Data $\longrightarrow$ Models

NCAR Summer School, Mathematics & Climate
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University of Vermont

Lyapunov Exponents
Atmosphere
Solar System
Data Assimilation
Toy Climate Models
Model Error
Another Toy
Global Models

Outline

Lyapunov Exponents
- Atmosphere
- Solar System

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- Toy Climate
- Models

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Lorenz, Tellus, 1960

- "If we should observe a hurricane, we might ask ourselves, 'Why did this hurricane form?' If we could determine the exact initial conditions at an earlier time, and if we should feed these conditions, together with a program for integrating the exact equations, into an electronic computer, we should in due time receive a forecast from the computer, which would show the presence of a hurricane.

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Lorenz, Tellus, 1960

- "If we should observe a hurricane, we might ask ourselves, 'Why did this hurricane form?' If we could determine the exact initial conditions at an earlier time, and if we should feed these conditions, together with a program for integrating the exact equations, into an electronic computer, we should in due time receive a forecast from the computer, which would show the presence of a hurricane.
- We then might still be justified in asking why the hurricane formed. The answer that the physical laws required a hurricane to form from the given antecedent conditions might not satisfy us, since we were aware of that fact even before integrating the equations."

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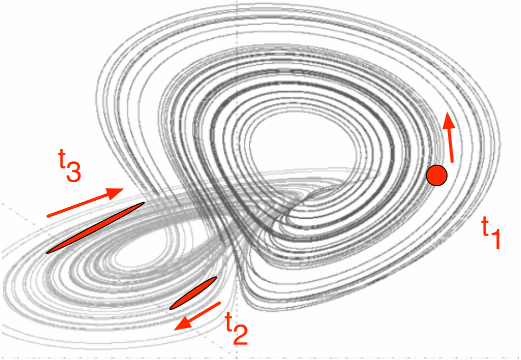
Lorenz (1963) System

$$\frac{dx}{dt} = \sigma(y - x)$$
$$\frac{dy}{dt} = \rho x - y - xz$$
$$\frac{dz}{dt} = xy - \beta z$$



Data → Models

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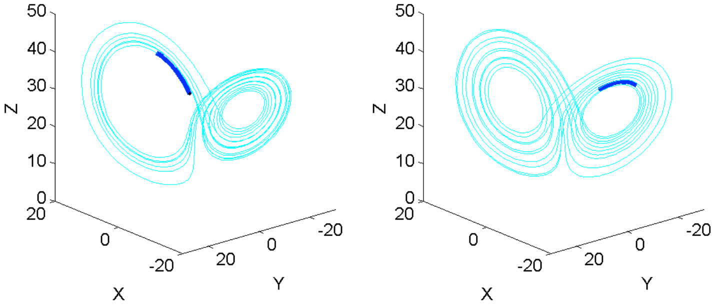
Lorenz (1963) System



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Lorenz (1963) System

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Patil et. al., PRL, 2001

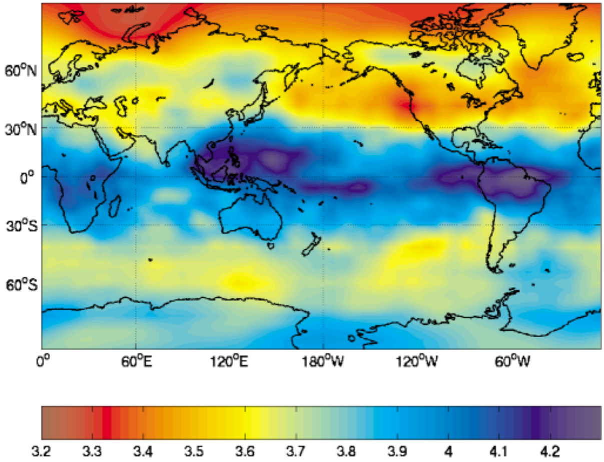

FIG. 3 (color). Average locations of regions with low BV dimensions are shown through the pointwise time average of the BV dimension calculated from ensemble forecasts every 12 h from 10 February 2000 to 30 July 2000. Red (blue) depicts regions in which the BV dimension tends to be low (high).

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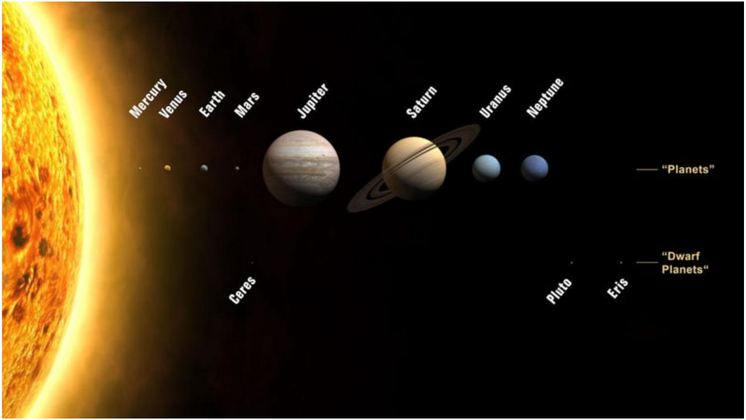
Question: Is our Solar System Chaotic?



