Modeling Arctic Sea Ice

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Collaborators

Oksana Guba (post doc, now at Sandia National Labs)
Ed Love (post doc, now at Sandia National Labs)
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Lynn Monday (student, now at Univ of Maryland/ARL)
Kara Peterson (student, now at Sandia National Labs)
Jason Sanchez (student)
Howard Schreyer (UNM Mechanical Engineering)
Ron Kwok (JPL)
Max Coon (NWRA, Seattle)
Gad Levy (NWRA, Seattle)
Matt Prius (NWRA, Seattle)

Funding: NSF, MMS, NASA

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Selected Historical Highlights

1893-1896 Nansen's Voyage: The FRAM drifted between 20-45 degrees to the right of the wind direction.

1902 Ekman layer, Ekman spiral

1928 Sverdrup: Added internal forces proportional to ice velocity but opposite in direction.

1965 Cambell: Viscous fluid model.

1970-1978 Arctic Ice Dynamics Joint Experiment: Elastic-plastic model, thermodyanamics, thickness distribution

1979 Hibler: Viscous-plastic model.

Dedicated to Max Coon

Colleague and Friend

Outline

- Introduction
    - Satellite Observations
    - Motivation
- Ice Model
- Kinematics
- Numerical Simulations
- Conclusions

Why Model Sea Ice?

- Forcasts: shipping, safety and environmental remediation.

  Where is the ice?

  How fast is it moving?

  Where is it going?

- Climate: global climate models

  How thick is the ice?

  What is its extent?

  How much is new?

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Equations of Motion

$$\underbrace{(\rho h)\frac{d\mathbf{v}}{dt}}_{\text{Inertia}} - \mathbf{t}_a - \mathbf{t}_w + \underbrace{(\rho h) f_c (\mathbf{e}_3 \times \mathbf{v})}_{\text{Coriolis}} - \underbrace{\nabla \cdot (\boldsymbol{\sigma} h)}_{\text{Stress div}} = 0$$

Air Drag:

$$\mathbf{t}_a = c_a \rho_a \|\mathbf{v}_a\| \mathbf{R}_a \mathbf{v}_a \qquad \mathbf{R}_a = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$$

Water Drag:

$$\mathbf{t}_w = c_w \rho_w \|\mathbf{v} - \mathbf{v}_w\| \mathbf{R}_w (\mathbf{v} - \mathbf{v}_w) \qquad \mathbf{R}_w = \begin{bmatrix} \cos\beta & -\sin\beta \\ \sin\beta & \cos\beta \end{bmatrix}$$

Equations, continued

Thermodynamics:

$$\rho c \frac{\partial T}{\partial t} = \frac{\partial}{\partial z} k \frac{\partial T}{\partial z} + \kappa I_0 e^{-\kappa z}$$

fixed temp at ice-ocean interface, flux grows ice
flux balance at air-ice interface

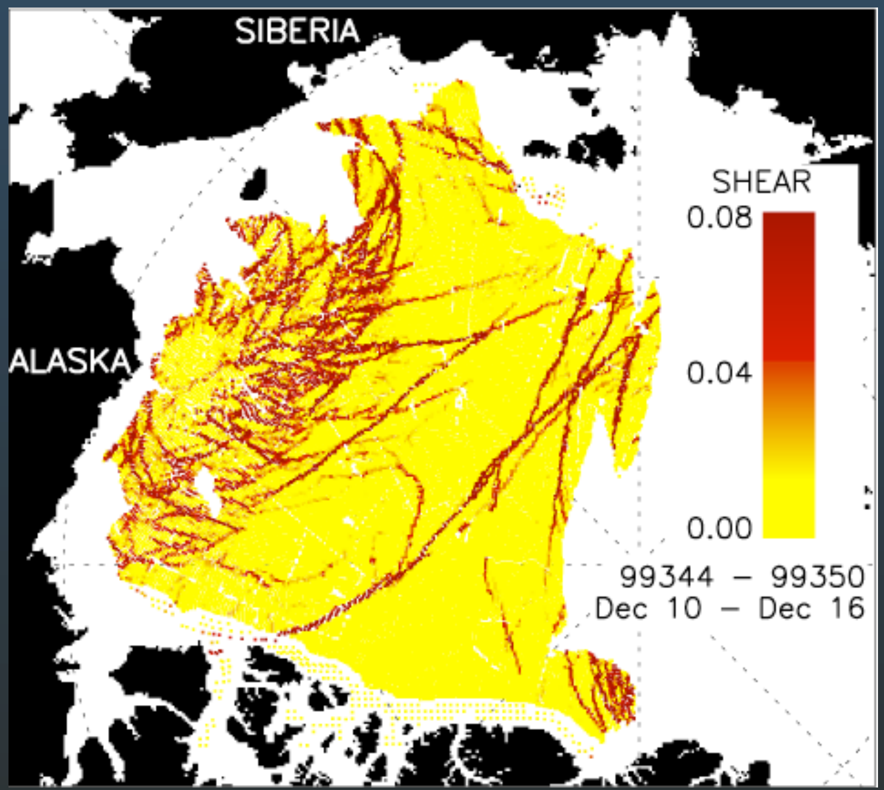
Thickness Distribution

$$\frac{Dg}{Dt} = -\frac{\partial}{\partial h}(fg) - g\nabla \cdot \mathbf{v} + \psi$$

Why A New Ice Model?

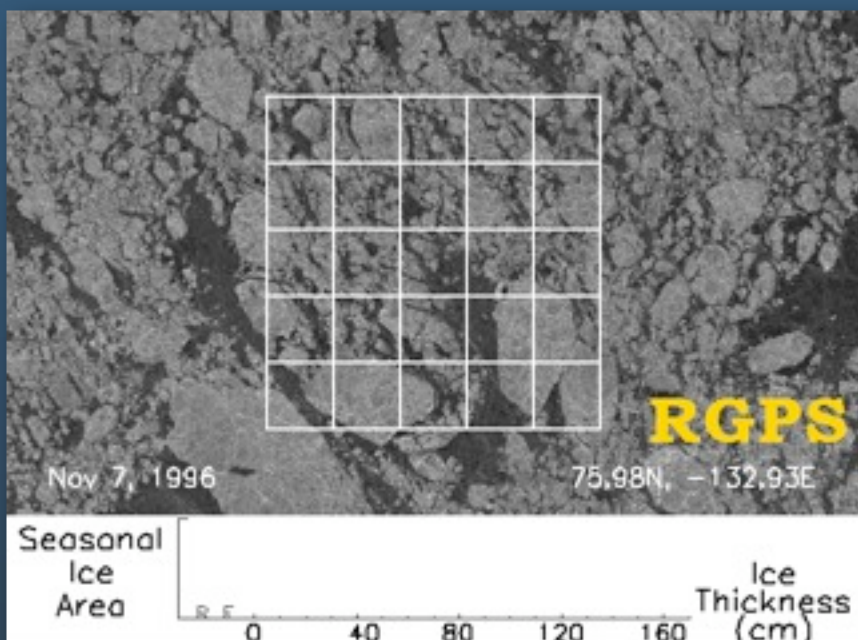
The viscous-plastic model is an isotropic model based on a 100 km scale in which it was assumed that cracks, ridges and leads were randomly distributed.

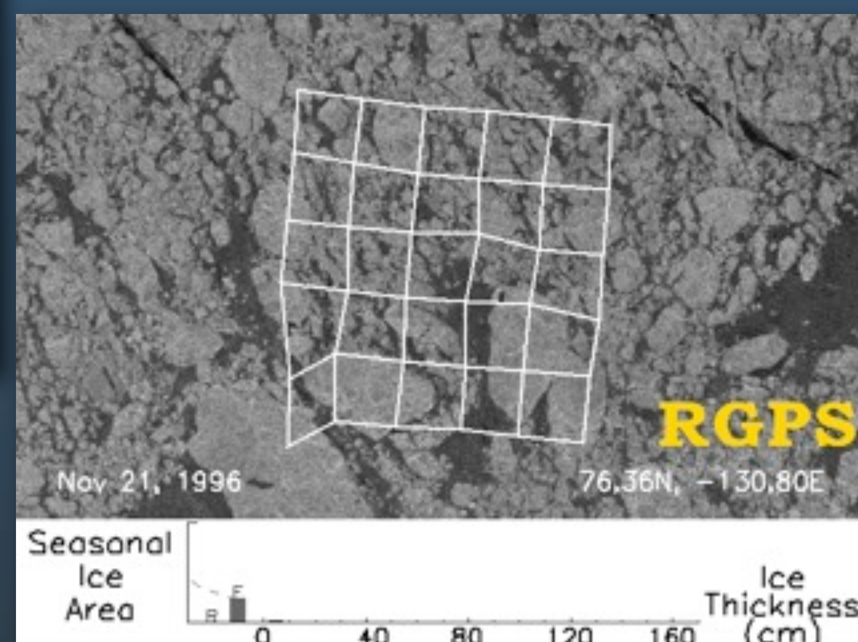
RGPS analysis of satellite images shows large ice deformation events occurring in long-lasting linear features that appear to correspond to displacement (or velocity)discontinuities in the deformation field due to leads.

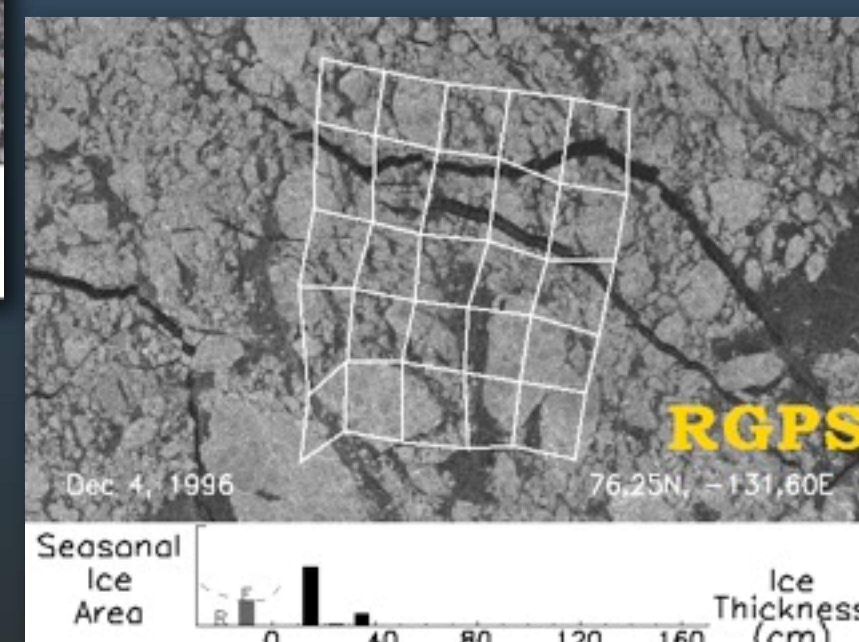


RGPS

Radarsat Geophysical Processor System
at JPL







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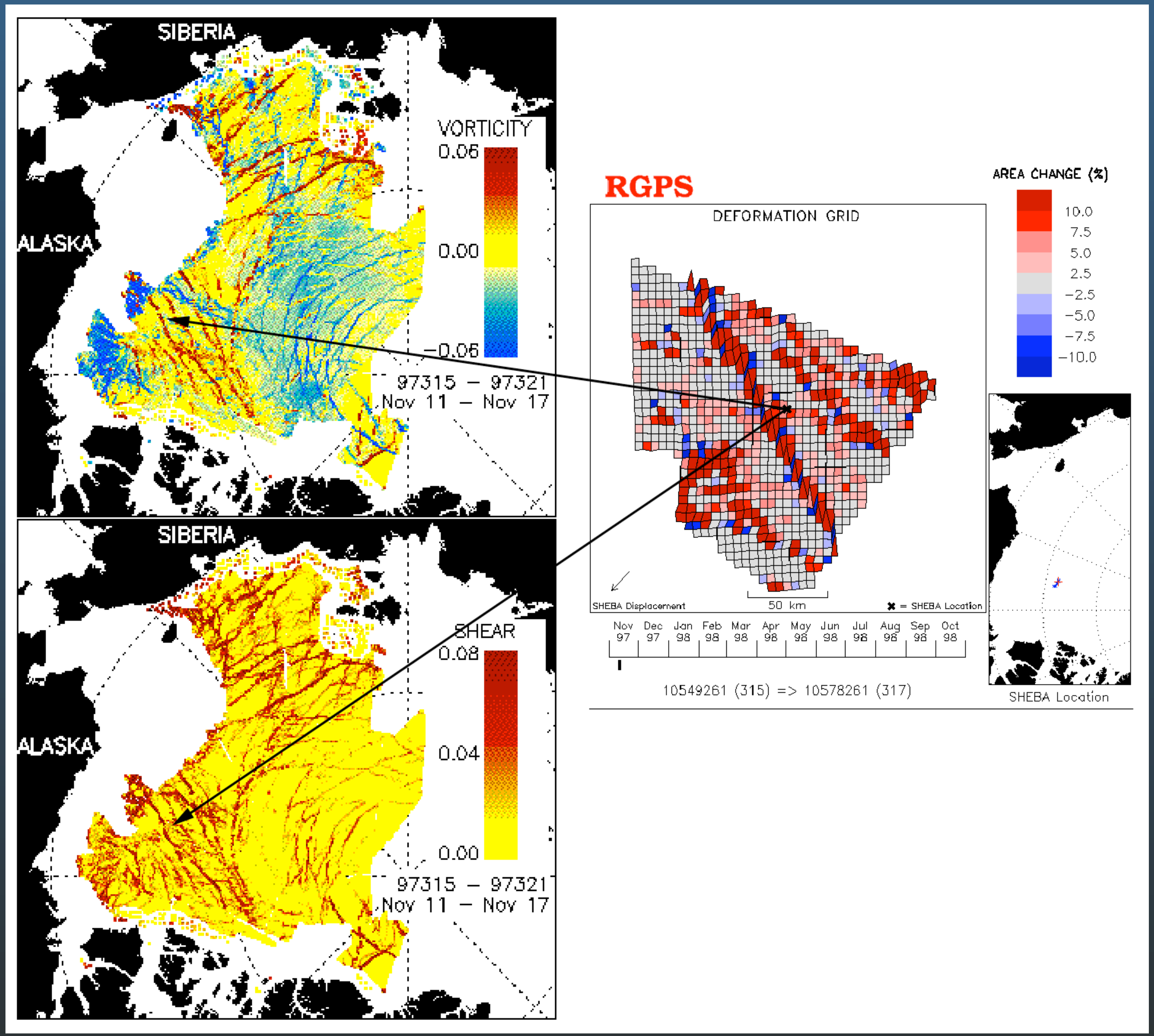
RGPS Cells



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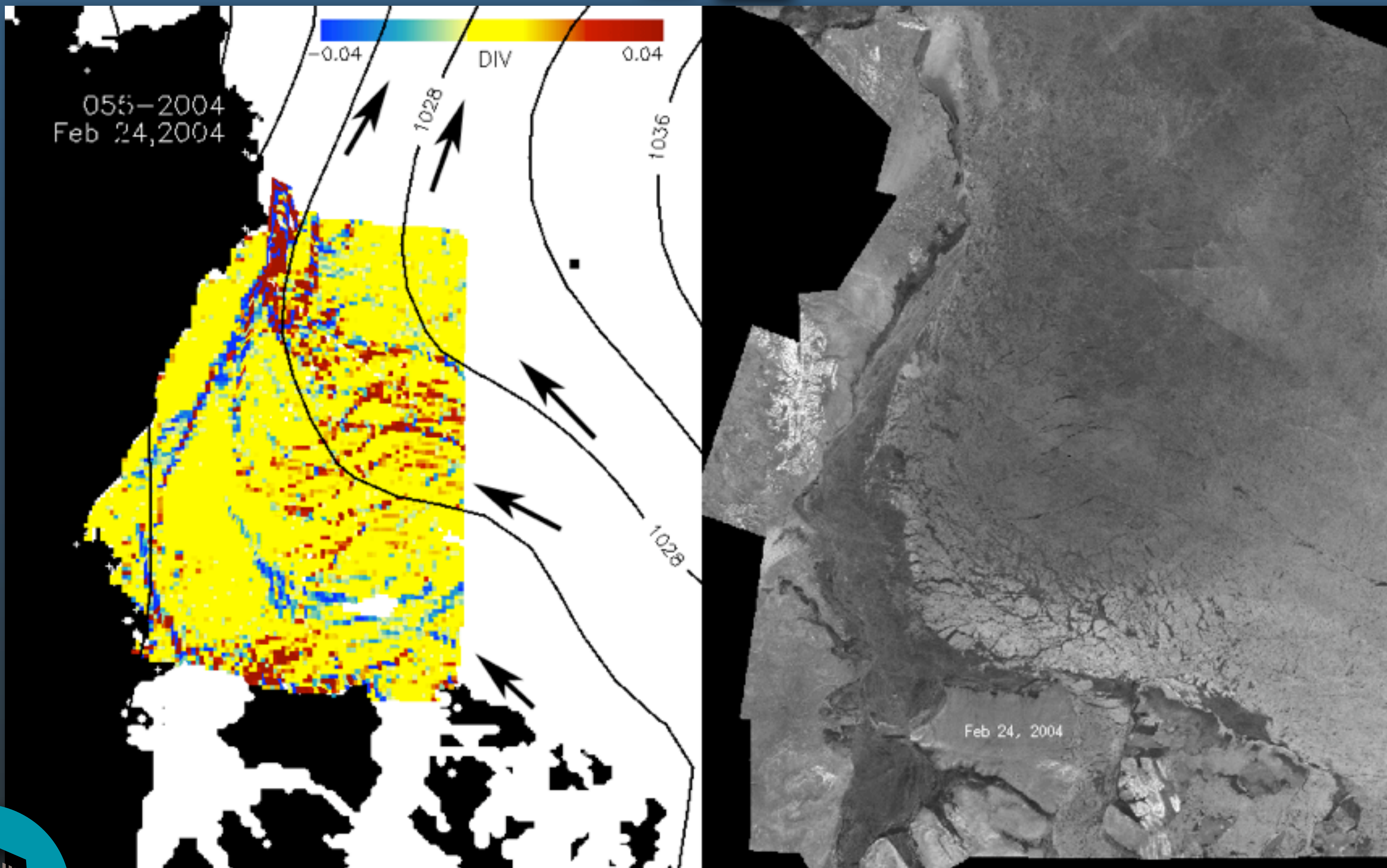
Divergence



