Time Series Analysis: Decomposition and Models of Data

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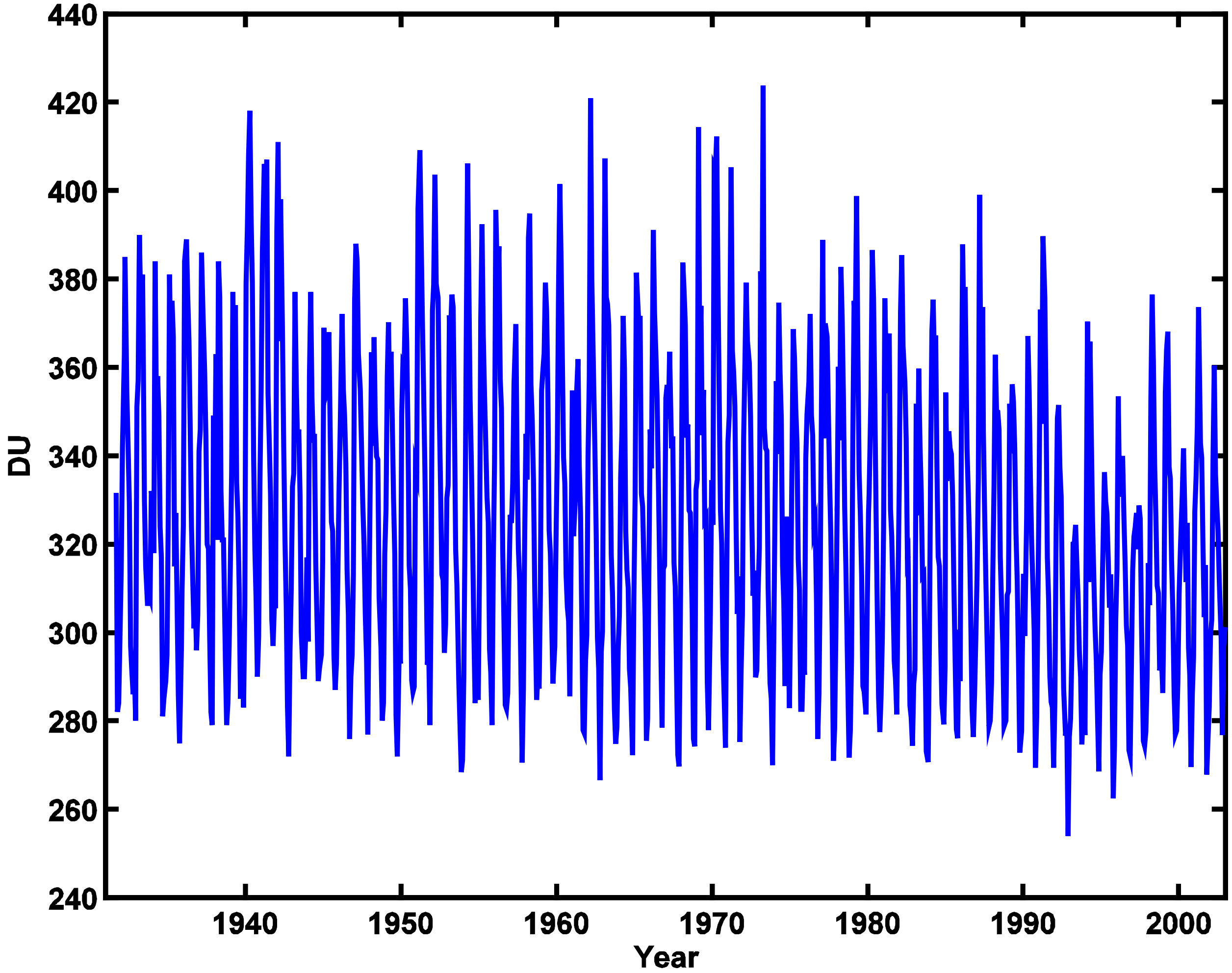
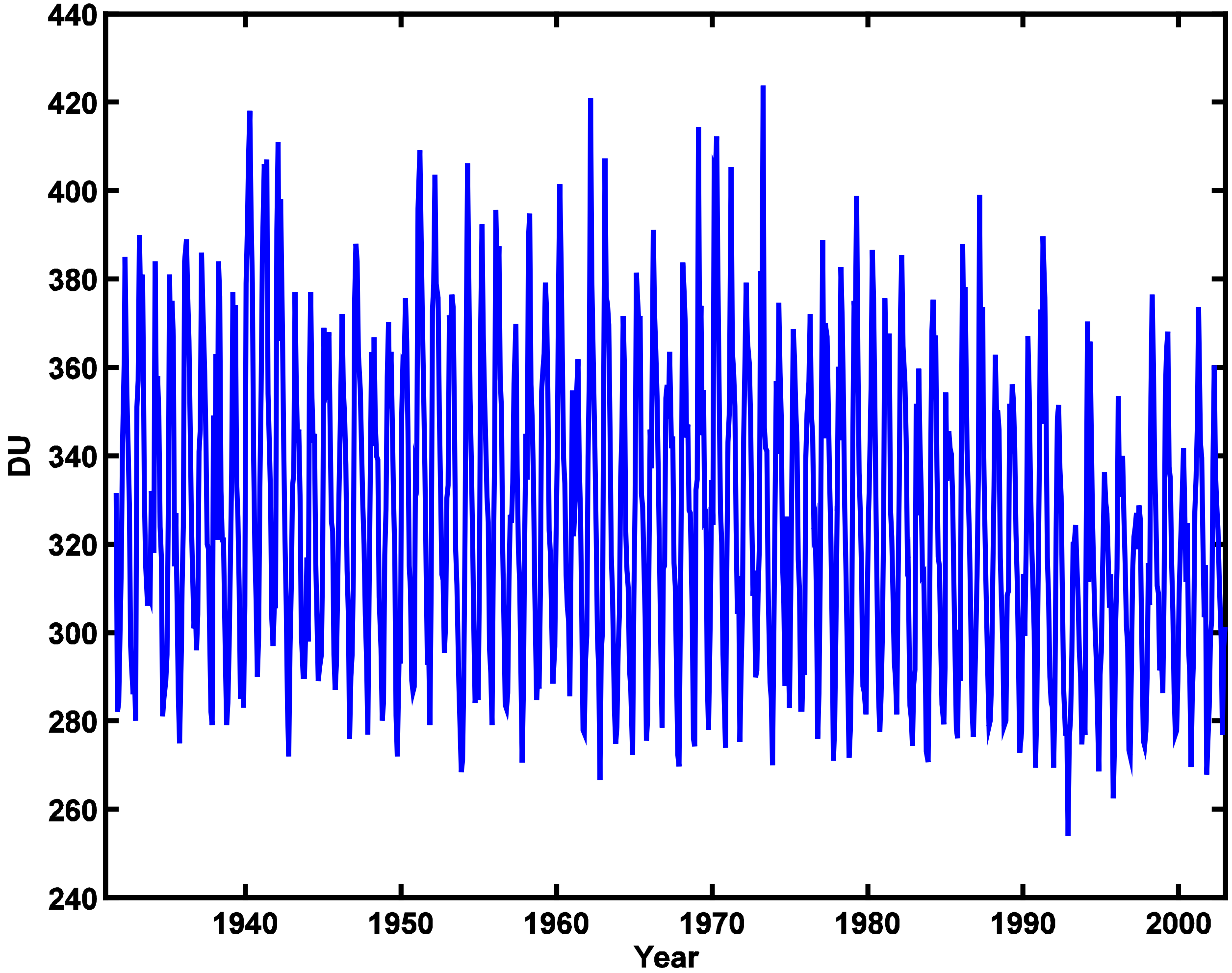
Motivation

- Properties of Climatic Time Series
  - Short: records are often short with respect to time scales of interest.
  - Noisy: many process acting on many time scales
- Components of variability often difficult to isolate
- Goal: reduce the complexity (degrees of freedom)
  - Decomposition: partition into multiple time series
  - Models: capture “important” components

Total Column Ozone: Arosa, Switzerland

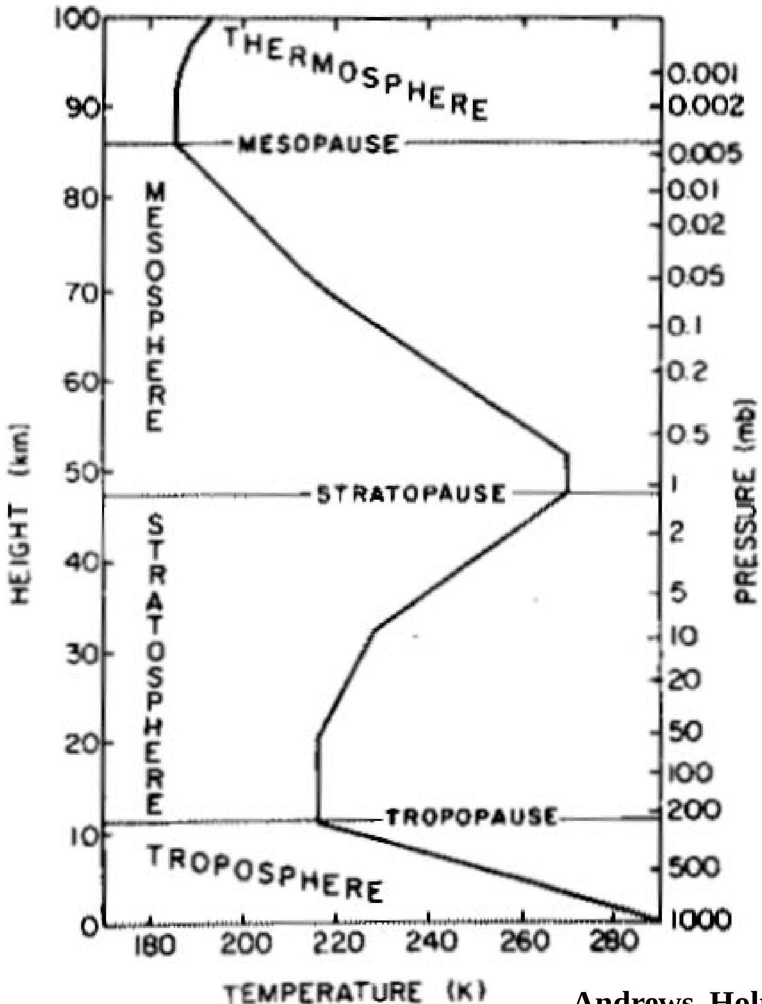


Total Column Ozone: Arosa, Switzerland: Key Features



- **Annual Cycle**
- **Interannual Variability**
- **Nonlinear Trend**
- **Non-stationary**

Part I: Structure of the Stratosphere



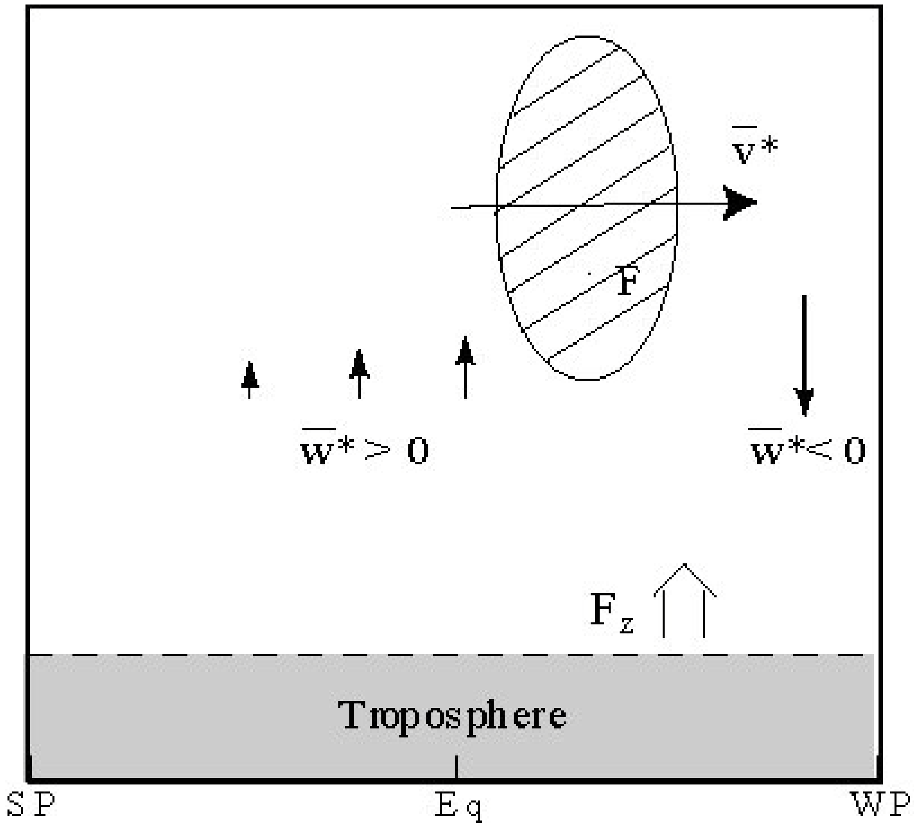
- Troposphere:
  - Decreasing temperature with height
  - vertically mixed
- Stratosphere:
  - Increasing temperature with height
  - Stably stratified
  - Relatively quiescent compared to troposphere
- Mid-latitude Mean Profile

Andrews, Holton, Leovy, 'Middle Atmosphere Dynamics', 1987

Some Features of the Stratosphere Sources of Variability for TCO

- Dynamical Features:
  - Brewer-Dobson (Meridional) Circulation
    - Overturning equator-to-pole flow
  - Quasi-biennial Oscillation (QBO)
    - Equatorial zonal (east-west) wind
  - Polar Night (Winter) Vortex
- Other sources of IAV (interannual variability):
  - 11-yr Solar Cycle
  - El-Nino Southern Oscillation (?)
- Interactions between processes