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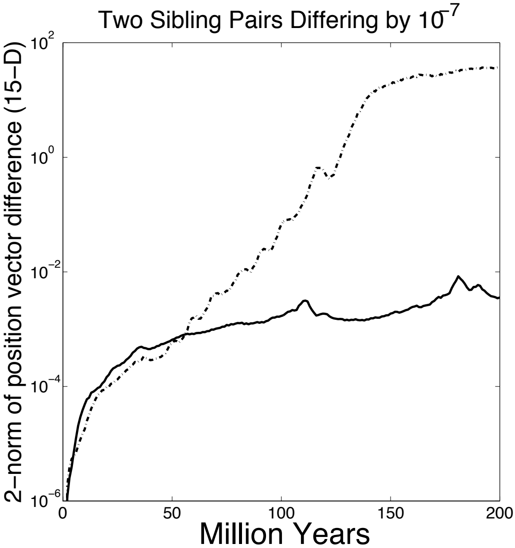
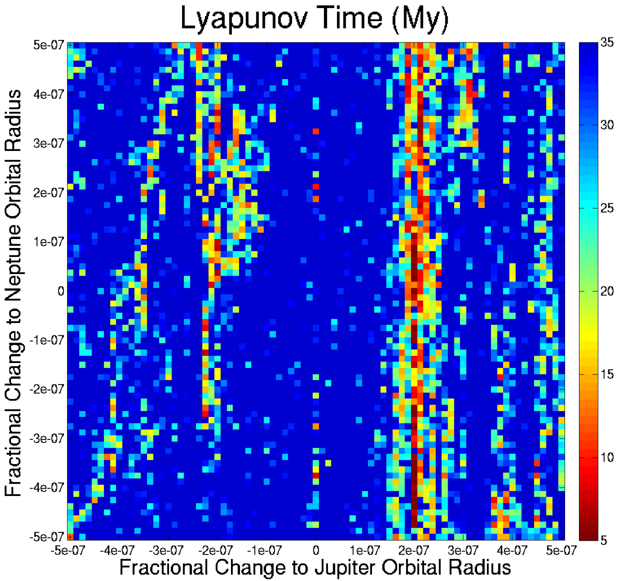
Answer: Chaos/Order Separatrix Passes Directly Through the Current Observational Error Ball



Stability of the Solar System

Hayes et. al. 2010 MNRAS in press



Stability of the Solar System

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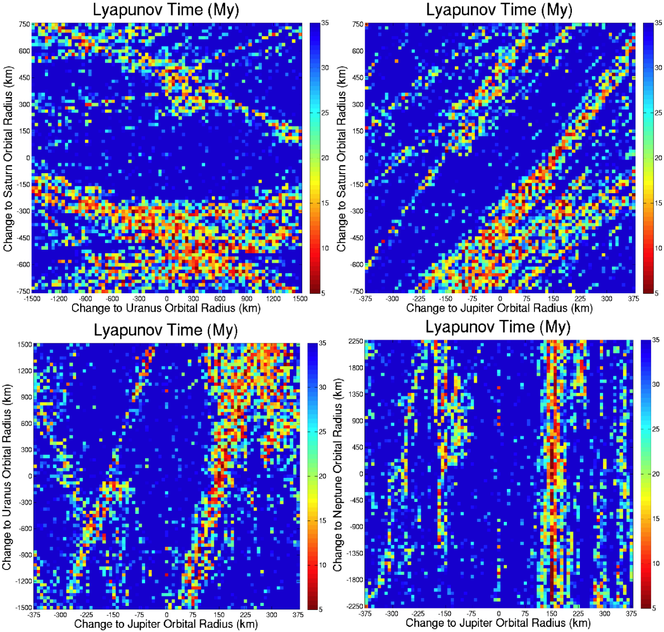
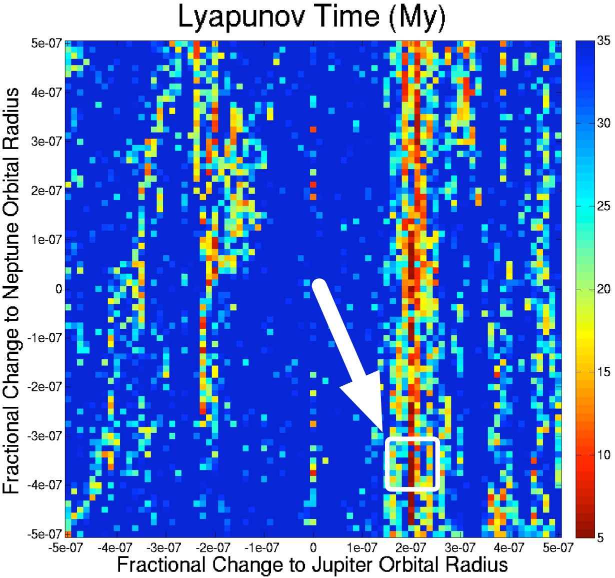


