

Development of a Petascale Conservative Dynamical Core for Climate Simulation

Ram Nair

Computational Mathematics Group
Institute for Mathematics Applied to the Geosciences (IMAGe)
National Center for Atmospheric Research
Boulder CO 80305, USA.
(rnair@ucar.edu)

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Overview

- Motivation
- Discontinuous Galerkin Methods (DGM)
 - 1 1-D Case
 - 2 Extension to 2-D
- The DG Baroclinic Model (HOMME)
 - 1 Vertical aspects (Lagrangian Dynamics, Remapping)
 - 2 Horizontal Aspects (DGM, Discretization)
- Validation
 - 1 2D Results
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- Summary

Motivation

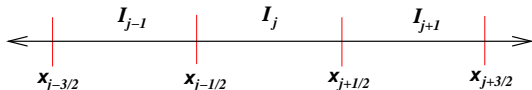
- Why do we need a new climate model?
- Because, the existing models have serious limitations to satisfy all of the following properties:
 - 1 Local and global conservation
 - 2 High-order accuracy
 - 3 High parallel efficiency
 - 4 Geometric flexibility (“Local” method)
 - 5 Monotonic (non-oscillatory) advection
- **Discontinuous Galerkin Method (DGM)** based model has the potential to address all of the above issues
- Recently, the Spectral Element (SE) model in HOMME shown to efficiently scale $\mathcal{O}(32,000)$ processors on IBM BG/L.

Discontinuous Galerkin Method (DGM) in 1D

- 1D scalar conservation law:

$$\begin{aligned}\frac{\partial U}{\partial t} + \frac{\partial F(U)}{\partial x} &= 0 \quad \text{in } \Omega \times (0, T), \\ U_0(x) &= U(x, t = 0), \quad \forall x \in \Omega\end{aligned}$$

- The domain Ω (periodic) is partitioned into N_x **non-overlapping** elements (intervals) $I_j = [x_{j-1/2}, x_{j+1/2}]$, $j = 1, \dots, N_x$, and $\Delta x_j = (x_{j+1/2} - x_{j-1/2})$



DGM-1D: Weak Formulation

A **weak formulation** of the problem for the approximate solution U_h is obtained by multiplying the PDE by a *test function* $\varphi_h(x)$ and integrating over an element I_j :

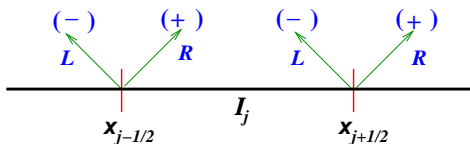
$$\int_{I_j} \left[\frac{\partial U_h}{\partial t} + \frac{\partial F(U_h)}{\partial x} \right] \varphi_h(x) dx = 0, \quad U_h, \varphi_h \in \mathcal{V}_h$$

Integrating the second term by parts \implies

$$\int_{I_j} \frac{\partial U_h(x, t)}{\partial t} \varphi_h(x) dx - \int_{I_j} F(U_h(x, t)) \frac{\partial \varphi_h}{\partial x} dx + F(U_h(x_{j+1/2}, t)) \varphi_h(x_{j+1/2}^-) - F(U_h(x_{j-1/2}, t)) \varphi_h(x_{j-1/2}^+) = 0,$$

where $\varphi(x^-)$ and $\varphi(x^+)$ denote "left" and "right" limits

DGM-1D: Flux term (“Gluing” the discontinuous element edges)



- Flux function $F(U_h)$ is **discontinuous** at the interfaces $x_{j\pm 1/2}$
- $F(U_h)$ is replaced by a **numerical flux** function $\hat{F}(U_h)$, dependent on the left and right limits of the discontinuous function U . At the interface $x_{j+1/2}$,

$$\hat{F}(U_h)_{j+1/2}(t) = \hat{F}(U_h(x_{j+1/2}^-, t), U_h(x_{j+1/2}^+, t))$$

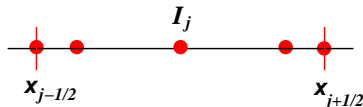
- Typical flux formulae (**Approx. Riemann Solvers**): Gudunov, Lax-Friedrichs, Roe, HLLC, etc.