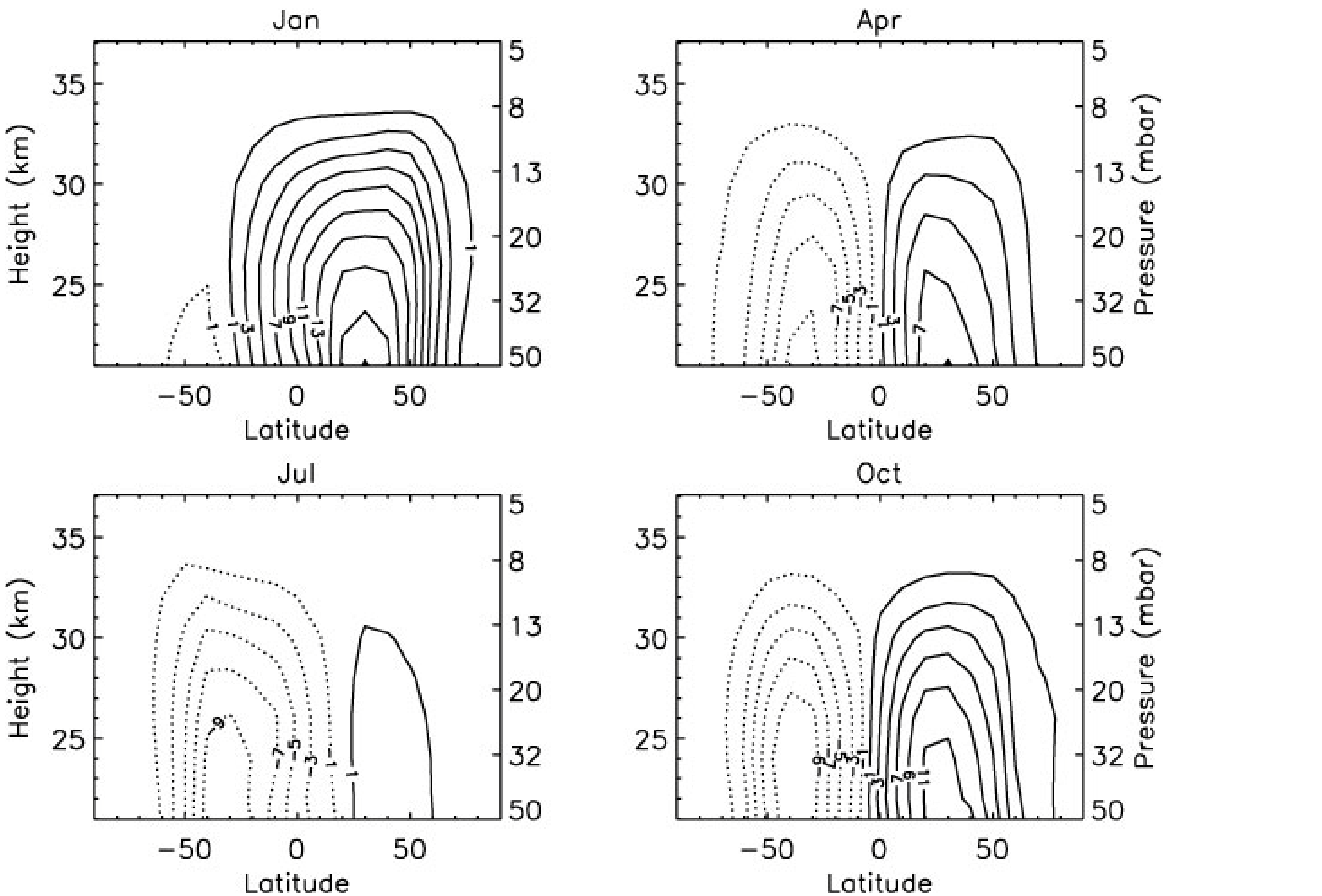
Brewer-Dobson Circulation (BDC) and Planetary Waves



Courtesy of M. Salby

- Upwelling planetary waves break in shaded region
- Drives the Brewer-Dobson circulation
- Transports heat to the polar vortex
- Effect on the strength of the polar night vortex
- Not steady: extreme events - SSWs

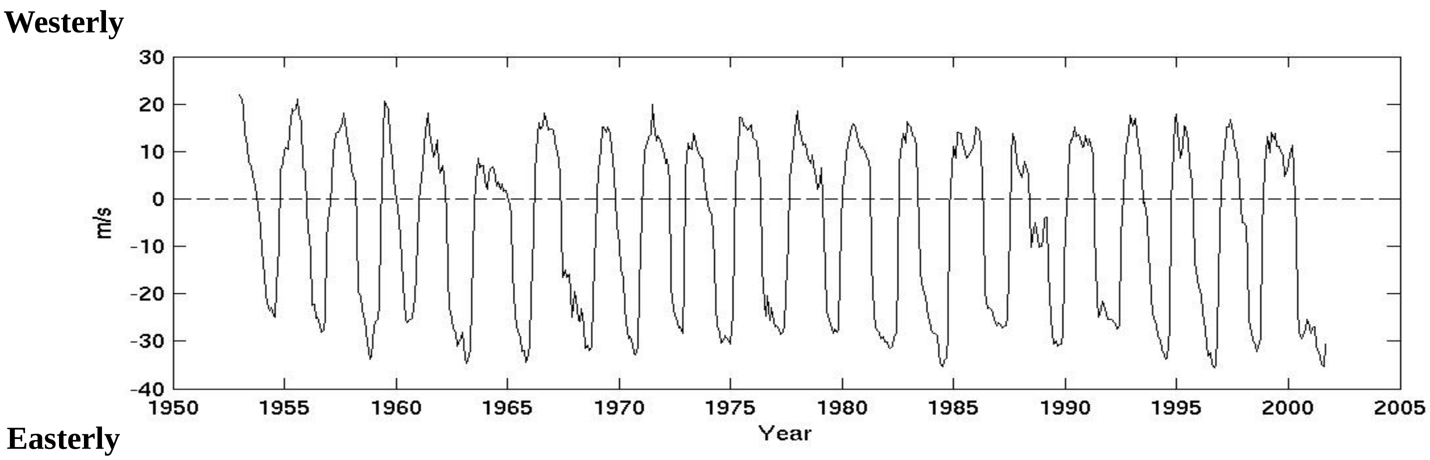
Meridional (Brewer-Dobson) Circulation: Mean Seasonal Cycle



Units: 10^9 kg/s

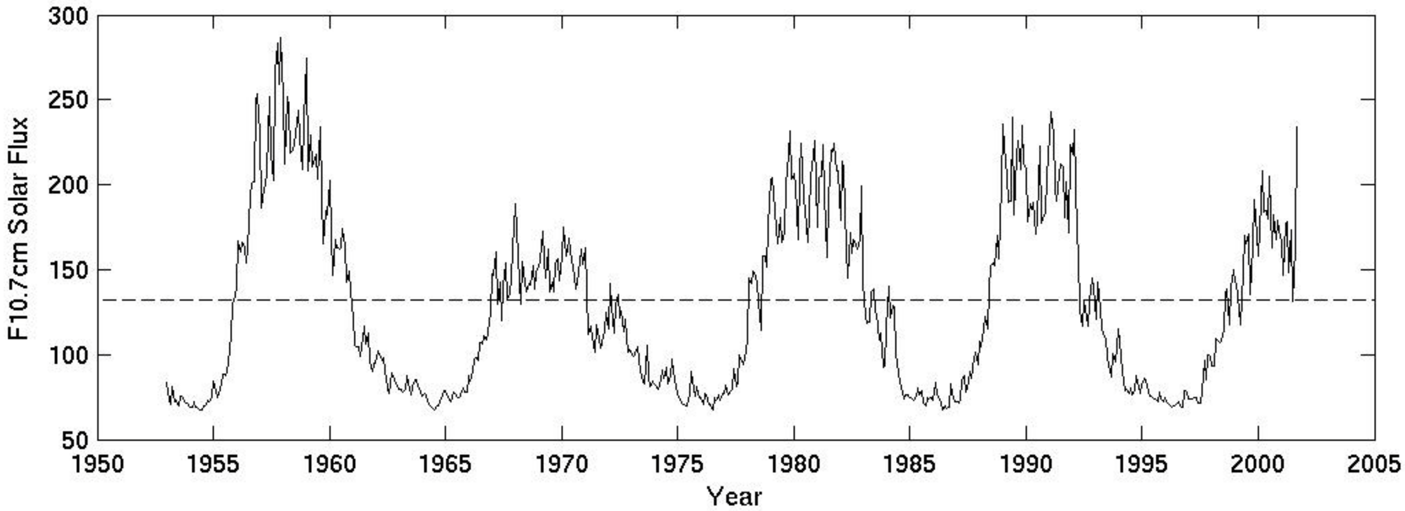
NCEP/NCAR Reanalysis: Isentropic Mass Stream Function

Quasi-biennial Oscillation (QBO):



- Oscillation in the equatorial zonal (east-west) wind.
- Average Period: 28 mo.
- Downwelling Anomaly
- 30 hPa Singapore zonal wind
- Effects the strength of the Brewer-Dobson (meridional) circulation

Cycle in Solar Irradiance



- F10.7cm (2800MHz) Solar Flux: Monthly Means
- Variability < 1% for Total Solar Irradiance
- But ~5% for UV, interacts with stratospheric ozone
- Related to the sunspot cycle

Part II:
Time Series Analysis

- Single Time Series
- Decomposition: Trend + Oscillation

$$x(t) = x_T(t) + x_O(t)$$

- Further decomposition of oscillatory component:, e.g.. Fourier Series

$$x(t) = x_T(t) + \sum_{k=-N/2}^{N/2} c_k e^{i2\pi kt/N}$$

Use of a priori knowledge

- If there is a known (or hypothesized) behavior, specifically isolate it first, e.g., annual cycle

$$x(t) = x_T(t) + x_A(t) + \sum_{k=-N/2}^{N/2} c_k e^{i2\pi kt/N}$$

- Instead of sequential decompositions

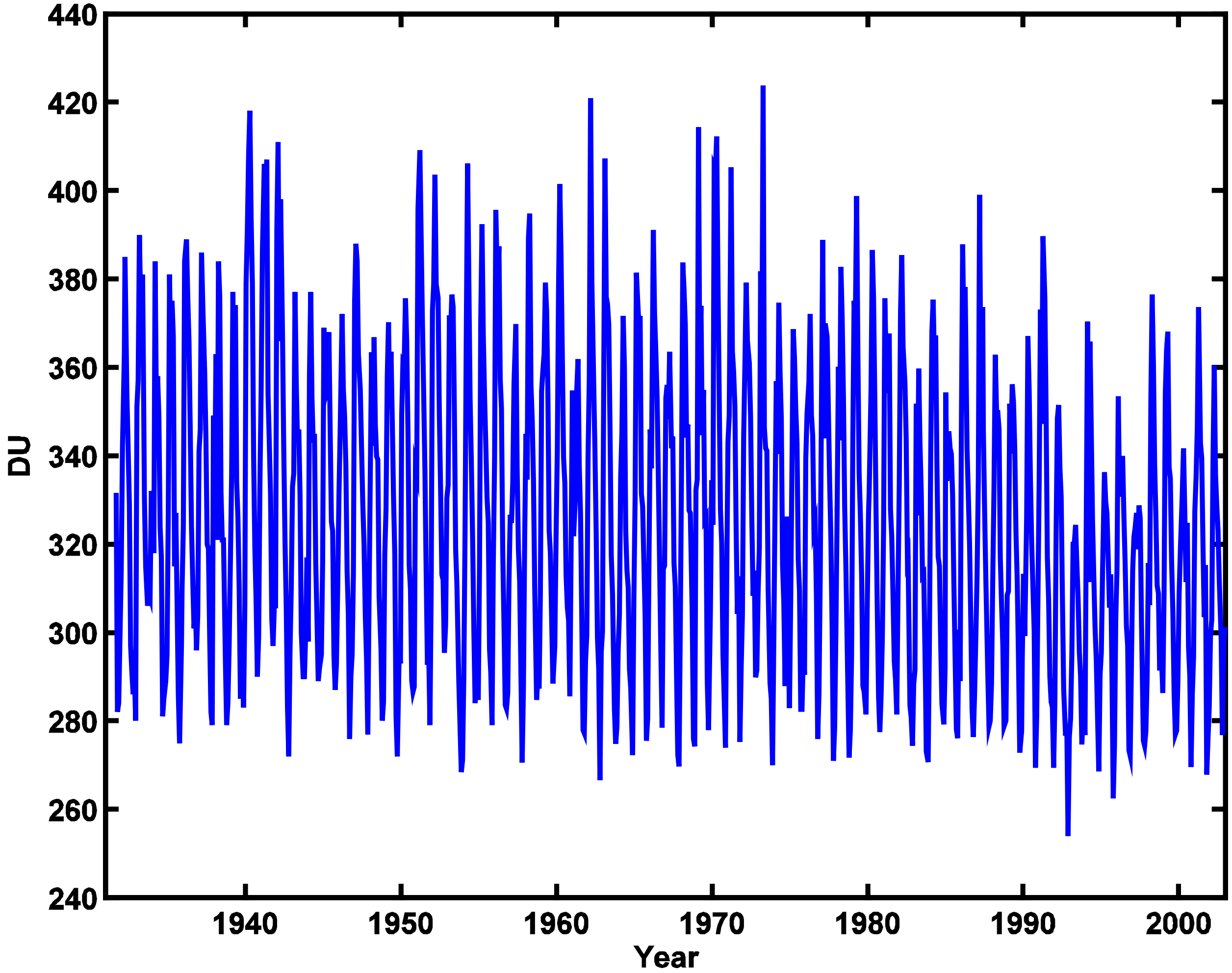
  (where order maybe important),

  we can *model* the data:

  eg. multiple regression:

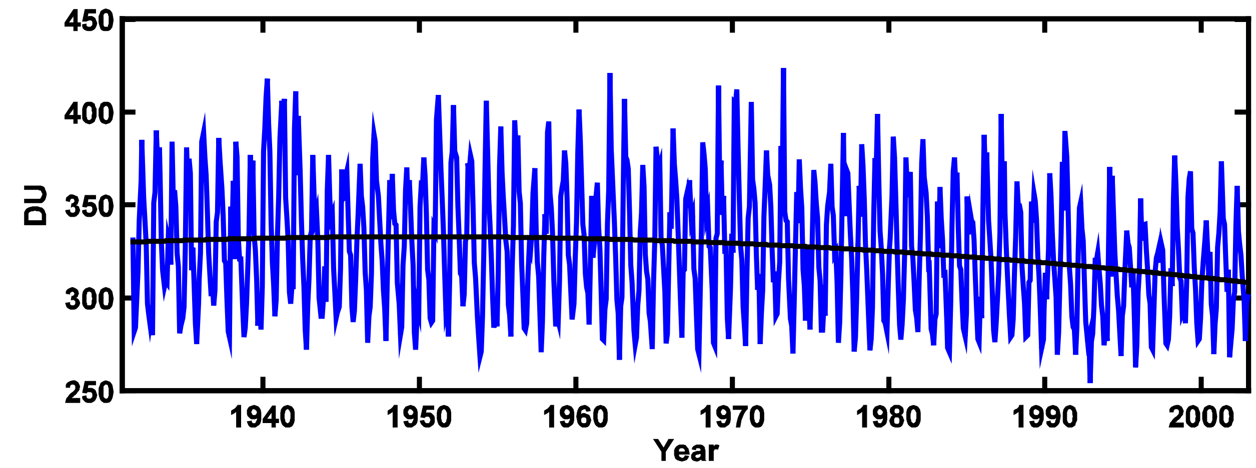
$$x(t) = \mathrm{Y}\overline{c} + \varepsilon(t) = \sum_n c_n y_n(t) + \varepsilon(t)$$

Total Column Ozone: Arosa, Switzerland: Fit a trend

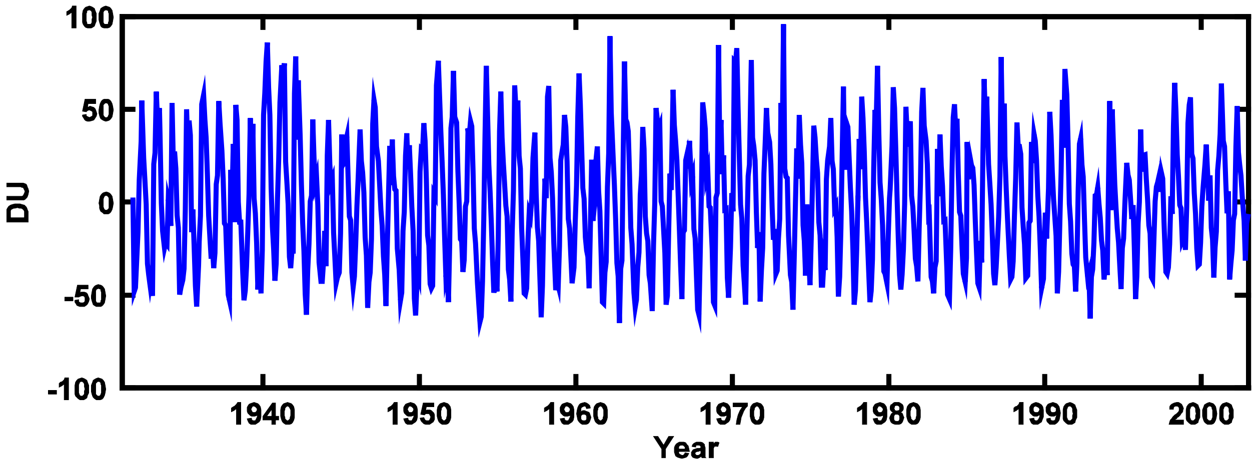


- **Annual Cycle**
- **Interannual Variability**
- **Nonlinear Trend**
- **Non-stationary**