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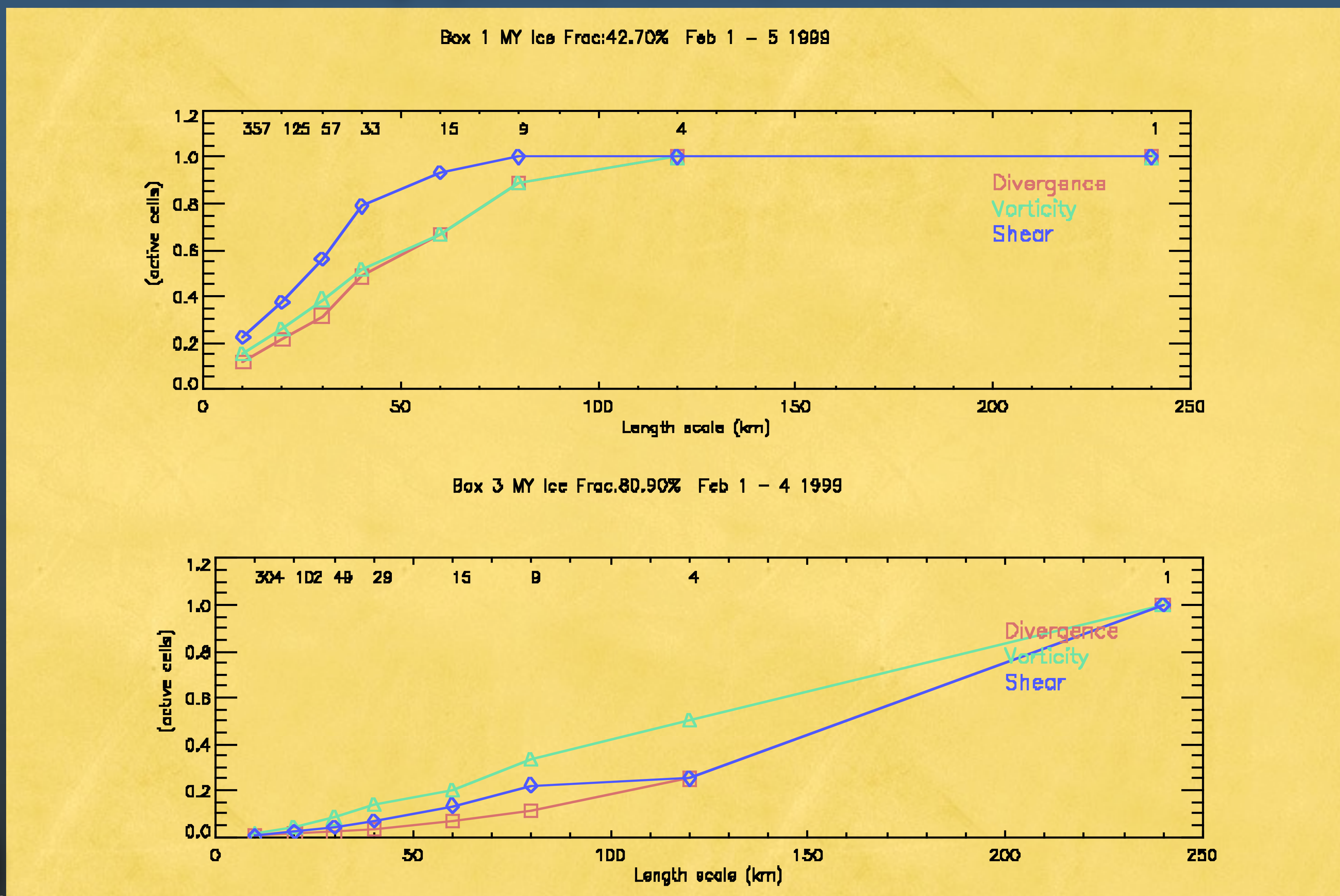
Why a New Ice Model?

Most 10 km Lagrangian cells do not have permanent deformation during the year

(R. Kwok, J. Geophys. Res., Vol. 111, No. C11, C11S22, 2006)



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Elastic-Decohesive Sea Ice Model

Overall Objective: Numerically simulate "linear kinematic features" (eg. leads and ridges)

Initial Focus: Prediction and appearance of leads

- Dominant feature of the Arctic
- Source of new ice production
- Allow ice motion
- Key to describing forces in sea ice

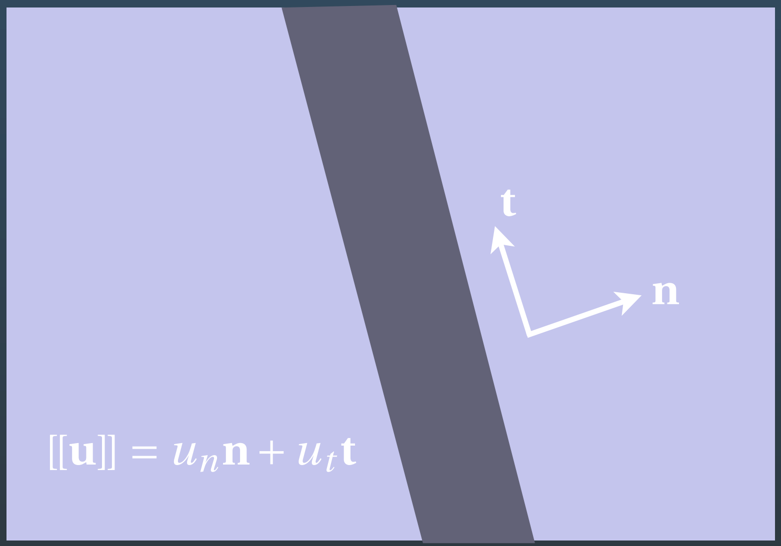
Proposed Approach: Elastic-Decohesive Model

For thick first-year ice and multi-year ice, we assume most deformation occurs due to discontinuities in the displacement field.

Ice is quasibrittle so we can borrow from models of concrete and rock.

Elastic-Decohesive Model

- Intact ice modeled as elastic
- Leads modeled as discontinuities
- Model predicts initiation of a lead and its orientation
- Traction is reduced with lead opening until a complete fracture forms



Schreyer, H., L. Monday, D. Sulsky, M. Coon, R. Kwok (2006), Elastic-decohesive Constitutive Model for Sea Ice, J. of Geophys. Res., 111, C11S26, doi:10.1029/2005JC003334.

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Elastic-Decohesive Model

- Similar to elastic-plastic model:
  - damage surface ~ yield surface
  - damage surface gives stress state at which a lead begins
  - damage surface gives orientation of lead
  - behavior is anisotropic after failure
  - damage surface constructed from empirical data (Schulson*) and *in situ* data (Coon†)
- Goal: capture essential properties:
  - correct energy dissipation
  - correct peak stress
  - keep method numerical tractable

*Schulson, Brittle Failure of Ice. Engineering Fracture Mechanics, 68:1839-1887, 2001.

†Coon, Knoke, Eckert, and Pritchard, JGR, 103(C10), 21,915– 921,925, 1998.

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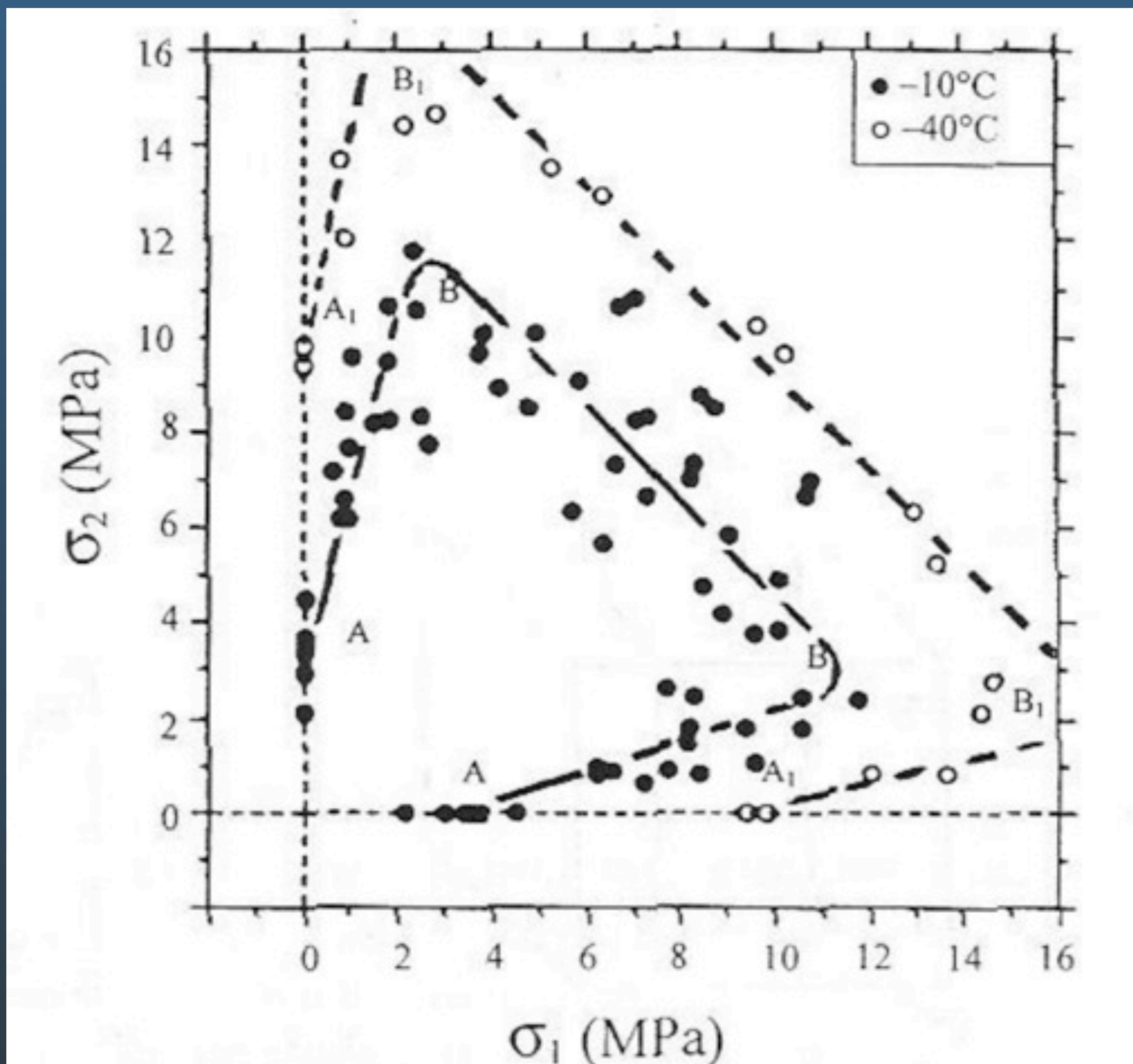
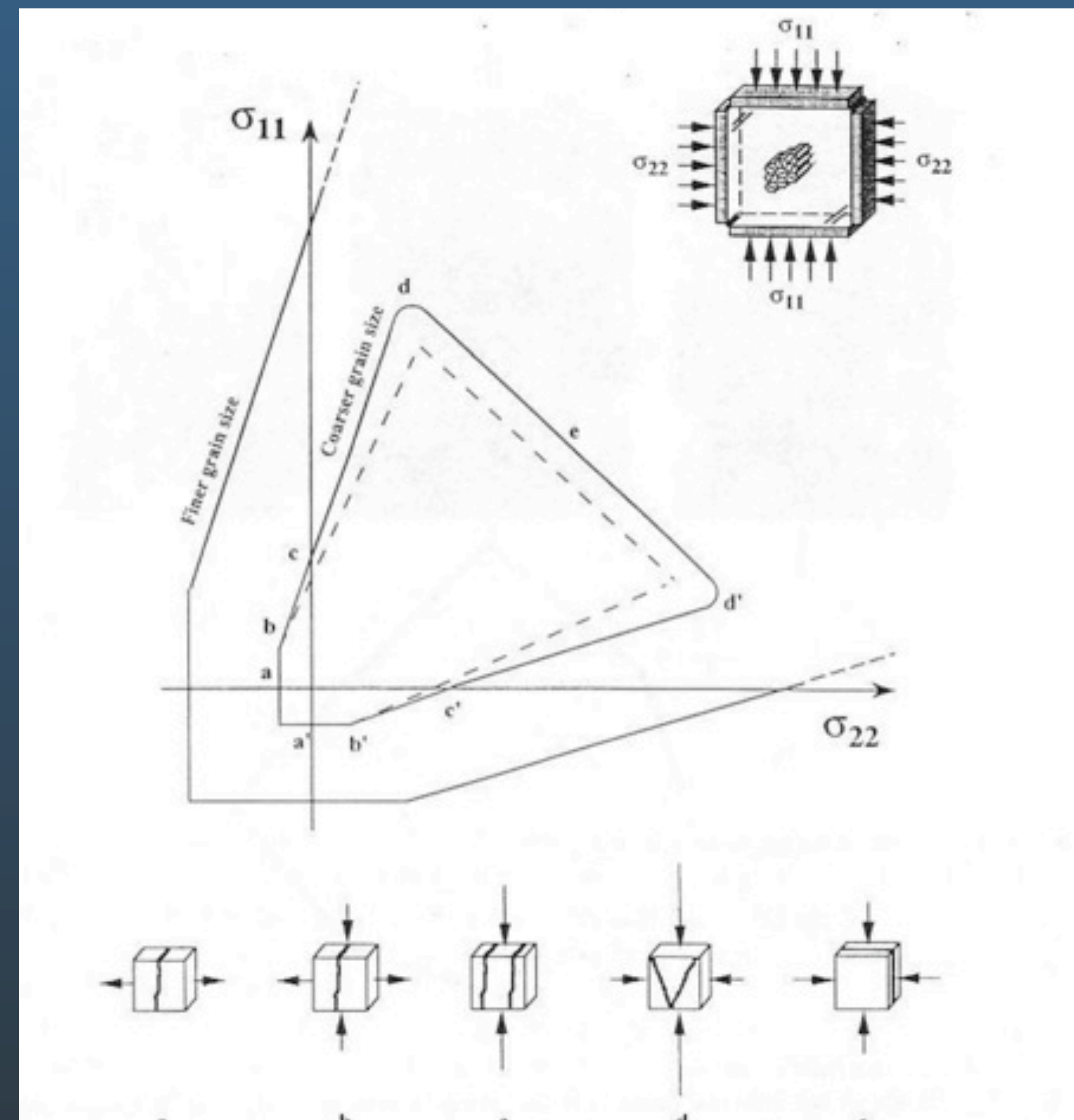
Laboratory Data





Schulson, E. M. (2001) Brittle failure of ice,
Engng. Fract Mech., **68**:1879-1887

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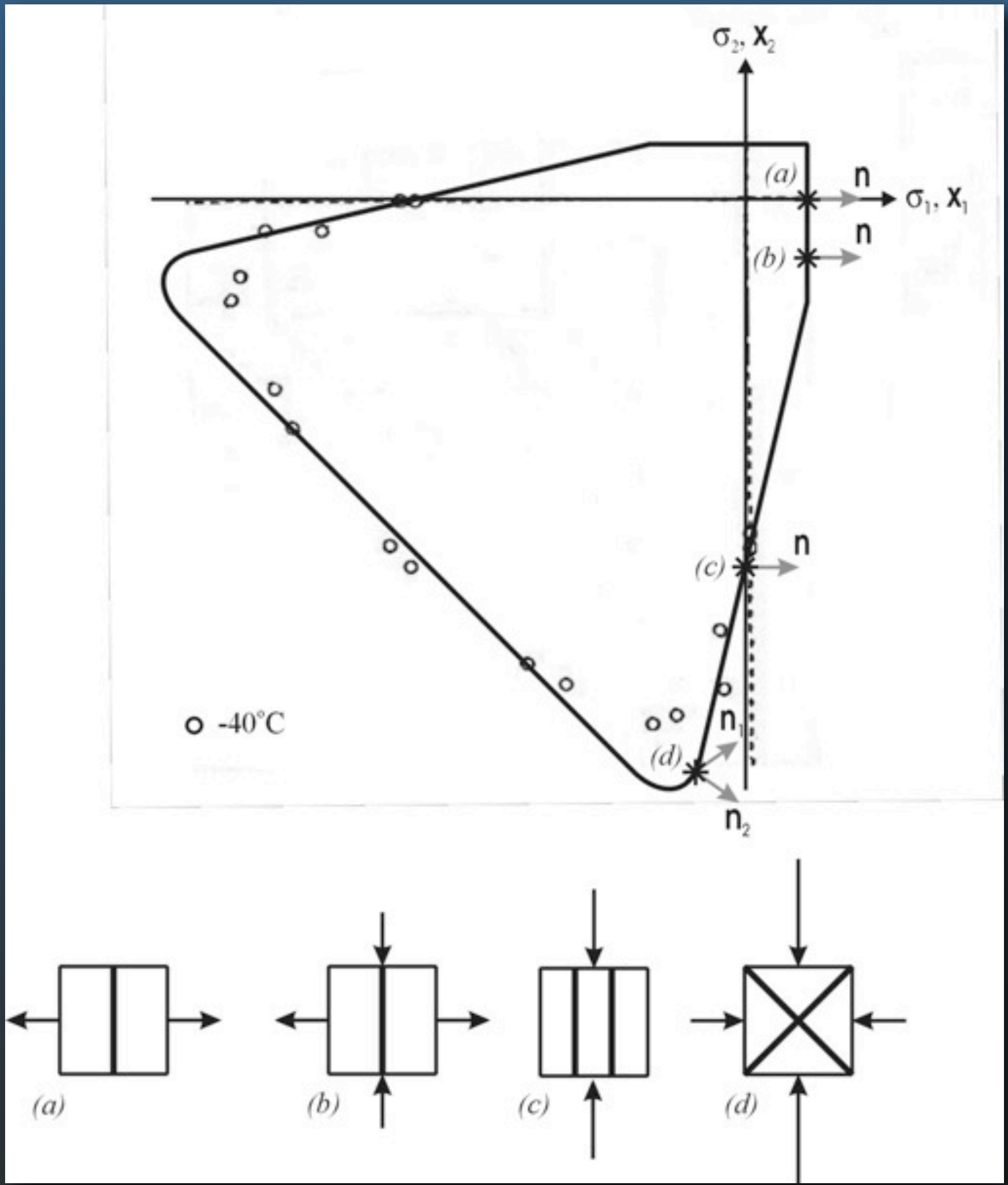
Stress at Failure - Failure Initiation

The failure envelope in stress space that describes initiation of failure is

$$F(\boldsymbol{\sigma}) = 0$$

What is *F* ?

Schulson, E. Brittle Failure of Ice. Engineering Fracture Mechanics, 68:1839-1887, 2001.



