Progress toward dynamical paleoclimate state estimation

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Plan

- Motivation & goals
  - paleo state estimation challenges
  - hypothesis: current weather DA sufficient
- Efficiently assimilating time-integrated obs
  - Results for a simple model
  - Results for a less simple model
- Optimal networks
  - where to site future obs conditional on previous

Motivation

- Reconstruct past climates from proxy data.
- Statistical methods (observations)
  - time-series analysis
  - multivariate regression
  - no link to dynamics
- Modeling methods
  - spatial and temporal consistency
  - no link to observations
- State estimation
  - This talk & workshop.

Long-term Goals

- Reconstruct last 1-2K years
  - Expected value and error covariance
  - Unique dataset for decadal variability
  - Basis for rational regional downscaling
- Test paleo network design ideas
  - Where to take highest impact **new** obs?

Challenges for paleo state estimation

No shortage of excuses for not trying!

1) proxy data are time integrated

cf. weather assimilation of "instantaneous" obs

2) long-time periods

computational expense

predictability "horizon"

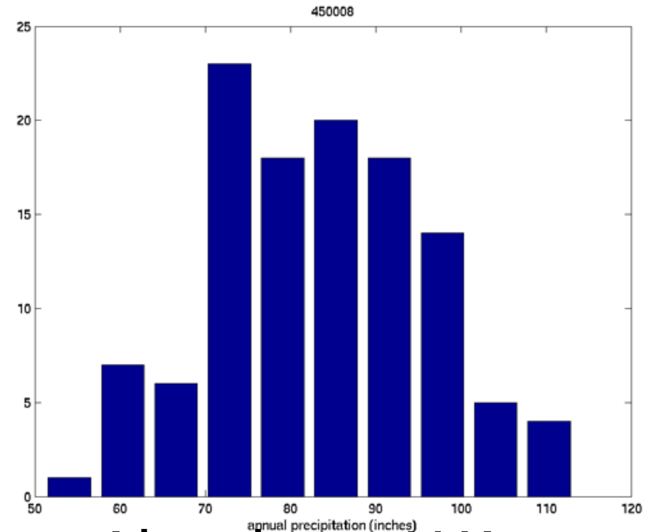
3) Proxies often chemical or biological

forward model problem (tree rings)

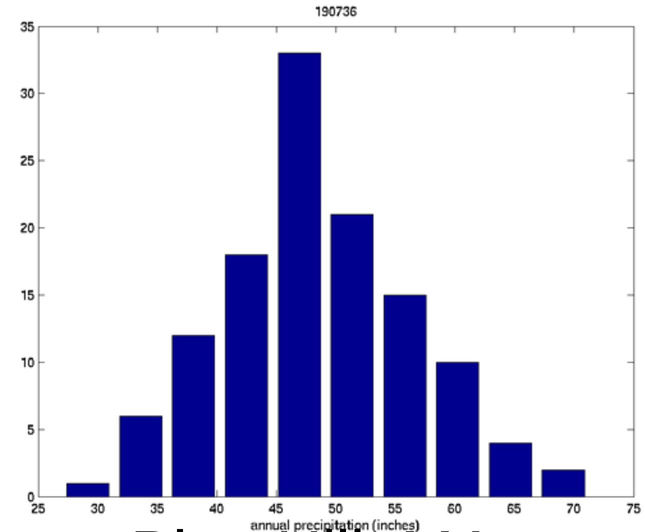
4-n) nonlinearity; non-Gaussianity; bias; proxy timing; external forcing, etc.

[similar problems in weather DA haven't stopped decades of progress]

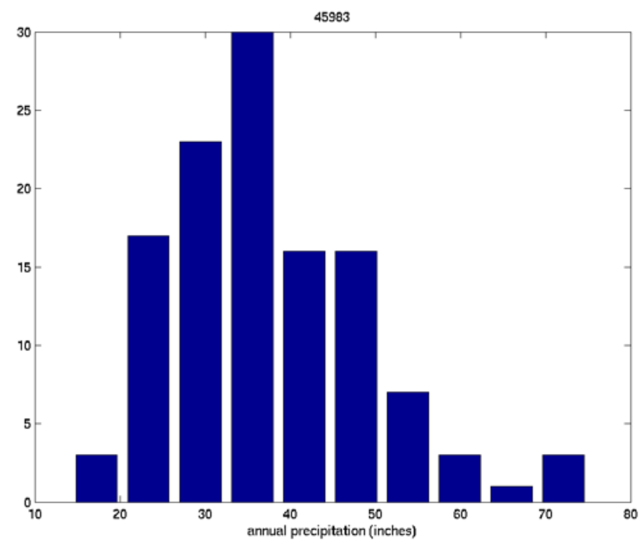
Is Precipitation Gaussian?



Aberdeen, WA



Blue Hill, MA



Mt. Shasta, CA

Annual Precipitation
(100+ years)

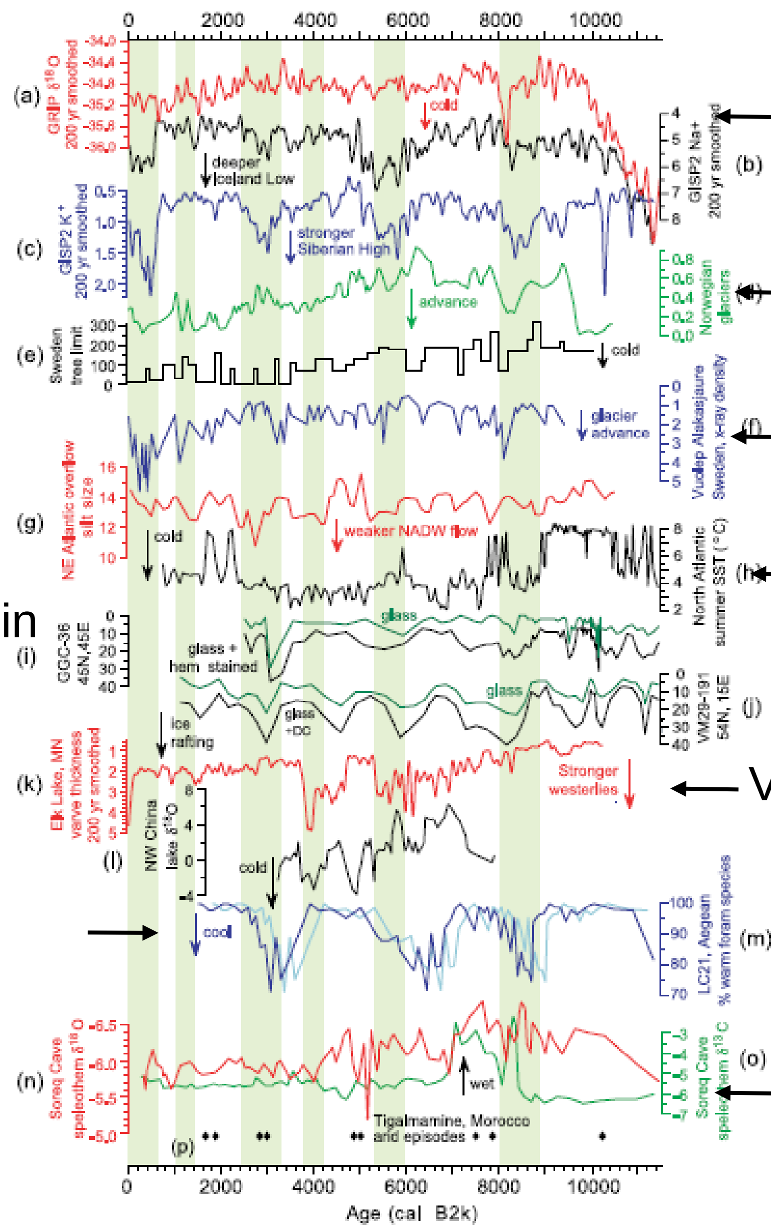
Approach

- Develop a method as close to “classical” as possible
- Assume (until proven otherwise) that:
  - Errors are Gaussian distributed
  - Dynamics are ~linear in appropriate sense
  - I.e., Kalman filtering is a reasonable first approximation
- Why? Relax one key aspect of statistical reconstruction:
  - stationary statistics (leading EOFs; proxy regression)
- Key challenge topics addressed here:
  1. Efficient Kalman filtering for time-averaged observations
  2. Simplified models; assess predictability on proxy timescales
  3. Physical proxies only: ice core accumulation & isotopes

Paleoclimate

- Historical record of Earth's climate.
- Benchmark for future climate change.
  - E.g., dynamics of decadal variability.
- "observations" are by proxy.
  - Examples:
    - Ice cores (accumulation, isotopes)
    - Tree rings.
    - Corals.
    - Sediments (pollen, isotopes).
  - Typically, related to climate variables, then analyzed.

Climate variability: a qualitative approach



North

GRIP $\delta^{18}O$ (temperature)

GISP2 K^+ (Siberian High)

Swedish tree line limit shift

Sea surface temperature from planktonic foraminiferals

hematite-stained grains in sediment cores (ice rafting)

Varve thickness (westerlies)

foraminifera

Cave speleotherm isotopes (precipitation)

Mayewski et al., 2004

The estimation problem

Observe & estimate a low-frequency signal in the presence of large amplitude high frequency noise.

Kalman filtering on high frequency timescale is problematic

Traditional Kalman Filtering

$$\mathbf{x}^a = \mathbf{x}^b + \mathbf{K}(\mathbf{y} - \mathbf{H}\mathbf{x}^b)$$

$$\mathbf{K} = \mathbf{B}\mathbf{H}^{\mathrm{T}}[\mathbf{H}\mathbf{B}\mathbf{H}^{\mathrm{T}} + \mathbf{R}]^{-1}$$

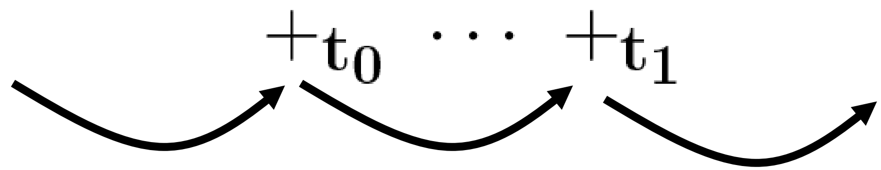
$$\mathbf{H}\mathbf{x} = \frac{1}{t_1 - t_0}\int_{t_0}^{t_1} \hat{\mathbf{H}}\mathbf{x}\,dt \equiv \mathbf{y}^e$$

$$\mathbf{B}\mathbf{H}^{\mathrm{T}} = \mathbf{x}\mathbf{y}^{e\mathrm{T}} = \mathrm{cov}(\mathbf{x}, \mathbf{y}^e)$$ **fast noise**

sequential filtering $+\mathbf{t_0} \cdots +\mathbf{t_1}$

Observations have little effect on the ***averaged*** state.

Affecting the Time-Averaged State



To filter the t_0 --> t_1 time mean:

1. Perform n assimilation steps over the interval.
   - Expensive: scales linearly with n.

2. Only update the time-mean (Dirren and Hakim 2005).
   - No more expensive than traditional KF.

Time-Averaged Assimilation

$$\mathbf{x} = \overline{\mathbf{x}} + \mathbf{x}' \qquad \overline{\mathbf{x}} = \frac{1}{t_1 - t_0}\int_{t_0}^{t_1} \mathbf{x}\, dt \qquad \boxed{\mathbf{H}\mathbf{x} = \hat{\mathbf{H}}\overline{\mathbf{x}}}$$

$$\overline{\mathbf{x}}^a = \overline{\mathbf{x}}^b + \mathbf{K}_{\mathrm{A}}(\mathbf{y} - \hat{\mathbf{H}}\overline{\mathbf{x}}^b) \quad \mathbf{K}_{\mathrm{A}} = \overline{\mathbf{x}}\mathbf{y}^{e\mathrm{T}}[\mathbf{y}^e\mathbf{y}^{e\mathrm{T}} + \mathbf{R}]^{-1}$$

$$\mathbf{x}^{a\prime} = \mathbf{x}^{b\prime} + \mathbf{K}_{\mathrm{P}}(\mathbf{y} - \hat{\mathbf{H}}\overline{\mathbf{x}}^b) \quad \mathbf{K}_{\mathrm{P}} = \mathbf{x}'\mathbf{y}^{e\mathrm{T}}[\mathbf{y}^e\mathbf{y}^{e\mathrm{T}} + \mathbf{R}]^{-1}$$

$\mathbf{x}'\mathbf{y}^{e\mathrm{T}} \approx 0 \rightarrow \mathbf{x}^{a\prime} = \mathbf{x}^{b\prime}$ Cost savings: just update time-mean

EnKF Algorithm

1. Advance full ensemble from t_0 to t_1.
2. Compute time mean, perturbations.
   - observation estimate.
3. Update ensemble mean and perturbations.
   - Time-averaged fields only!
4. Add time perturbations to the updated mean.

- Time-mean can be accumulated while running the model
- Existing code requires only minor modification.