Fractal Basin Boundaries

Hayes et. al. 2010 MNRAS in press



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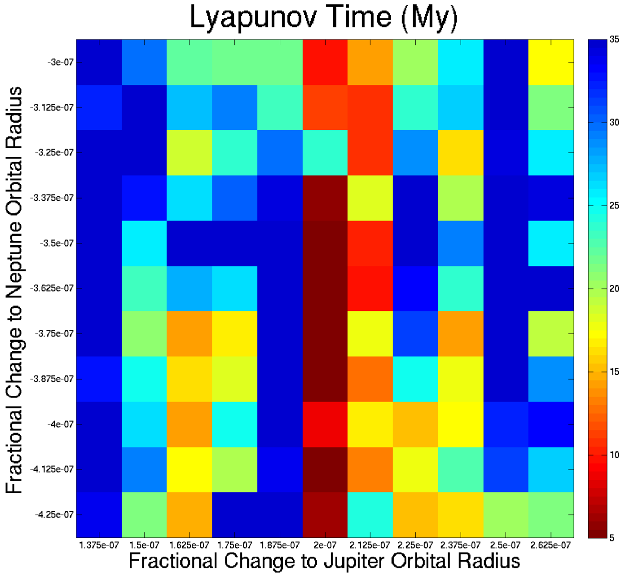
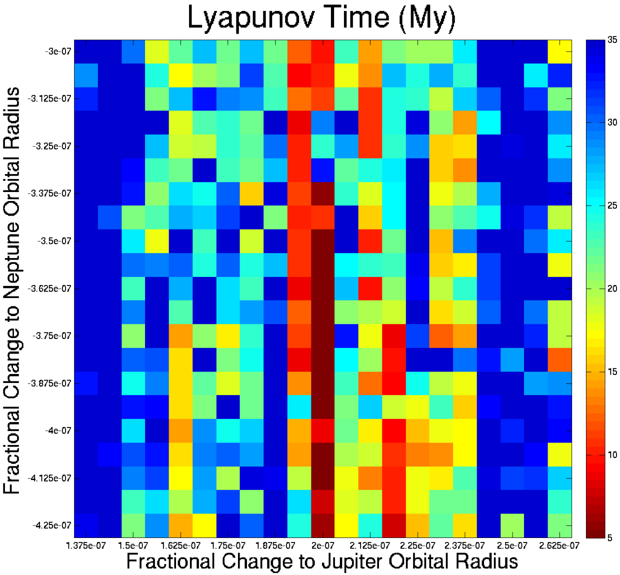


Fractal Basin Boundaries

Hayes et. al. 2010 MNRAS in press



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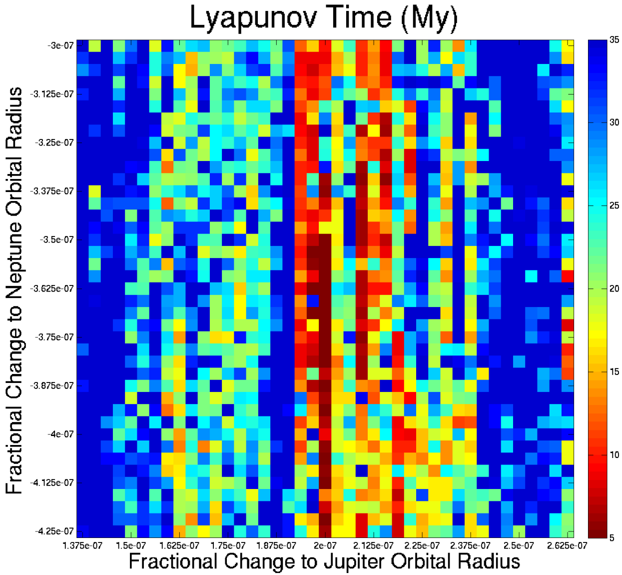

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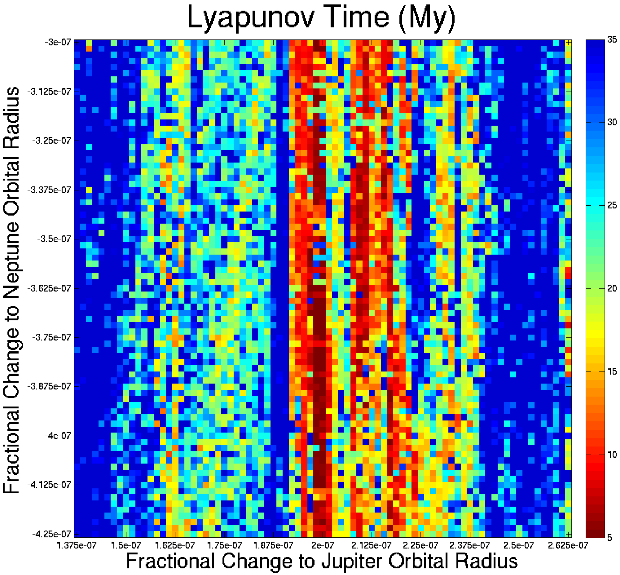

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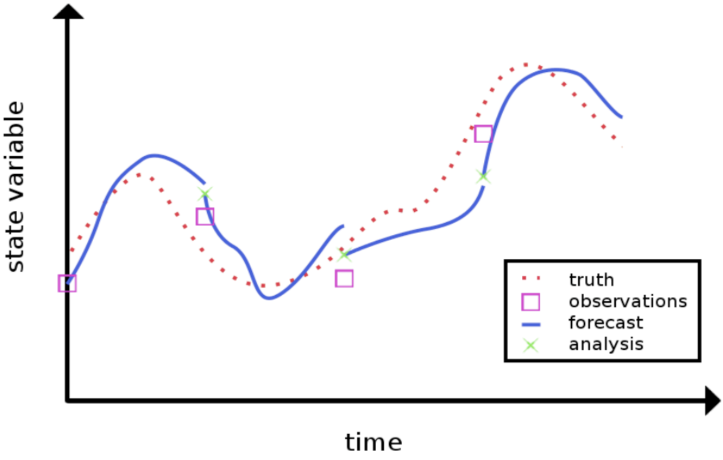
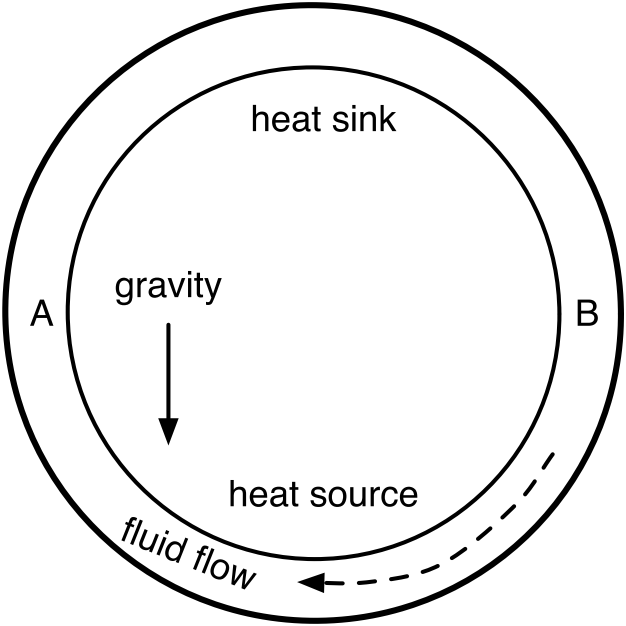