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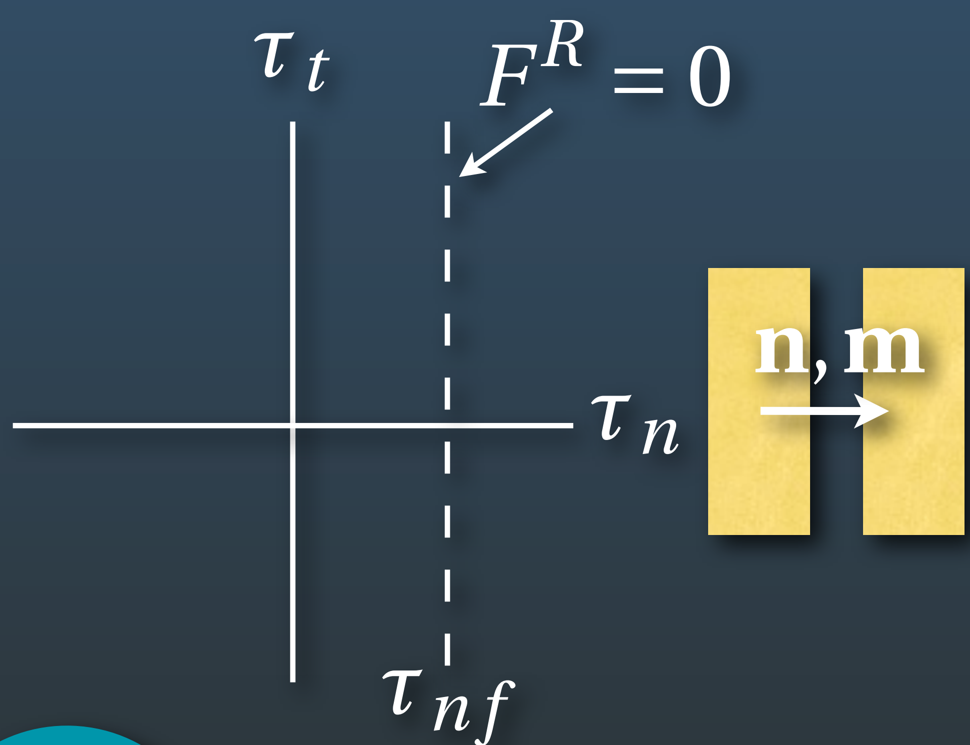
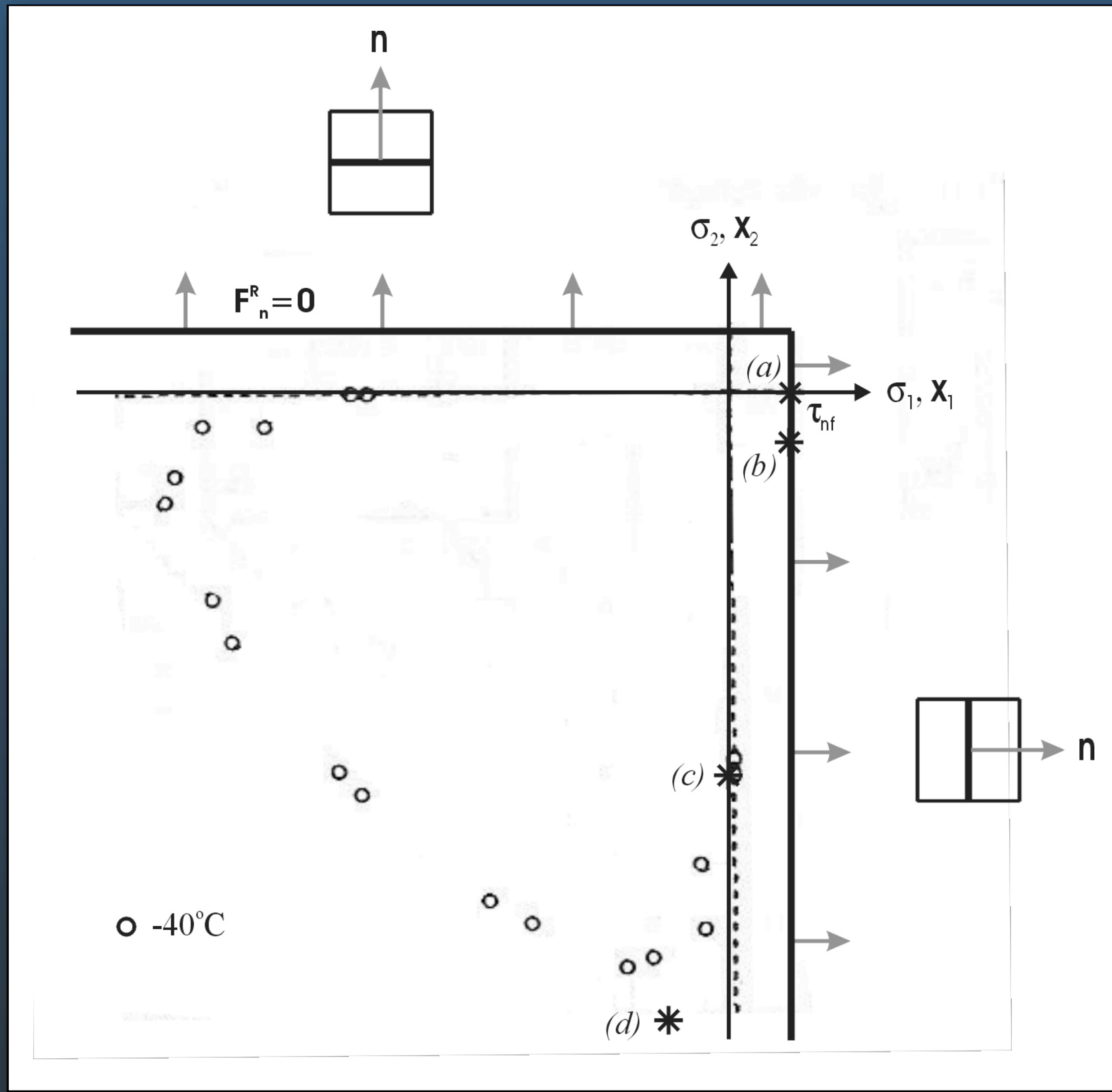
Rankine Criterion

$$F_n^R = \frac{\tau_n}{\tau_{nf}} - 1$$

τ_{nf} = tensile strength





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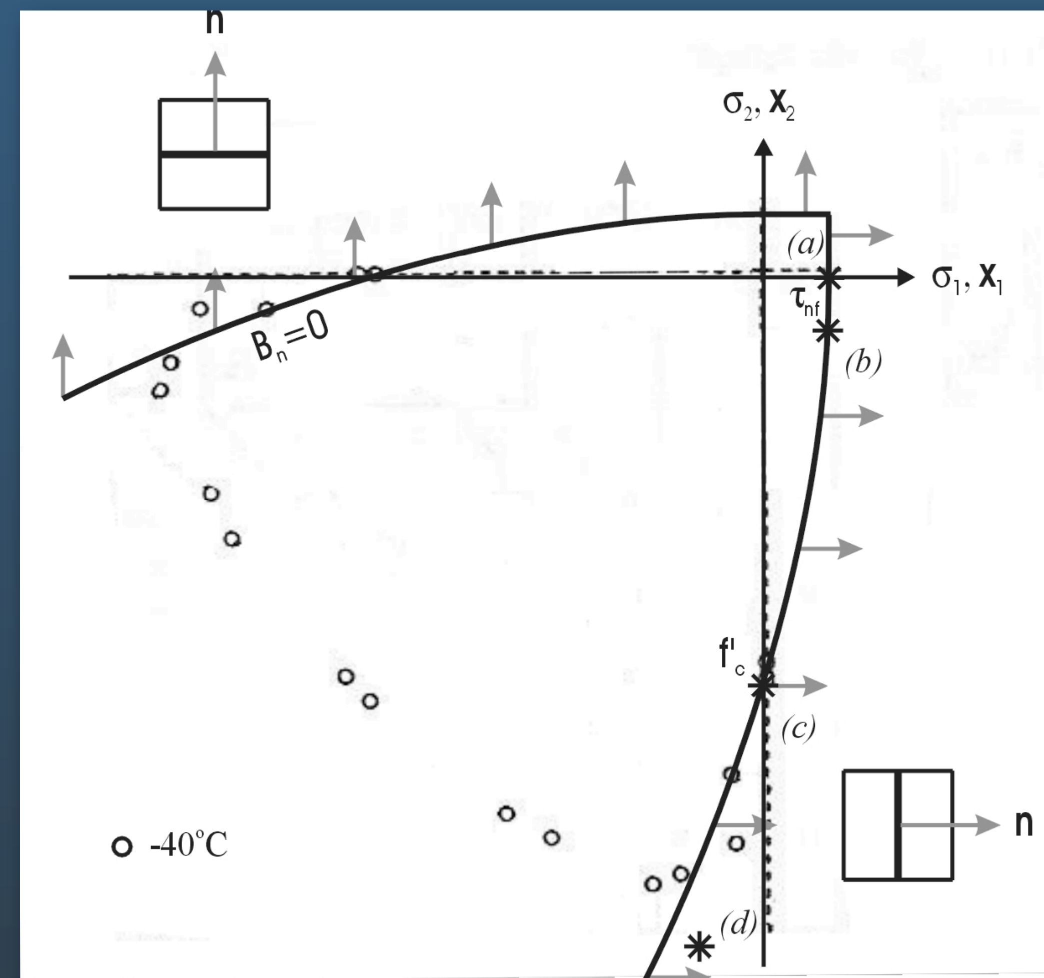
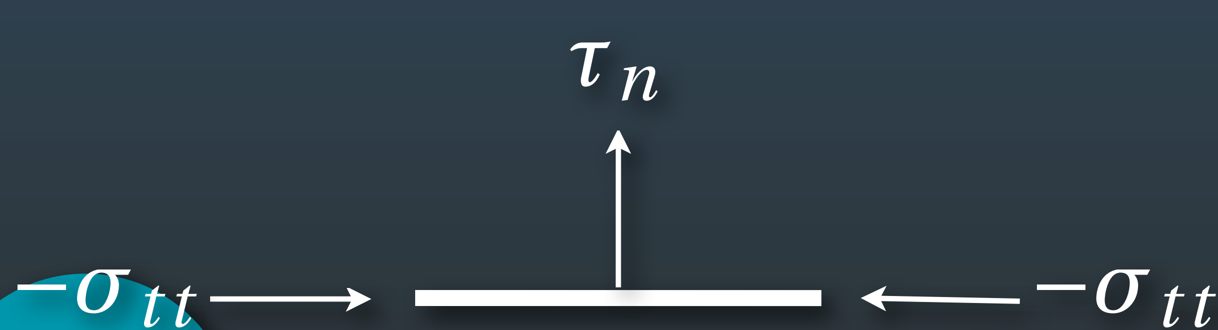
Brittle Decohesion Criterion

$$B_n = \frac{\tau_n}{\tau_{nf}} + \frac{\langle -\sigma_{tt} \rangle^2}{f_c'^2} - 1$$

$$\langle x \rangle = \begin{cases} x, & \text{if } x \geq 0 \\ 0, & \text{if } x < 0 \end{cases}$$

f_c' = compressive strength

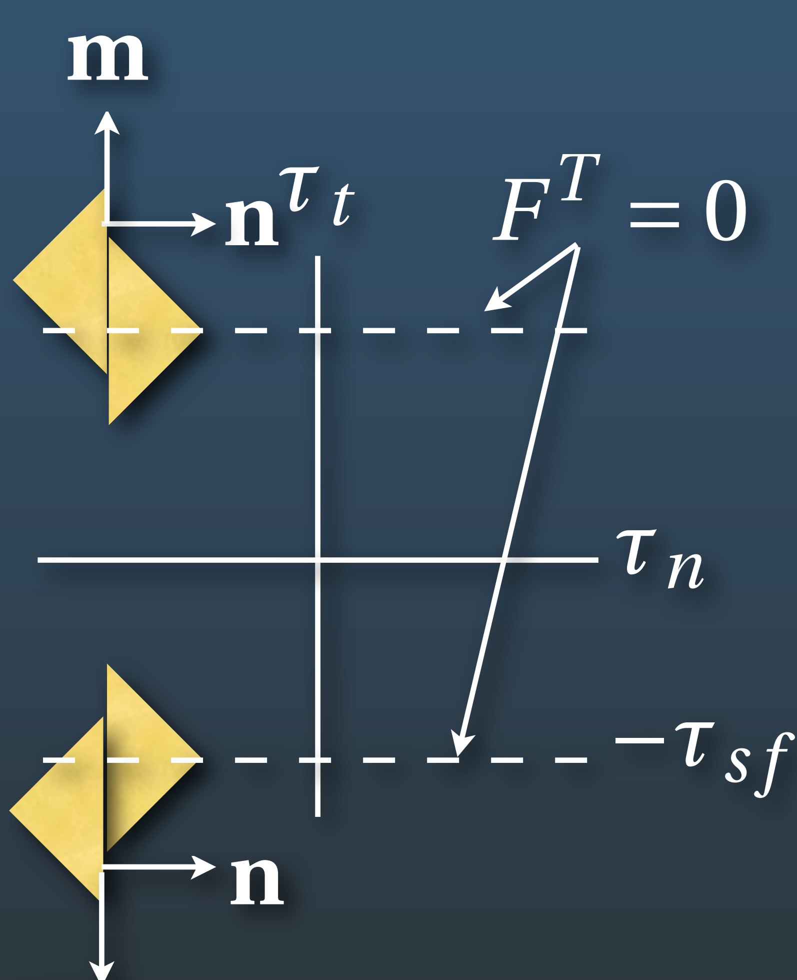


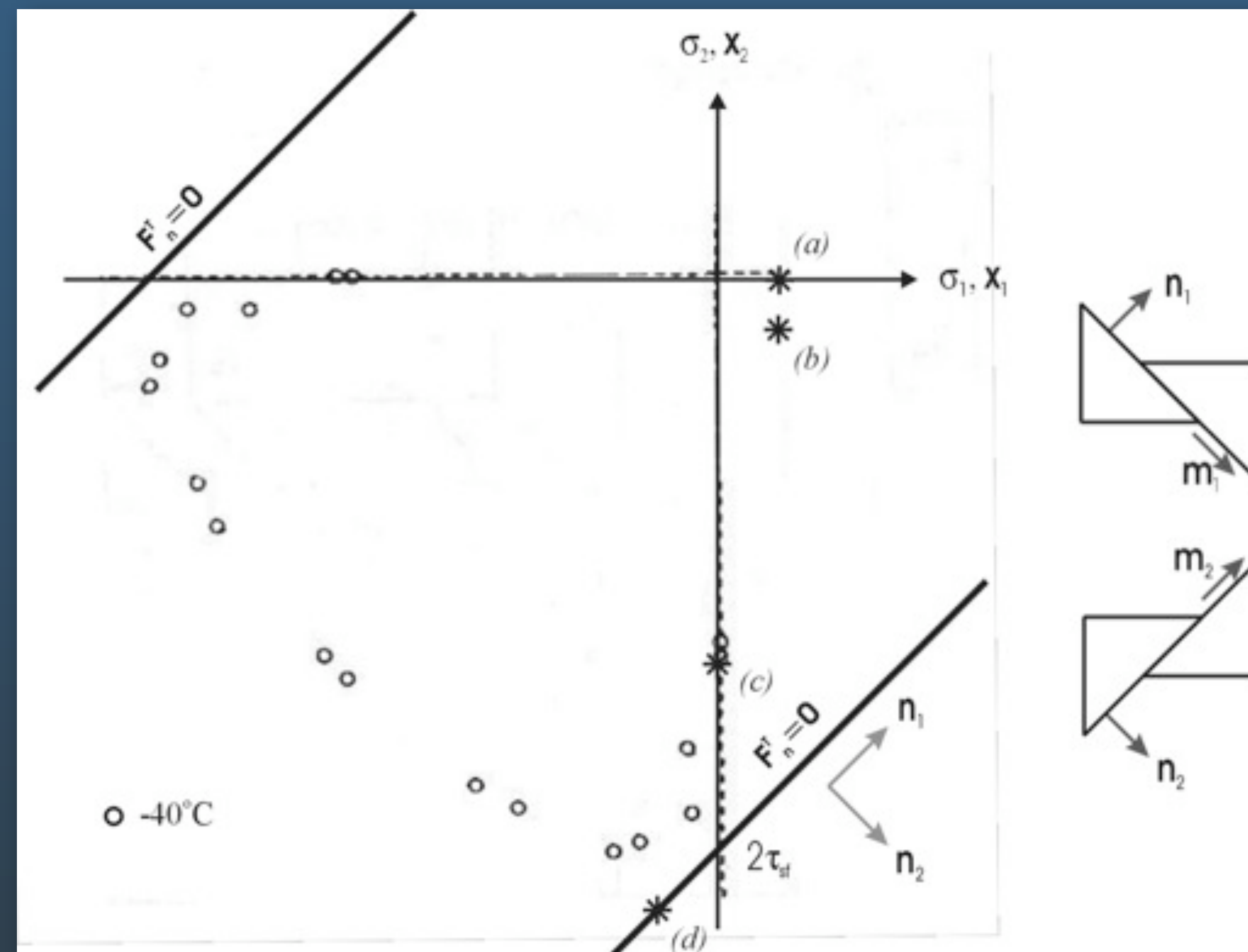


Tresca Criterion

$$F_n^T = \left(\frac{\tau_t}{\tau_{sf}}\right)^2 - 1$$

τ_{sf} = shear strength





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Stress at Failure - Failure Initiation - Orientation

Schulson, E. M. (2001) Brittle failure of ice, Engng. Fract Mech., **68**:1879-1887

$$F_n = \left(\frac{\tau_t}{s_m \tau_{sf}}\right)^2 + e^{\kappa B_n} - 1$$

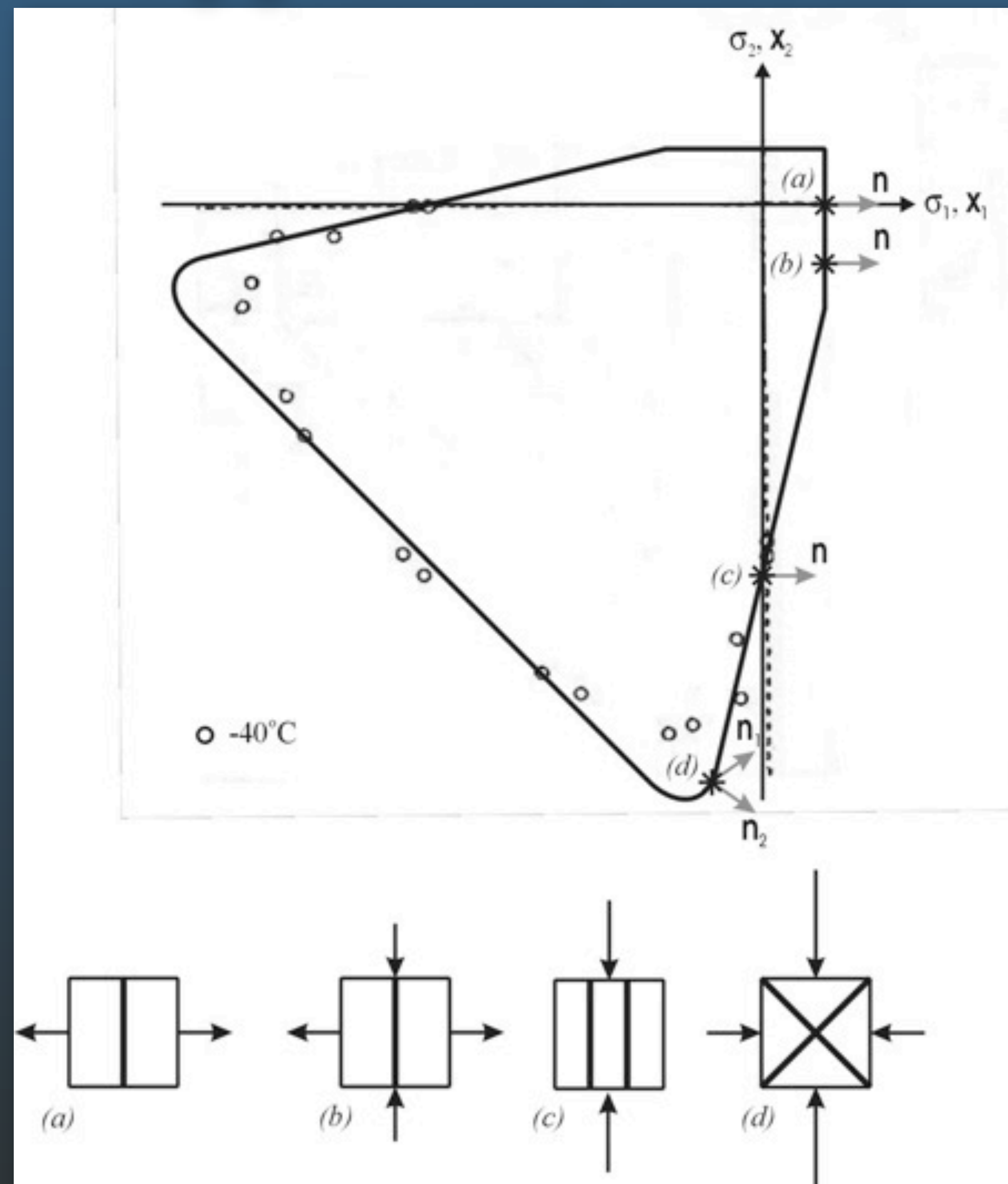
$$B_n = \frac{\tau_n}{\tau_{nf}} + \frac{\langle -\sigma_{tt} \rangle^2}{f_c'^2} - 1$$