DGM-1D: Space Discretization

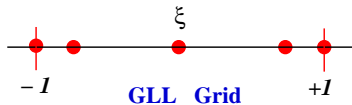
- Choose an orthogonal basis set \mathcal{B} spanning the space V_h^k , s.t., approx. solution U_h and φ_h are in V_h^k .
- Use a high-order Gaussian quadrature such as the **Gauss-Lobatto-Legendre (GLL)** quadrature rule
- Map every element I_j onto the reference element $[-1, +1]$ by introducing a local coordinate $\xi \in [-1, +1]$ s.t.,

$$\xi = \frac{2(x - x_j)}{\Delta x_j}, \quad x_j = (x_{j-1/2} + x_{j+1/2})/2 \Rightarrow \frac{\partial}{\partial x} = \frac{2}{\Delta x_j} \frac{\partial}{\partial \xi}.$$

Regular Element



Reference Element



DGM-1D: Orthogonal Basis Set (Modal Vs Nodal)

- The **modal** basis set consists of Legendre polynomials, $\mathcal{B} = \{P_\ell(\xi), \ell = 0, 1, \dots, k\}$

$$U_j(\xi, t) = \sum_{\ell=0}^k U_j^\ell(t) P_\ell(\xi) \quad \text{for } -1 \leq \xi \leq 1, \quad \text{where}$$
$$U_j^\ell(t) = \frac{2\ell + 1}{2} \int_{-1}^1 U_j(\xi, t) P_\ell(\xi) d\xi \quad \ell = 0, 1, \dots, k.$$

- The **nodal** basis set \mathcal{B} is constructed using Lagrange-Legendre polynomials $h_i(\xi)$ with roots at Gauss-Lobatto quadrature points.

$$U_j(\xi) = \sum_{j=0}^k U_j h_j(\xi) \quad \text{for } -1 \leq \xi \leq 1,$$
$$h_j(\xi) = \frac{(\xi^2 - 1) P_k'(\xi)}{k(k+1) P_k(\xi_j) (\xi - \xi_j)}.$$

- In any case, the **mass matrix** is diagonal

DGM: Explicit Time Integration

- Final Semi-discretized form \implies

$$\frac{d}{dt} U_j = \mathcal{L}(U_j) \quad \text{in } (0, T)$$

- Strong Stability Preserving third-order Runge-Kutta (SSP-RK) scheme (*Gottlieb et al., 2001*)

$$\begin{aligned} U^{(1)} &= U^n + \Delta t \mathcal{L}(U^n) \\ U^{(2)} &= \frac{3}{4} U^n + \frac{1}{4} U^{(1)} + \frac{1}{4} \Delta t \mathcal{L}(U^{(1)}) \\ U^{n+1} &= \frac{1}{3} U^n + \frac{2}{3} U^{(2)} + \frac{2}{3} \Delta t \mathcal{L}(U^{(2)}). \end{aligned}$$

where the superscripts n and $n + 1$ denote time levels t and $t + \Delta t$, respectively

- Note: The Courant number for the DG scheme is estimated to be $1/(2k + 1)$, where k is the degree of the polynomial, however, no theoretical proof exists when $k > 1$ (*Cockburn and Shu, 1989*).

DG-2D Spatial Discretization for an Element Ω

2D Scalar conservation law

$$\frac{\partial U}{\partial t} + \nabla \cdot \mathbf{F}(U) = S(U), \quad \text{in } \Omega \times (0, T); \forall (x^1, x^2) \in \Omega$$

where $U = U(x^1, x^2, t)$, $\nabla \equiv (\partial/\partial x^1, \partial/\partial x^2)$, $\mathbf{F} = (F, G)$ is the flux function, and S is the source term.

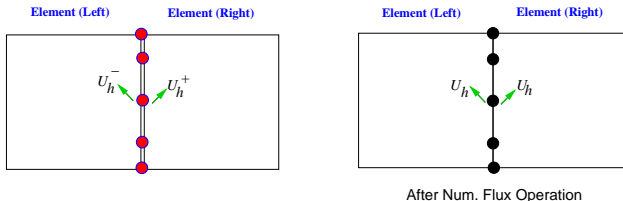
- **Weak Galerkin formulation:** Multiplication of the basic equation by a *test function* $\varphi_h \in \mathcal{V}_h$ and integration over an element Ω .

$$\frac{\partial}{\partial t} \int_{\Omega} U_h \varphi_h d\Omega - \int_{\Omega} \mathbf{F}(U_h) \cdot \nabla \varphi_h d\Omega + \int_{\Gamma} \mathbf{F}(U_h) \cdot \vec{n} \varphi_h d\Gamma = \int_{\Omega} S(U_h) \varphi_h d\Omega$$

where U_h is an approximate solution in \mathcal{V}_h .