Arosa TCO: Trend Fit and Detrending

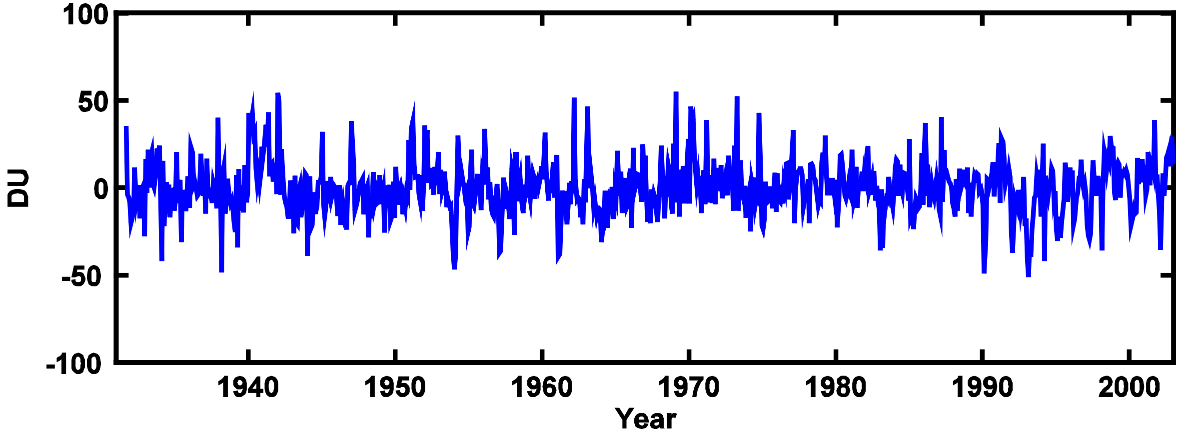


- **Quadratic Trend: Least Squares Fit**



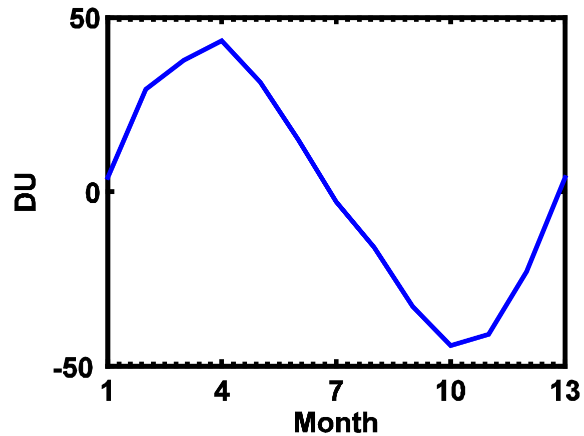
- **Detrended Data (Anomaly)**

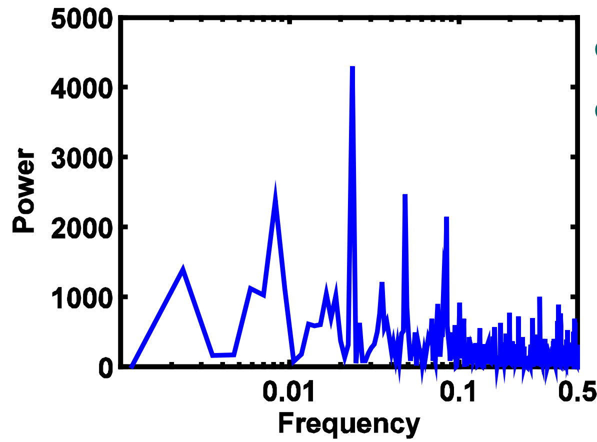
Arosa TCO: Mean Seasonal Cycle, Deseasonalized Data, Fourier Spectrum



- **Mean Annual Cycle: Growth: Fall/Winter, Decay: Spring/Summer Mean BDC**
- **Power Spectrum:**

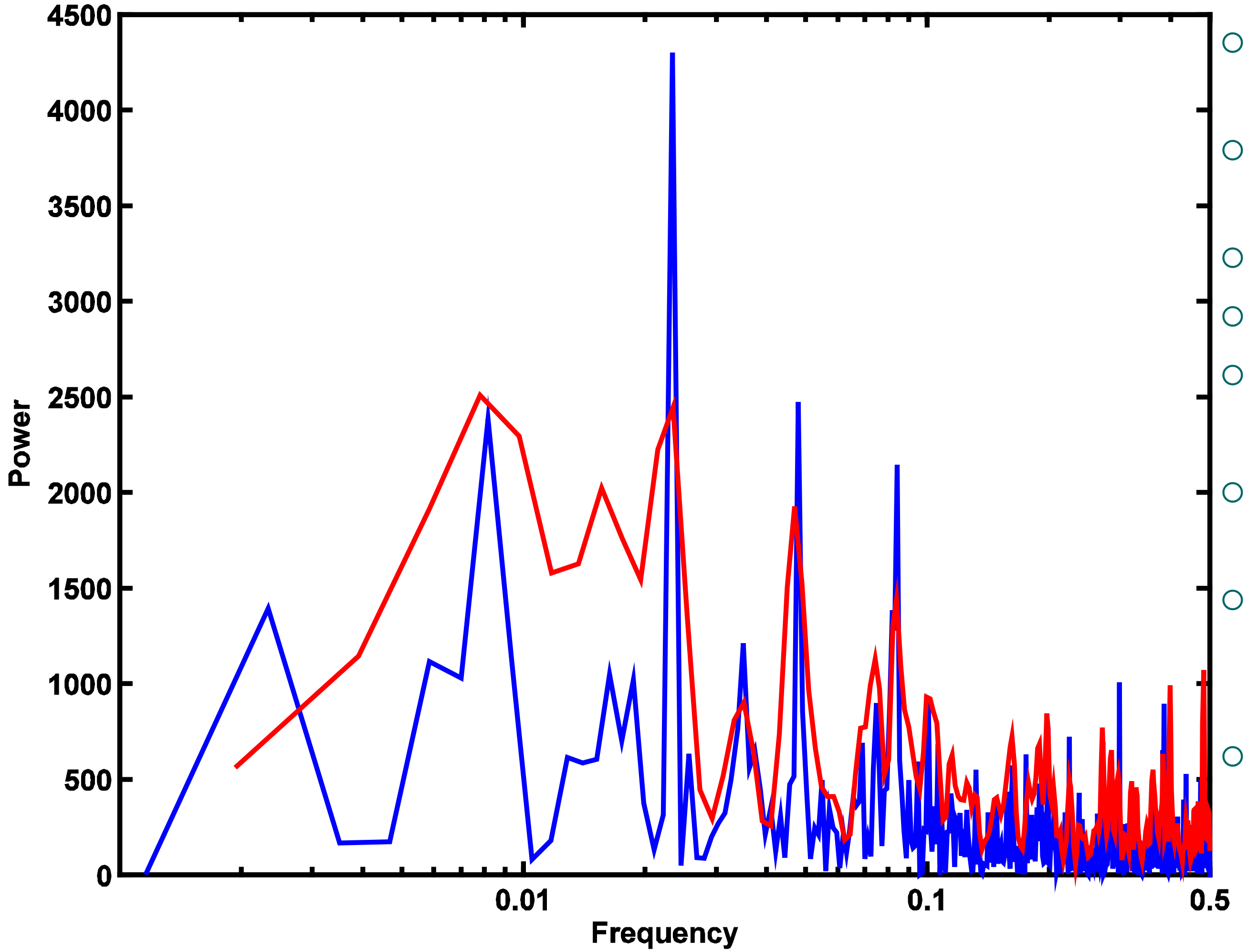
  Squared Coefficients of Fourier Transform of Deseasonalized data





- **Very poor statistically**
- **New Topic: Power Spectral Estimation**

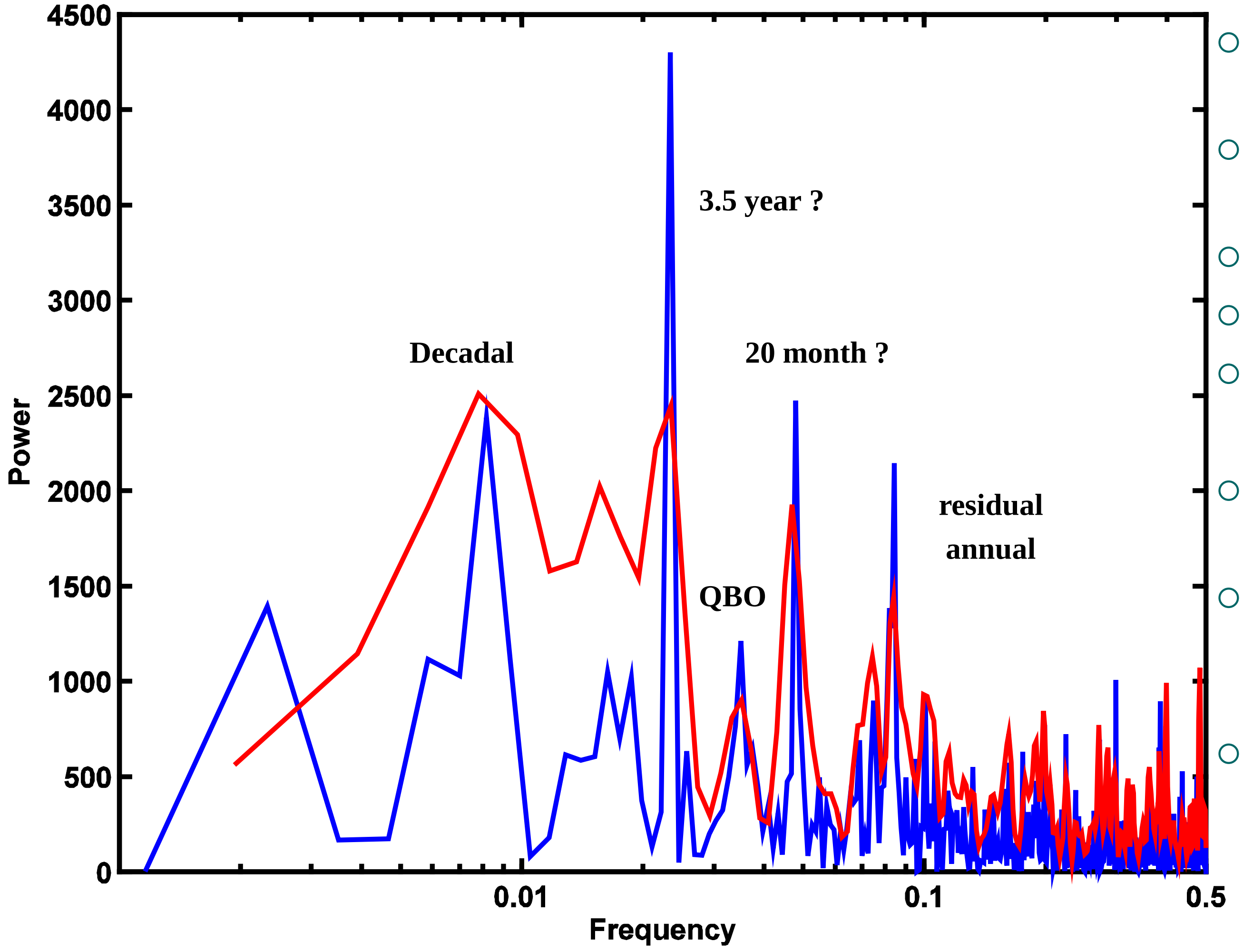
Arosa TCO: Power Spectral Estimate



- **Blue: Raw Spectrum**
- **Red: Welch Spectral Estimate**
- **3 segments**
- **50% overlap**
- **Hamming filter**

- **Average spectrum from segments**
- **Poorer spectral resolution and range**
- **Improved statistics**

Arosa TCO: Power Spectral Estimate



- Blue: Raw Spectrum
- Red: Welch Spectral Estimate
- 3 segments
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- Average spectrum from segments
- Poorer spectral resolution and range
- Improved statistics

Properties of Fourier Spectral Analysis

- Global:
  - Energy - Frequency analysis
  - Problem for non-stationary data
- Basis functions:
  - Harmonic
  - Orthogonal
  - A priori choice
- Other power spectral estimates: autoregressive, multi-taper

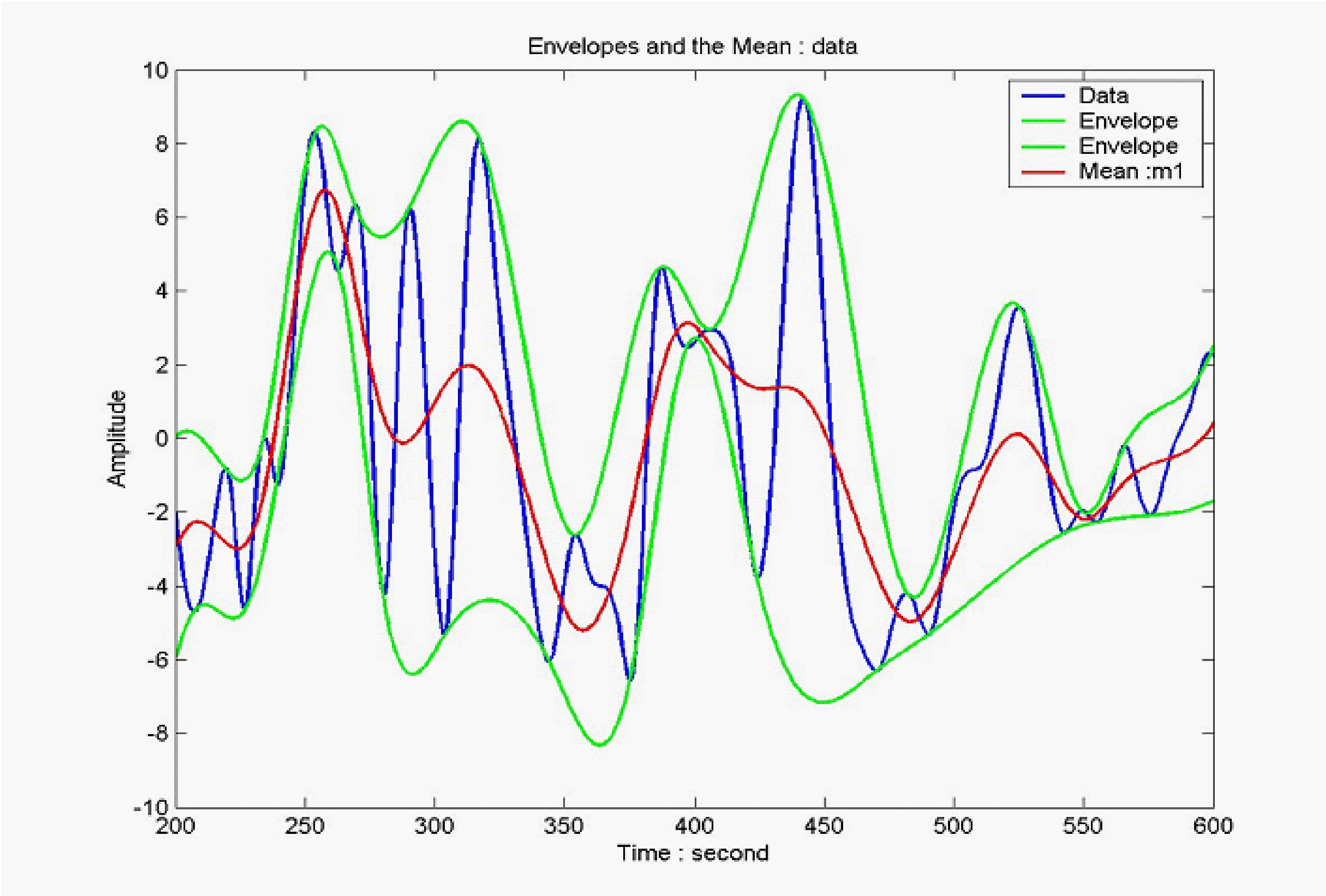
A couple other analysis techniques

- Wavelets:
  - Local: Energy-time-frequency
  - A priori basis functions: compact support
  - Not a decomposition
- Empirical Mode Decomposition:
  - Local: Energy-time-frequency
  - Data-adaptive: no a priori basis functions
  - Decomposition