τ_{nf} = tensile strength

τ_{sf} = shear strength

f_c' = compressive strength

s_m = shear magnification



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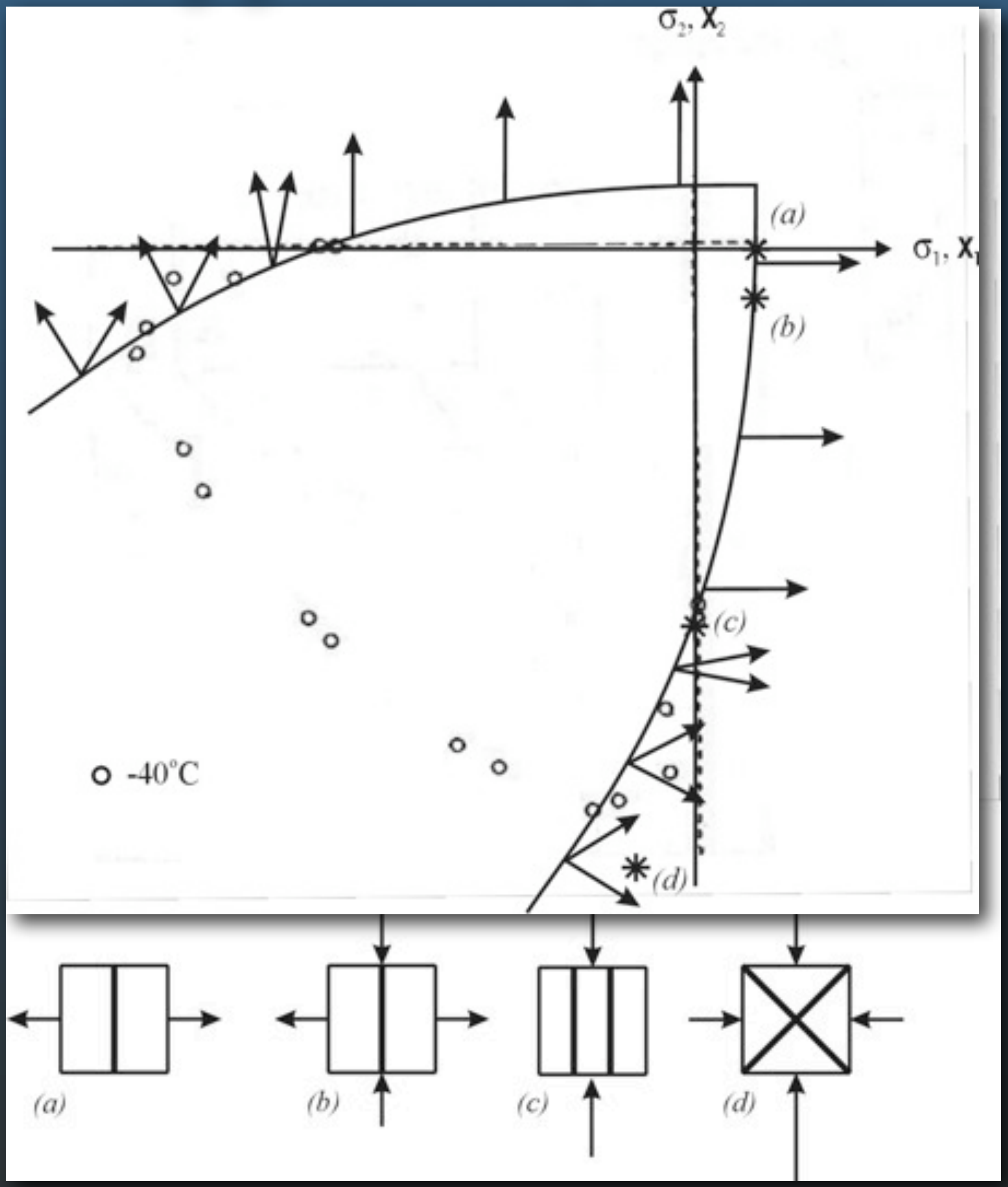

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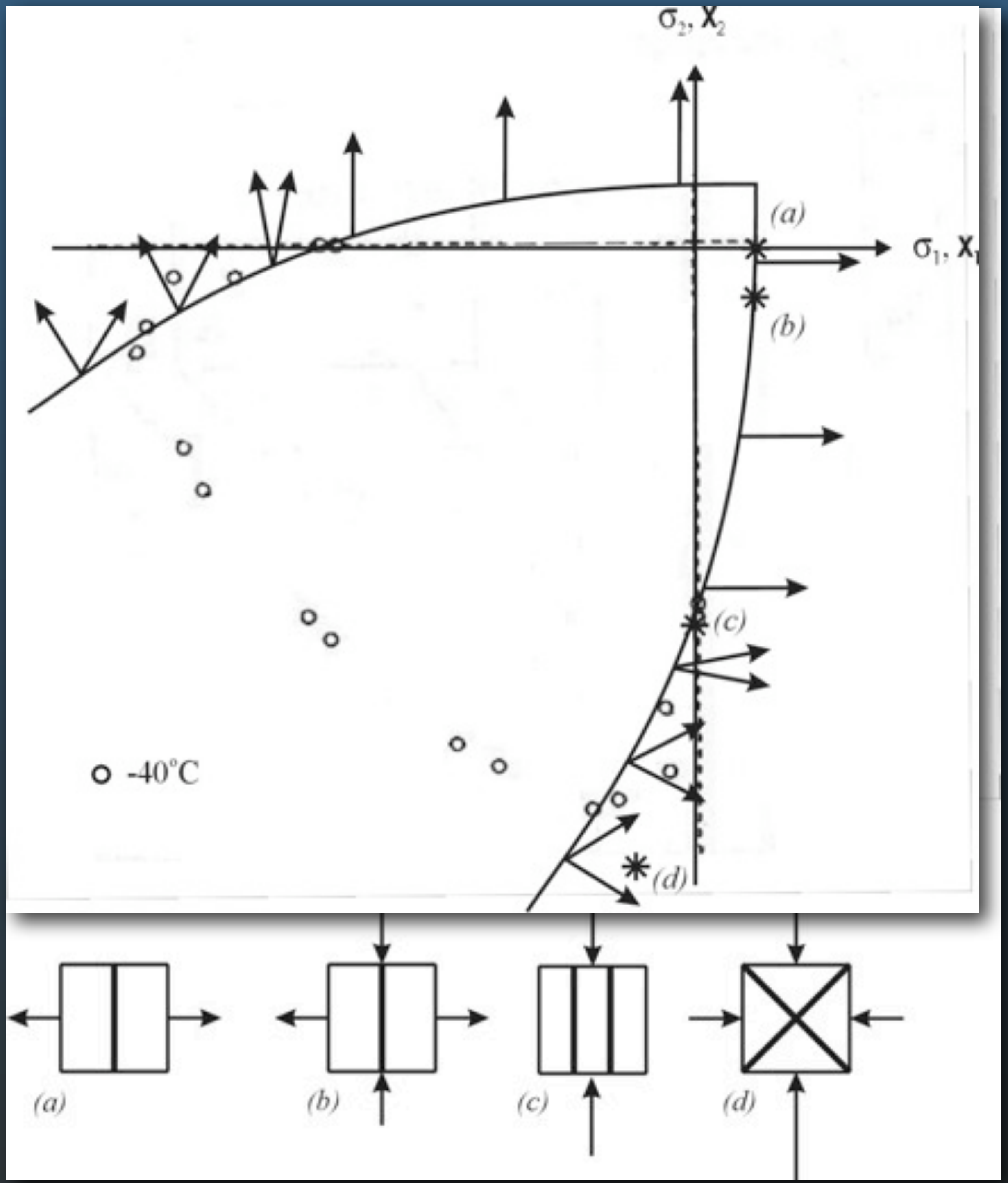

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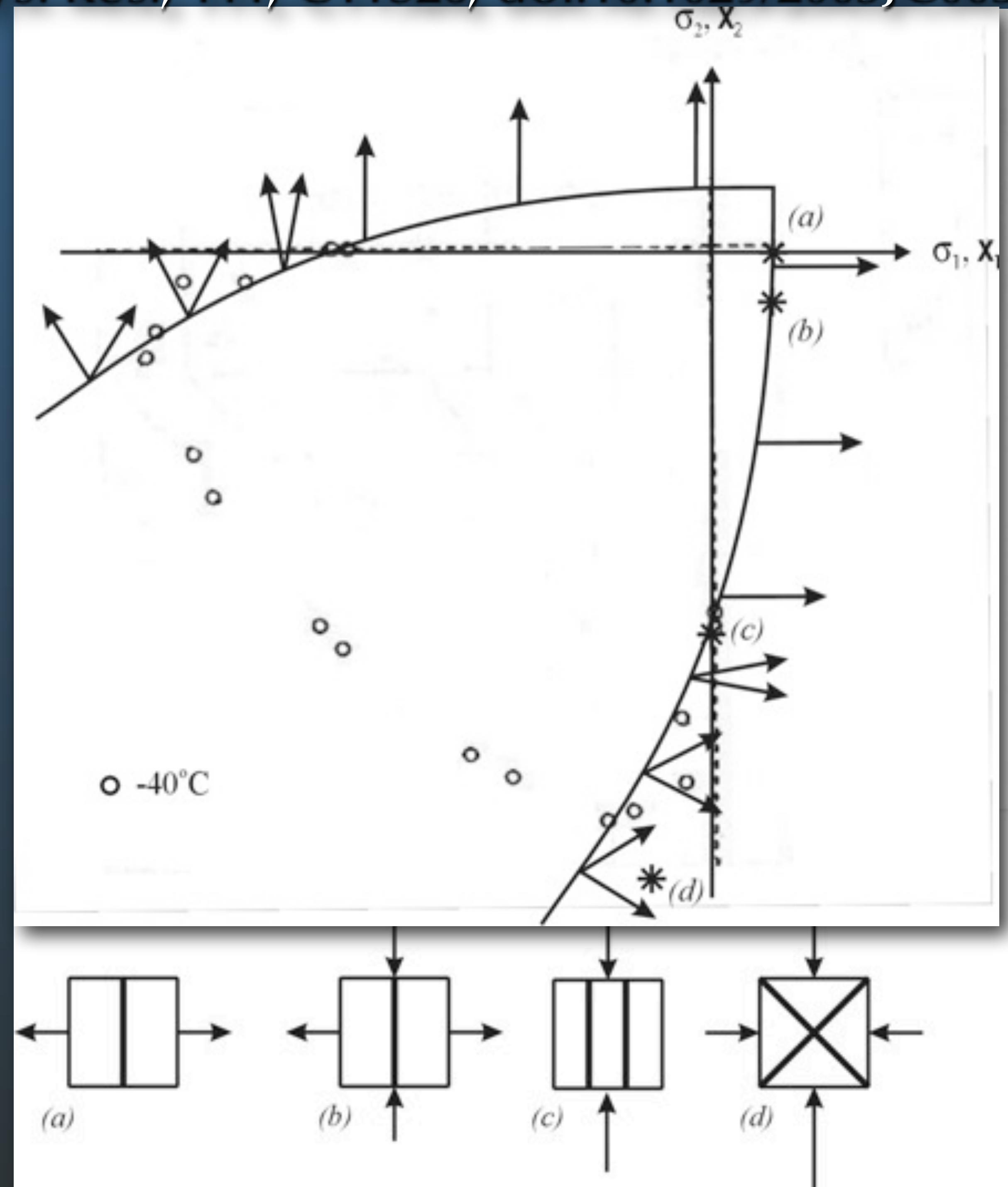

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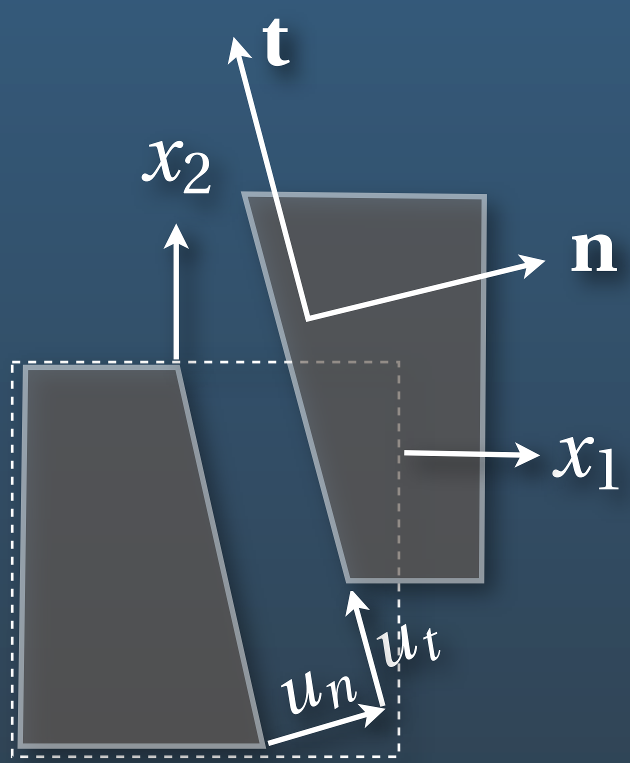
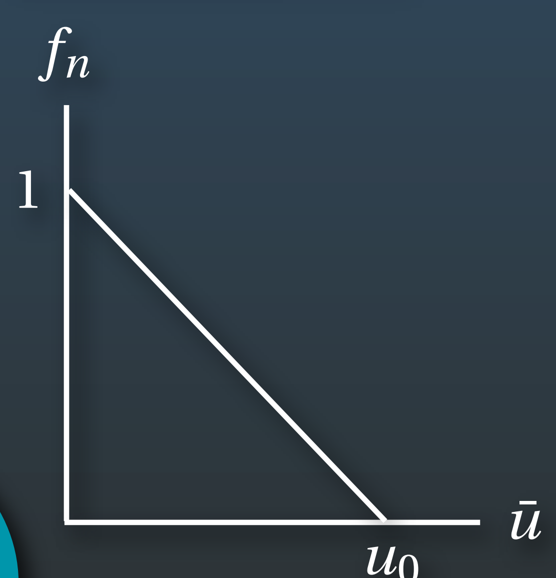
Failure Evolution

As decohesion occurs material becomes weaker.





$$[\mathbf{u}] = u_n \mathbf{n} + u_t \mathbf{t}$$

$$\bar{u} = u_n$$

$$f_n = \left\langle 1 - \frac{\bar{u}}{u_0} \right\rangle$$

$$B_n = \frac{\tau_n}{\tau_{nf}} + f_n \left[\frac{\langle -\sigma_{tt} \rangle^2}{f_c'^2} - 1 \right]$$

$$F_n = \left(\frac{\tau_t}{s_m \tau_{sf}} \right)^2 + e^{\kappa B_n} - 1$$

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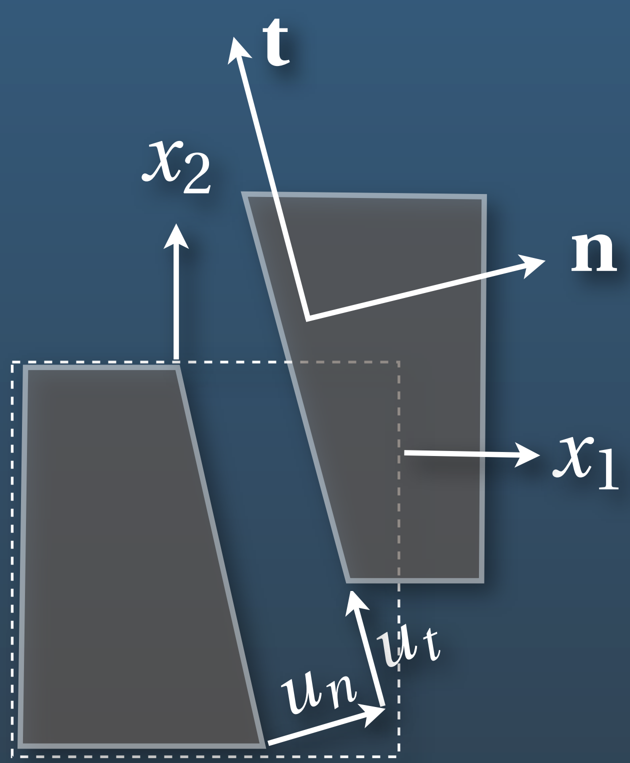

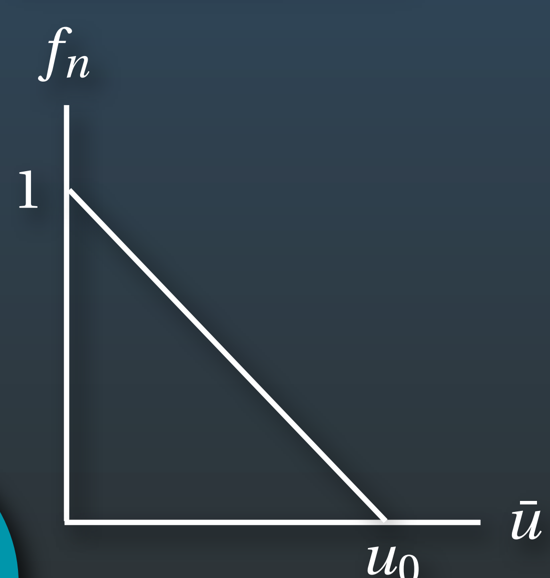

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$$F_n = \left(\frac{\tau_t}{s_m \tau_{sf}} \right)^2 + e^{\kappa B_n} - 1$$