Testing on idealized models

- 1-D Lorenz (1996) system
- Idealized mountain--storm-track interaction
- QG model coupled to a slab ocean
- Analytical stochastic energy-balance model

Illustrative Example #1

Dirren & Hakim (2005)

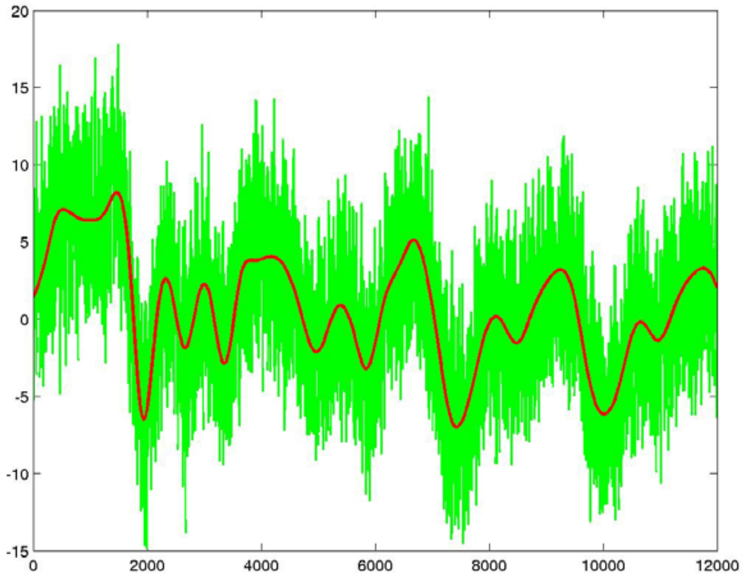
Lorenz & Emanuel (1998): Linear combination of fast & slow processes

"high-freq." $\tau_{hf} \approx 3-4$

"low-freq." $\tau_{lf} \approx 450-600$

$$\boxed{X_m(j,t) = X_{hf}(j,t) + X_{lf}(j,t)} \quad j = 1 : N_x$$

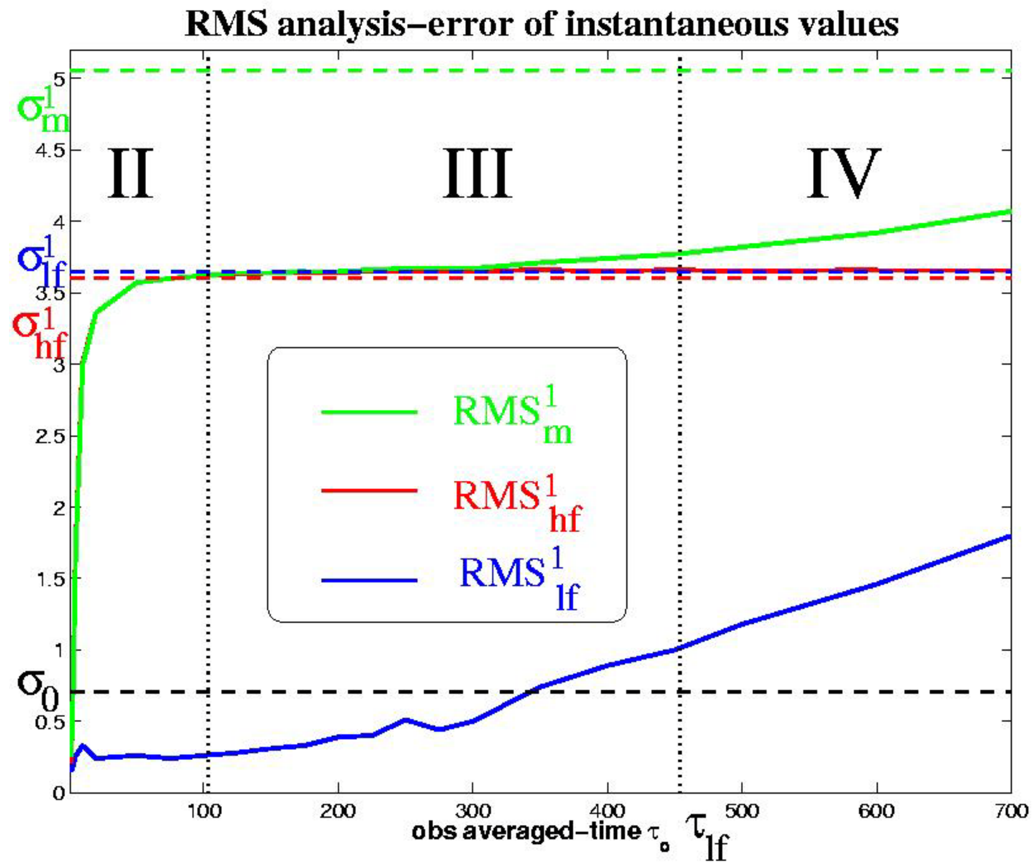
$$\frac{dX_i}{dt}(j,t) = \frac{1}{\alpha_i}[(X_i(j+1,t) - X_i(j-2,t)) \cdot X_i(j-1,t) - X_i(j,t) + F] \quad i = \text{hf,lf}$$



- LE ~ a scalar discretized around a latitude circle.

- LE has elements of atmos. dynamics:
  chaotic behavior, linear waves, damping, forcing

Observe all d.o.f.



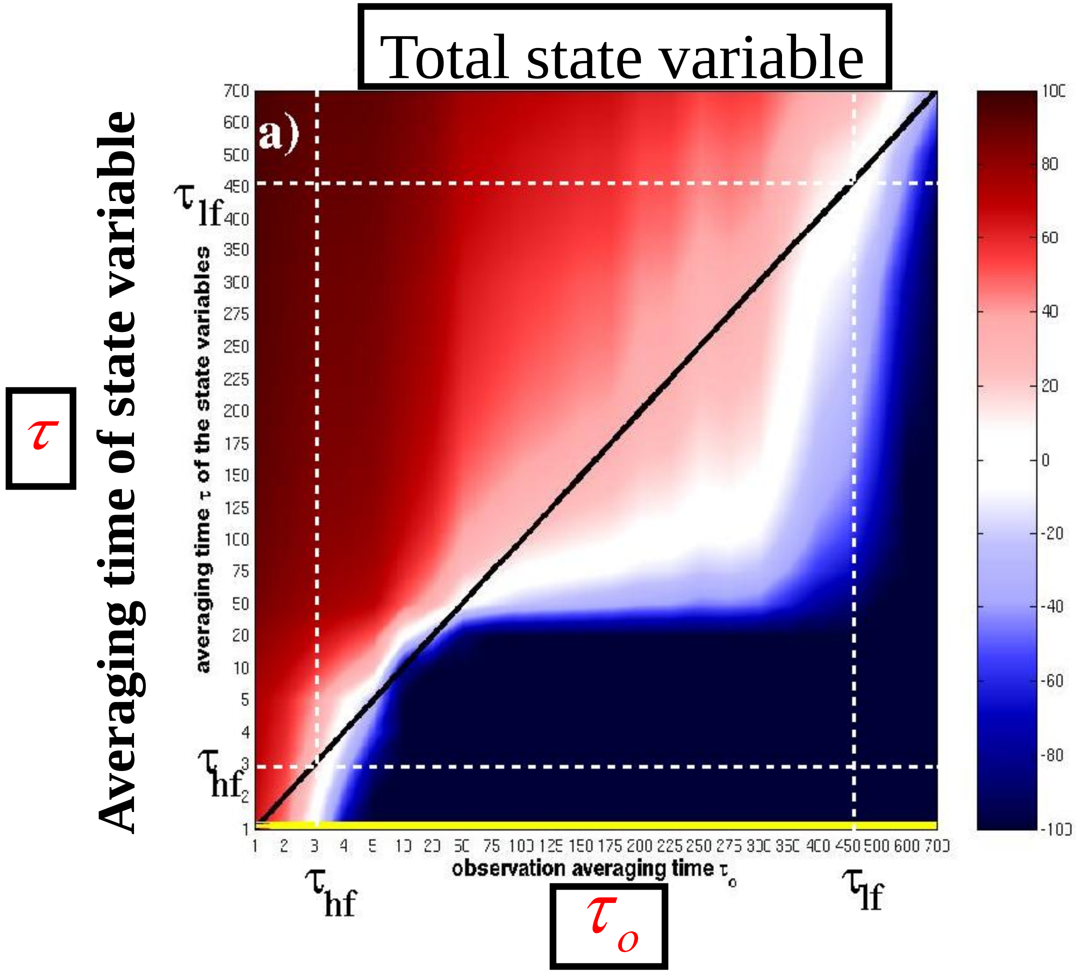
(dashed : clim)

- Low-frequency variable well constrained.
- Instantaneous states have large errors.

Improvement Percentage of RMS errors

$$p(\tau,\tau_o) = \min\left(\frac{\sigma_o - RMS_m^\tau}{\sigma_o}, \frac{\sigma^{\tau_o} - RMS_m^\tau}{\sigma^{\tau_o}}\right)*100.$$

Obs uncertainty Climatology uncertainty



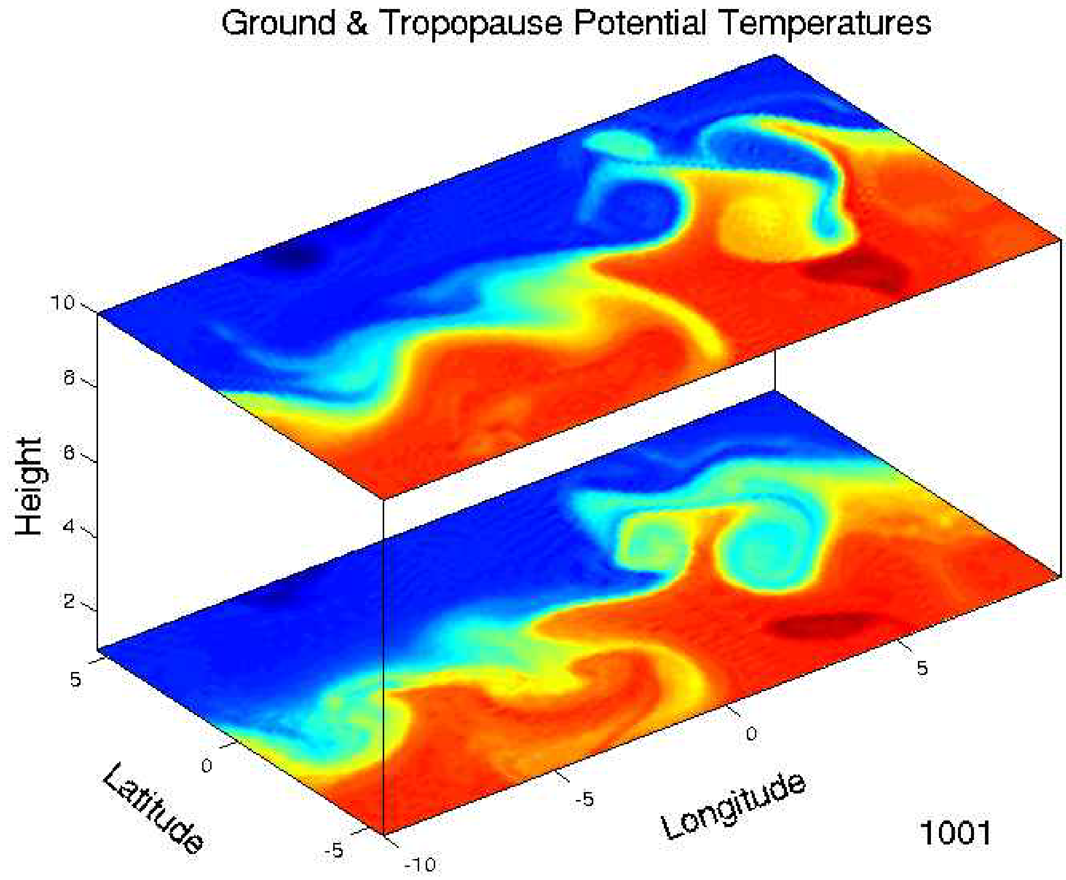
Constrains signal at higher freq.than the obs themselves!