- Can be extended to a system of equations

DG-2D: The Flux Term



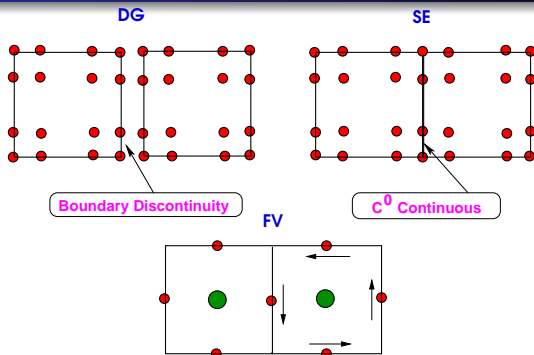
- Along the boundaries (Γ) of an element the solution U_h is **discontinuous** (U_h^- and U_h^+ are the left and right limits).
- Therefore, the analytic flux $\mathbf{F}(U_h) \cdot \vec{n}$ must be replaced by a numerical flux such as the **Lax-Friedrichs Flux**:

$$\mathbf{F}(U_h) \cdot \vec{n} = \frac{1}{2} \left[(\mathbf{F}(U_h^-) + \mathbf{F}(U_h^+)) \cdot \vec{n} - \alpha(U_h^+ - U_h^-) \right].$$

- For the SW system, α is the upper bound on the absolute value of eigenvalues of the **flux Jacobian** $\mathbf{F}'(U)$; (*Nair et al., 2005*)

$$\alpha^1 = \max \left(|u^1| + \sqrt{\Phi G^{11}} \right), \quad \alpha^2 = \max \left(|u^2| + \sqrt{\Phi G^{22}} \right)$$

The DG, SE & FV Methods



DGM is a hybrid approach ($DG \leftarrow SE + FV$)

- The domain \mathcal{D} is partitioned into non-overlapping elements Ω_{ij} such that the element boundaries are **discontinuous**.
- Based on **conservation laws** but exploits the spectral expansion of SE method and treats the element boundaries using FV "tricks."

DG-2D: Spatial Discretization

High-order nodal basis set

- The **nodal basis set** is constructed using a tensor-product of Lagrange-Legendre polynomials ($h_i(\xi)$) with roots at **Gauss-Lobatto** quadrature points.

$$h_i(\xi) = \frac{(\xi^2 - 1) P'_N(\xi)}{N(N+1) P_N(\xi_i) (\xi - \xi_i)}; \quad \int_{-1}^1 h_i(\xi) h_j(\xi) = w_i \delta_{ij}.$$

where $P_N(\xi)$ is the N^{th} order Legendre polynomial, and w_i weights associated with the Gauss quadrature.

- The approximate solution (U_h) and test function (φ_h) are represented in terms of nodal basis set.

$$U_{ij}(\xi, \eta) = \sum_{i=0}^N \sum_{j=0}^N U_{ij} h_i(\xi) h_j(\eta) \quad \text{for} \quad -1 \leq \xi, \eta \leq 1,$$

- The nodal version was shown to be more computationally efficient than the **modal** in (Dennis et al., 2006).

DG-3D Model: Explicit Time Integration

- Final form for the nodal discretization leads to the ODE:

$$\frac{d}{dt} U_{ij}(t) = \frac{4}{\Delta x_i^1 \Delta x_j^2 w_i w_j} [I_{Grad} + I_{Flux} + I_{Source}],$$

- For a system of conservation laws, solve the ODE system:

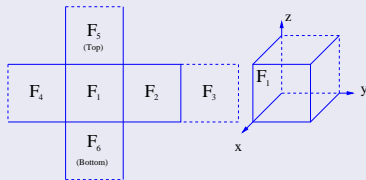
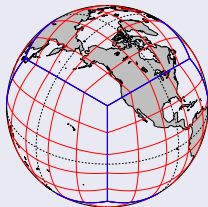
$$\frac{d}{dt} \mathbf{U} = L(\mathbf{U}) \quad \text{in} \quad (0, T) \times \Omega$$

- Time integration: Explicit third-order Runge-Kutta (SSP) scheme (*Gottlieb et al., 2001*)
- Options for explicit diffusion (∇^2 or ∇^4).
- Boyd-Vandeevan spatial Filter
- Optional Monotonic Limiter (for scalar fields)

HOMME (High-Order Method Modeling Environment)

