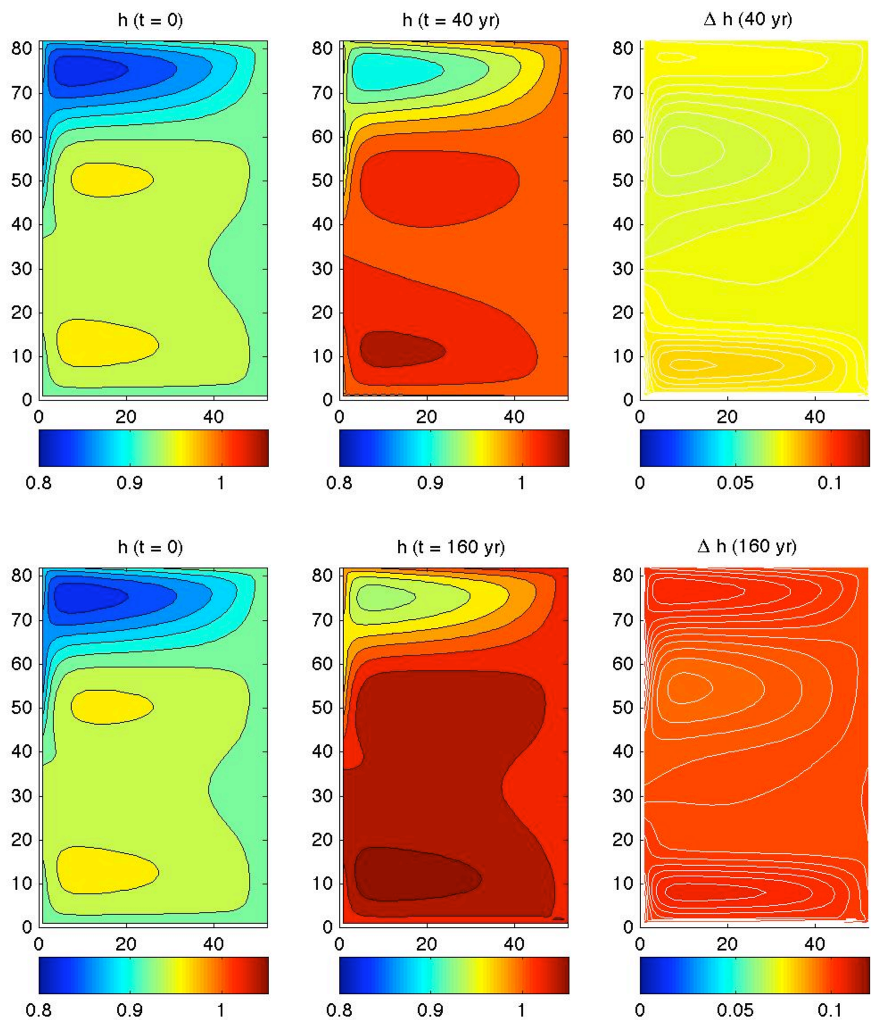

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