Image: Kameron Harris

A Toy Climate Experiment



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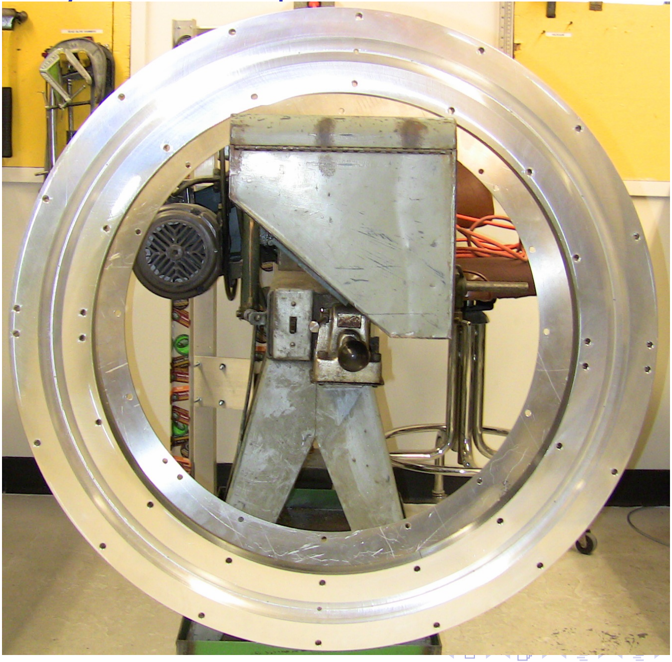

A Toy Climate Experiment

Credit: Glenn Russell

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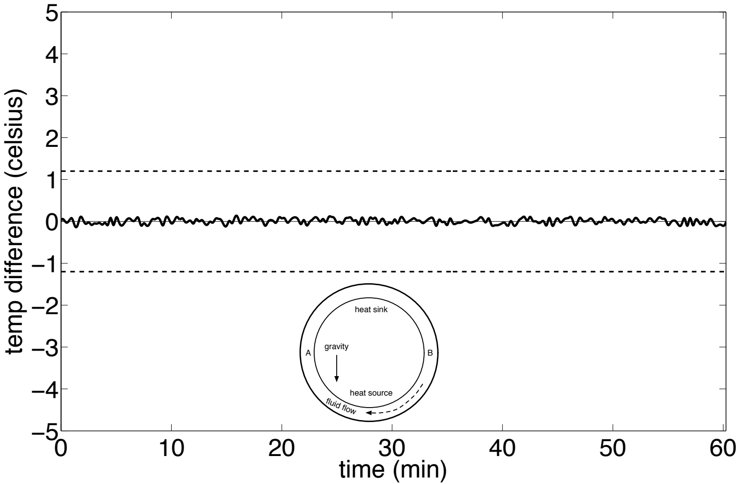
Typical Observations of Delta Temp (A-B)
Forcing: Small
Stable Conduction



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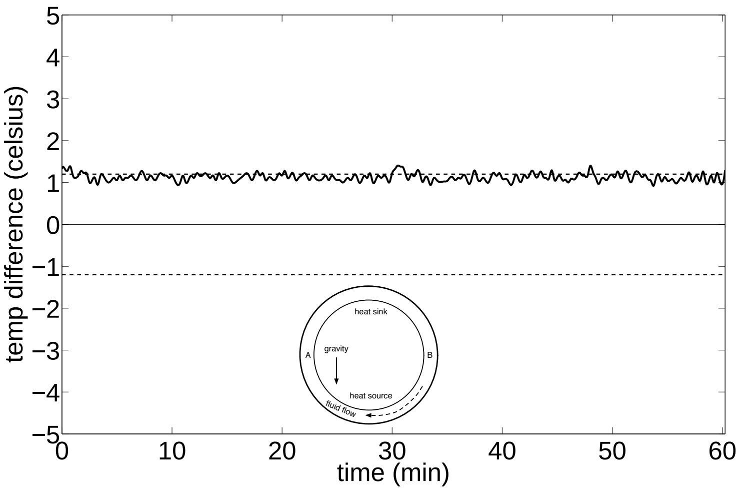
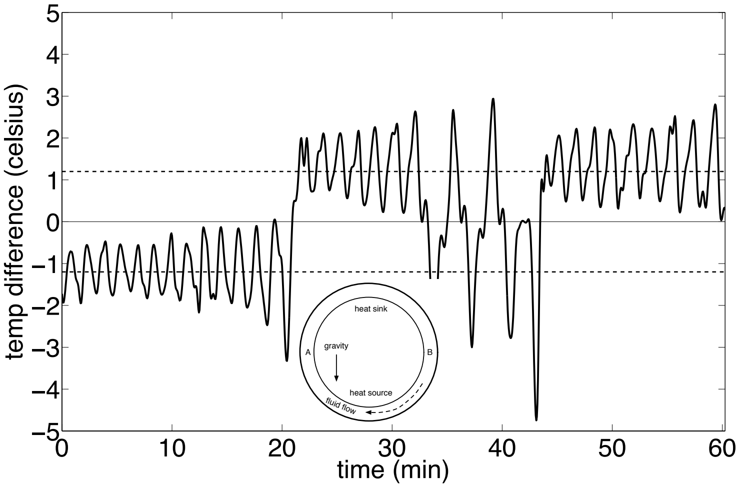
Typical Observations of Delta Temp (A-B)
Forcing: Medium
Stable Convection