Empirical Mode Decomposition (EMD)

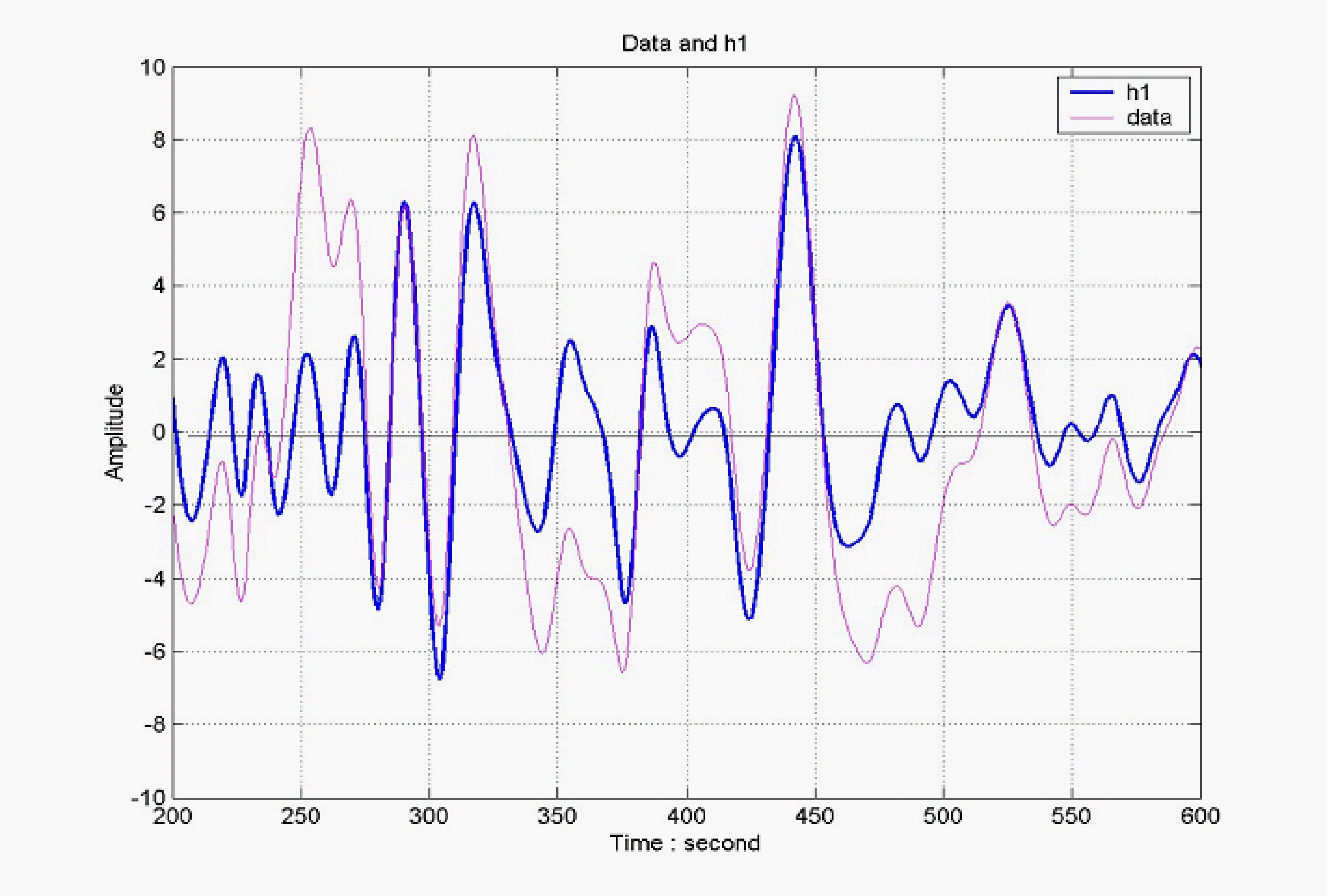
- Decomposition: set of time series
  - IMFs: Intrinsic Mode Functions
  - Interlaced zero-crossings and extrema
  - Symmetric envelope
  - Ideally, “local” mean is zero
- For each IMF
  - Find upper and lower envelope of data by spline fit to the extrema
  - Find estimate of the “local” mean: average of upper and lower envelopes
  - Remove “local” mean estimate
  - Iterate until stopping criteria met
  - Final residual is the IMF
- Iterate for each IMF
  - Subtract IMF from data set
  - Repeat process with the resulting new data set

EMD: 1st IMF, 1st Iteration
Envelope, 1st "local" mean estimate



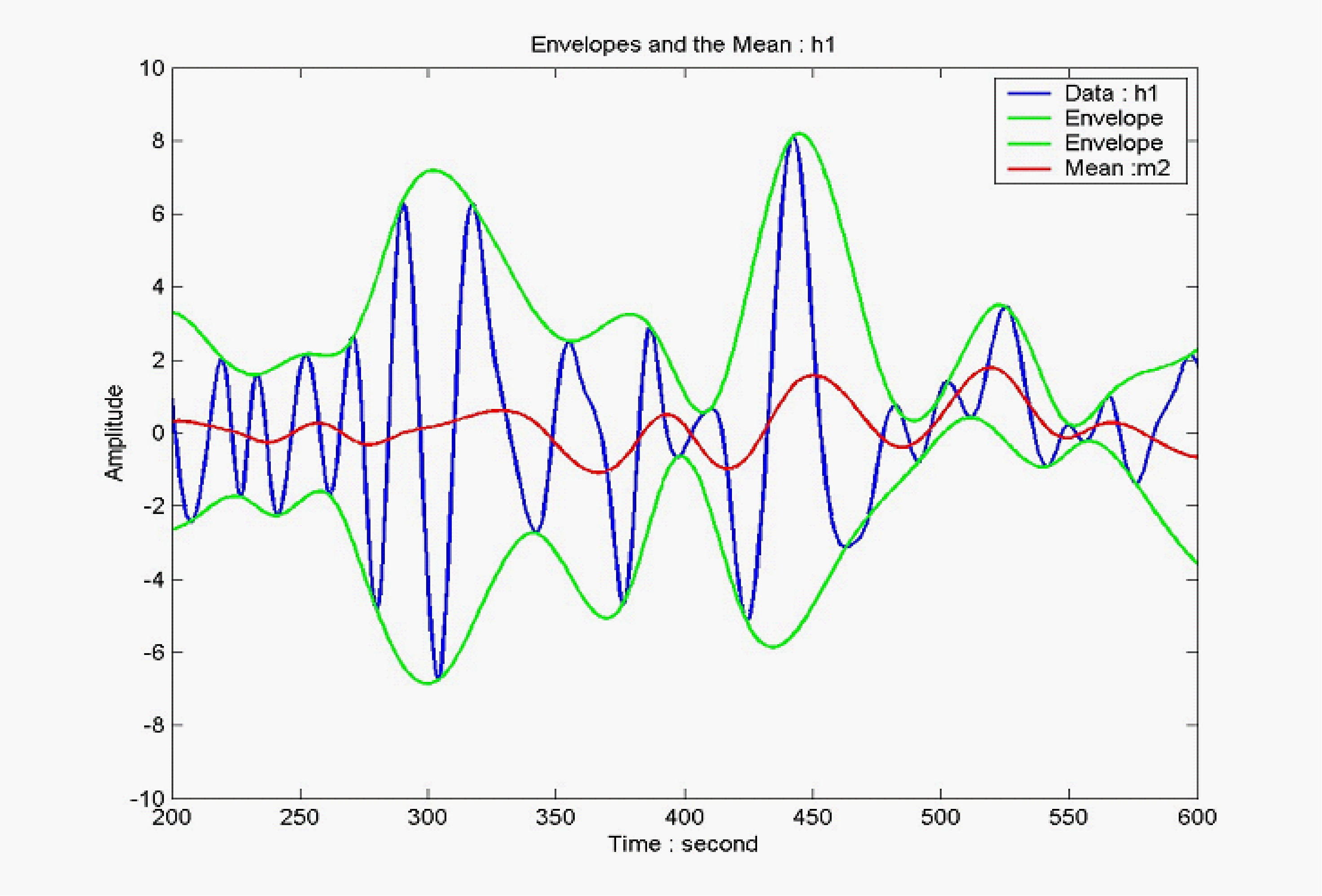
N.E.Huang, *Hilbert-Huang Transform and Its Applications, 2005*

EMD: 1st IMF, 1st Iteration Residual



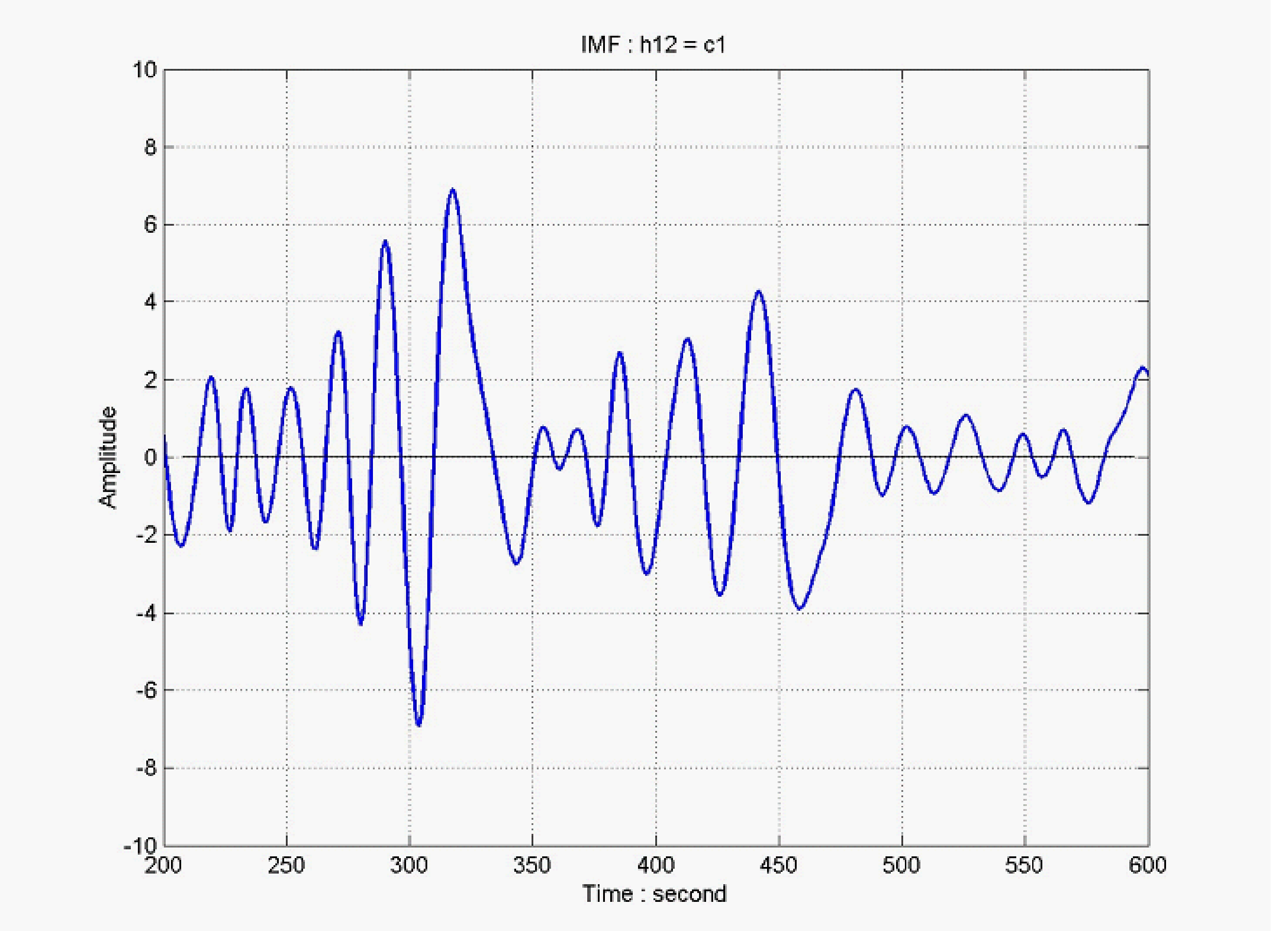
N.E.Huang, *Hilbert-Huang Transform and Its Applications, 2005*