A less simple model

Helga Huntley (University of Delaware)

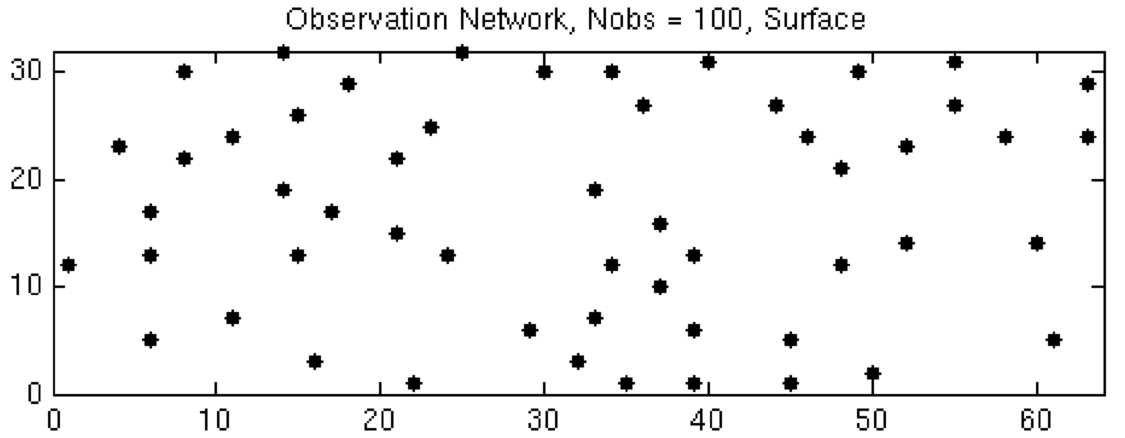
- QG "climate model"
  - Radiative relaxation to assumed temperature field
  - Mountain in center of domain
- Truth simulation
  - 100 observations (50 surface & 50 tropopause)
  - Gaussian errors
  - Range of time averages

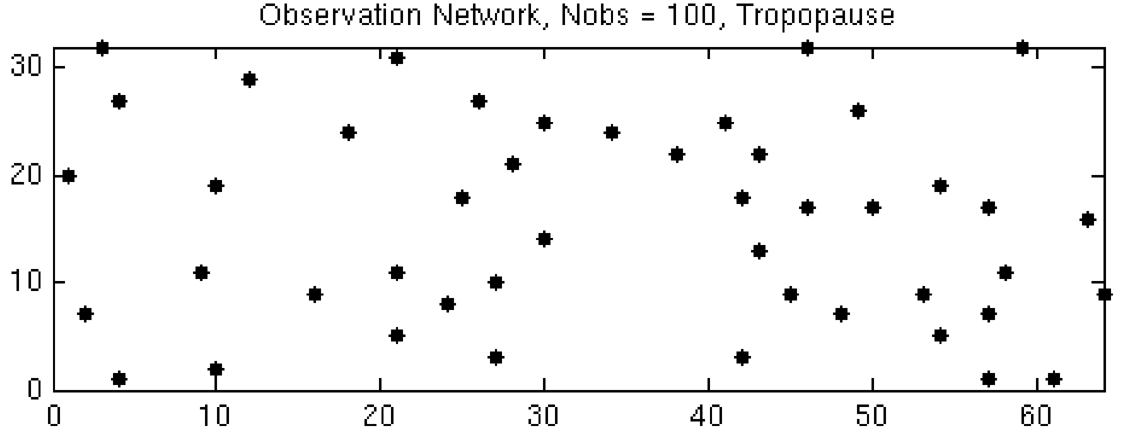
Snapshot



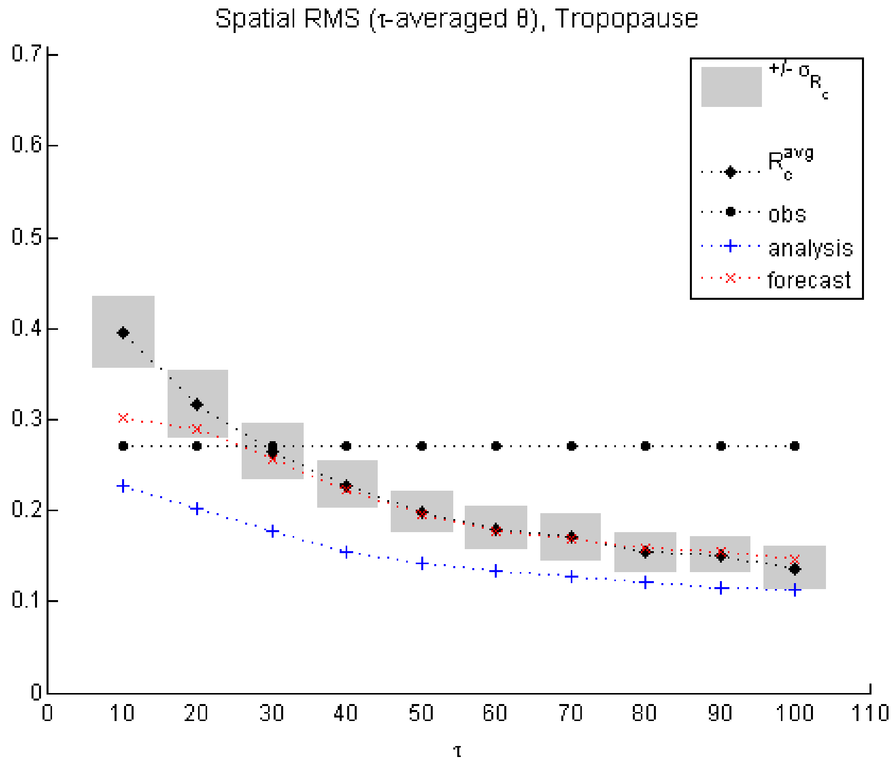
Ground & Tropopause Potential Temperatures
Height
10
8
6
4
2
5
0
-5
Latitude
-10
-5
0
5
Longitude
1001

Observation Locations





Average Spatial RMS Error



Ensemble compared against an ensemble control

Implications

- Mean state is well constrained by few, noisy, obs.
- Forecast error saturates at climatology for tau ~ 30.
- For longer averaging times, model adds little.
  - Equally good results are obtained by assimilating with an ensemble drawn from climatology:
    - cheap (no model runs).
    - reduced sampling error (huge ensembles easy).
    - but, no flow-dependence to corrections.
    - subject to model error.

QG model coupled to a slab ocean

and it’s approximation by an energy balance model

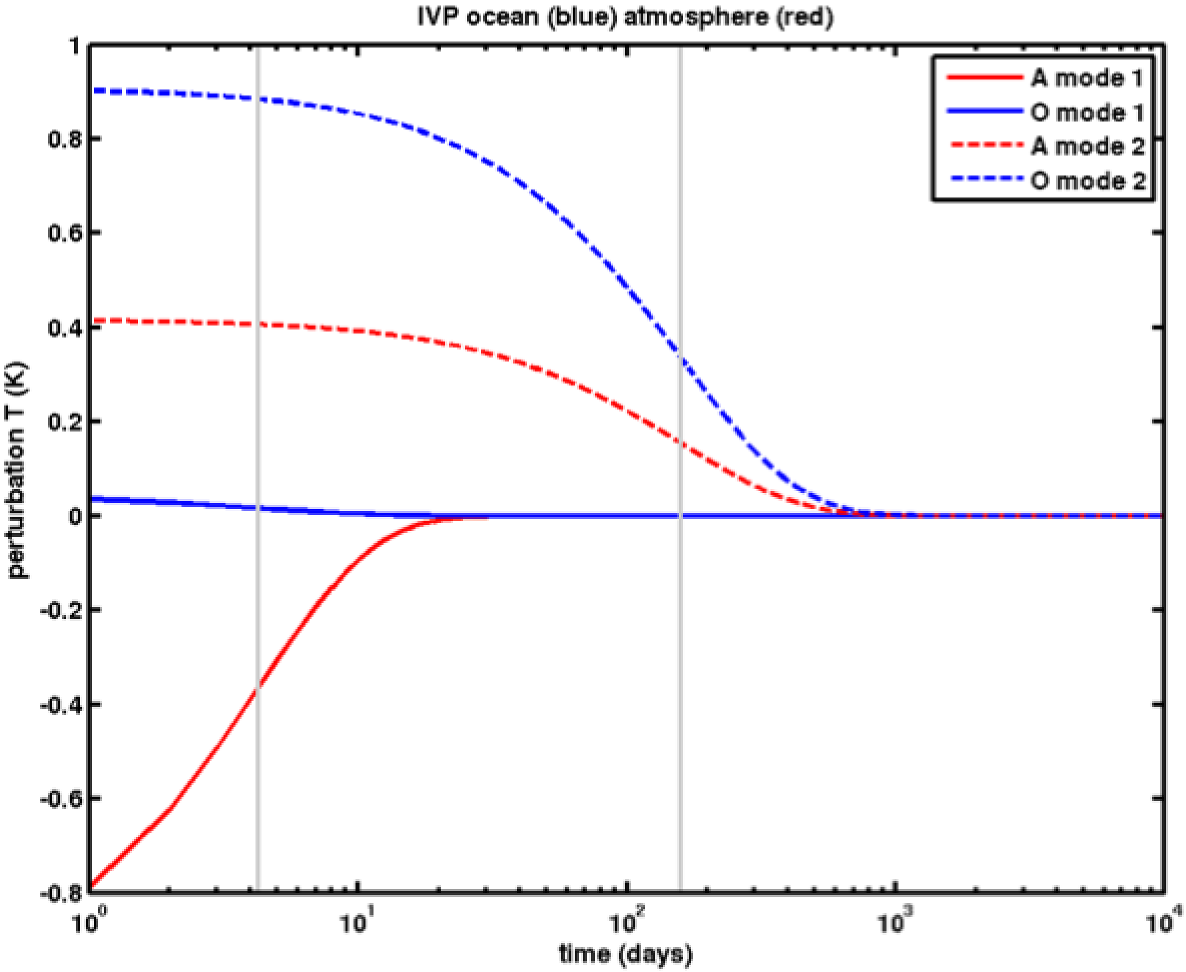
With A. Pendergrass, G. Roe, & D. Battisti

Barsugli & Battisti (1998) energy balance model

$$\frac{d\vec{T}}{d\hat{t}} = \begin{bmatrix} -a & b \\ \frac{c}{\beta} & -\frac{d}{\beta} \end{bmatrix} \vec{T} + \begin{bmatrix} N & 0 \\ 0 & 0 \end{bmatrix} \vec{W} = \mathsf{A}\vec{T} + \mathsf{N}\vec{W}$$

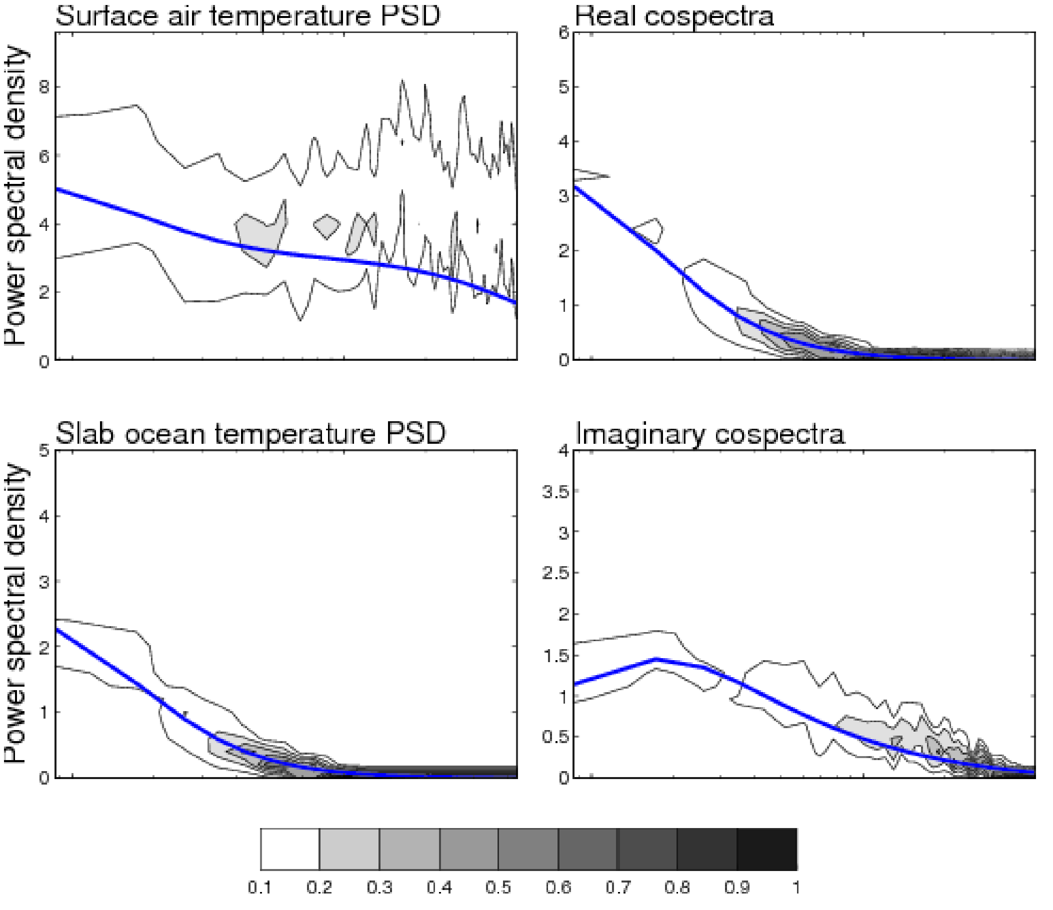
- a, d : damping parameters (radiation)
- b, c : coupling coefficients
- β : ratio of heat capacities
- N : noise forcing