Typical Observations of Delta Temp (A-B)
Forcing: Large
Chaotic Convection



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CFD Simulation

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Ridouane et. al. International Journal of Heat & Mass Transfer 2010

Data Assimilation

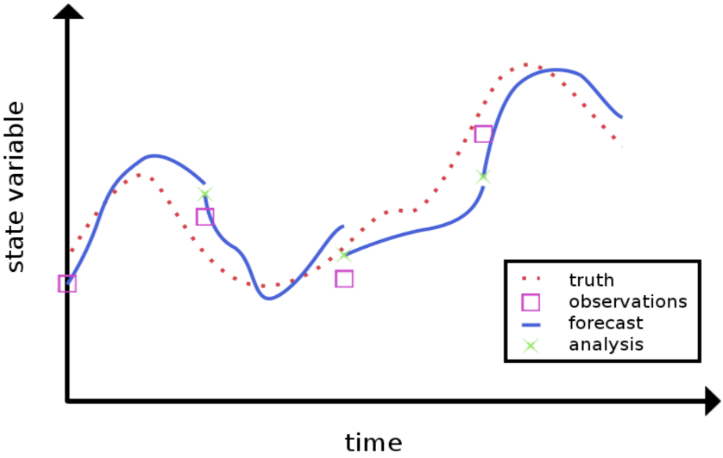
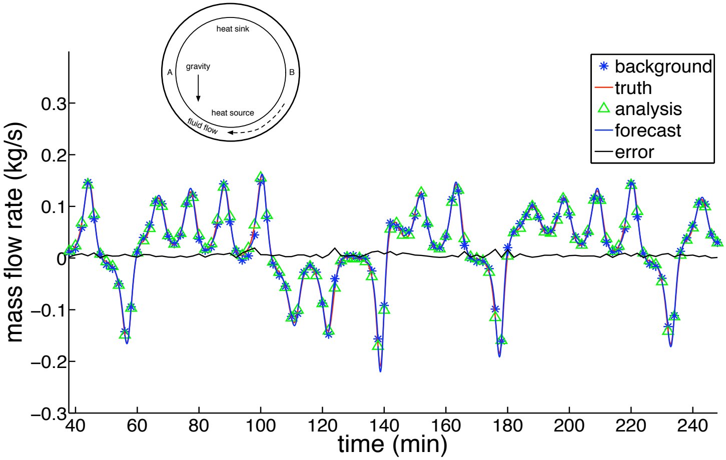


Image: Kameron Harris

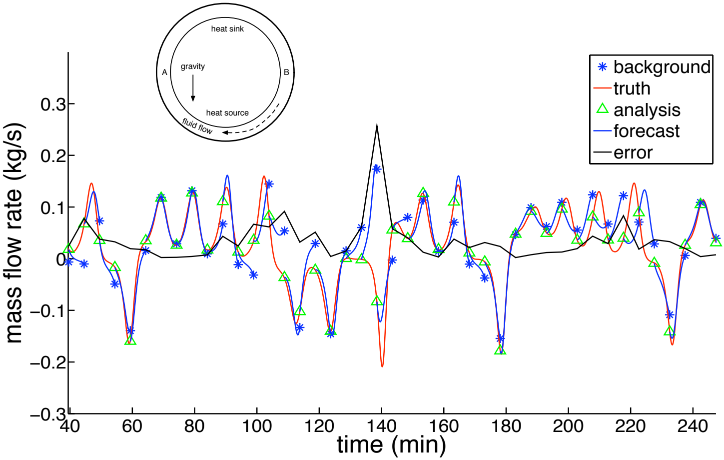
Data Assimilation



(a)assimilation window = 120 s

Image: Kameron Harris

Data Assimilation



(b)assimilation window = 300 s

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Toy Climate Storm

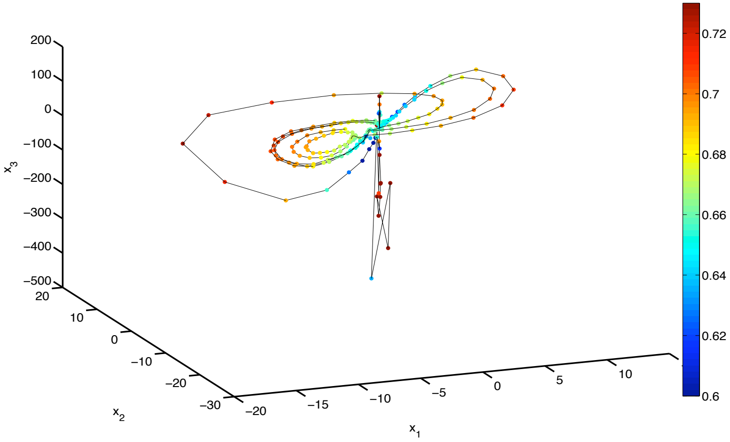
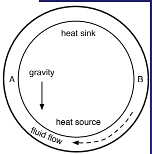