EMD: 1st IMF, 2nd Iteration Envelope, 2nd "local" mean estimate



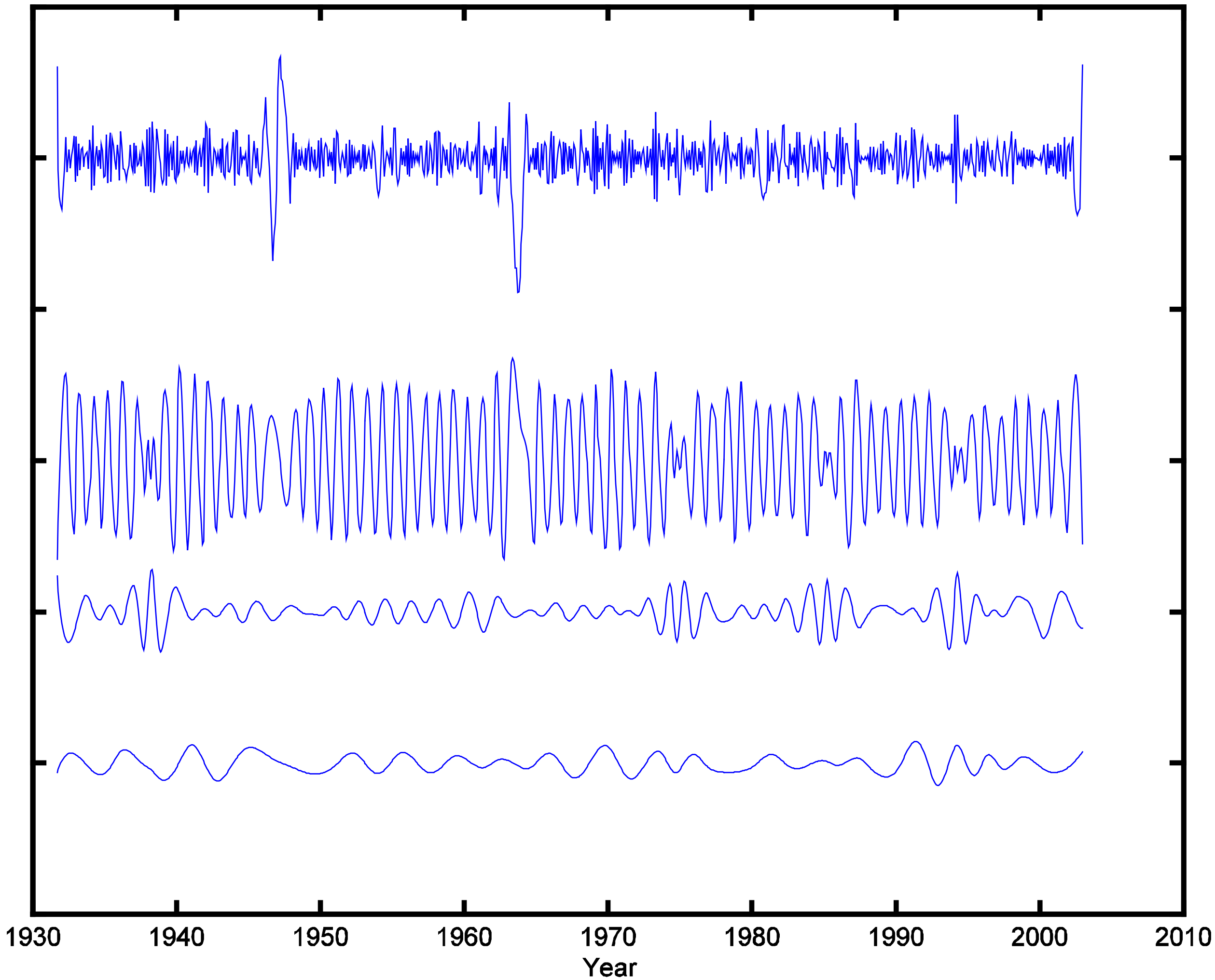
N.E.Huang, *Hilbert-Huang Transform and Its Applications, 2005*

EMD: 1st IMF
Residual after 12 iterations



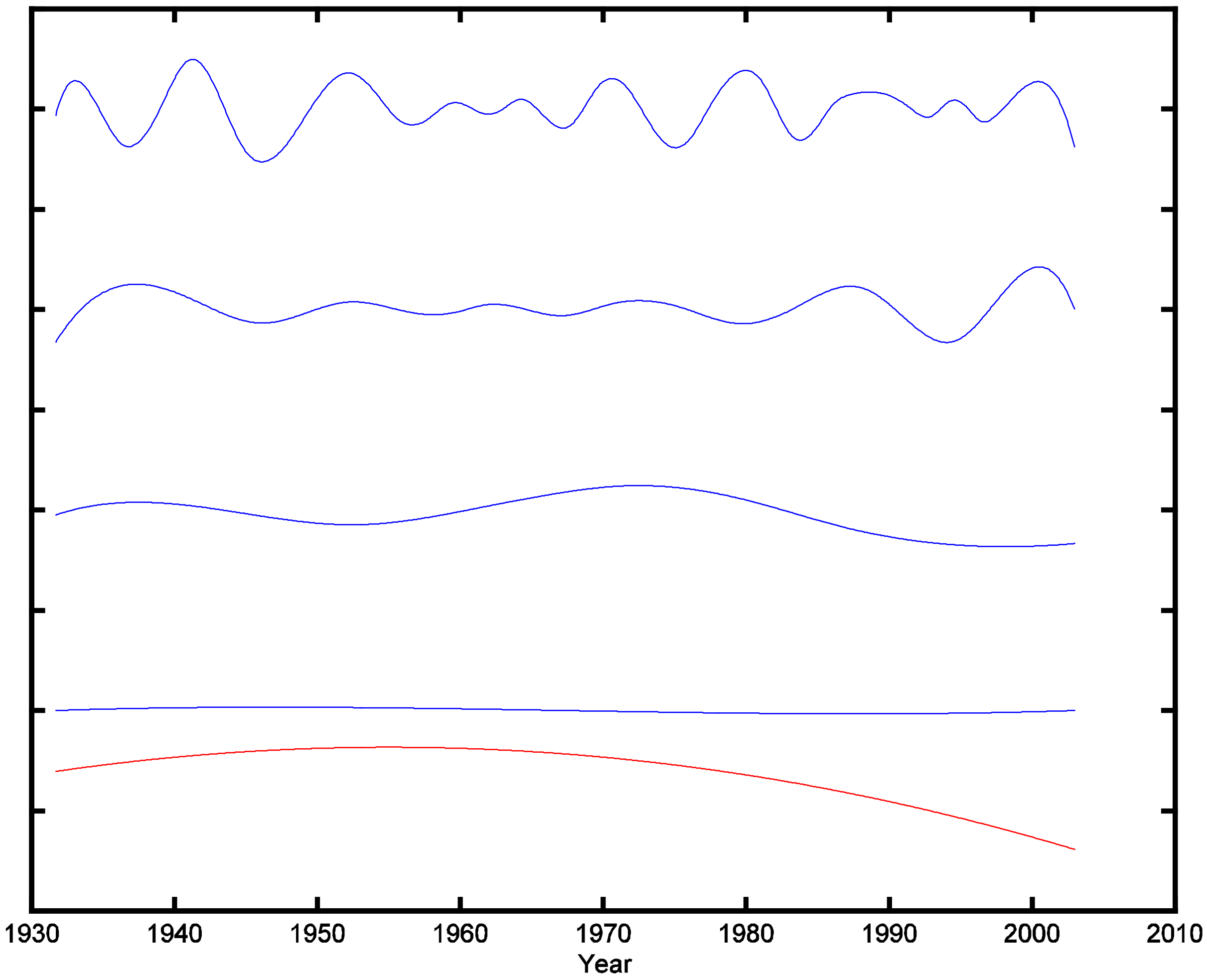
N.E.Huang, *Hilbert-Huang Transform and Its Applications, 2005*

Arosa TCO: EMD Decomposition IMFs 1-4



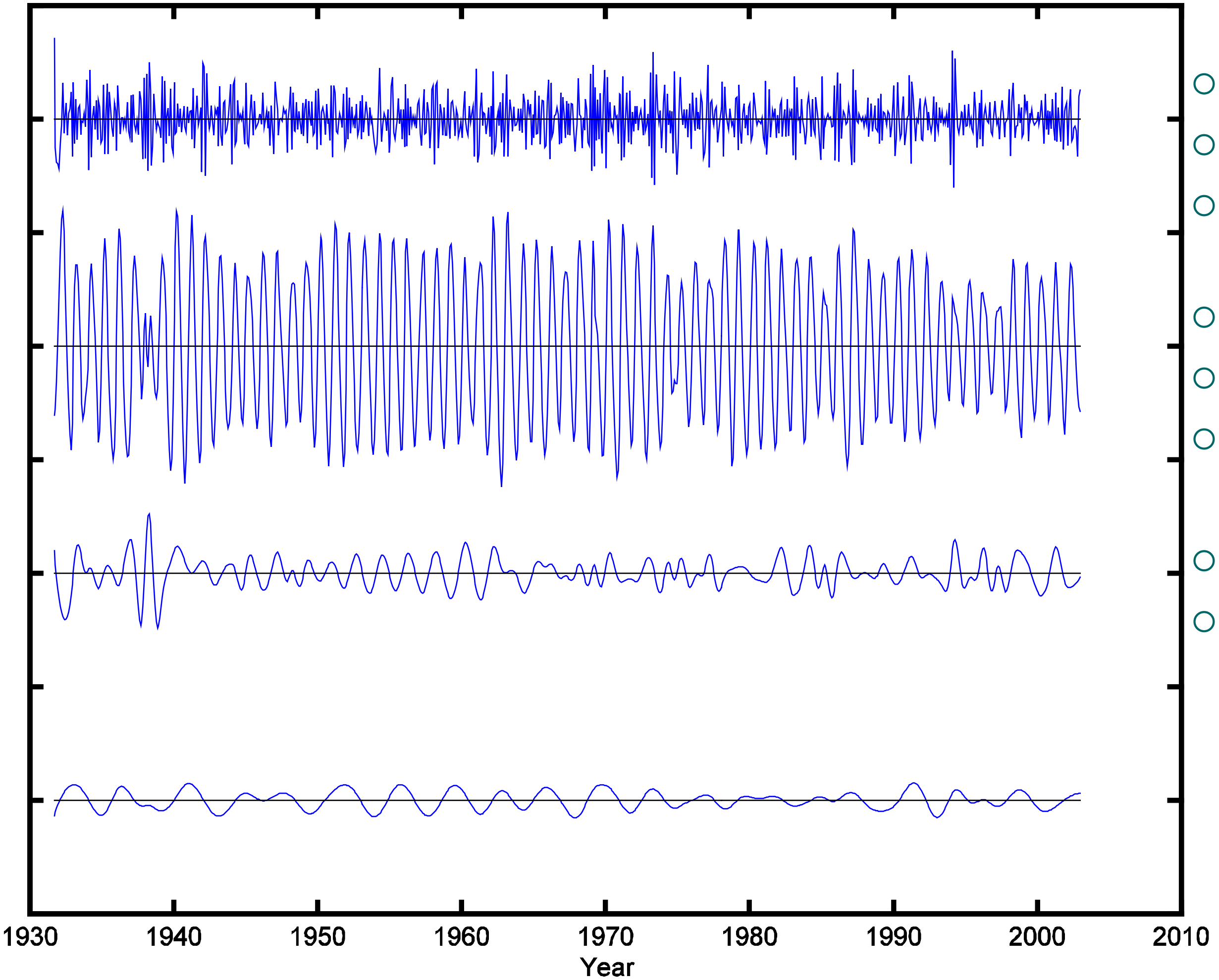
- Intra-annual
- Annual
- 20-30 month
- ~3.5 year
- Mode-mixing

Arosa TCO: EMD Decomposition IMFs 5-8, Residual



- Decadal
- Multi-decadal
- Trend (?)
- Edge Effects

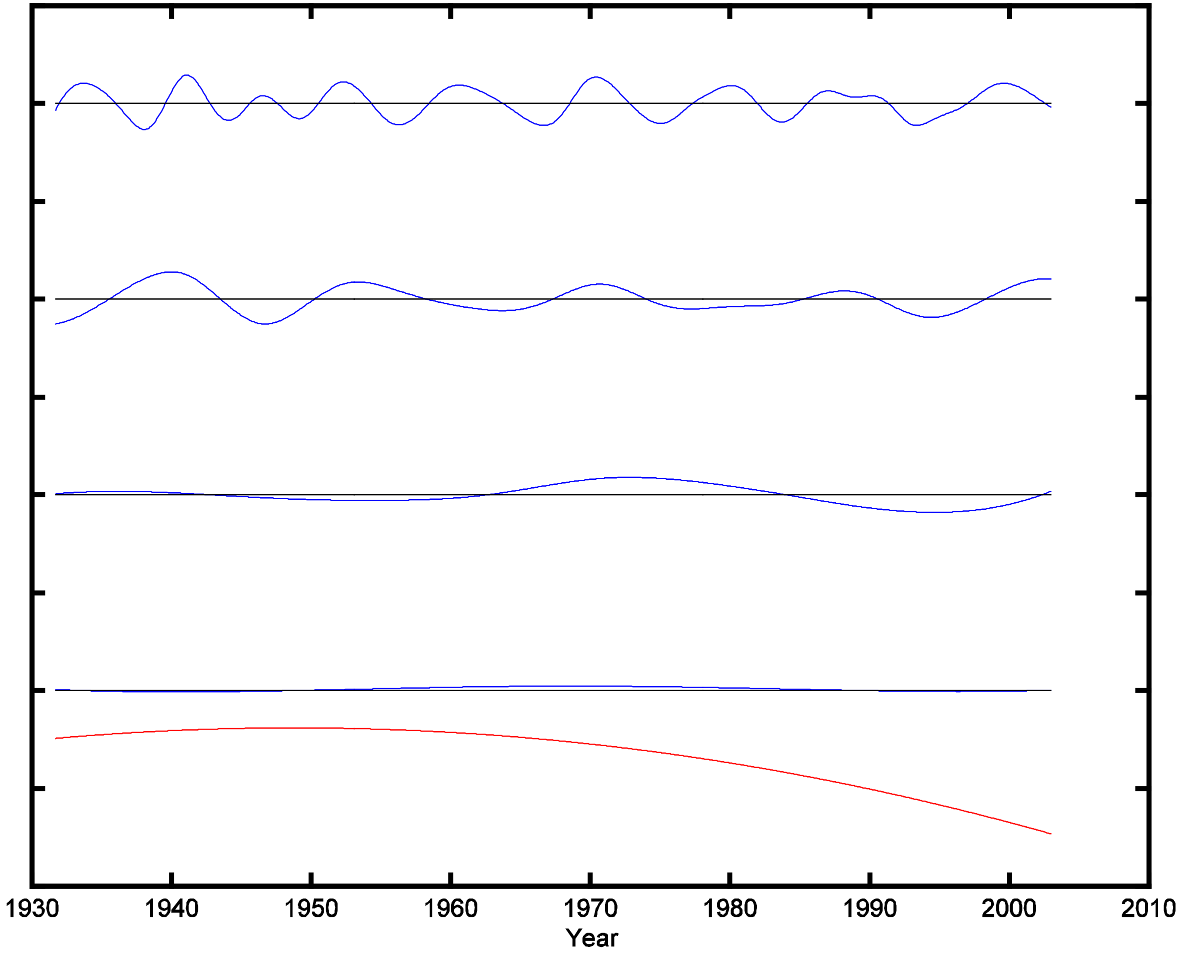
Arosa TCO: Ensemble EMD (EEMD) IMFs 1-4



- Add random noise
- Take EMD
- Repeat many times: ensemble of IMFs
- Average each IMF
- Added noise cancels
- Not a decomposition.