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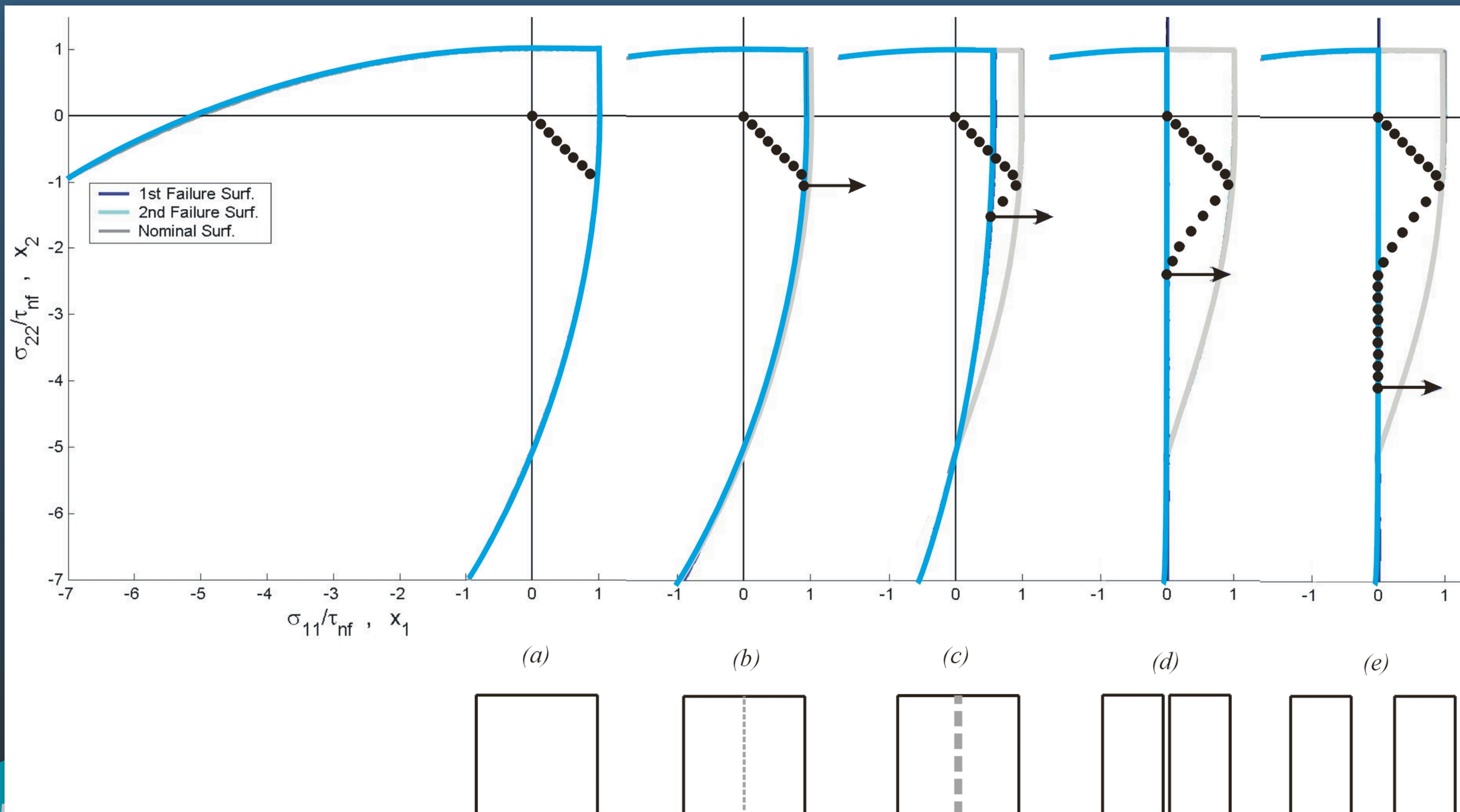
Failure Evolution

$$e_1 = -e_2$$



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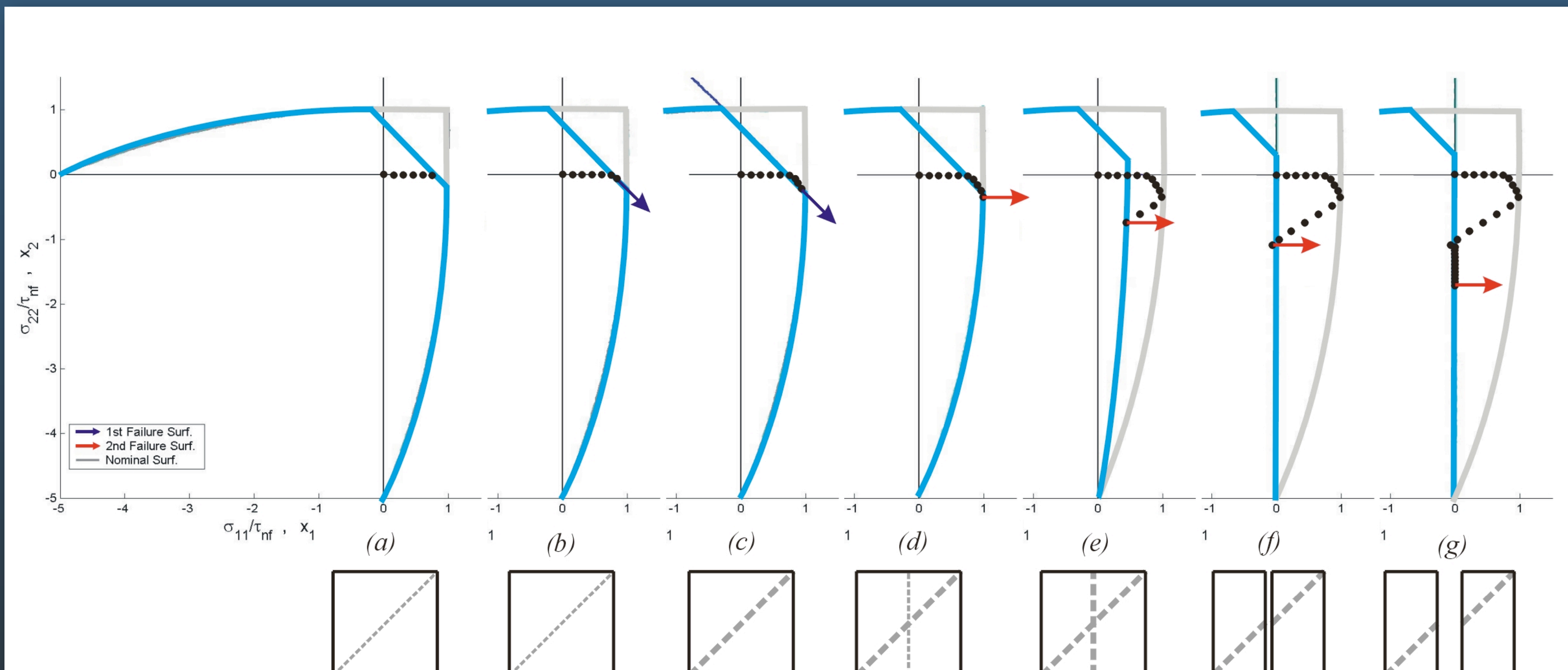
Initialized Weak Plane

$f_n = 0.5$ $\theta = 45^\circ$



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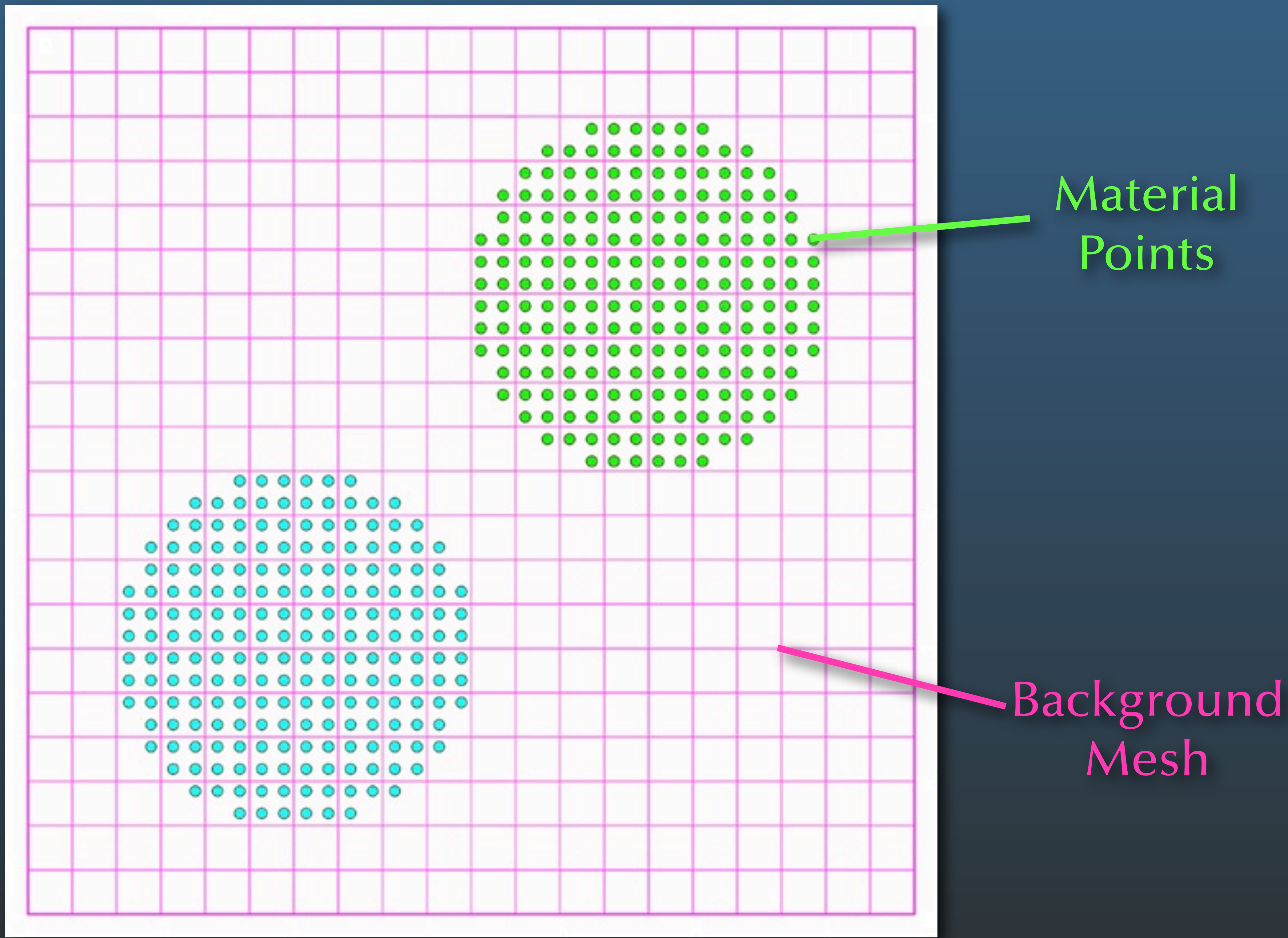
Elements of an MPM Simulation



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Computational Cycle

1. Interpolate material-point data to background mesh

$$m_i = \sum_p m_p N_i(\mathbf{x}_p)$$

$$m_i \mathbf{v}_i = \sum_p m_p \mathbf{v}_p N_i(\mathbf{x}_p)$$

$$\mathbf{f}_i^{\text{int}} = -\sum_p \mathbf{G}_{pi}^T \boldsymbol{\sigma}_p m_p / \rho_p$$

2. Solve equations of motion on mesh

$$m_i \mathbf{a}_i = \mathbf{f}_i$$

$$\mathbf{v}_i \leftarrow \mathbf{v}_i + \Delta t \mathbf{a}_i$$

3. Update material points

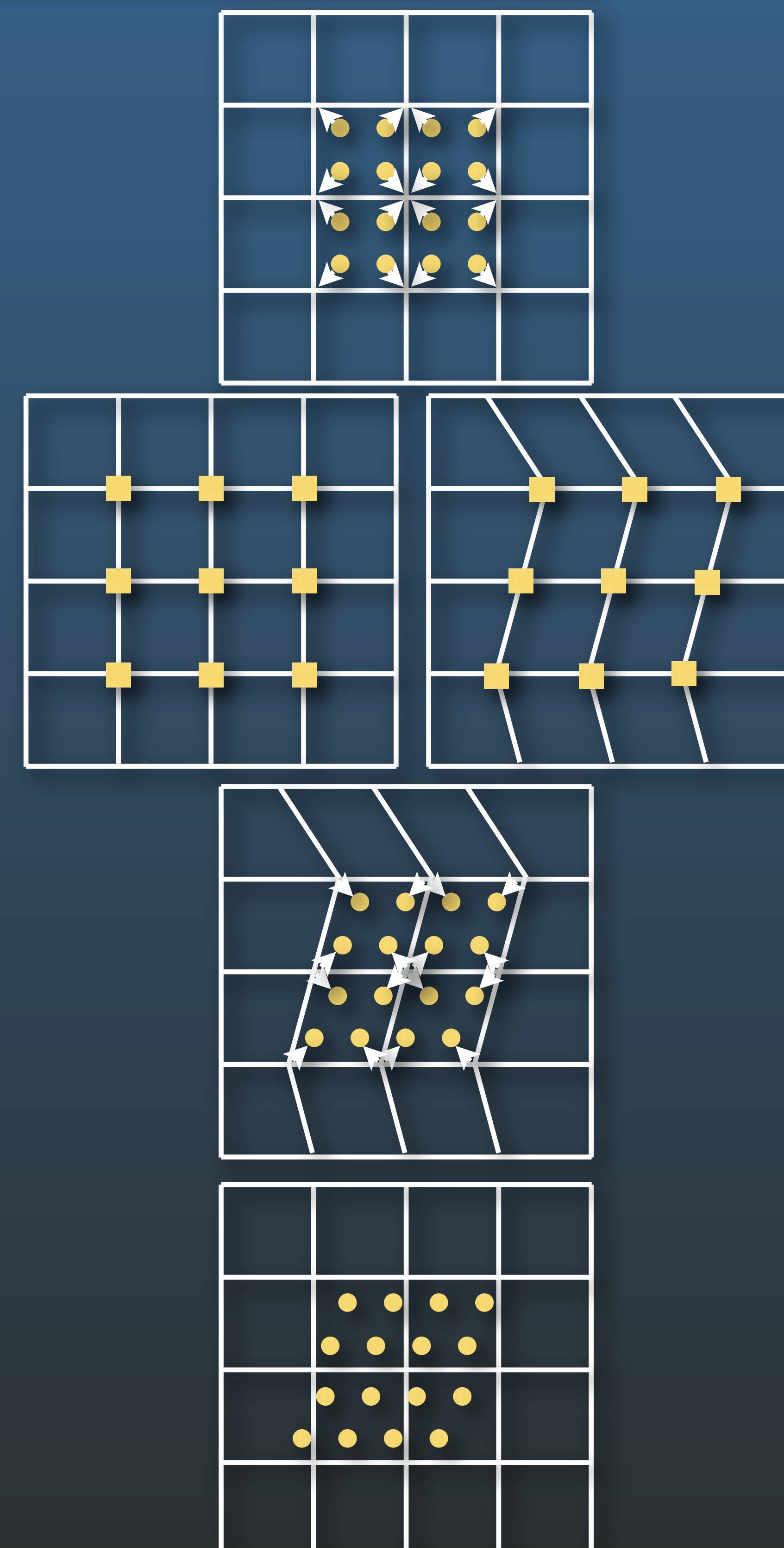
$$\mathbf{x}_p \leftarrow \mathbf{x}_p + \Delta t \sum_i \mathbf{v}_i N_i(\mathbf{x}_p)$$

$$\mathbf{v}_p \leftarrow \mathbf{v}_p + \Delta t \sum_i \mathbf{a}_i N_i(\mathbf{x}_p)$$

$$\mathbf{F}_p \leftarrow \mathbf{f}_p \mathbf{F}_p, \quad \mathbf{f}_p = \mathbf{I} + \Delta t \sum_i \mathbf{v}_i \nabla N_i(\mathbf{x}_p)$$

$$\boldsymbol{\sigma}_p = \ldots$$

4. Redefine the grid



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Features of MPM

Dual description of the continuum: material points and background computational mesh

The convective phase of the algorithm is performed by Lagrangian material points which carry position, mass, velocity…

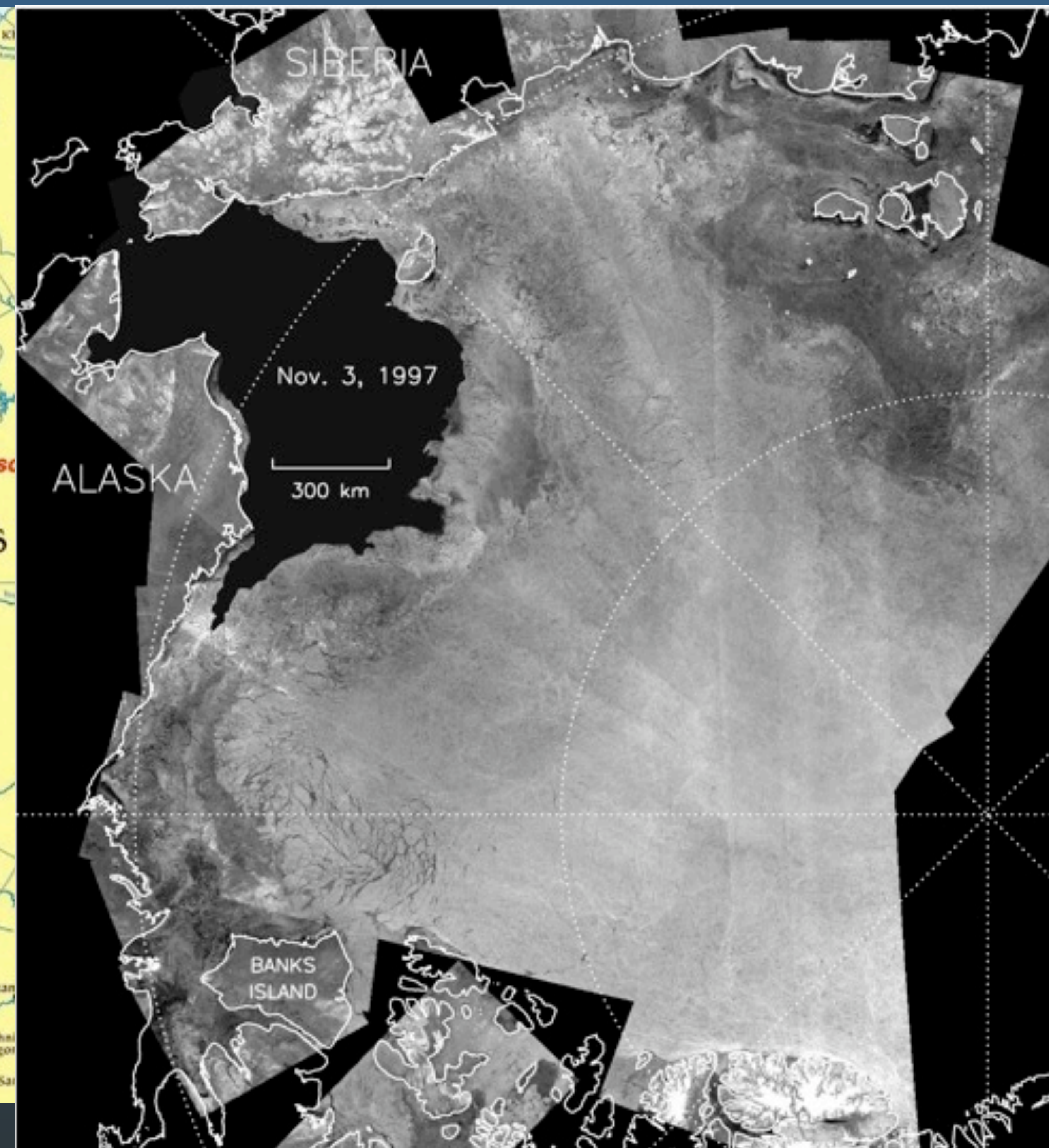
The interaction between material points is solved using a finite element or finite difference discretization on a mesh (cost is linear in the number of material points)