Eigenvectors



One fast mode and one slow mode

QG & BB spectral comparison



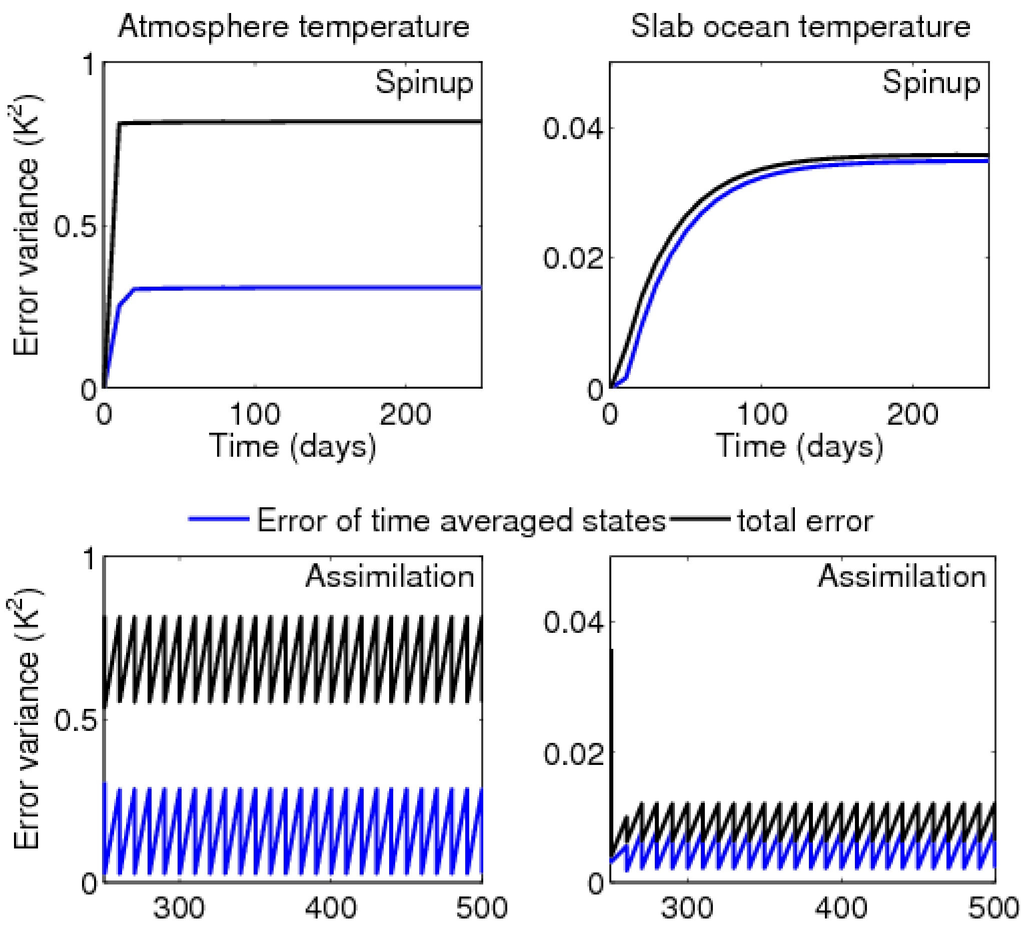
- Good agreement, particularly in phasing

Key to estimation: covariance propagation

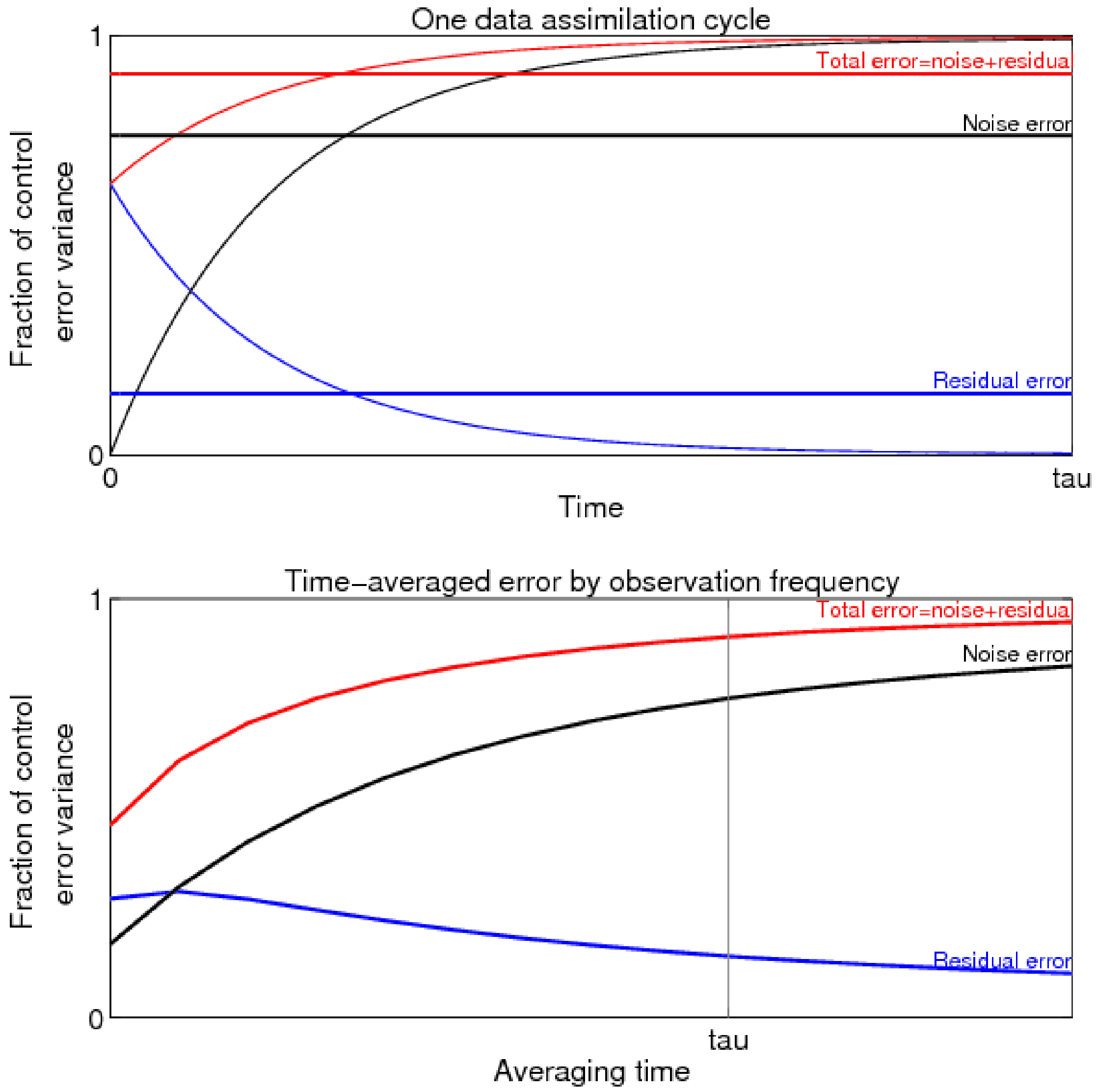
$$\langle \bar{\varepsilon}^* \bar{\varepsilon}^{*T} \rangle = \frac{1}{\tau^2} \int_0^\tau e^{\mathsf{A}t} \varepsilon^*(0) dt \left(\int_0^\tau e^{\mathsf{A}t} \varepsilon^*(0) dt \right)^T$$
$$+ \frac{1}{\tau^2} \int_0^\tau \left(\int_s^\tau e^{\mathsf{A}(t-s)} \mathsf{N} dt \right) \left(\int_s^\tau e^{\mathsf{A}(t-s)} \mathsf{N} dt \right)^T ds.$$

- First term: initial condition error (damped)
- Second term: accumulation of noise.

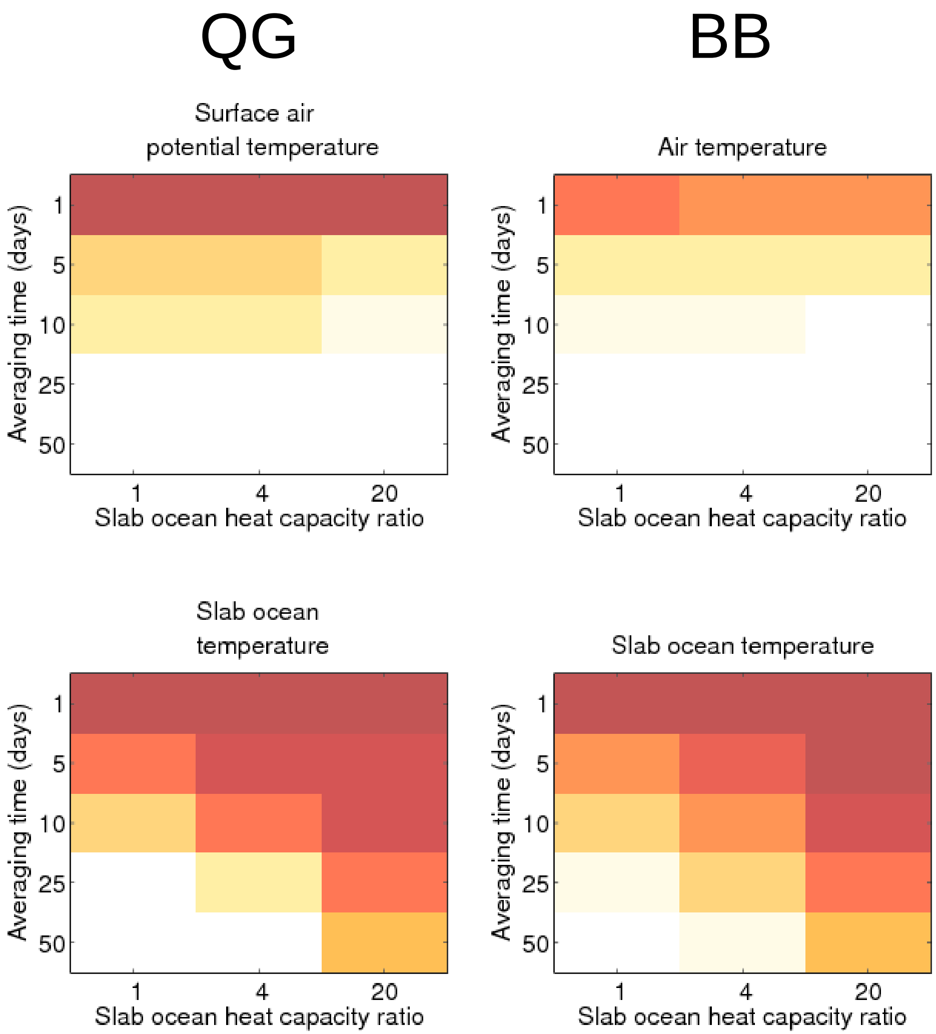
Energy model DA spinup



One time-averaged forecast cycle

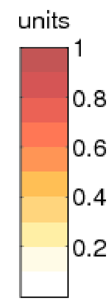


Sensitivity to Slab Depth



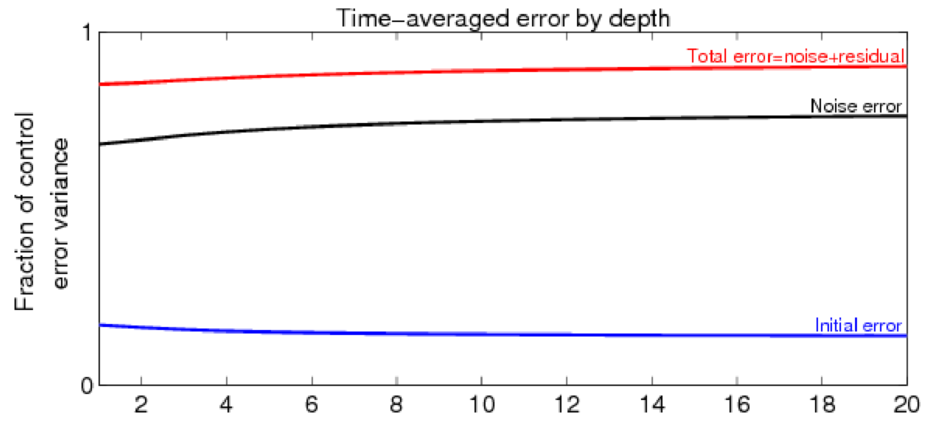
Increasing slab depth:

- Improves ocean
- <u>Degrades</u> atmosphere

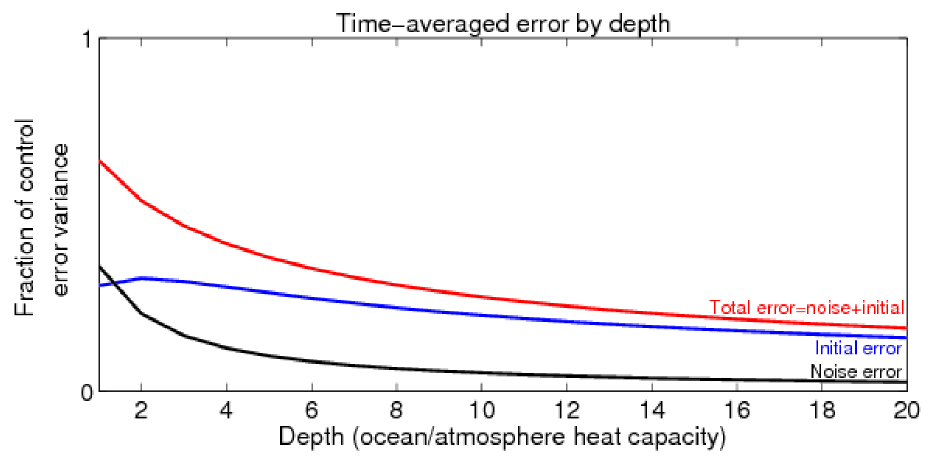


Why does depth degrade atmosphere?

atmosphere

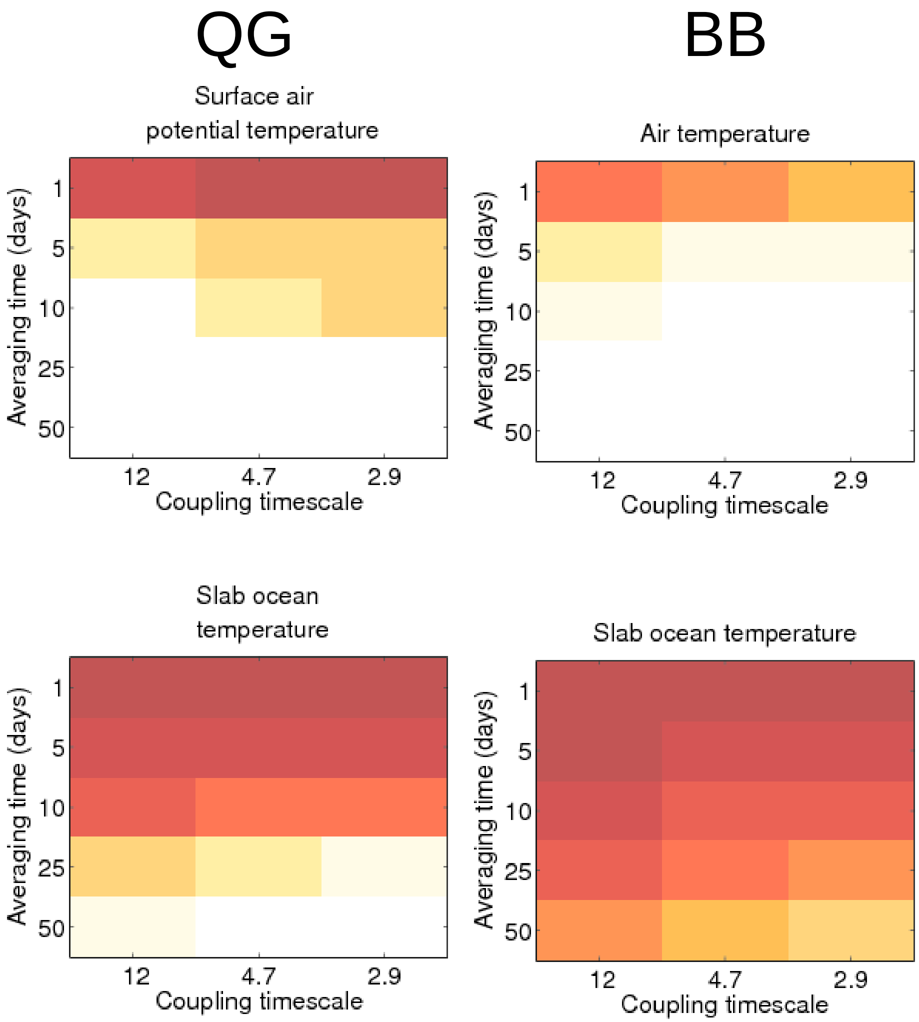


ocean



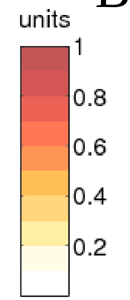
Noise “accumulates” in the atmosphere when the slab ocean is deeper

Sensitivity to Coupling



Increasing coupling:

- Atmosphere: QG & BB opposite sensitivity
- Ocean: tighter coupling degrades the analysis
- BB: Noise “recycles”



Observing Network Design

with Helga Huntley (U. Delaware)

Optimal Observation Locations

- Rather than use random networks, how to <u>optimally</u> site new observations?
  - choose locations with largest change in a metric.
  - theory based on ensemble sensitivity (Hakim & Torn 2005; Ancell & Hakim 2007; Torn and Hakim 2007; similar: Evans et al. 2002; Khare & Anderson 2006)
  - Here, metric = projection coefficient for first EOF
  - QG model with mountain

Ensemble Sensitivity

- Given metric *J*, find the observation that most reduces error variance.
- Find a second observation **conditional on first**.
- Let ***x*** denote the state (ensemble mean removed).
  - Analysis covariance $A = \mathrm{cov}(\mathbf{x}, \mathbf{x})$
  - Changes in metric given changes in state
  - Metric variance $\delta J = \left[\frac{\partial J}{\partial \mathbf{x}}\right]^T \delta \mathbf{x}$ + O(δx²)

$$\sigma = \frac{1}{M-1}\delta J \delta J^T = \left[\frac{\partial J}{\partial \mathbf{x}}\right]^T A \left[\frac{\partial J}{\partial \mathbf{x}}\right]$$

Sensitivity + State Estimation

- Estimate variance change for the i'th observation

$$\delta\sigma = \left[\frac{\partial J}{\partial \mathbf{x}}\right]^T (A_{i-1} - A_i) \left[\frac{\partial J}{\partial \mathbf{x}}\right]$$

- Kalman filter theory gives $\mathbf{A}_i$:

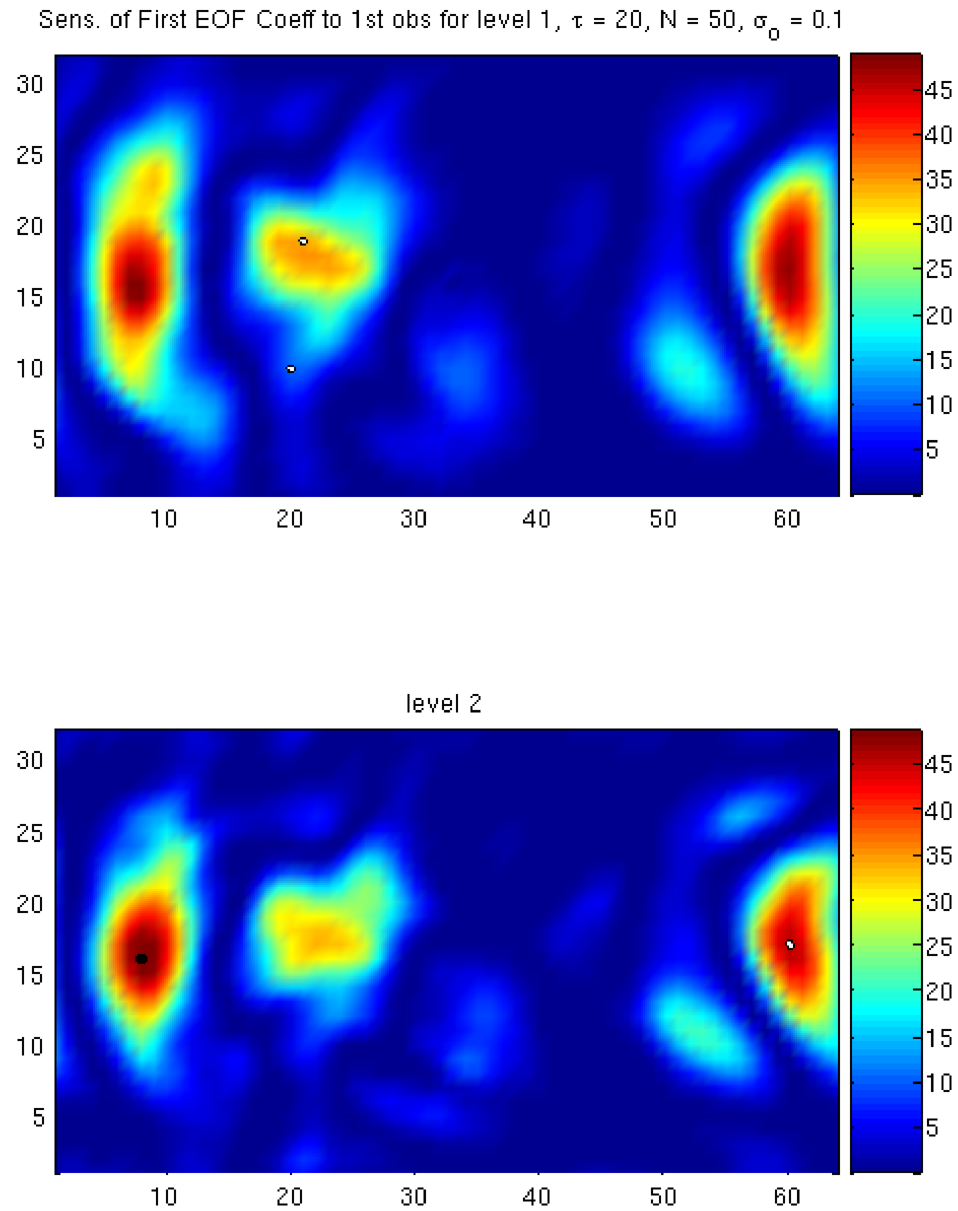
$$A_i = (I - K_i H_i) A_{i-1}$$

where
$$K_i = A_{i-1} H_i^T [H_i A_{i-1} H_i^T + R_i]^{-1}$$

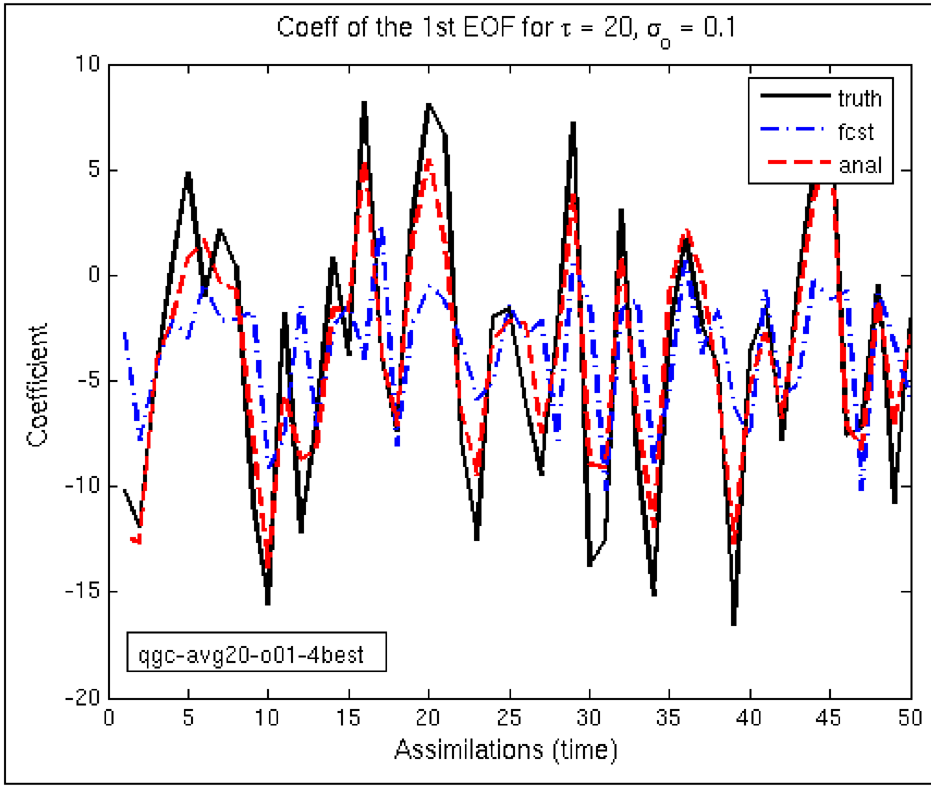
- Given δσ at each point, find largest value.

Results for tau = 20

- The four most sensitive locations, conditional on previous point.