- The **Discontinuous Galerkin (DG)** model is the conservative option in the HOMME framework
- HOMME Grid: The sphere is decomposed into 6 identical regions, using the equiangular projection (*Sadourny, 1972*)
 - Local coordinate systems are free of singularities
 - Creates a non-orthogonal curvilinear coordinate system

Cubed Sphere Geometry: Logical cube-face orientation



HOMME Grid System

Metric Tensor G_{ij} , [Cubed-Sphere \Rightarrow Sphere] Transform

Central angles $x^1, x^2 \in [-\pi/4, \pi/4]$ are the independent variables.

$$G_{ij} = \frac{R^2}{\rho^4 \cos^2 x^1 \cos^2 x^2} \begin{bmatrix} 1 + \tan^2 x^1 & -\tan x^1 \tan x^2 \\ -\tan x^1 \tan x^2 & 1 + \tan^2 x^2 \end{bmatrix}$$

where $\rho^2 = 1 + \tan^2 x^1 + \tan^2 x^2$, $i, j \in \{1, 2\}$

- Metric tensor in terms of longitude-latitude (λ, θ) :

$$G_{ij} = A^T A; \quad A = \begin{bmatrix} R \cos \theta \partial \lambda / \partial x^1 & R \cos \theta \partial \lambda / \partial x^2 \\ R \partial \theta / \partial x^1 & R \partial \theta / \partial x^2 \end{bmatrix}$$

- The matrix A is used for transforming spherical velocity (u, v) to the **covariant** (u_1, u_2) and **contravariant** (u^1, u^2) vectors.

Hydrostatic Prognostic Equations in Flux Form (Curvilinear coordinates)

$$\frac{\partial u_1}{\partial t} + \nabla_c \cdot \mathbf{E}_1 + \dot{\eta} \frac{\partial u_1}{\partial \eta} = \sqrt{G} u^2 (f + \zeta) - R T \frac{\partial}{\partial x^1} (\ln p)$$

$$\frac{\partial u_2}{\partial t} + \nabla_c \cdot \mathbf{E}_2 + \dot{\eta} \frac{\partial u_2}{\partial \eta} = -\sqrt{G} u^1 (f + \zeta) - R T \frac{\partial}{\partial x^2} (\ln p)$$

$$\frac{\partial}{\partial t} (m) + \nabla_c \cdot (\mathbf{U}^i m) + \frac{\partial(m\dot{\eta})}{\partial \eta} = 0$$

$$\frac{\partial}{\partial t} (m\Theta) + \nabla_c \cdot (\mathbf{U}^i \Theta m) + \frac{\partial(m\dot{\eta} \Theta)}{\partial \eta} = 0$$

$$\frac{\partial}{\partial t} (mq) + \nabla_c \cdot (\mathbf{U}^i q m) + \frac{\partial(m\dot{\eta} q)}{\partial \eta} = 0$$

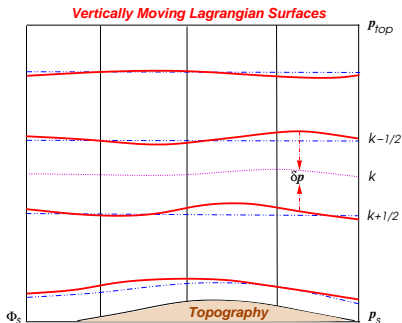
$$m \equiv \sqrt{G} \frac{\partial p}{\partial \eta}, \nabla_c \equiv \left(\frac{\partial}{\partial x^1}, \frac{\partial}{\partial x^2} \right), \eta = \eta(p, p_s), G = \det(G_{ij}), \frac{\partial \Phi}{\partial \eta} = -\frac{R T}{p} \frac{\partial p}{\partial \eta}.$$

Where m is the mass function, Θ is the potential temperature and q is the moisture variable. $\mathbf{U}^i = (u^1, u^2)$, $\mathbf{E}_1 = (E, 0)$, $\mathbf{E}_2 = (0, E)$; $E = \Phi + \frac{1}{2} (u_1 u^1 + u_2 u^2)$ is the energy term. Φ is the geopotential, ζ is the relative vorticity, and f is the Coriolis term.

Vertical Lagrangian Coordinates (*Starr, 1945*)

A “vanishing trick” for vertical advection terms!

- Terrain-following Eulerian surfaces are treated as material surfaces.
- The resulting **Lagrangian surfaces** are free to move up or down direction.



3D Prognostic Equations with Vertical Lagrangian Coordinates

- Lagrangian treatment of the Vertical coordinates results in $\dot{\eta} = 0$ and the mass function $m = \sqrt{G}\delta p = \Delta p$ (pressure thickness).
- Contravariant formulation preserves the familiar “vector invariant” form for the momentum equations.