Information is transferred between the material points and the mesh by interpolation (only changes are interpolated, keeping numerical dissipation relatively small)

Material points move in a continuous velocity field providing a natural no-slip contact algorithm

Can use any constitutive model

Arctic Sea Ice



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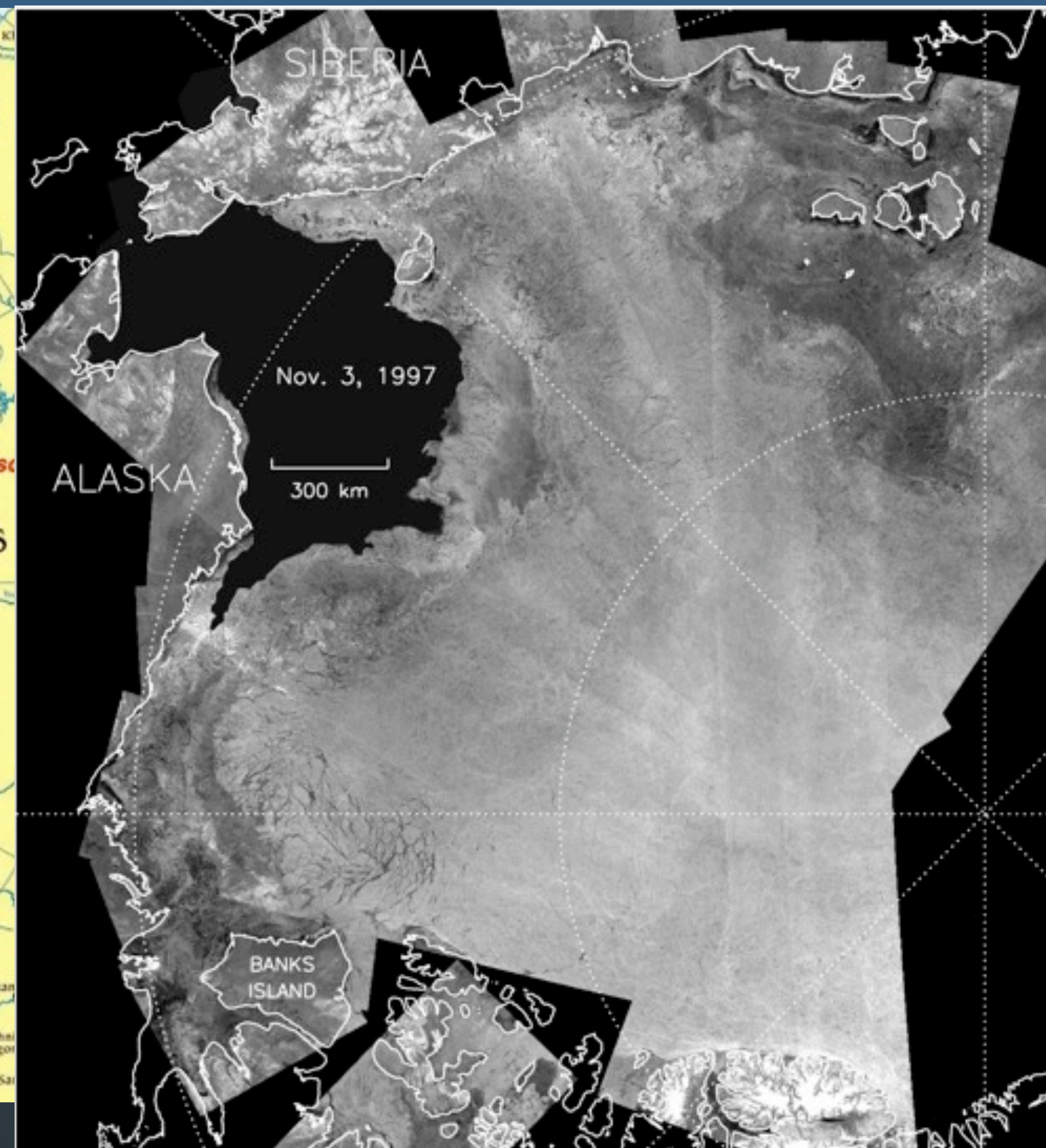
Arctic Sea Ice



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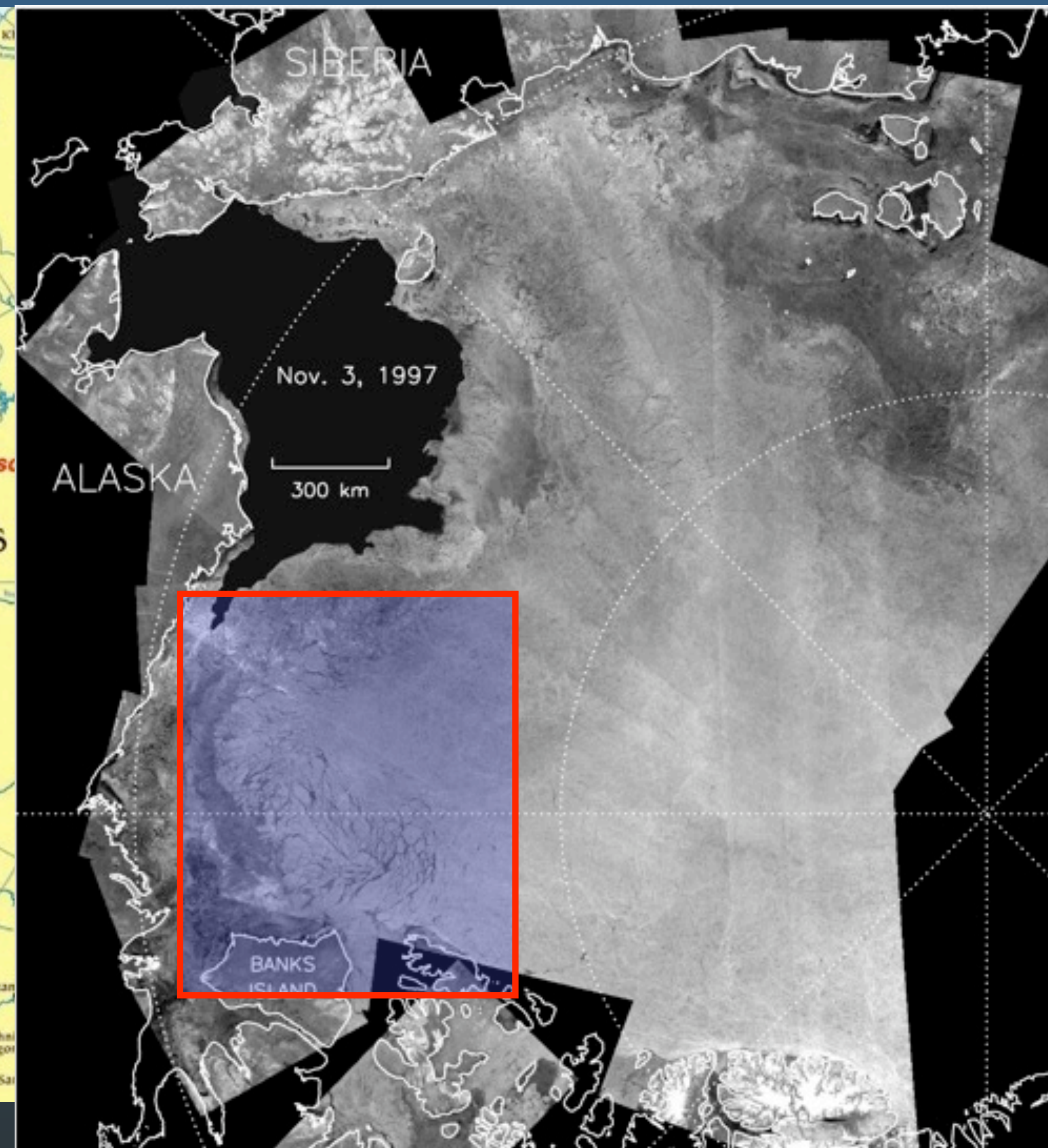
Arctic Sea Ice



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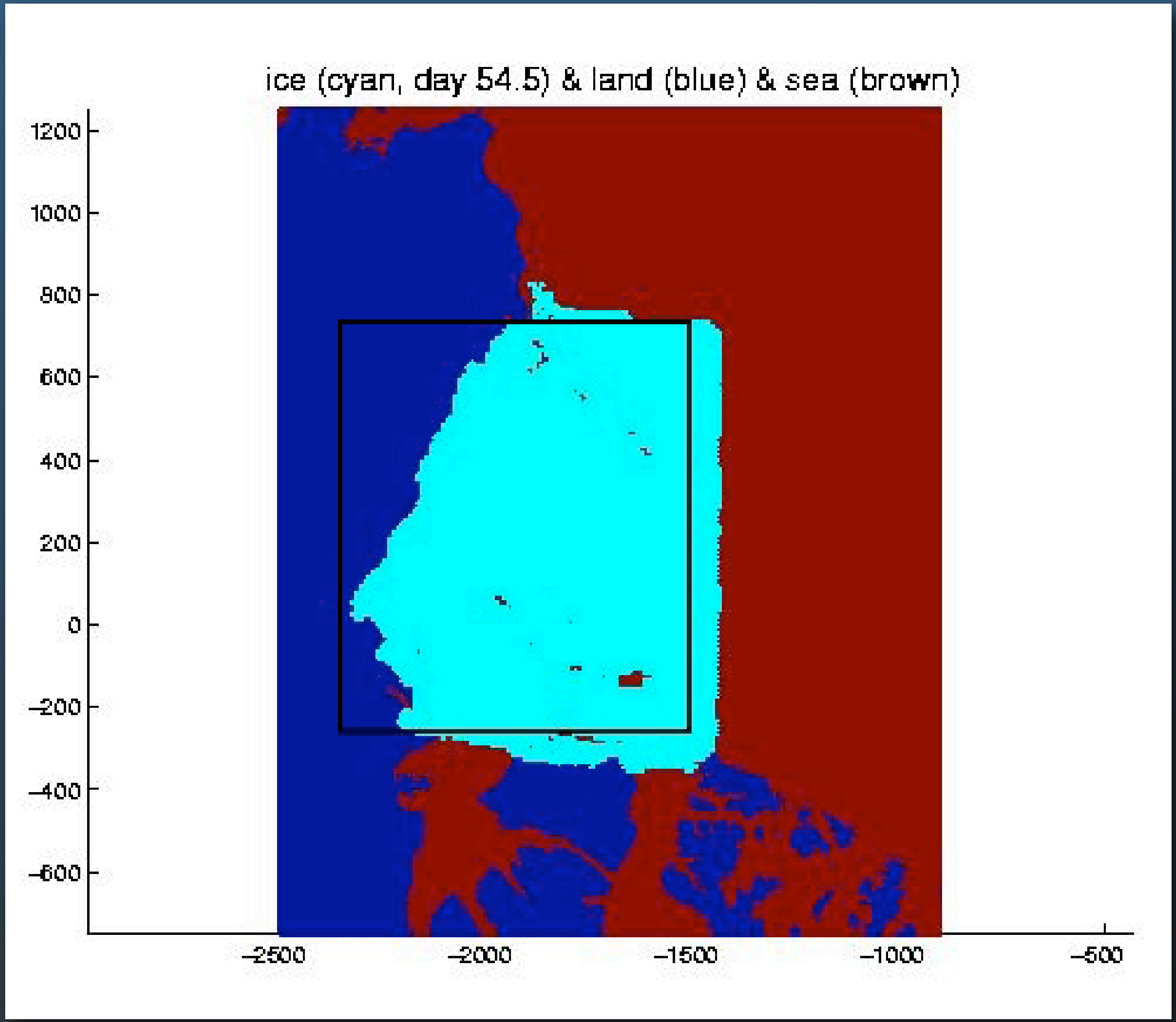
Problem Set Up

Simulate 16 days in Feb/Mar, 2004



Set up:

- 10 km square background grid
- 4 material points per element
- rigid material points for land
- include wind, ocean, and Coriolis forces
- Right, top, bottom boundary conditions from RGPS displacements

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Simulation Parameters

symbol	name	value
ρ	sea ice density (mass/vol)	917.0
h	sea ice thickness	3.0
f_c	coriolis parameter	1.46×10^{-4}
c_a	air drag coefficient	0.0012
ρ_a	air density	1.20
$\mathbf{v}_a$	wind velocity	from data
α	air turning angle	0.50
c_w	sea water drag coefficient	0.00536
ρ_w	sea water density	1026.0
β	sea water turning angle	0.0
$\mathbf{v}_w$	sea water velocity	from data
G	(linearized) elastic shear modulus	3.6765×10^5
K	(linearized) elastic bulk modulus	11.905×10^5
$\mathbf{e}_3$	out of plane vector	$(0,0,1)^T$
$\boldsymbol{\sigma}$	ice stress	$---$
$\mathbf{v}$	ice velocity	$---$

Units: m, kg, sec

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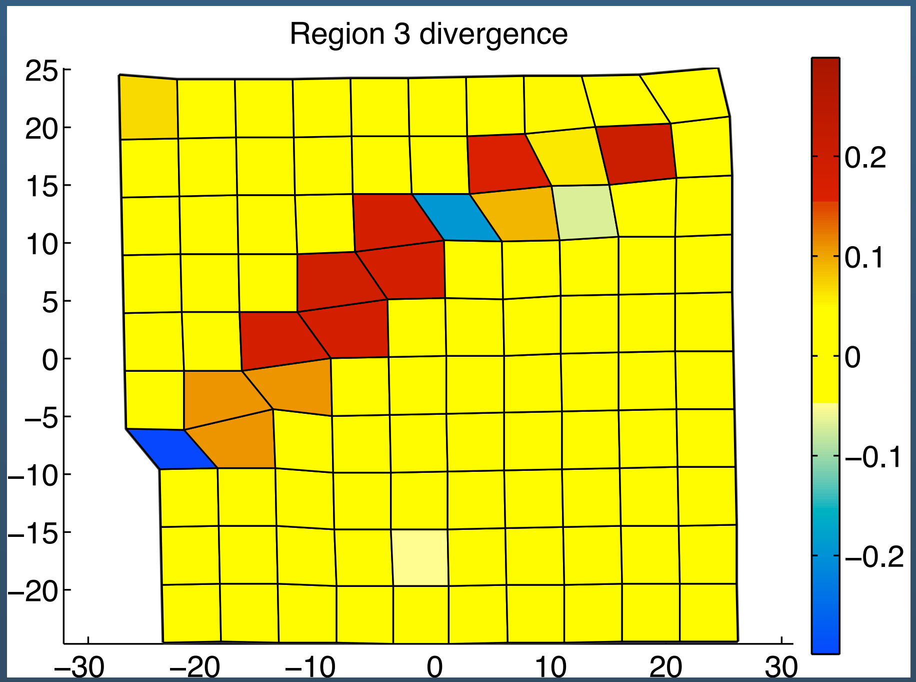
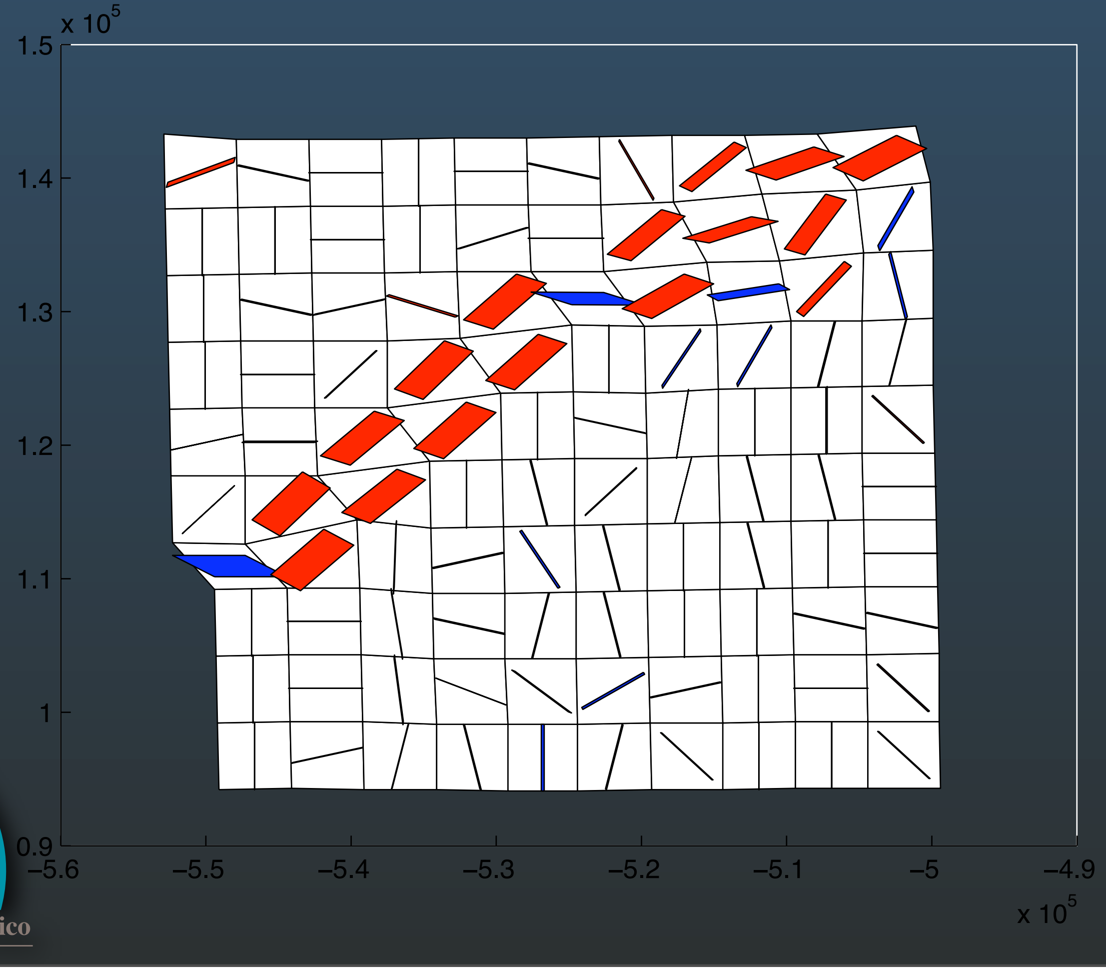
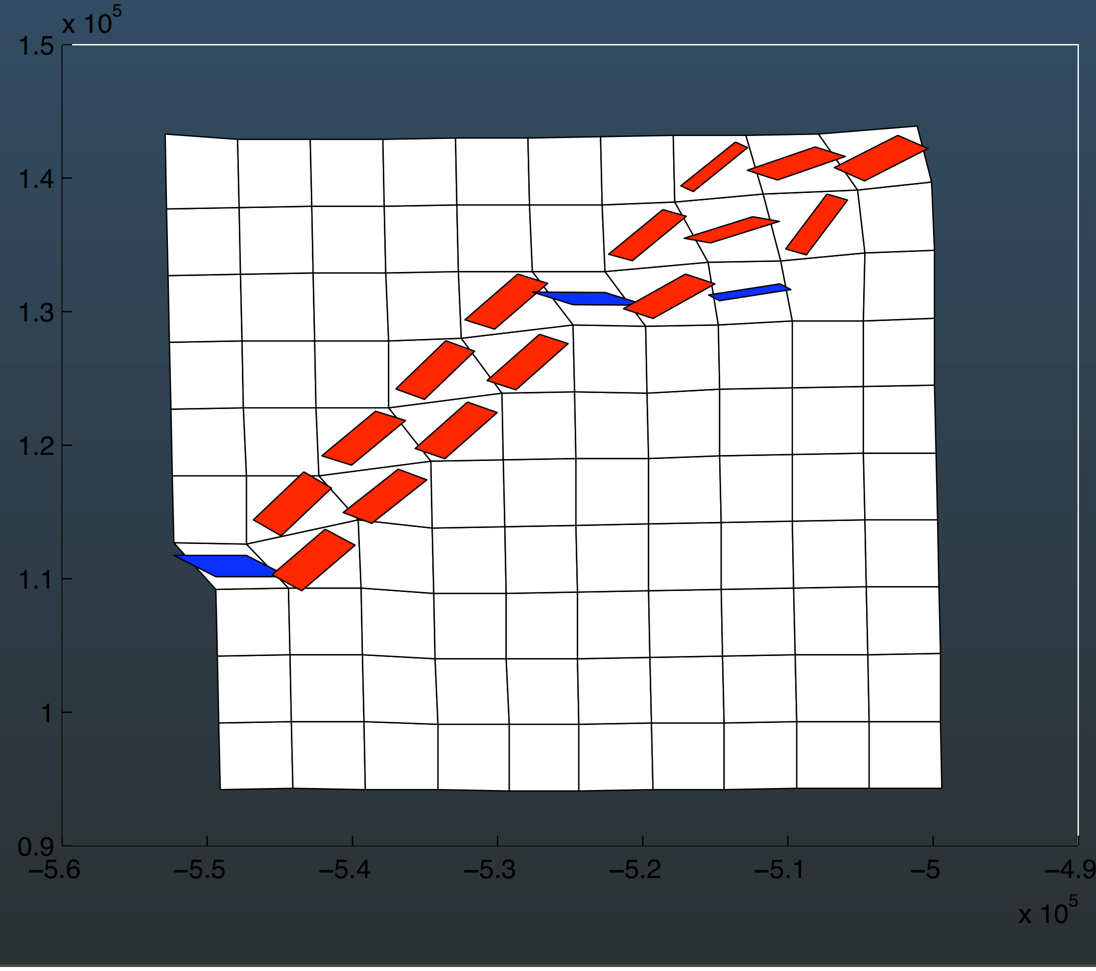
Saturday, February 13, 2010