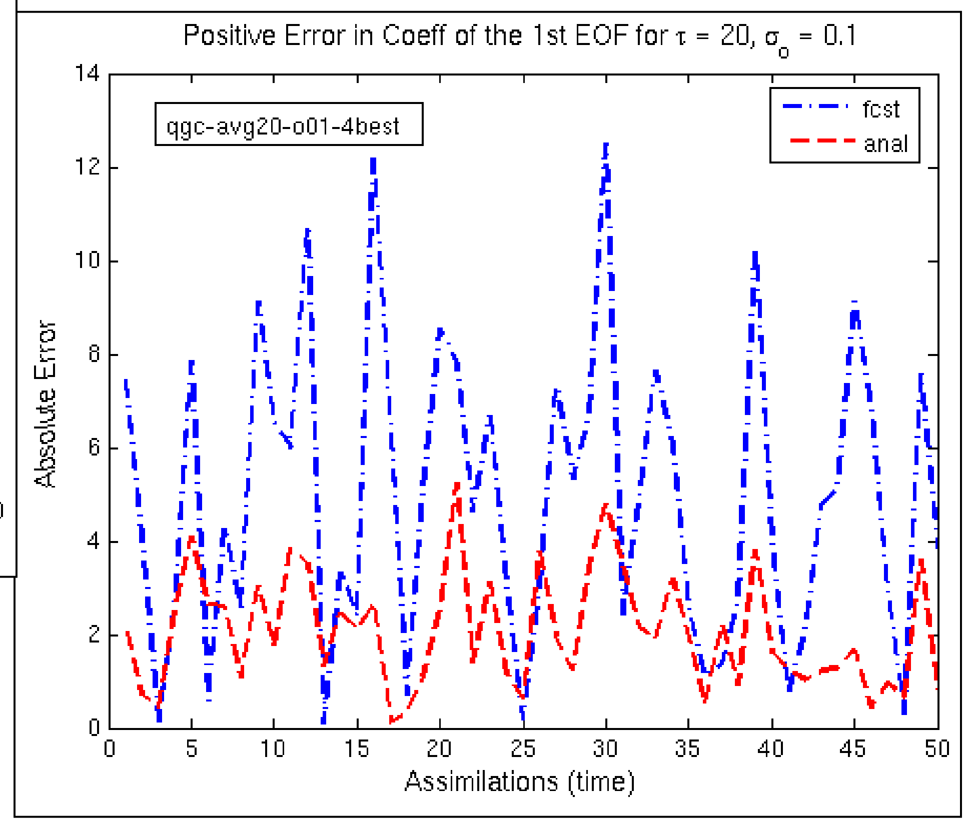


4 Optimal Observation Locations



Avg Error - Anal = 2.0545

- Fcst = 4.8808



Summary

- Time for paleo assimilation of select proxy data.
  - ensemble filters
  - ice-core accumulation & isotopes
- Modified Kalman filter approach
  - Update time mean
  - Easy, works well in existing EnKF.
- Filter corrects time scales shorter than proxy timescale.
  - Dynamics scatter information.
- Beyond predictability time scale, random samples drawn from model climate work well.
  - Model error problematic.

Ensemble Sensitivity (cont'd)

- For identity *H*, choose the point maximizing:

$$\boxed{\delta\sigma = \frac{[\mathrm{cov}(\delta J, \delta x_i)]^2}{\mathrm{var}(\delta x_i) + R}}$$

- Choose second point conditional on first:

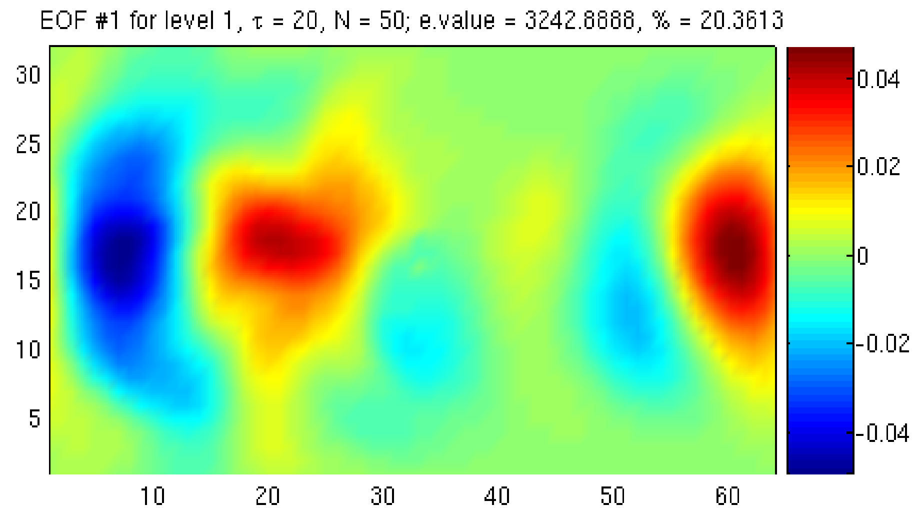
$$\boxed{\delta\sigma = \frac{\left[\mathrm{cov}(\delta J, \delta x_i) - \frac{\mathrm{cov}(\delta J, \delta x_1)\mathrm{cov}(\delta x_1, \delta x_i)}{\mathrm{var}(\delta x_1) + R}\right]^2}{\mathrm{var}(\delta x_i) - \frac{\mathrm{cov}(\delta x_1, \delta x_i)}{\mathrm{var}(\delta x_1) + R} + R}}$$

- Etc.

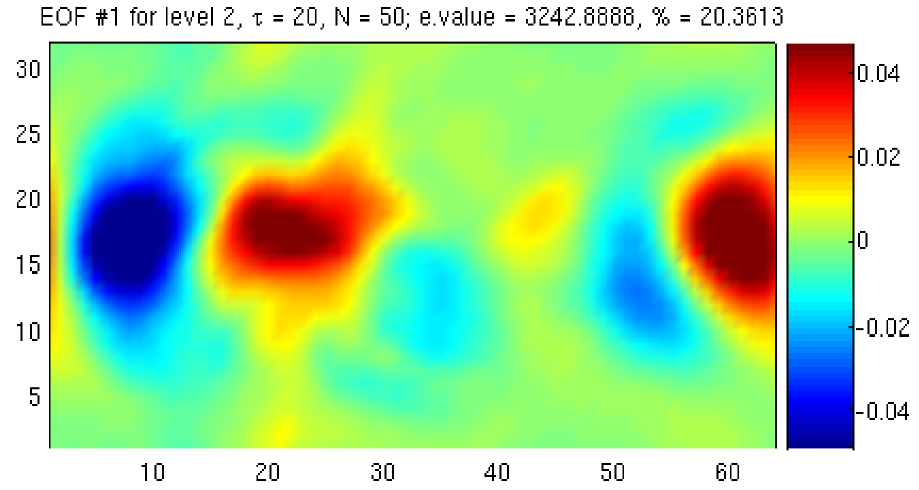
Ensemble Sensitivity (cont'd)

- *A recursive formula, which requires the evaluation of just* <u>*k+3*</u> *lines (1 covariance vector + (k+6) entry-wise mults/divs/adds/subs) for the k'th point.*