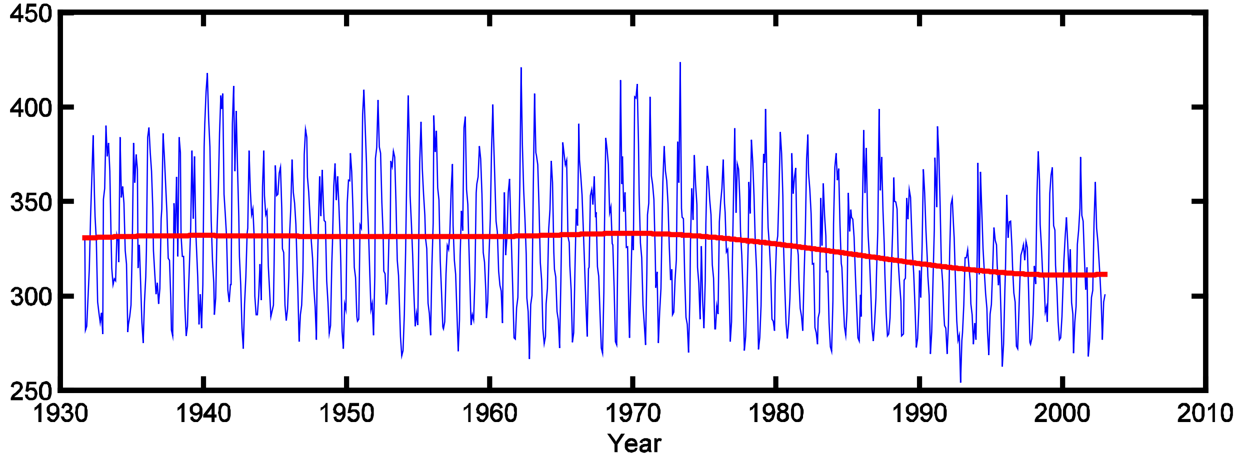
- Reduced mode mixing
- More coherent signals

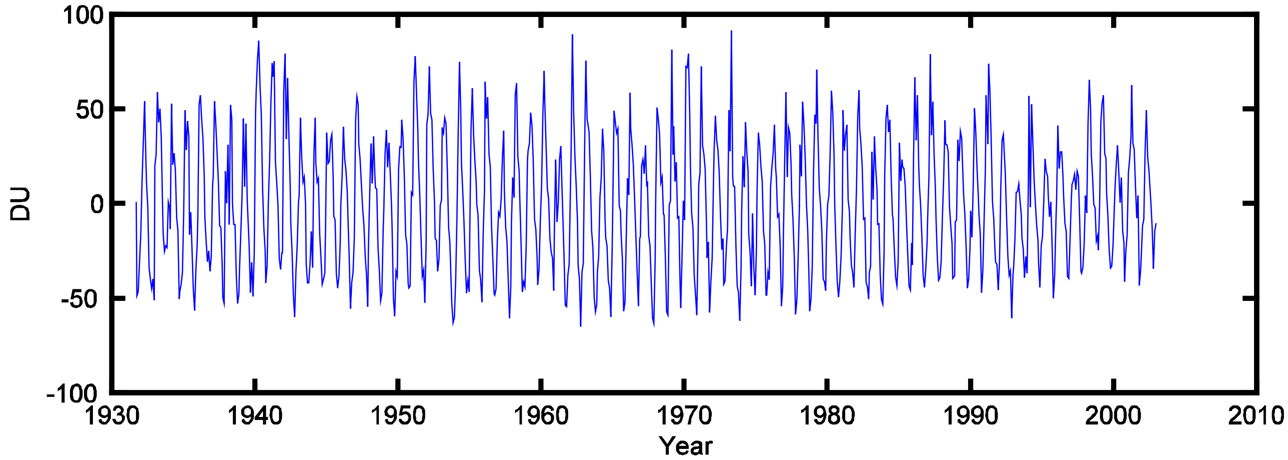
Arosa TCO: Ensemble EMD (EEMD) IMFs 5-8, Residual



- Somewhat reduced edge effect

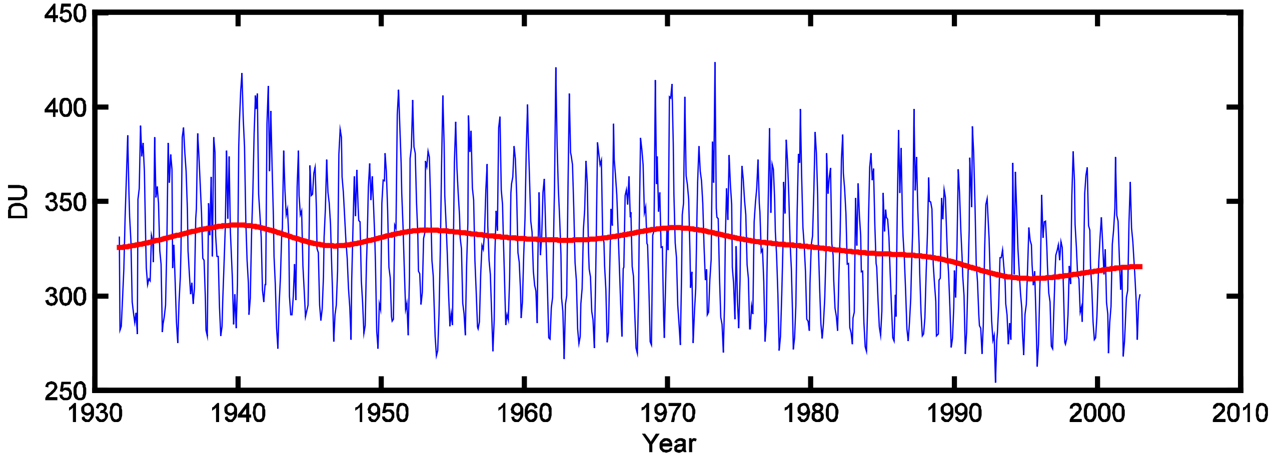
Arosa TCO: Detrending Trend: Residual + IMFs 6 & 7



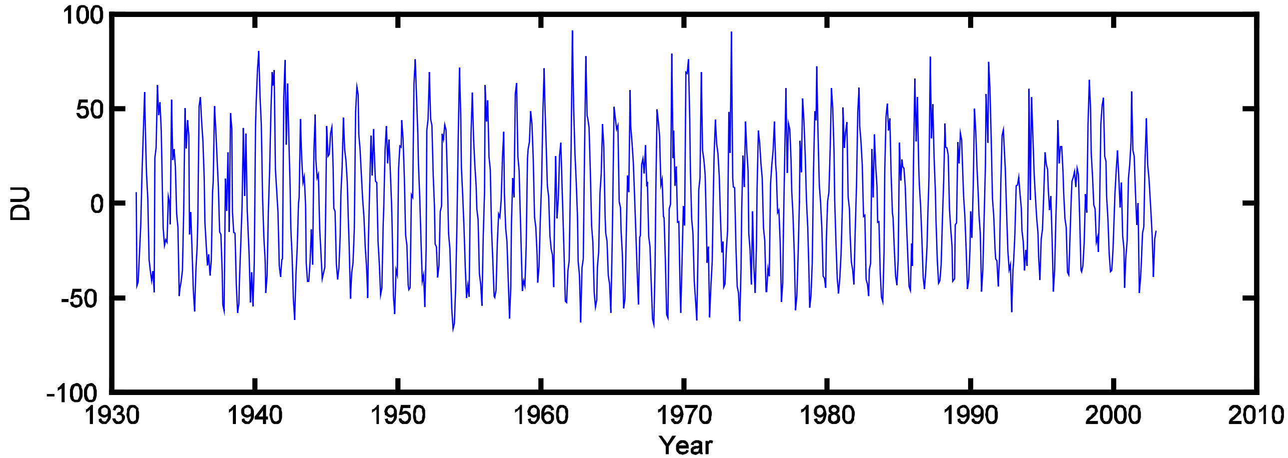


- **Data-adaptive trend**

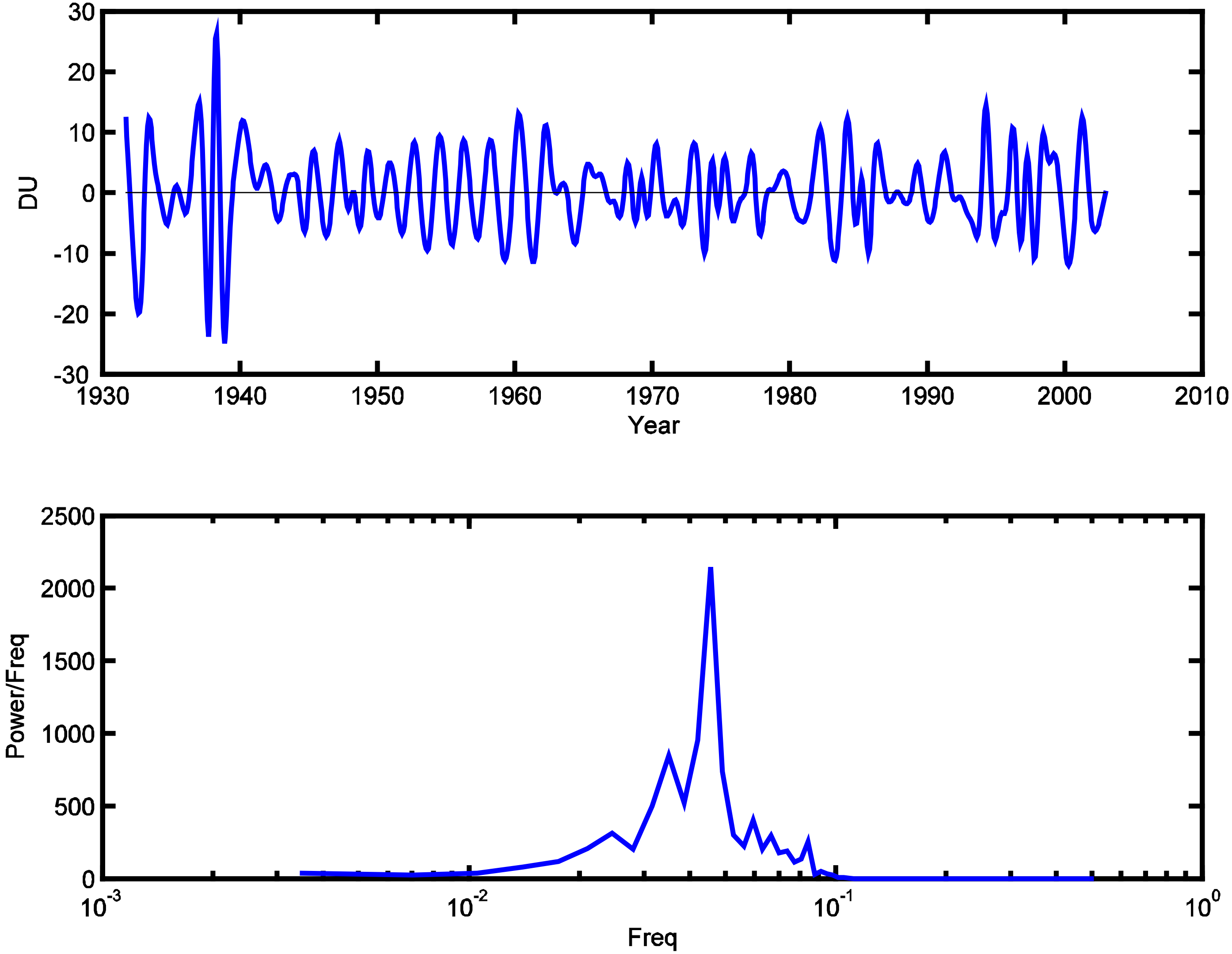
Arosa TCO: Detrending Trend: Residual + IMFs 5,6 & 7



- **Data-adaptive trend**



Arosa TCO: IMF 3



- **QBO**
- **20 mo.**

A Limitation of Univariate Analysis

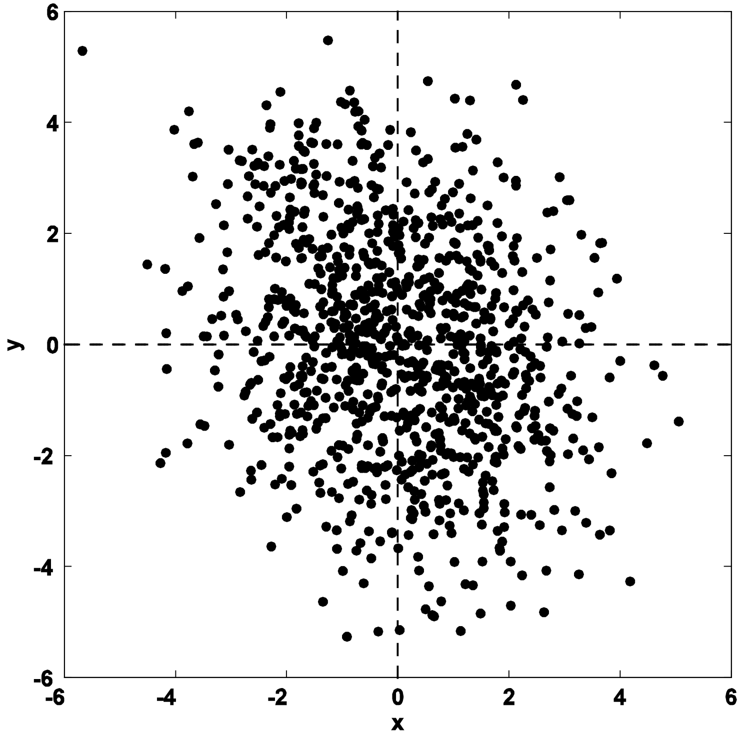
- A single time series may not have enough information to robustly establish (statistically) the existence of the underlying processes.
- Many records are multivariate, e.g. Surface temperature records on a lat-lon grid. We can use the full information to better isolate fundamentals patterns of variability.

Some Properties of Multivariate Data

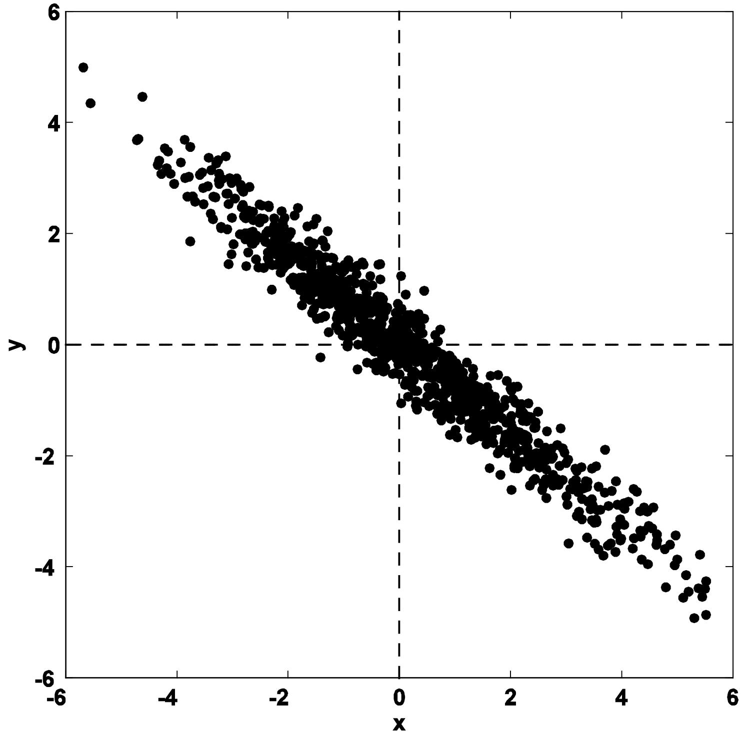
- Multiple time series, many measurements taken at the same set of times
- Often high correlations between time series
- Goals:
  - Isolate underlying processes
  - Data reduction: find a smaller set of variables which still capture behavior.

2 variable example (synthetic data)

Weakly Correlated Variables



Strongly Correlated Variables



Principal Component Analysis

- Rotation into a new orthogonal basis
- New basis is ordered: 1^{st} basis vector captures more of the data's temporal variance than other vector.