Where are the leads?







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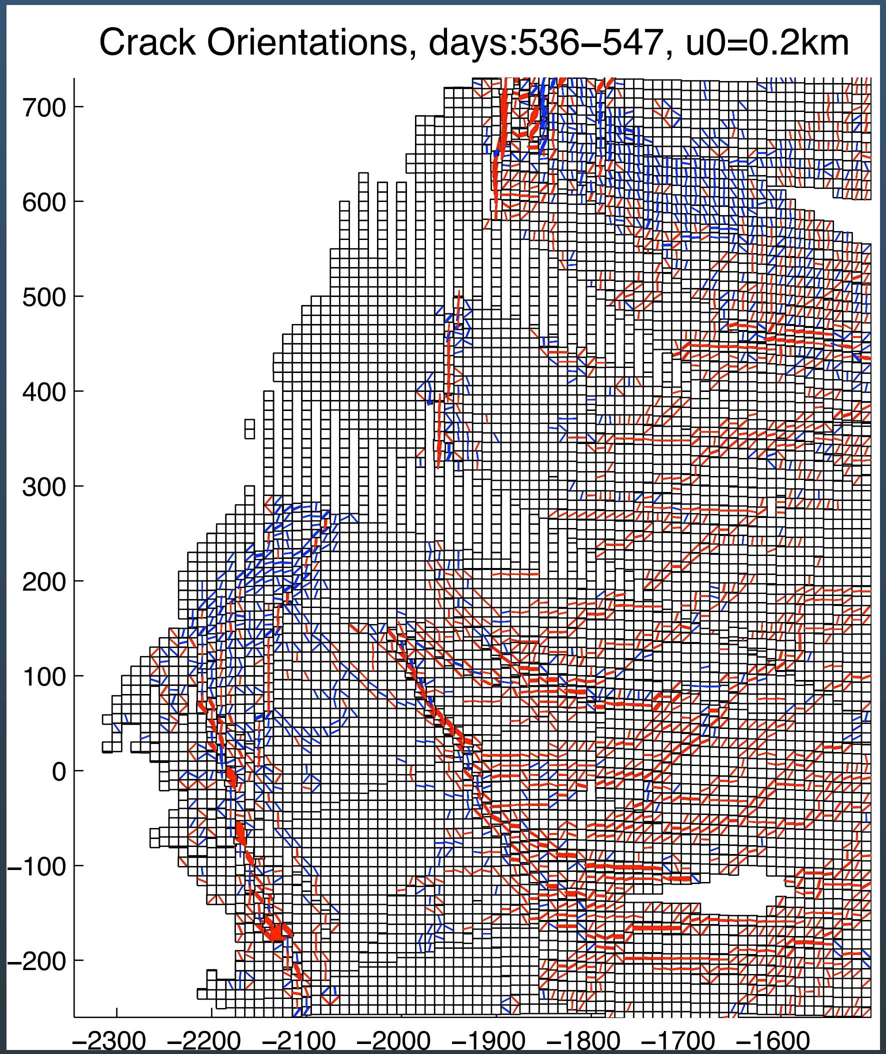
Initialization

Use a kinematic analysis of satellite data to find existing leads



Day 54 (Feb. 23)

$E = 1$ MPa

$\nu = 0.36$

$\tau_{nf} = 25$ KPa

$\tau_{sf} = 15$ KPa

$f'_c = 125$ KPa

$u_0 = 400$ m

$s_m = 4$

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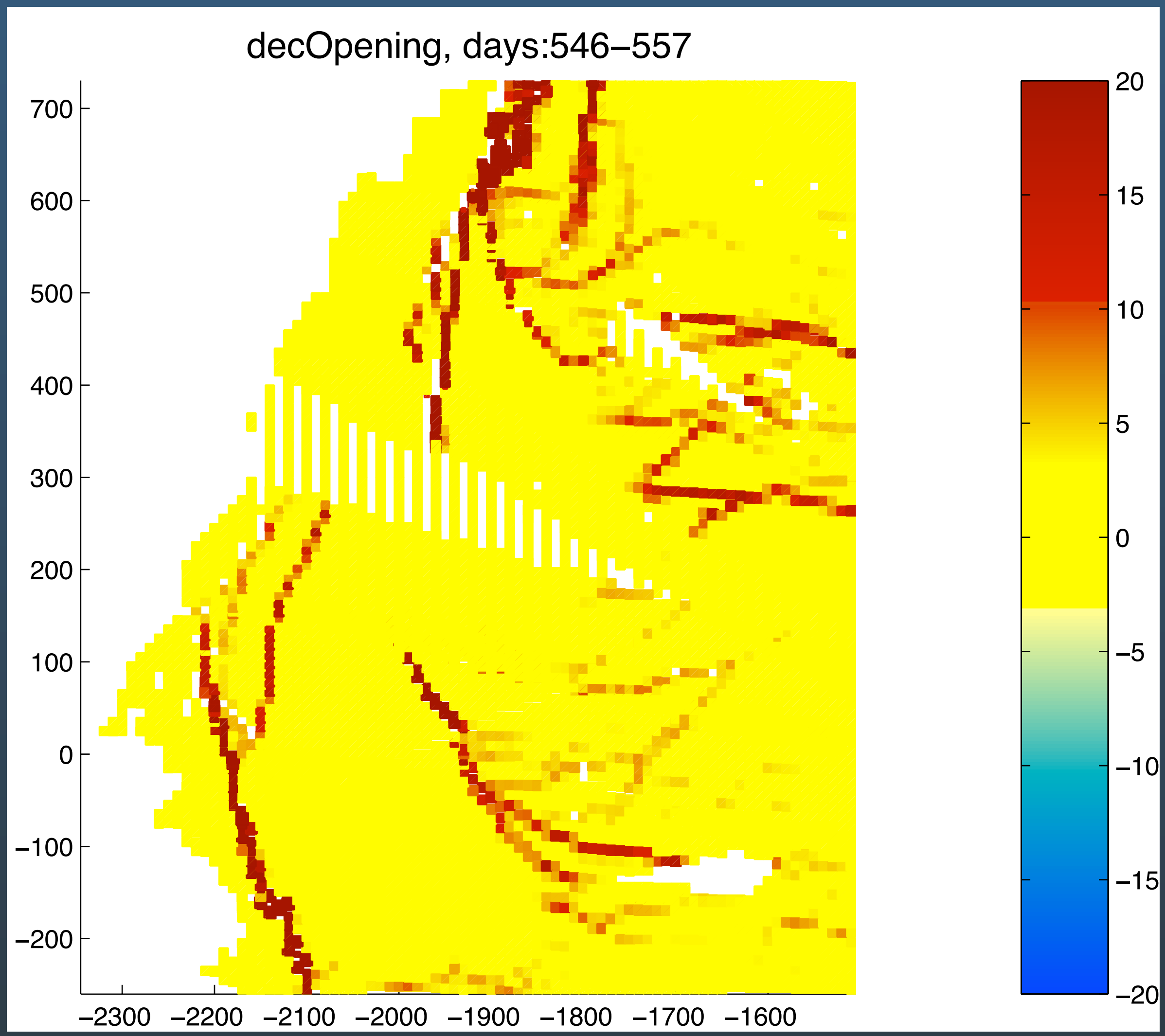
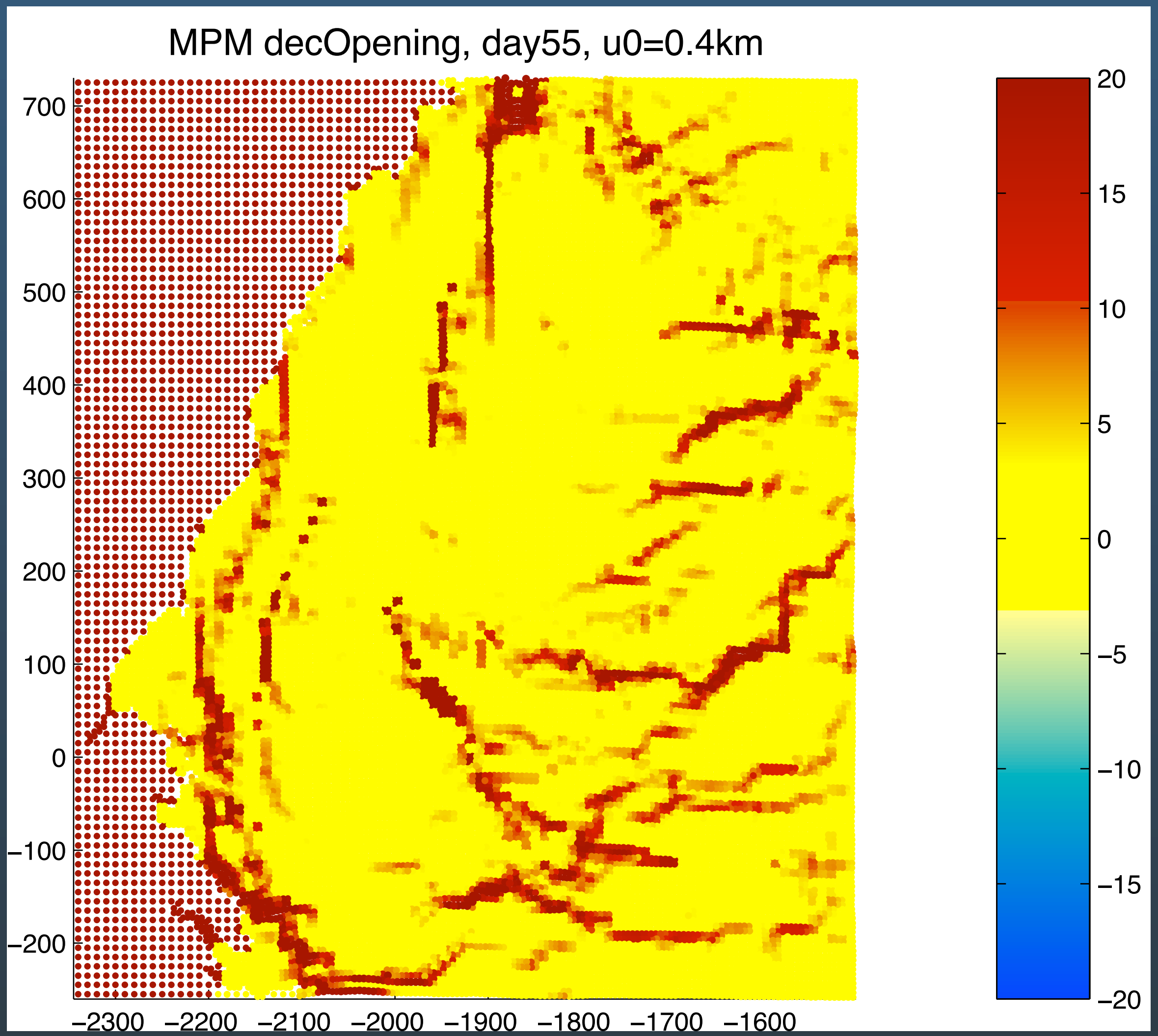
Fracture Patterns in the Beaufort

day 55



Observation



Simulation

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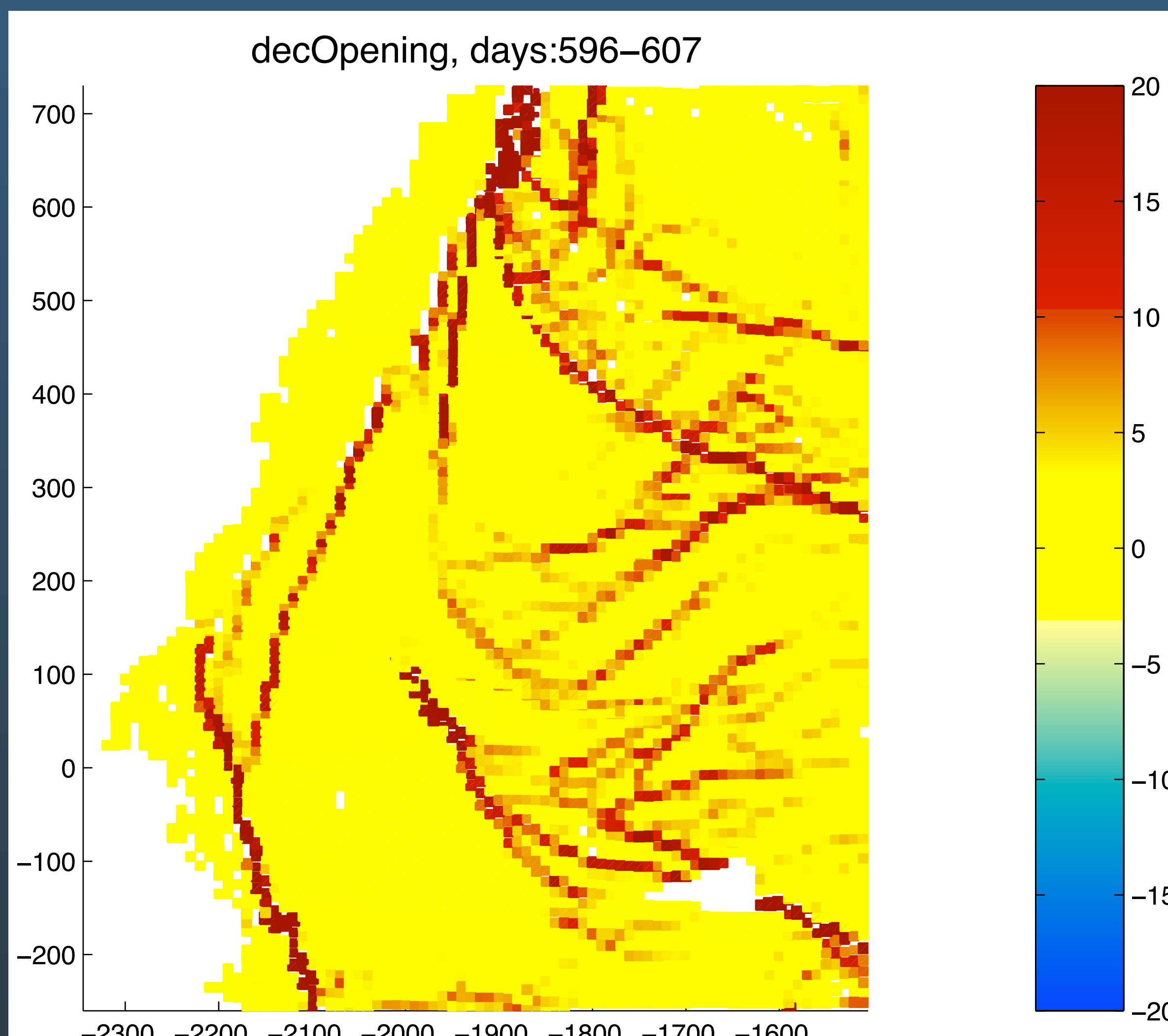
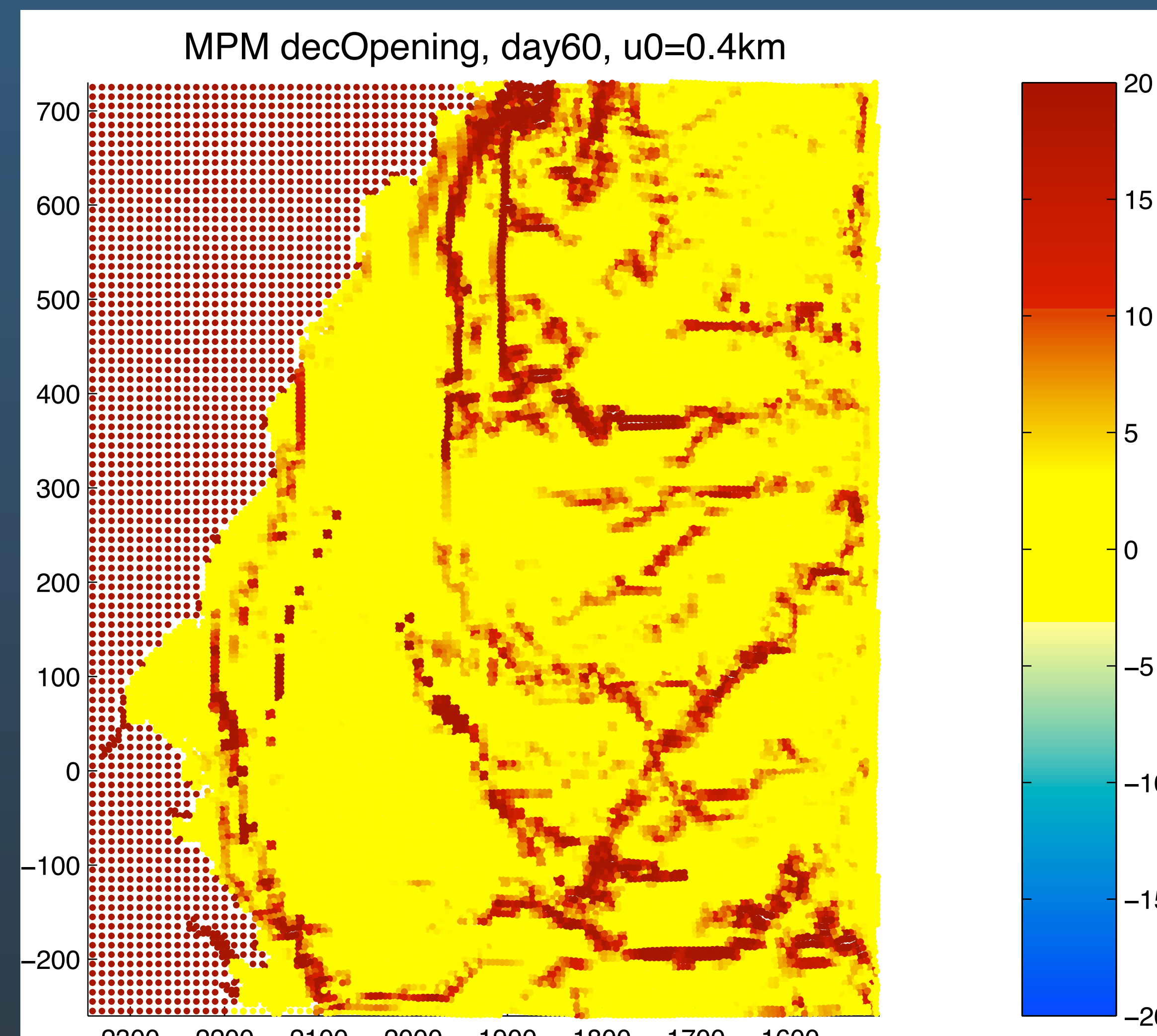
Fracture Patterns in the Beaufort