Results for tau = 20

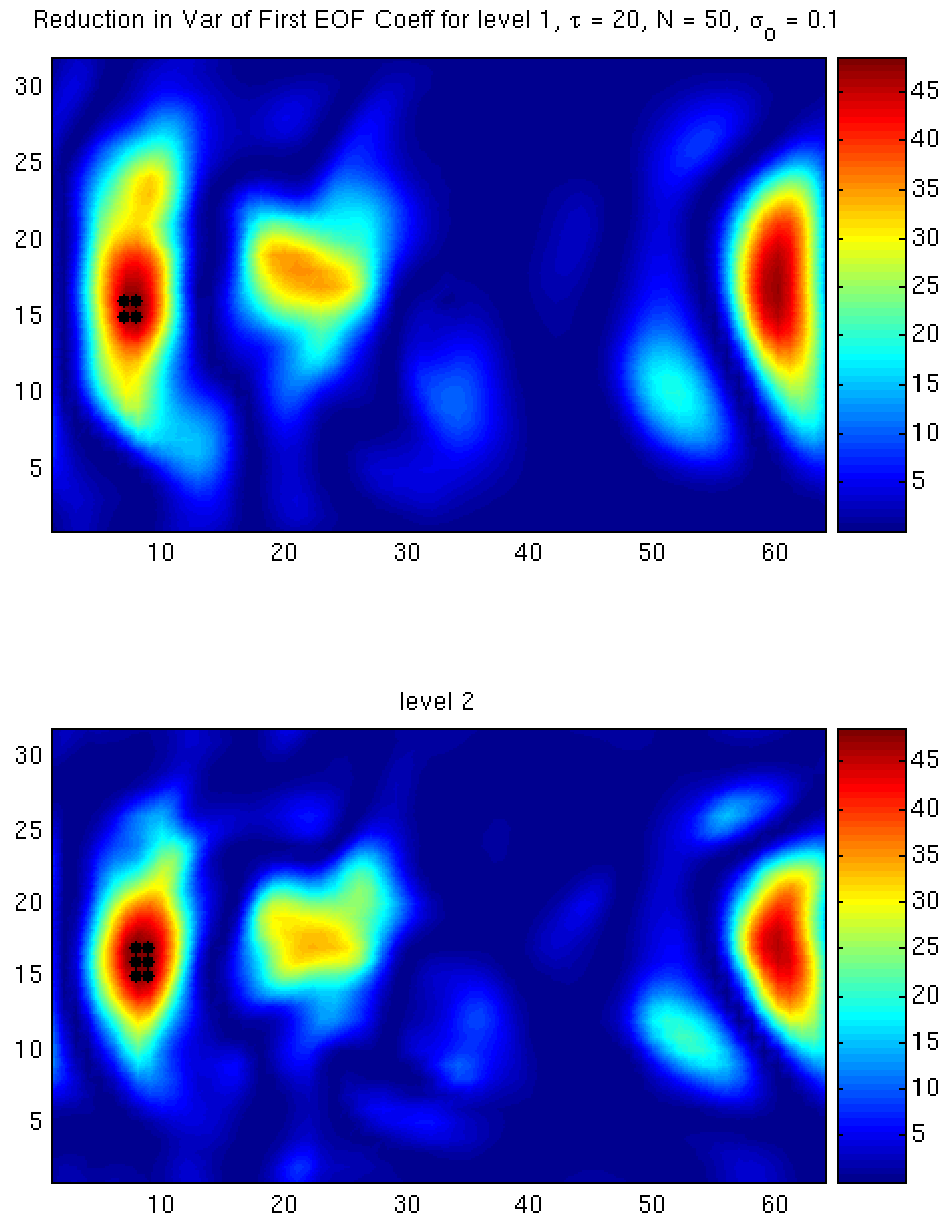


First EOF

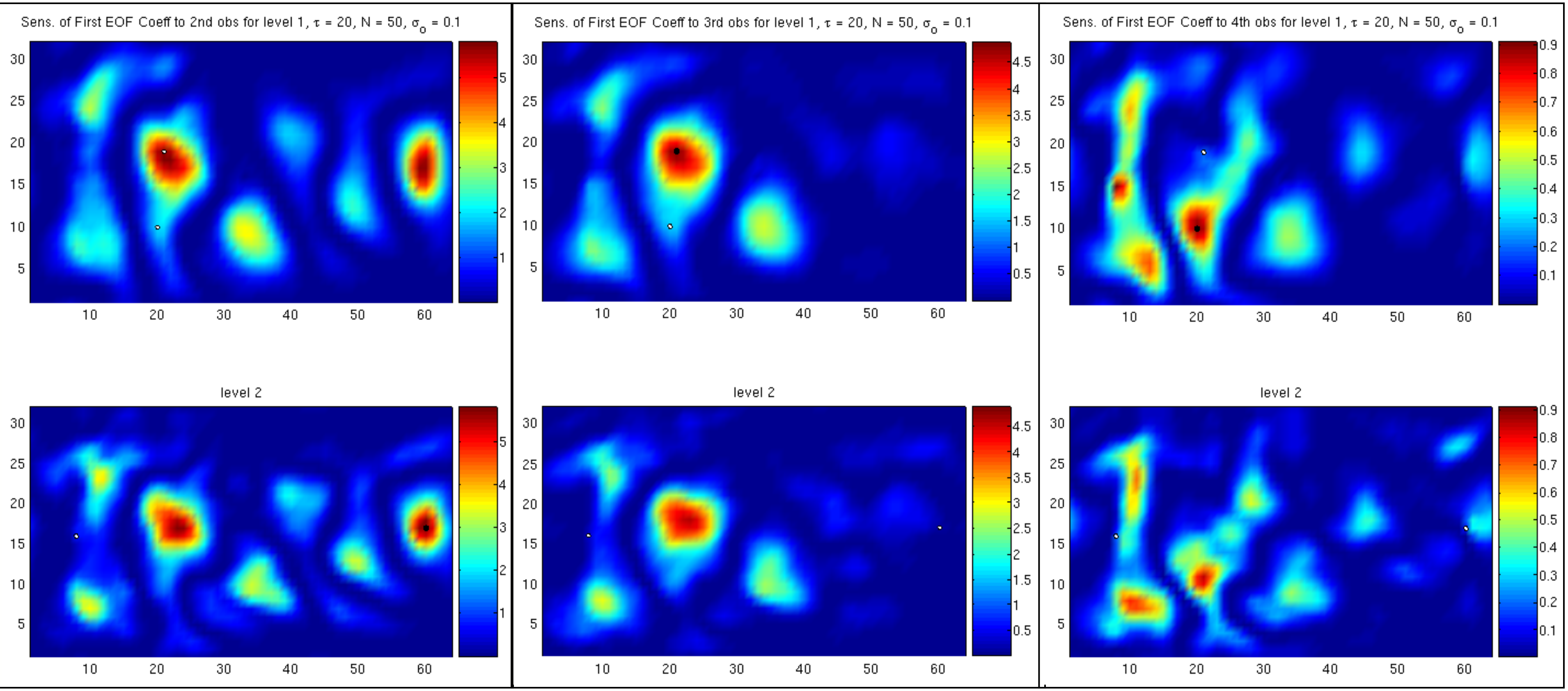


Results for tau = 20

- The ten most sensitive locations (unconditional)
- $\sigma_o = 0.10$

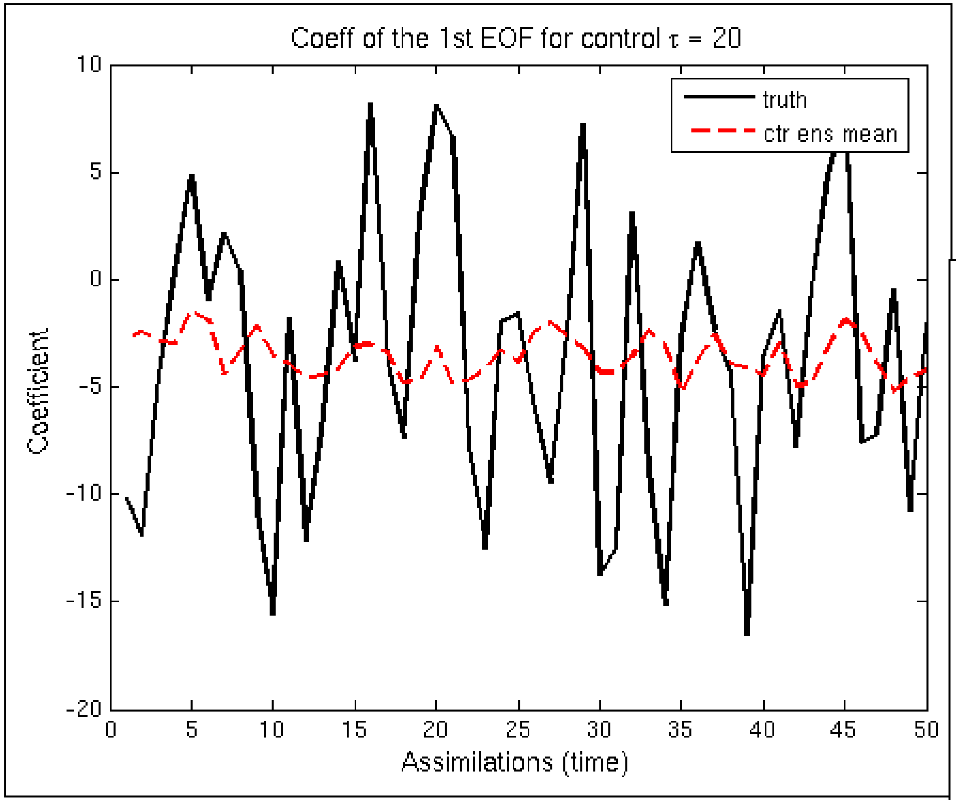


Results for tau = 20; σ_o = 0.10

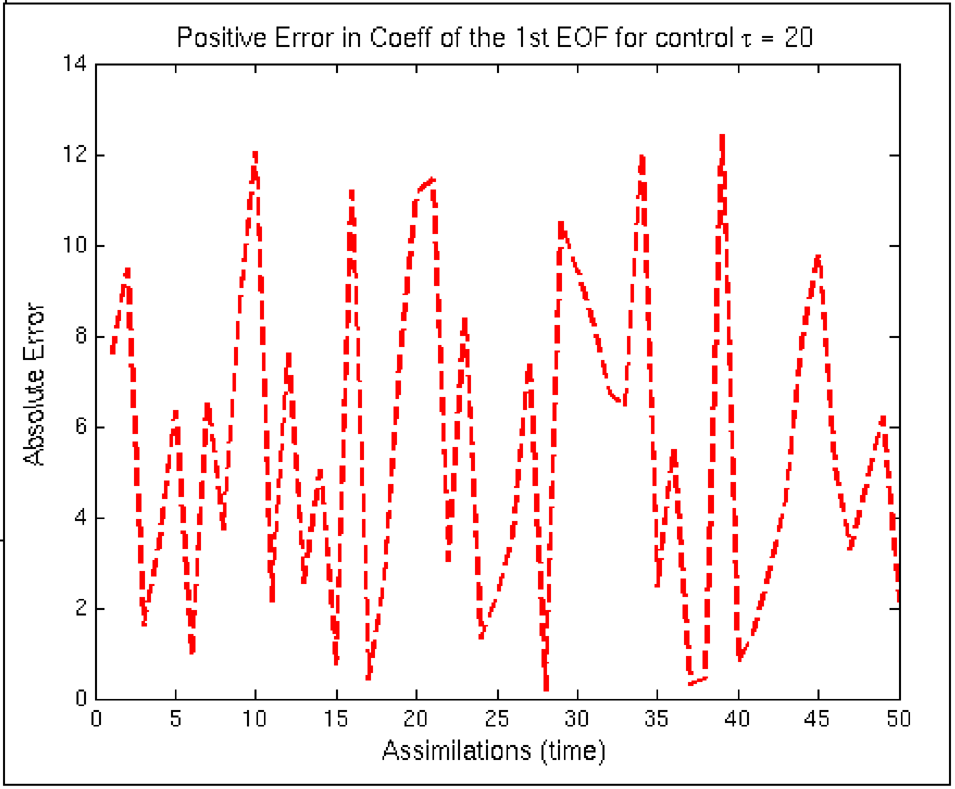


Note the decreasing effect on the variance.