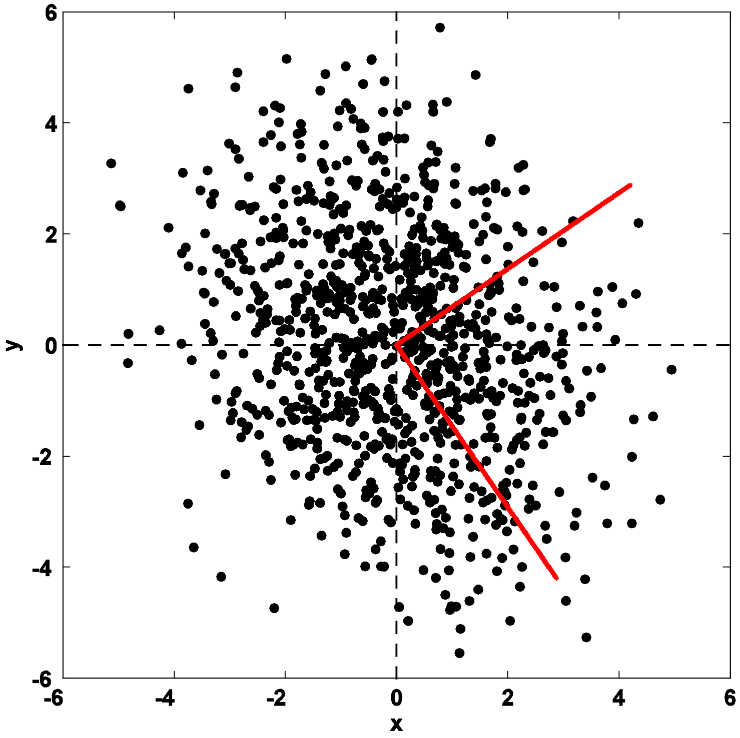
- PCA (as applied here): decomposes a multivariate time series into:
  - Orthogonal spatial patterns

    aka. Empirical Orthogonal Functions (EOF)
  - Associated time-varying amplitudes

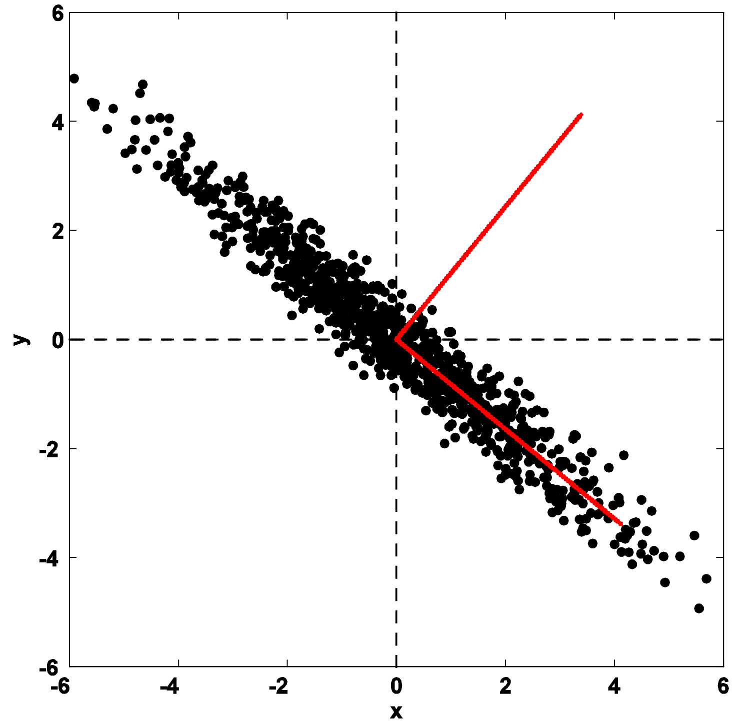
    aka. Principal Component time series (PCs)
- EOFs are sorted by order of the temporal variance captured by the oscillation of that pattern
- Analysis focuses on the spatially coherent patterns with the largest temporal variance.

PCA: 2 variable example

Weakly Correlated Variables



Strongly Correlated Variables



PCA algorithm

Given a data set (with centered columns)

$$D\left(t,x\right)=\left[\cdots\ \mathbf{d}_k\left(t\right)\ \cdots\right]\in\mathbb{R}^{m\times n}\qquad k=1,\ldots,n$$

consisting of n time series, each of length m;
define $r\equiv\operatorname{rank}\left(D\right)\le\min\left(m,n\right)$.

The covariance matrix of D is:

$$C=\frac{1}{m-1}D^TD=Q\Lambda Q^T\in\mathbb{R}^{n\times n}$$

where Λ is a diagonal matrix containing the sorted eigenvalues

$$\Lambda=diag(\lambda_k)\in\mathbb{R}^{n\times n}\qquad k=1,\ldots,n$$
$$\lambda_1\ge\lambda_2\ge\cdots\ge\lambda_r>\lambda_{r+1}=\cdots=\lambda_n=0$$

PCA algorithm, cont.

The EOF's are then defined by the columns of the orthogonal matrix Q, *i.e.* the sorted eigenvectors,

$$Q = [\cdots \; \mathbf{e}_k \; \cdots] \in \mathbb{R}^{n \times n} \qquad k = 1, \ldots, n$$
$$Q^T Q = I_n.$$

and the PC time series are defined by projection of the data onto the EOFs, *i.e.* the columns of

$$A = [\cdots \; \mathbf{a}_k \; \cdots] = D\,[\cdots \; \mathbf{e}_k \; \cdots] = DQ \in \mathbb{R}^{m \times n} \qquad k = 1, \ldots, n.$$

PCA algorithm, cont.

The data can be reconstructed

$$D = AQ^T = \sum_{k=1}^{n} \mathbf{a}_k\left(t\right)\mathbf{e}_k^T\left(x\right) \in \mathbb{R}^{m\times n}$$

Note that the PC time series are also orthogonal

$$A^T A = \Lambda = diag\left(\lambda_k\right) \in \mathbb{R}^{n\times n}$$

and an arbitrary scaling can be applied, *e.g.*

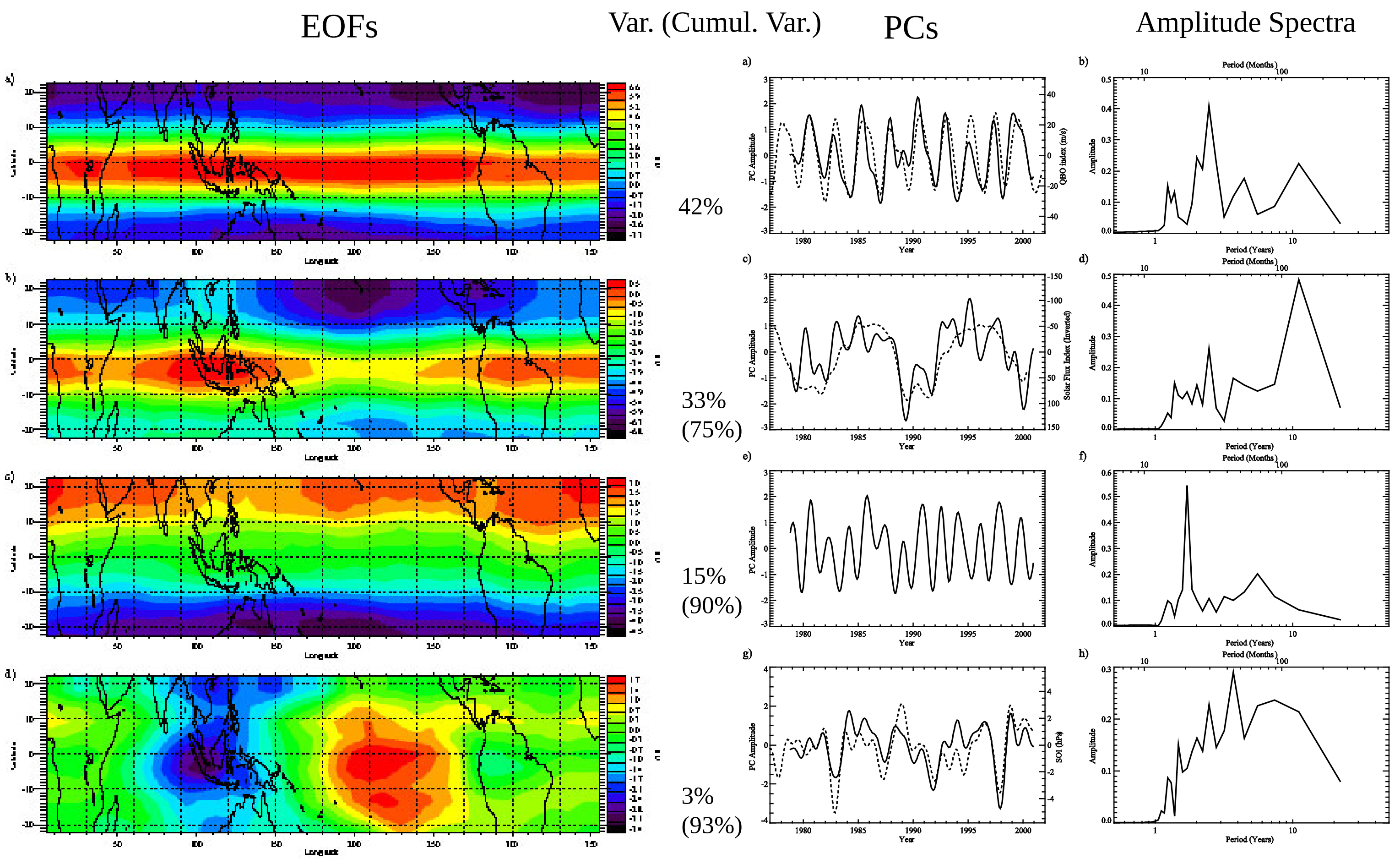
$$\tilde{\mathbf{e}}_k = \mathbf{e}_k\sqrt{\lambda_k} \qquad \tilde{\mathbf{a}}_k = \mathbf{a}_k/\sqrt{\lambda_k}$$

Here the EOFs have physical units
and the PC time series have unity standard deviations.

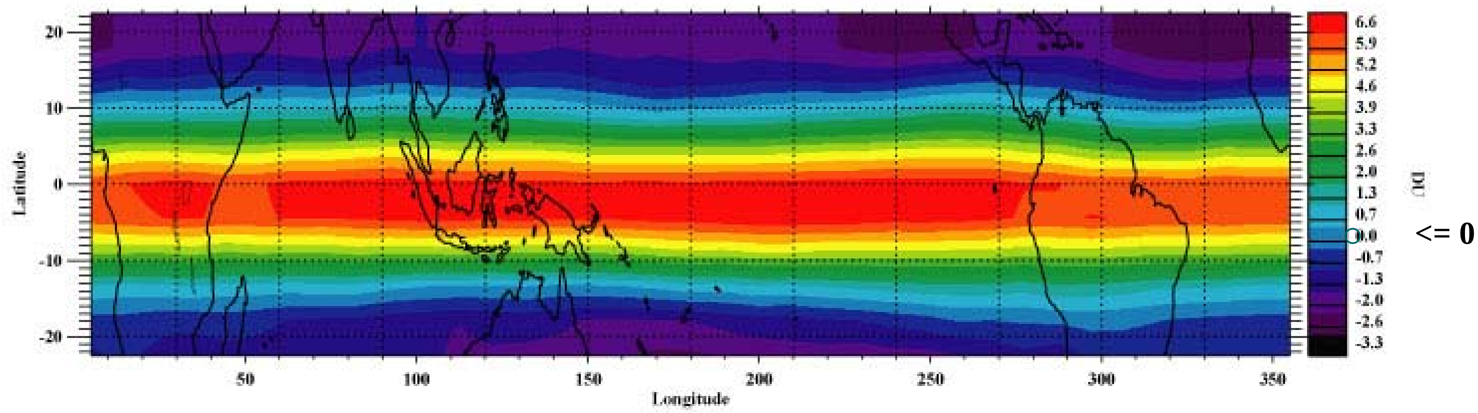
Features of PCA

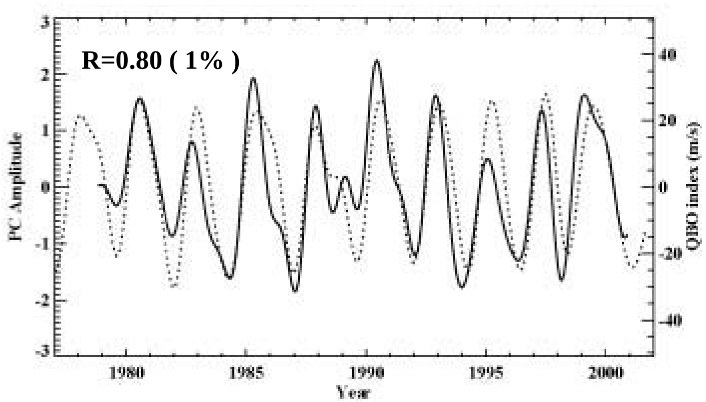
- EOFs must be mutually orthogonal; but physical modes need not be.
- Leads to mode-mixing, particularly in higher EOFs
- Decomposition is linear.
- PCA does not utilize *ordered* data (time), the data is considered to be a unordered set of observations; so the temporal order can be used to provide further analysis after performing the PCA.
- PCA are often calculated by an SVD of the centered data: EOFs (PCs) are proportional to the right (left) eigenvectors sorted by decreasing singular value.

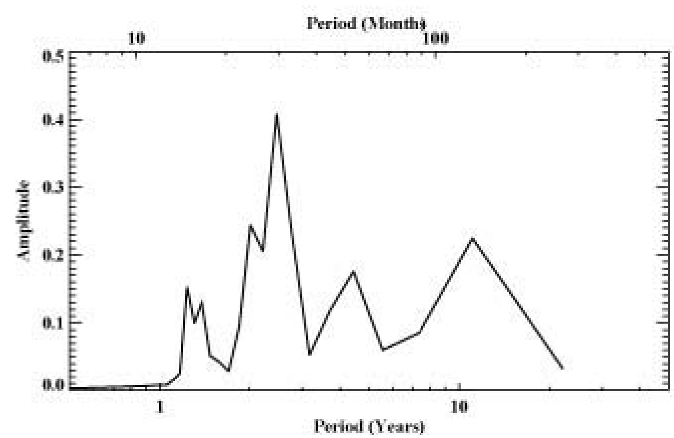
MOD EOF patterns and PC time series



MOD EOF 1: QBO (and Decadal)

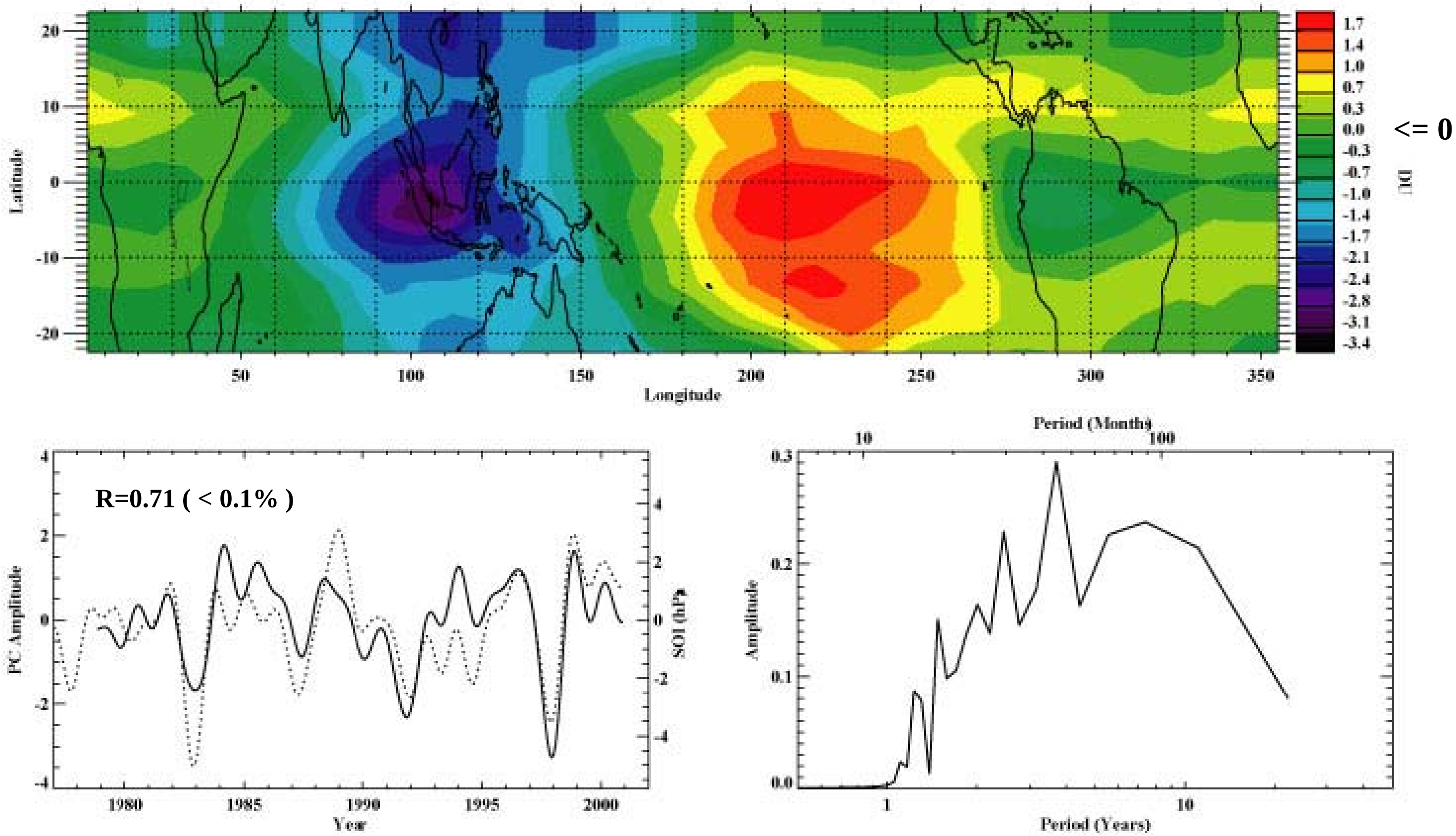






Captures 42% of the interannual variance.