Image: Kameron Harris



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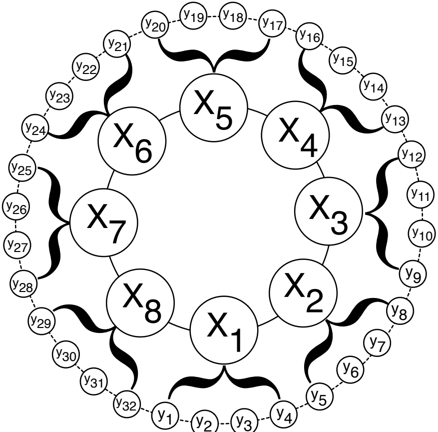
Lorenz and Emanuel, 1996

$$\frac{dx_i}{dt} = x_{i-1}(x_{i+1} - x_{i-2}) - x_i + F - \frac{hc}{b} \sum_{j=J(i-1)+1}^{iJ} y_j$$

$$\frac{dy_j}{dt} = -cby_{j+1}(y_{j+2} - y_{j-1}) - cy_j + \frac{hc}{b} x_{\mathrm{floor}[(j-1)/J]+1}$$

Schematic for $I = 8$ and $J = 4$:



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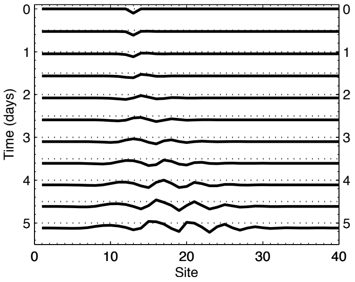
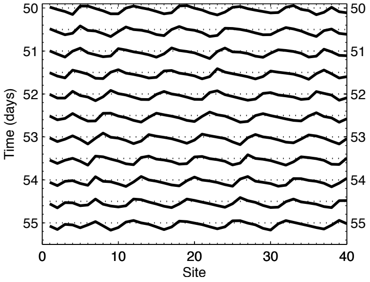
Dynamics

Credit: Ross Lieb-Lappen





Dynamics

- System

$$\frac{dx_i}{dt} = x_{i-1}(x_{i+1} - x_{i-2}) - x_i + F - \frac{hc}{b} \sum_{j=J(i-1)+1}^{iJ} y_j$$

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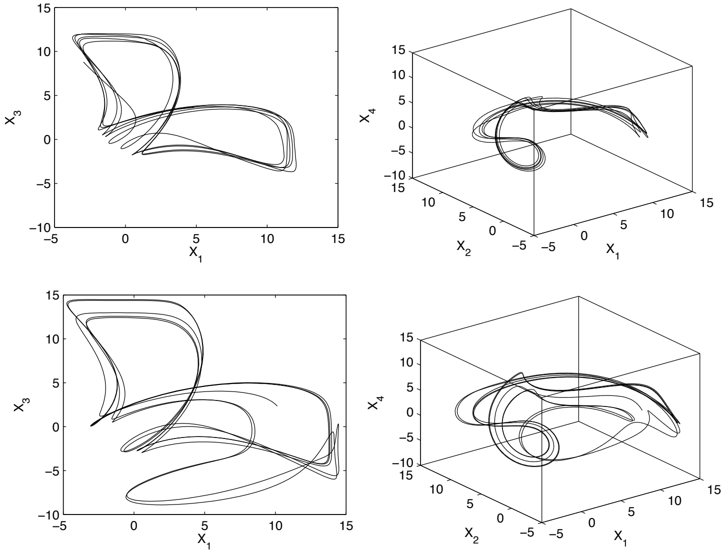
- Model