day 60



Observation

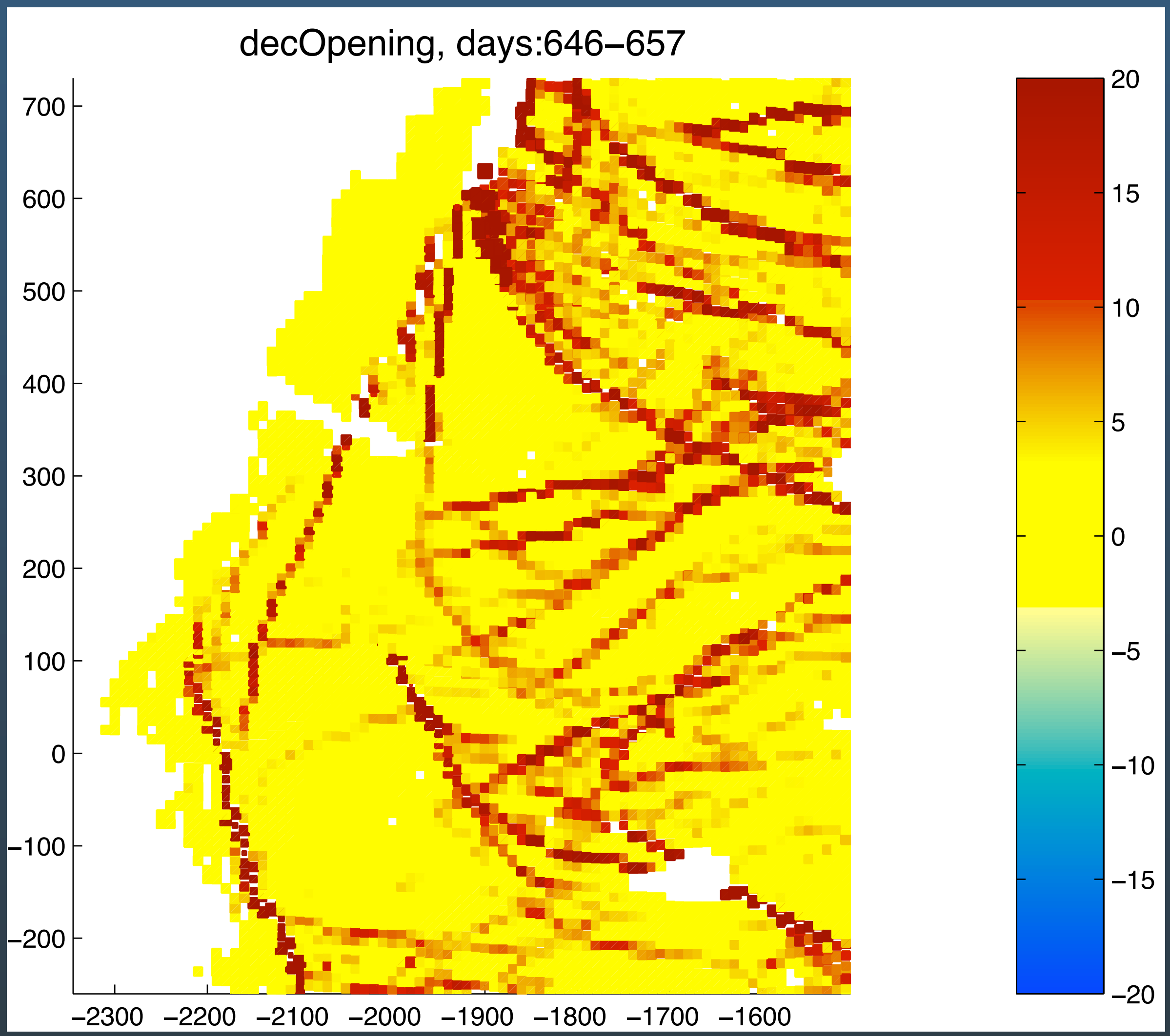


Simulation

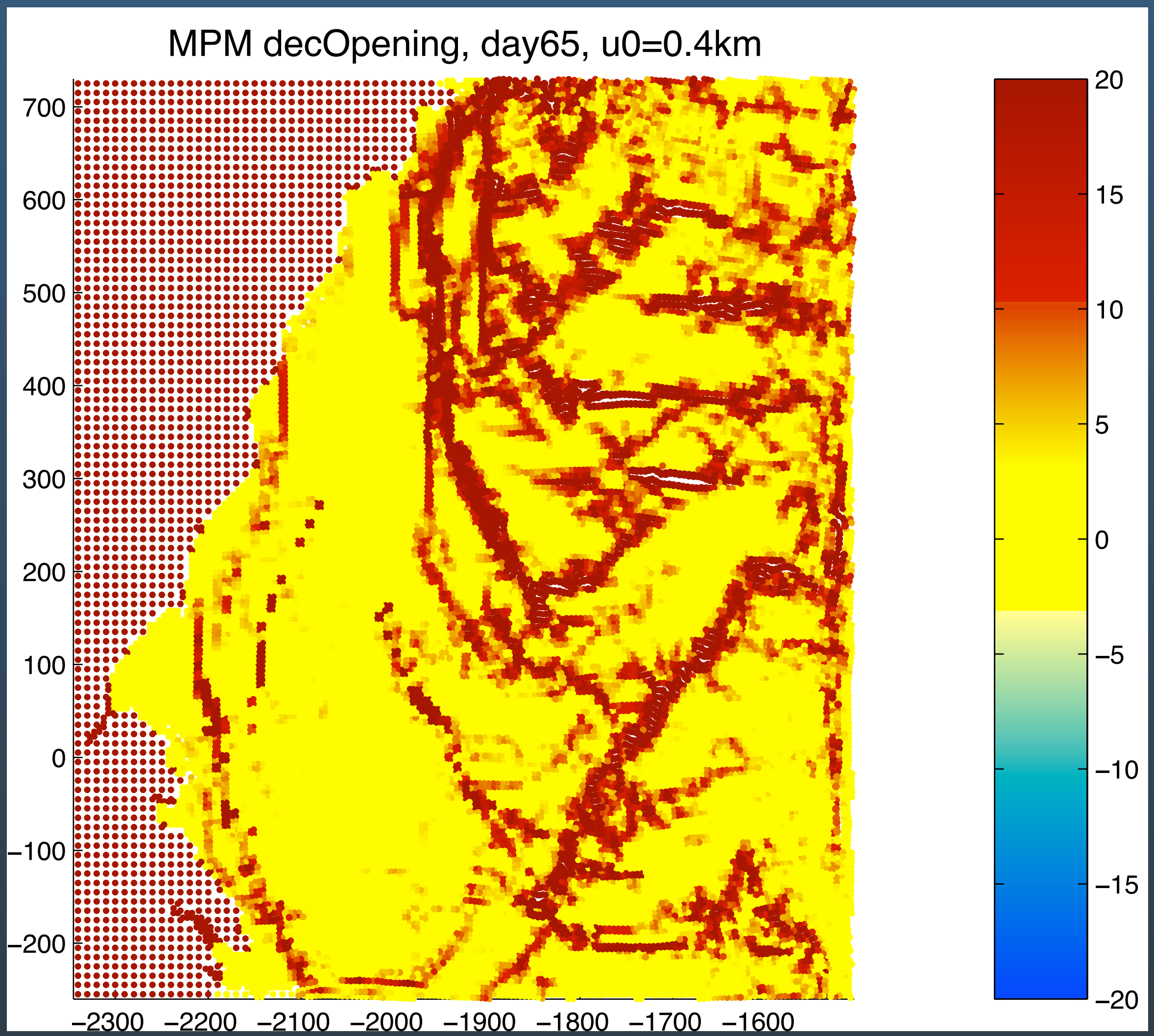
The University of New Mexico

Fracture Patterns in the Beaufort

day 65



Observation



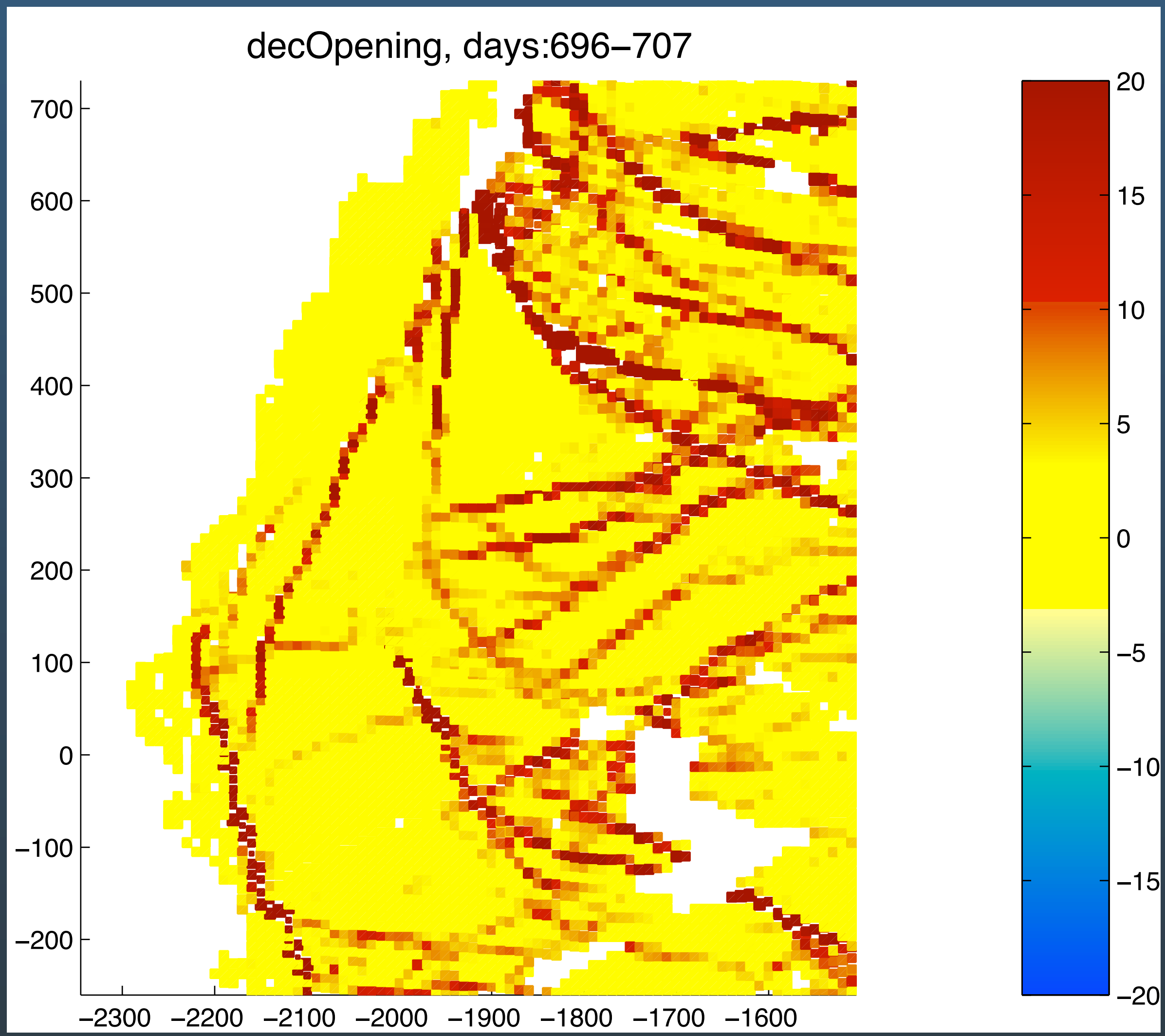
Simulation

The University of New Mexico

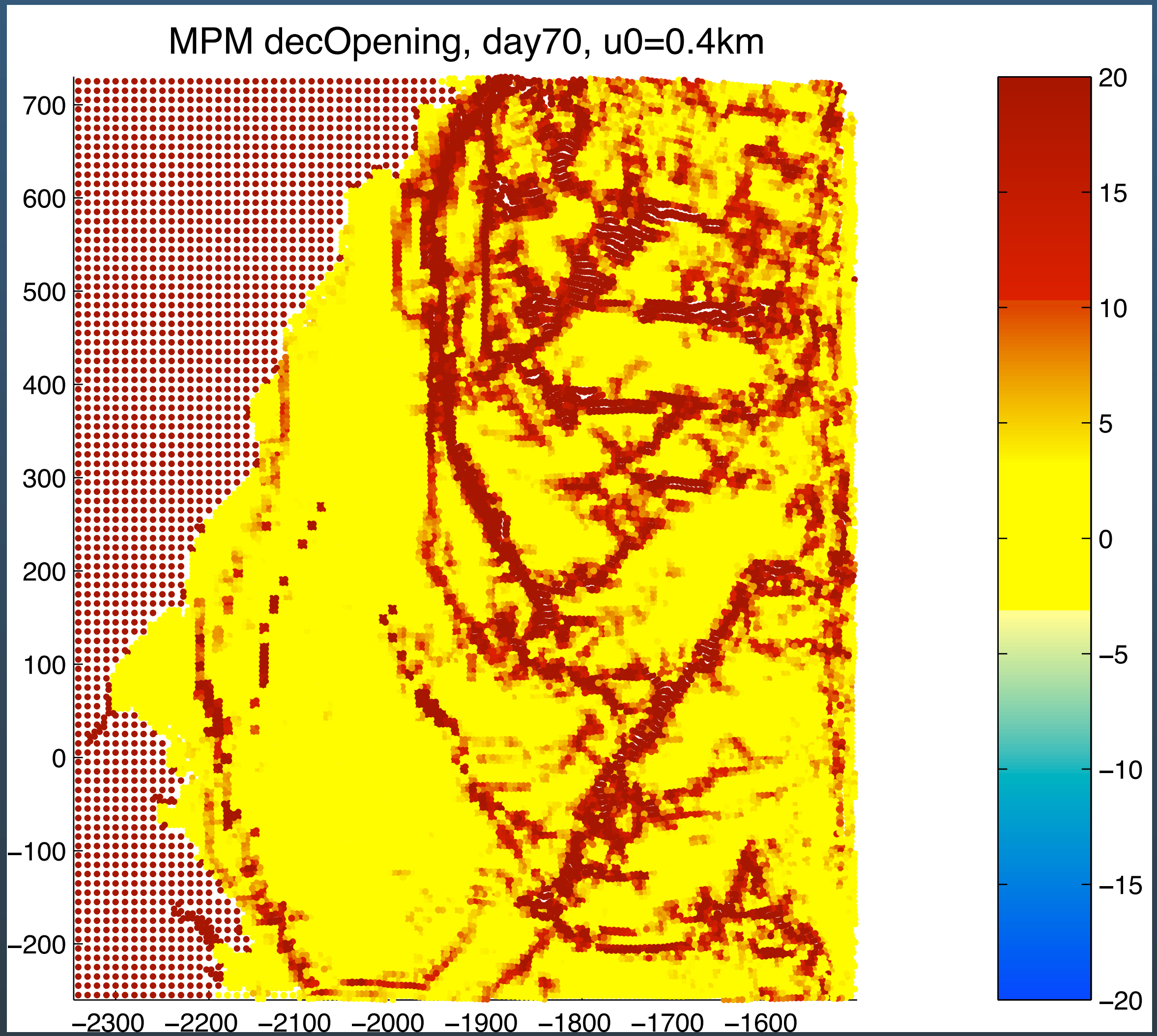
Saturday, February 13, 2010

Fracture Patterns in the Beaufort

day 70



Observation



Simulation

The University of New Mexico

Notes

Calculations were done

- without tuning parameters

- with crude initial guess
- with no refreezing of leads

- with errors from preprocessing satellite data

Summary

Features of elastic-decohesive model:

- Stress state at which leads initiate
- Orientation of lead at initiation
- Evolution of lead (softening)
- Existing material weakness
- Implemented in plasticity framework

Work in progress:

- Initial conditions
- Freezing model
- Coupling to ocean
- Metrics