Momentum Equations: No explicit vertical advection terms

$$\begin{aligned}\frac{\partial u_1}{\partial t} + \nabla_c \cdot \mathbf{E}_1 &= \sqrt{G}u^2 (f + \zeta) - R T \frac{\partial}{\partial x^1}(\ln p) \\ \frac{\partial u_2}{\partial t} + \nabla_c \cdot \mathbf{E}_2 &= -\sqrt{G}u^1 (f + \zeta) - R T \frac{\partial}{\partial x^2}(\ln p)\end{aligned}$$

$$\nabla_c \equiv \left(\frac{\partial}{\partial x^1}, \frac{\partial}{\partial x^2} \right), \quad \mathbf{E}_1 = (E, 0), \quad \mathbf{E}_2 = (0, E),$$

$$E = \Phi + \frac{1}{2} (u_1 u^1 + u_2 u^2)$$

3D Prognostic Equations: Flux-Form Continuity Equations

Temperature field is advected with the mass variable Δp

$$\begin{aligned}\frac{\partial}{\partial t} (\Delta p) + \nabla_c \cdot (\mathbf{U}^i \Delta p) &= 0 \\ \frac{\partial}{\partial t} (\Theta \Delta p) + \nabla_c \cdot (\mathbf{U}^i \Theta \Delta p) &= 0 \\ \frac{\partial}{\partial t} (q \Delta p) + \nabla_c \cdot (\mathbf{U}^i q \Delta p) &= 0\end{aligned}$$

where $\mathbf{U}^i = (u^1, u^2)$, $\Delta p = \sqrt{G} \delta p$, δp is the pressure thickness, and Θ is the potential temperature.

Vertical layers are coupled with the hydrostatic relations:

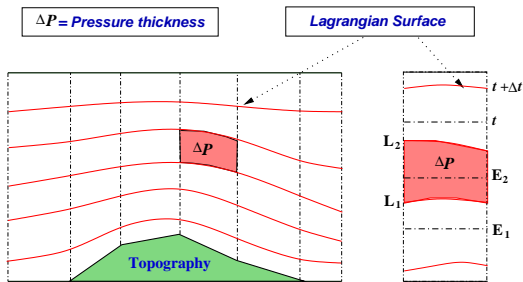
$$\Delta \Phi = -C_p \Theta \Delta \Pi, \quad \Delta \Phi = -RT \Delta \ln p$$

where $\Pi = (p/p_0)^\kappa$ and T Denotes the layer mean temperature.

The Remapping of Lagrangian Variables

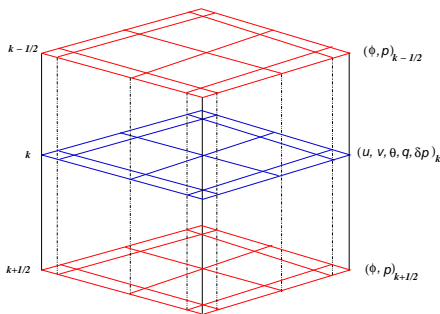
Vertically moving Lagrangian Surfaces

- Over time, Lagrangian surfaces deform and thus must be remapped.
- The velocity fields (u_1, u_2), and total energy (Γ_E) are **remapped** onto the reference coordinates using the 1-D conservative cell-integrated semi-Lagrangian (CISL) method (Nair & Machenhauer, 2002)



Terrain-following Lagrangian control-volume coordinates

Computational Grid Structure for DG-3D Model



GLL- Grid box

- The prognostic variables $u_1, u_2, \delta p, \Theta$ and q are staggered w.r.t p and ϕ .
- The remapping frequency is $\mathcal{O}(10) \times \Delta t$
- Potential temperature Θ is retrieved from the remapped total energy

$$\Gamma_E = c_p T + \frac{\delta(p\phi)}{\delta p} + K_E$$

The Vertical Lagrangian Dynamics

The hydrostatic pressure at Lagrangian surface, *Lin (MWR, 2004)*

$$p_\ell = p_{top} + \sum_{k=1}^{\ell} \delta p_k, \quad \ell = 1, 2, 3, \dots, N$$

where p_{top} represents the pressure at the model top, p_ℓ denotes the pressure at each Lagrangian surface. There are total $N + 1$ Lagrangian surfaces span N layers.