$$\frac{dx_i}{dt} = x_{i-1}(x_{i+1} - x_{i-2}) - x_i + F$$

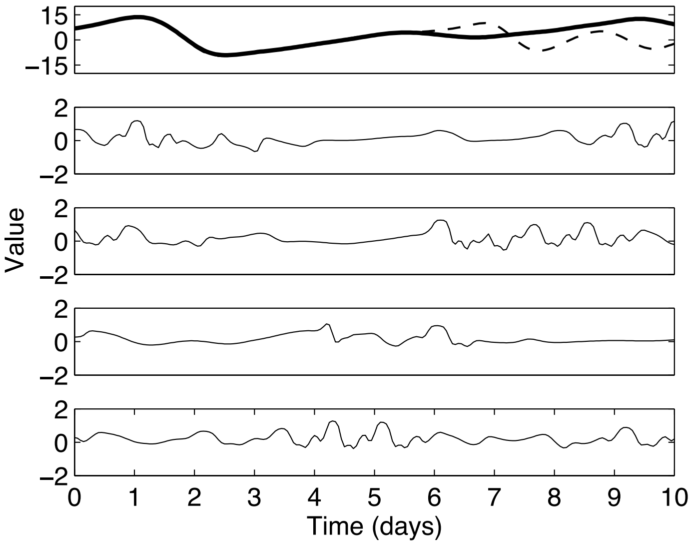
Model Dynamics

Credit: Ross Lieb-Lappen



Model Dynamics

Credit: Ross Lieb-Lappen



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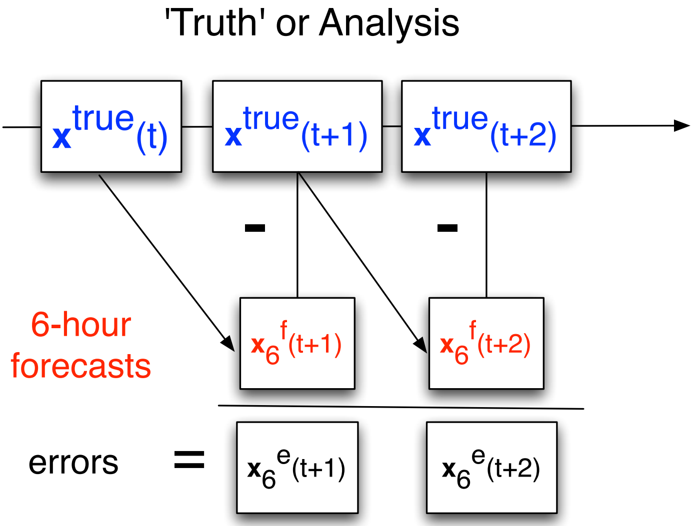
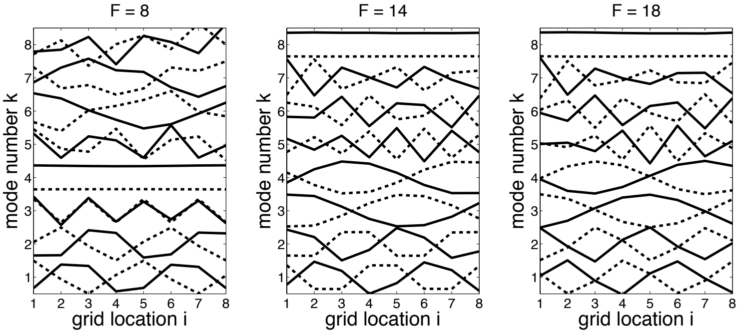
Model Error Estimation



Danforth and Kalnay, JAS, 2008

Model Error Estimation



Danforth and Kalnay, JAS, 2008

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$$\frac{dx_i}{dt} = x_{i-1}(x_{i+1} - x_{i-2}) - x_i + F - \frac{hc}{b} \sum_{j=J(i-1)+1}^{iJ} y_j$$

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- Model

$$\frac{dx_i}{dt} = x_{i-1}(x_{i+1} - x_{i-2}) - x_i + F$$

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$$\frac{dy_j}{dt} = -cby_{j+1}(y_{j+2} - y_{j-1}) - cy_j + \frac{hc}{b} x_{\mathrm{floor}[(j-1)/J]+1}$$

- Model

$$\frac{dx_i}{dt} = x_{i-1}(x_{i+1} - x_{i-2}) - x_i + F$$

- Model Error Correction

$$\frac{dx_i}{dt} = x_{i-1}(x_{i+1} - x_{i-2}) - x_i + F + G(\vec{x})$$

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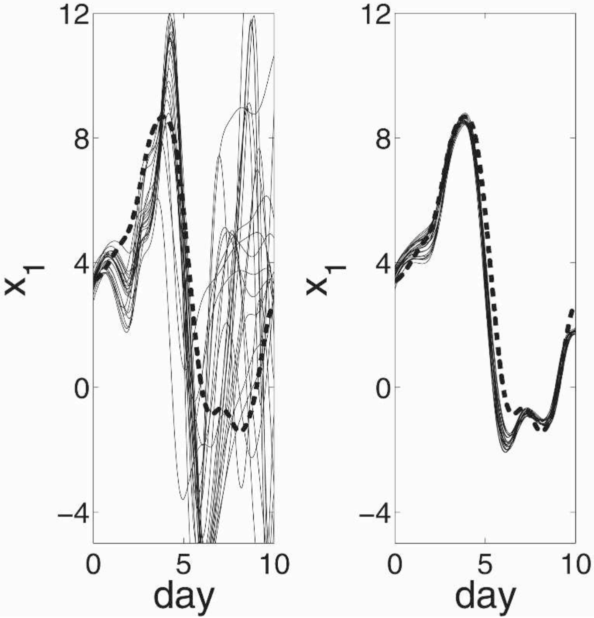
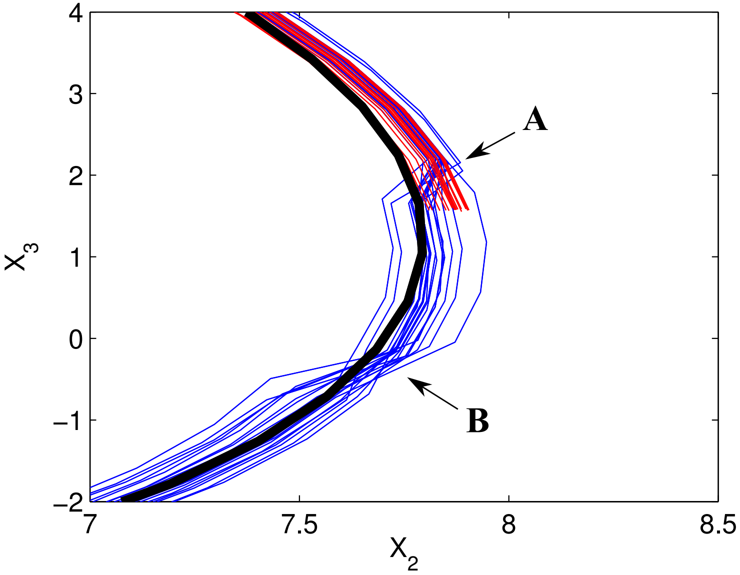
Model Error Correction



Danforth and Kalnay, JAS, 2008

Model Error Correction



Danforth and Kalnay, JAS, 2008

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National Center for Environmental Prediction Global Forecast Model