EOF 4: ENSO



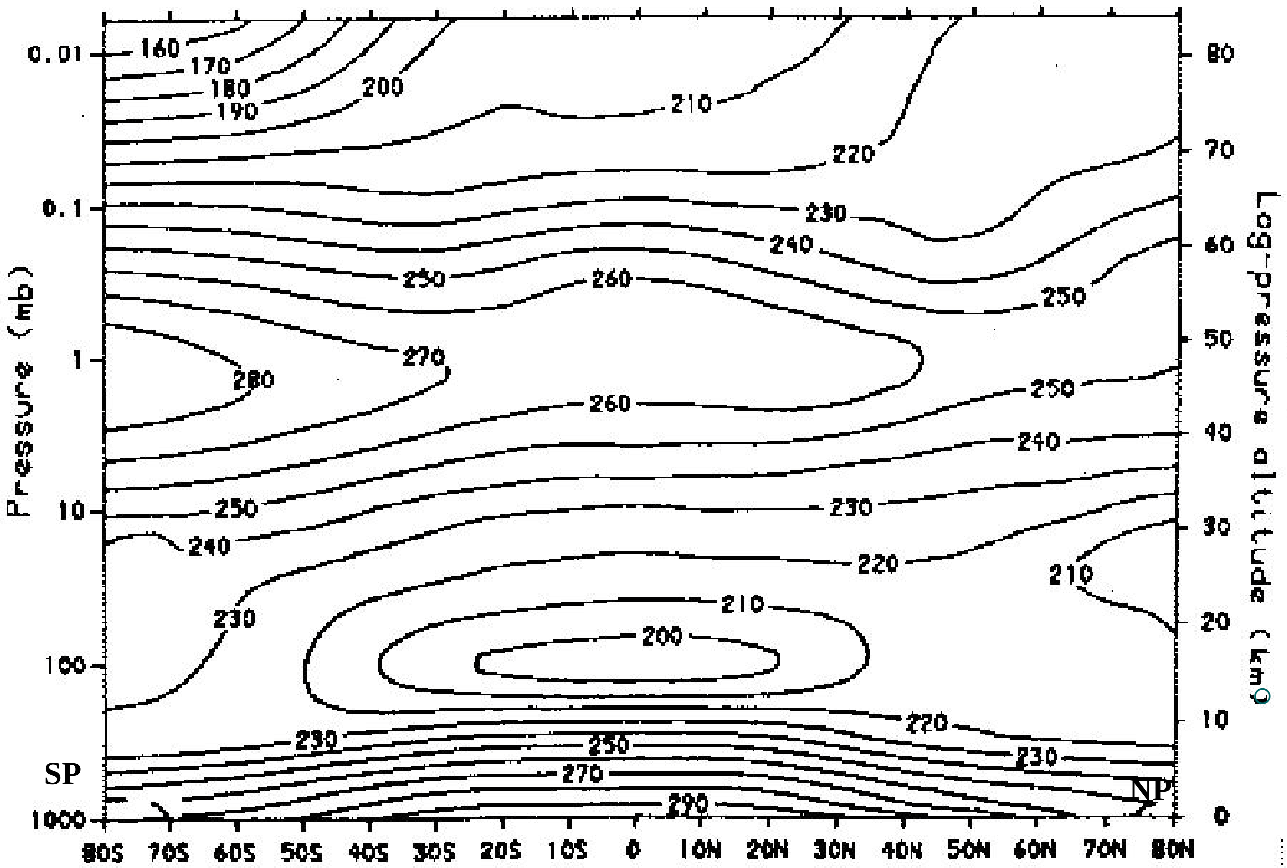
Captures 3% of the interannual variance.

PCA revisited

- Eigenvectors (spatial patterns) can be thought of as empirically derived normal modes.
- PC time series are the time-varying amplitudes of each mode: like a drum head.
- Truncated PCA is a good way to reduce the data set to subspace – least variability lost.
- Recent work developing nonlinear PCA techniques (e.g., Monahan & Fyfe – Neural Networks to find a lower dimensional manifold)

Extra Slides

Zonal Mean Temperature: Northern Hemisphere Winter



(a) Zonal mean temperature (K) January

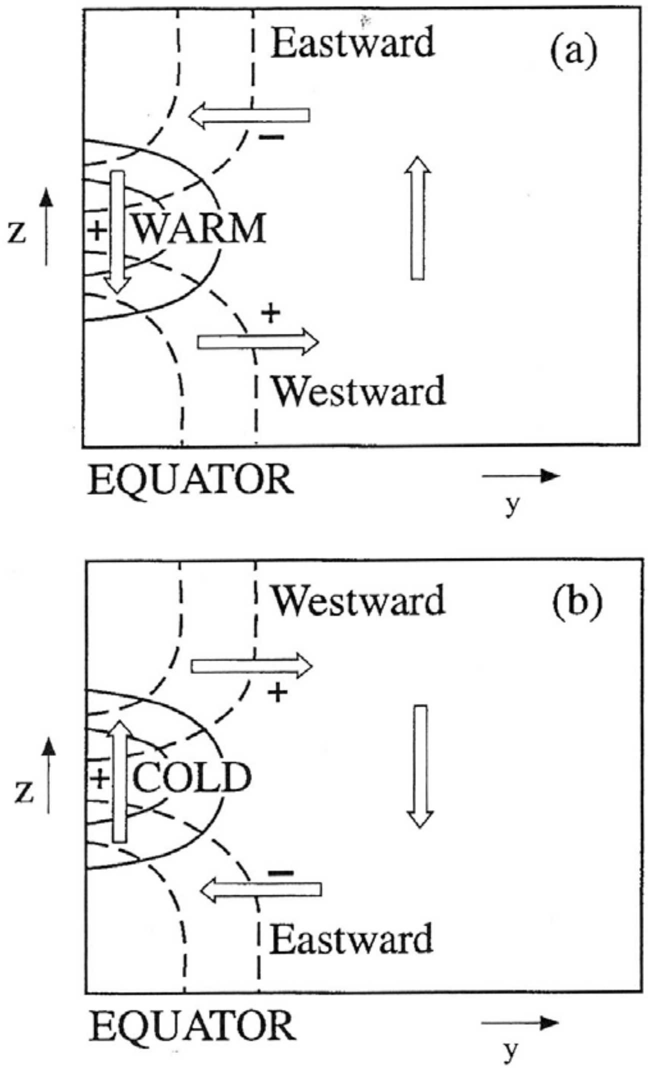
Mesosphere

Stratosphere

Troposphere

Andrews, Holton , Leovy, 'Middle Atmosphere Dynamics', 1987

QBO: Direct Meridional Circulation

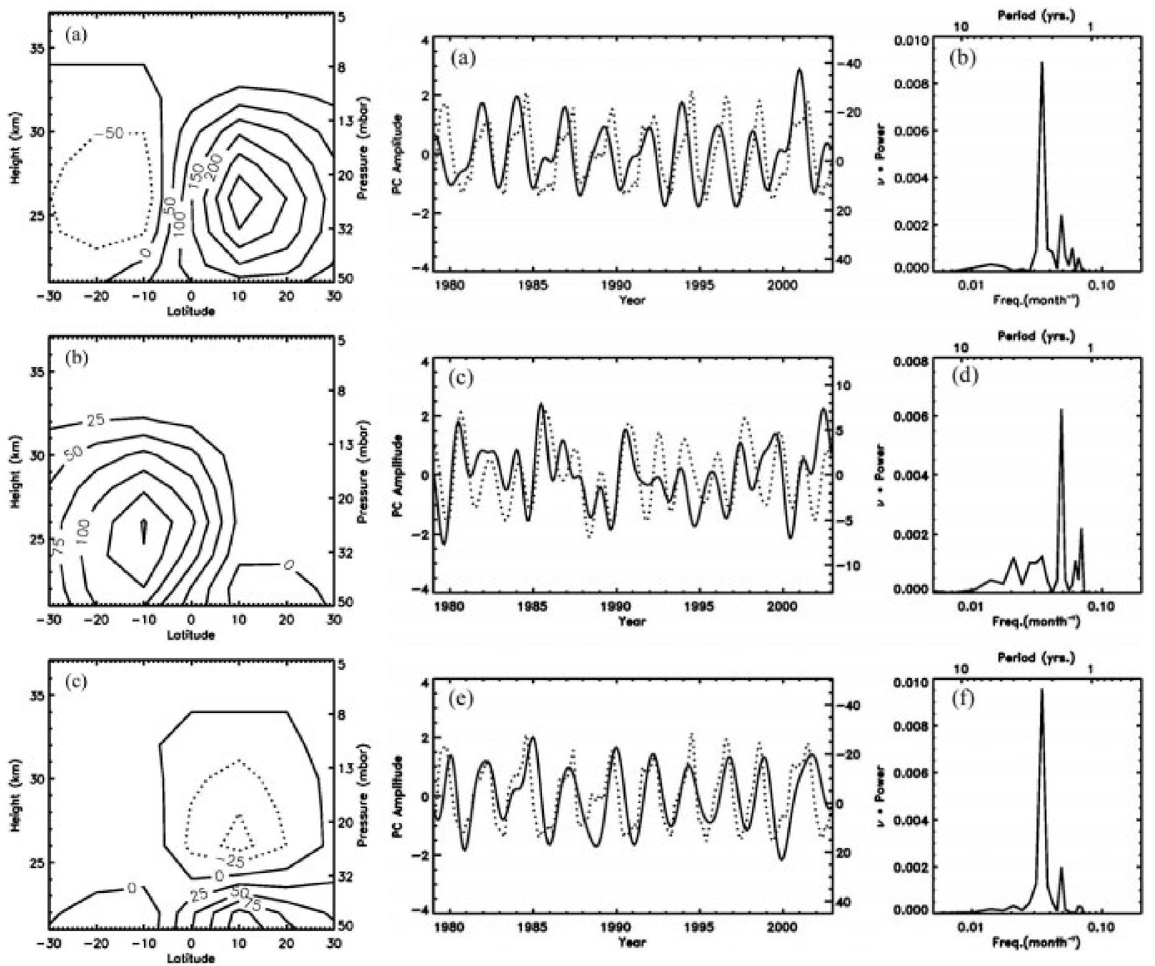


- Westerly shear zone associated with equatorial heating, requires compressional heating from downwelling
- Opposite in Easterly shear zone

Plumb, R.A. and R.C. Bell, QJRMS 108, 335-352, 1982

IAV of Isentropic Stream Function

EOFs **PCs** **Power Spectra** **Cumul. Variance**

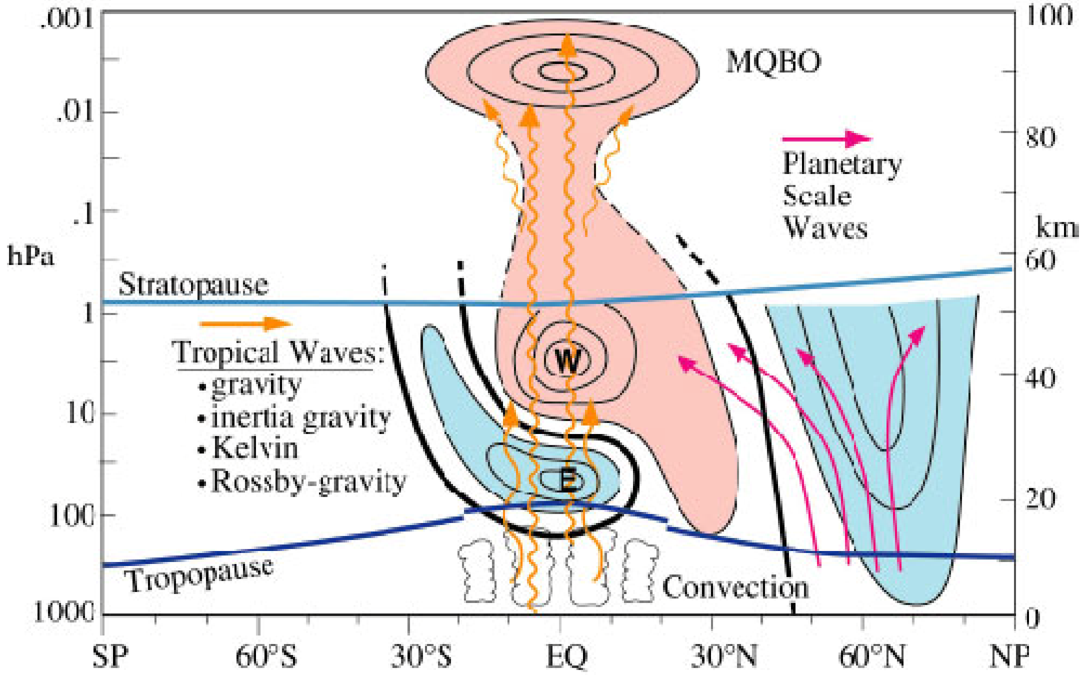


70%

88%

94%

Stratospheric Dynamics: Quasi-biennial Oscillation (QBO)



Baldwin, et. al., 'The Quasi-biennial Oscillation', Rev. Geo., 39, 179-229, 2001

- Oscillation in equatorial zonal wind with a mean period of 28 mo.
- Northern Winter
- Easterly phase QBO (40hPA)
- Positive (red) and negative (blue) zonal wind anomalies.

NCEP Geopotential Height: 30mb – 100mb Layer thickness