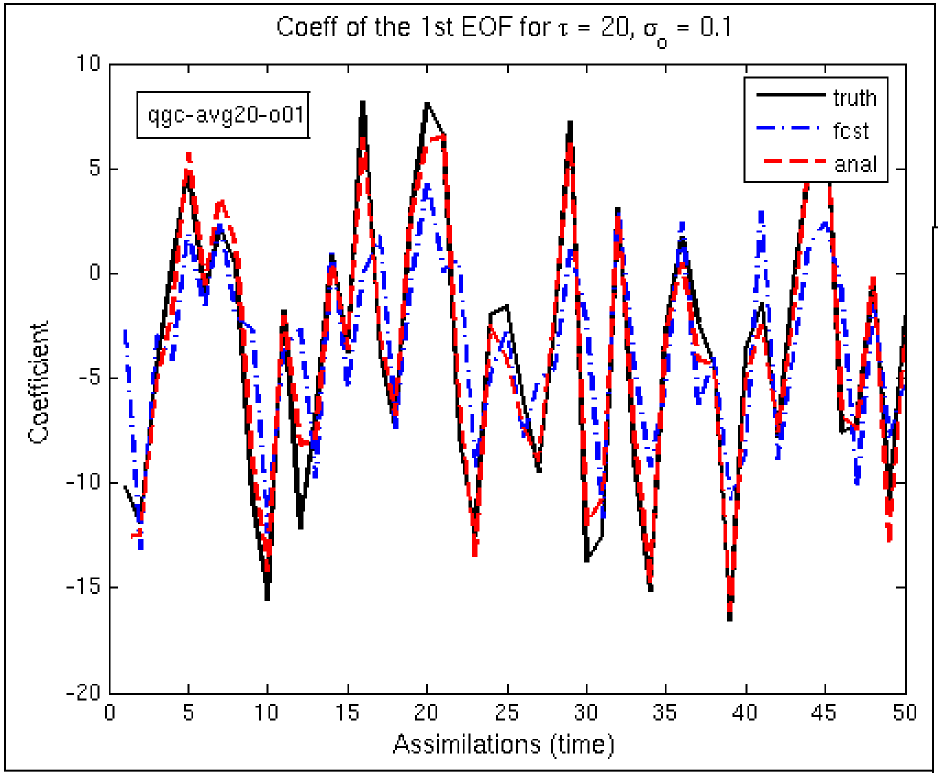
Control Case: No Assimilation



Avg error = 5.4484

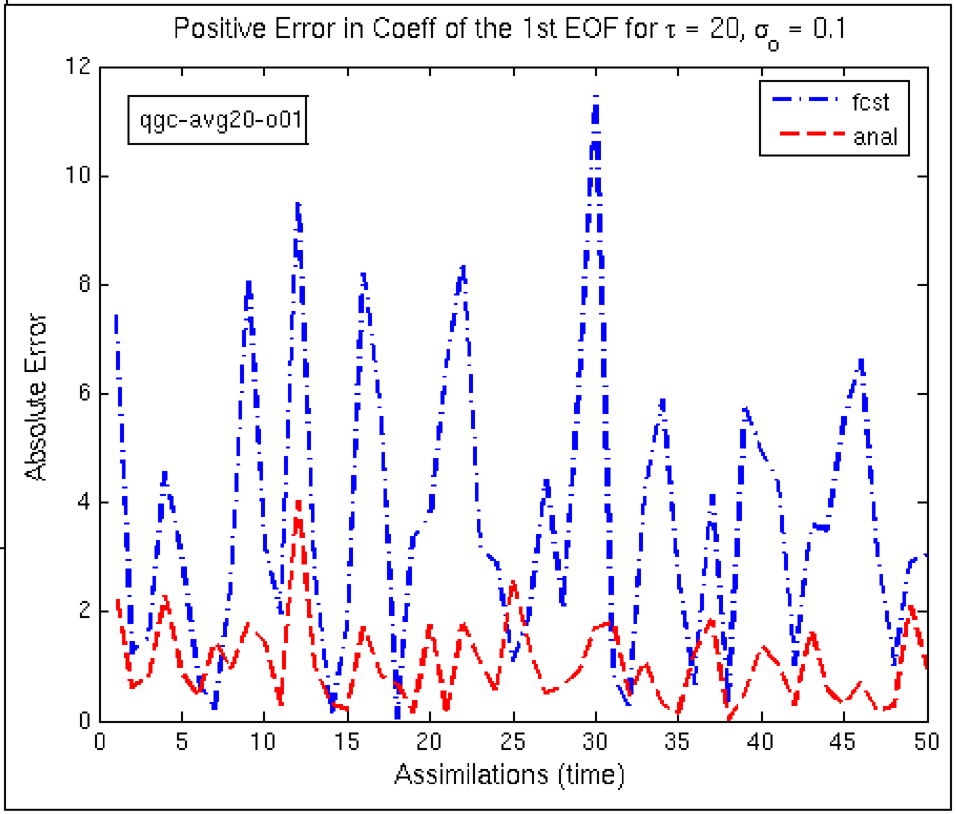


100 Random Observation Locations

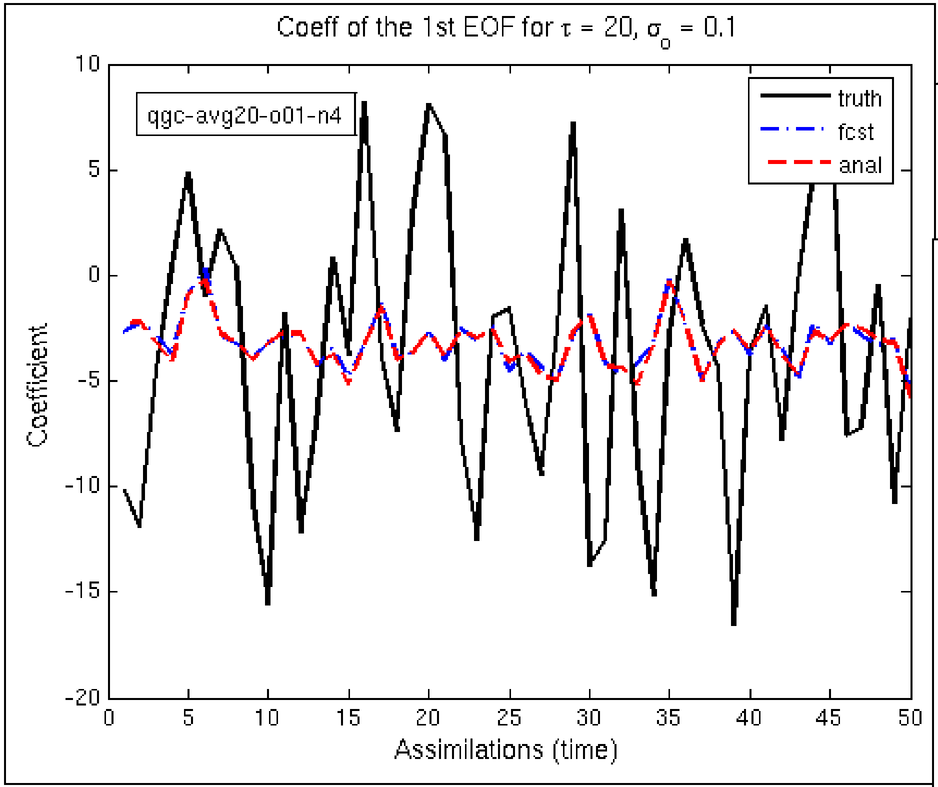


Avg Error - Anal = 1.0427

- Fcst = 3.6403

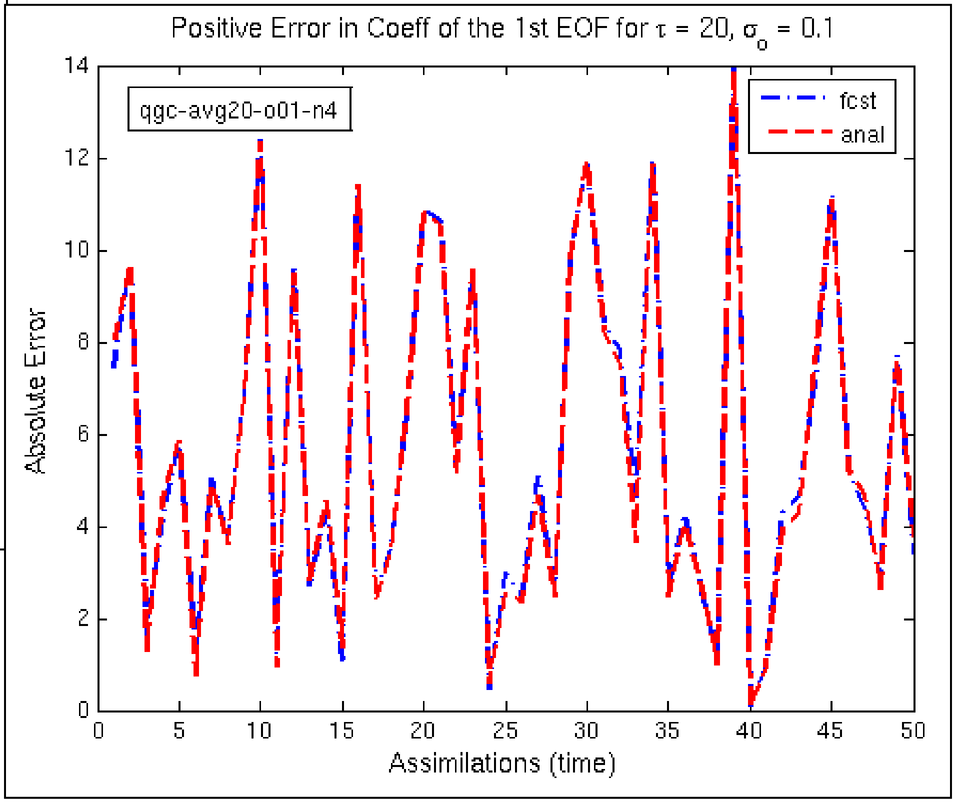


4 Random Observation Locations



Avg Error - Anal = 5.5644

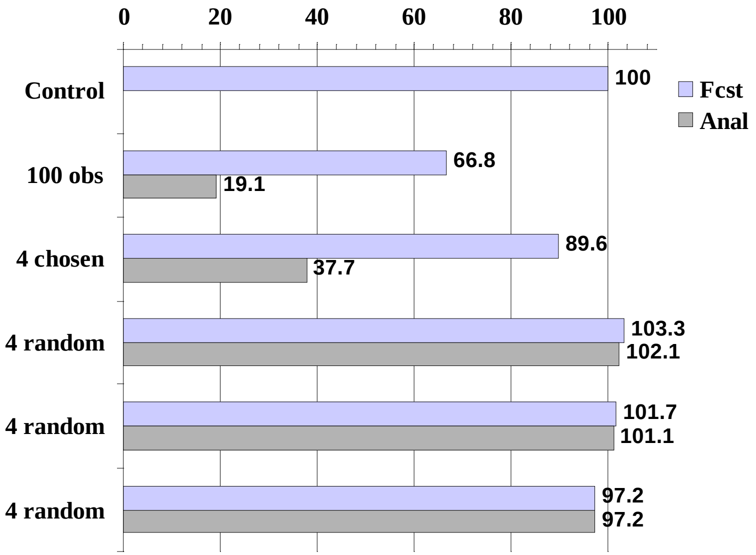
- Fcst = 5.6279



Summary

Avg Error	Fcst	Anal
Control	5.4484	
100 obs	3.6403	1.0427
4 chosen	4.8808	2.0545
4 random	5.6279	5.5644
4 random	5.5410	5.5091
4 random	5.2942	5.2953

Percent of ctr error

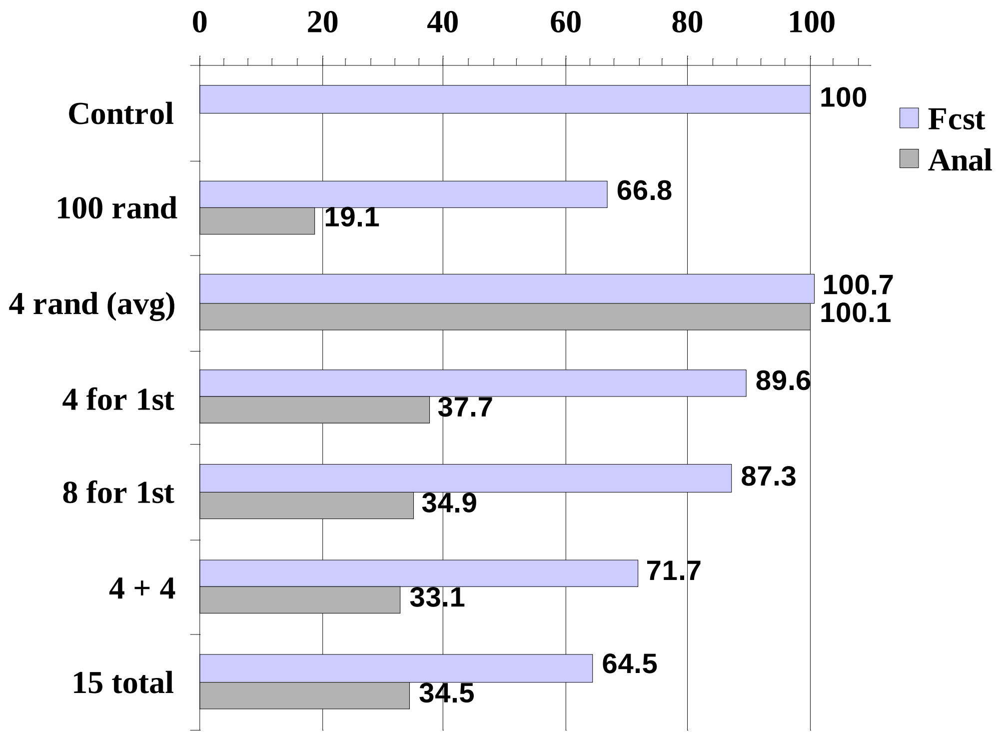


Assimilating just the 4 *chosen* locations yields a significant portion of the gain in error reduction in *J* achieved with 100 obs.

15 Chosen Observations

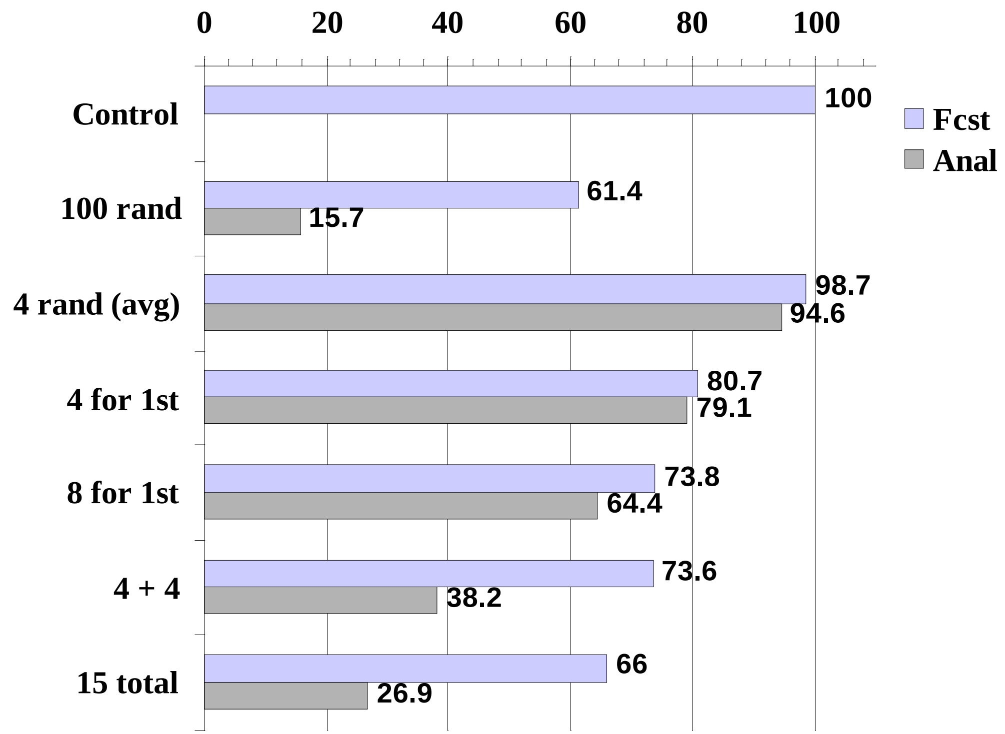
- For this experiment, take
  - 4 best obs to reduce variability in 1st EOF
  - 4 best obs to reduce variability in 2nd EOF
  - 2 best obs to reduce variability in 3rd EOF
  - 2□best obs to reduce variability in 4th EOF
  - 3 best obs to reduce variability in 5th EOF
- Number for each EOF chosen by $\delta\sigma < 0.7$
- All obs conditional on assimilation of previous obs.

15 Obs: Error in 1st EOF Coeff



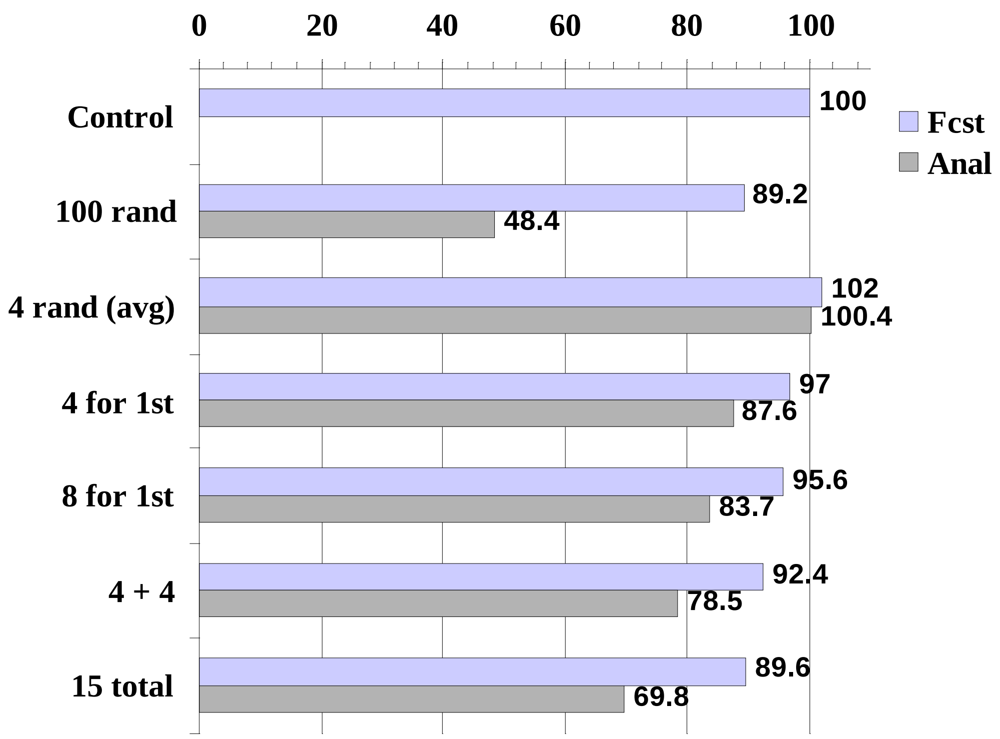
EOF1	Control	100R	4R	4O	8O	15 total
Fcst	5.4484	3.6403	5.4877	4.8808	4.7586	3.5138
Anal		1.0427	5.4563	2.0545	1.9020	1.8819

15 Obs: Error in 2nd EOF Coeff



EOF2	Control	100R	4R	4O	8O	15 total
Fcst	5.4114	3.3212	5.3394	4.3677	3.9937	3.5727
Anal		0.8478	5.1207	4.2796	3.4832	1.4563

15 Observations: RMS Error



RMS	Control	100 R	4 R	4 O	8 O	15 O
Fcst	0.2899	0.2586	0.2957	0.2810	0.2770	0.2596
Anal		0.1402	0.2912	0.2539	0.2425	0.2024

Current & Future Plans

Angie Pendergrass (UW)

- modeling on the sphere: SPEEDY
  - simplified physics
  - slab ocean
- ice-core assimilation
  - annual accumulation
  - oxygen isotopes