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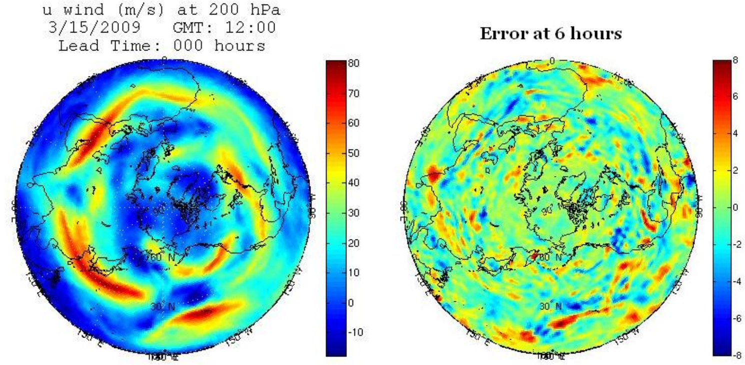
Credit: Nicholas Allgaier

Forecast Error



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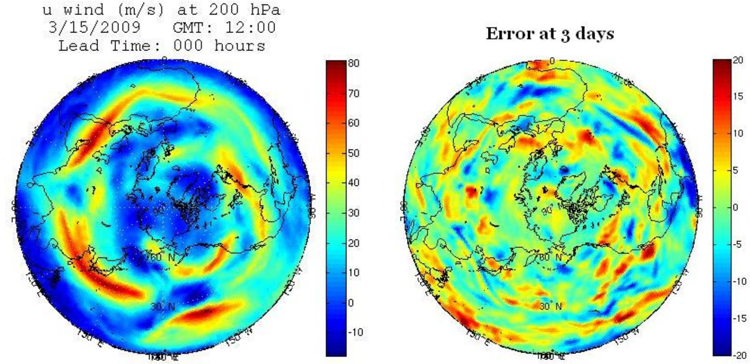

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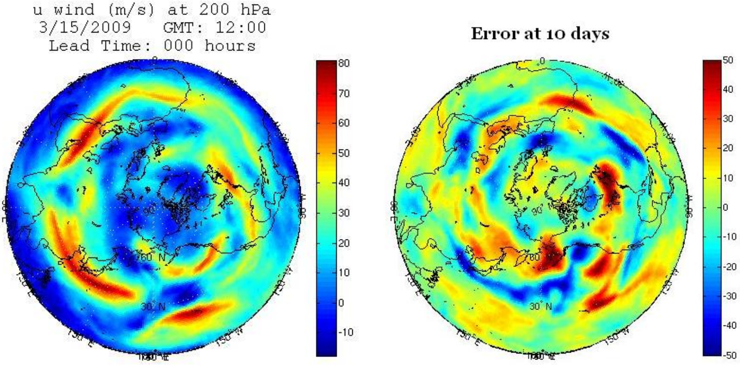
Forecast Error



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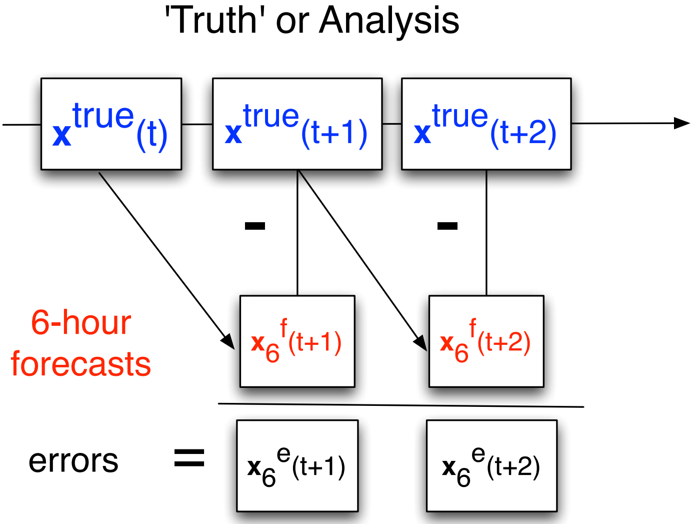
Estimating Model Error



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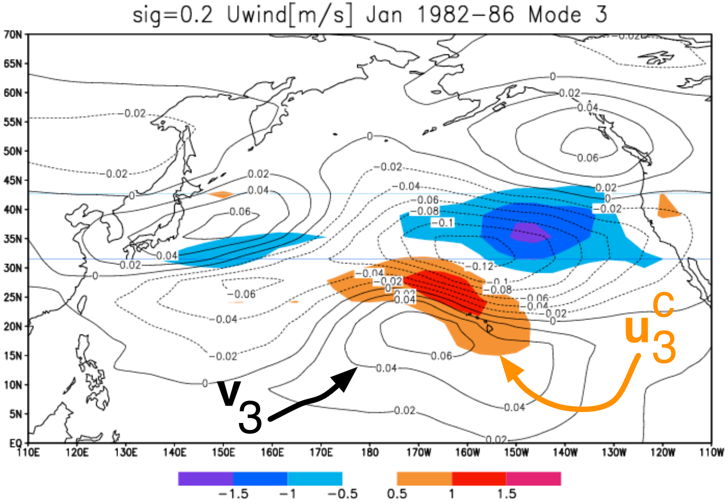
SVD Modes for Regression



Danforth, Kalnay, Miyoshi, MWR 2007

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Mark Semon & George Ruff, Physics

University of Vermont
Darren Hitt, Mechanical Engineering
Floyd Vilmont, Lab

Students
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Nicholas Allgaier, MS
Ross Lieb–Lappen, MS
Kameron D. Harris, MS

National Oceanic & Atmospheric Administration
U.S. Department of Commerce

National Science Foundation
WHERE DISCOVERIES BEGIN

National Aeronautics and Space Administration

EPSCoR
Experimental Program to Stimulate Competitive Research

VACC
VERMONT ADVANCED COMPUTING CENTER
UNIVERSITY OF VERMONT

COMPLEX SYSTEMS CENTER