The geopotential height at Lagrangian surface:

$$\Phi_\ell = \Phi_s + \sum_{k=N}^{\ell} \Delta\Phi_k, \quad \ell = N, N - 1, \dots, 1$$

where Φ_s represents the surface geopotential height at the model bottom and Φ_ℓ denotes the geopotential height at each Lagrangian surface.

DG-3D Model Horizontal Aspects: Shallow Water Model

Flux-form SW equations (Vector invariant form)

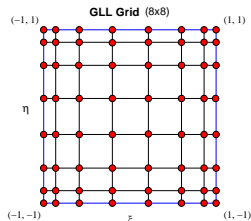
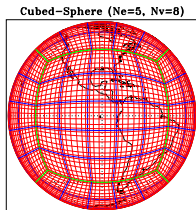
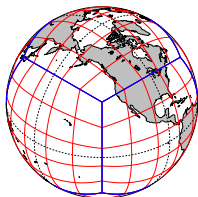
Nair et al. (MWR, 2005)

$$\begin{aligned}\frac{\partial u_1}{\partial t} + \frac{\partial}{\partial x^1} E &= \sqrt{G} u^2 (f + \zeta) \\ \frac{\partial u_2}{\partial t} + \frac{\partial}{\partial x^2} E &= -\sqrt{G} u^1 (f + \zeta) \\ \frac{\partial}{\partial t} (\sqrt{G} h) + \frac{\partial}{\partial x^1} (\sqrt{G} u^1 h) + \frac{\partial}{\partial x^2} (\sqrt{G} u^2 h) &= 0\end{aligned}$$

where $G = \det(G_{ij})$, h is the height, f Coriolis term; energy term and vorticity are defined as

$$E = \Phi + \frac{1}{2} (u_1 u^1 + u_2 u^2), \quad \zeta = \frac{1}{\sqrt{G}} \left[\frac{\partial u_2}{\partial x^1} - \frac{\partial u_1}{\partial x^2} \right].$$

DG-3D Model: Computational Domain



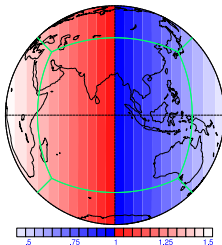
Cubed-Sphere ($N_e = 5$) with 8×8 GLL points

Flux is the only “communicator” at the element edges

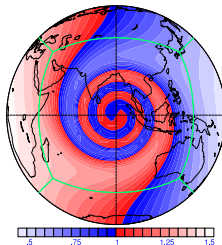
- Each face of the cubed-sphere is partitioned into $N_e \times N_e$ rectangular non-overlapping elements (i.e., total $6 \times N_e^2$).
- Each element is mapped onto the Gauss-Lobatto-Legendre (GLL) grid defined by $-1 \leq \xi, \eta \leq 1$, for integration.

Horizontal Advection: Moving Vortices on the Sphere

DG: Moving Vortex on the Sphere (HOMME/Nair)



DG: Moving Vortex on the Sphere (HOMME/Nair)



Initial field and DG solution after 12 days. Max error is $\mathcal{O}(10^{-5})$

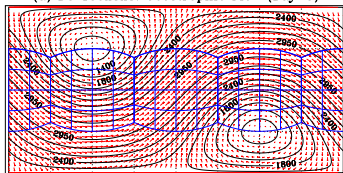
Deformational Flow Test: *Nair & Jablonowski (MWR, 2007)*

- The vortices are located at diametrically opposite sides of the sphere, the vortices deform as they move along a prescribed trajectory.
- Analytical solution is known and the trajectory is chosen to be a great circle along the NE direction ($\alpha = \pi/4$).

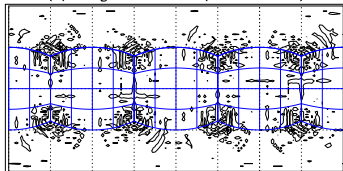
SW Test-2: Geostrophic Flow *(Nair et al., MWR 2005)*

- High-order accuracy and spectral convergence

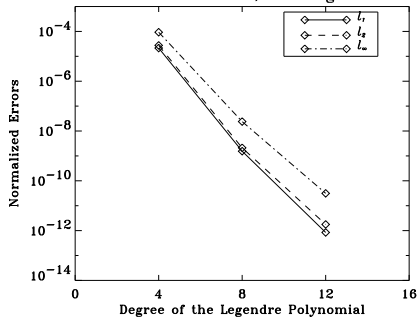
(a) DG 150x8x8: Geostrophic Flow (Day-5)



(b) Height Difference (Num - Exact)



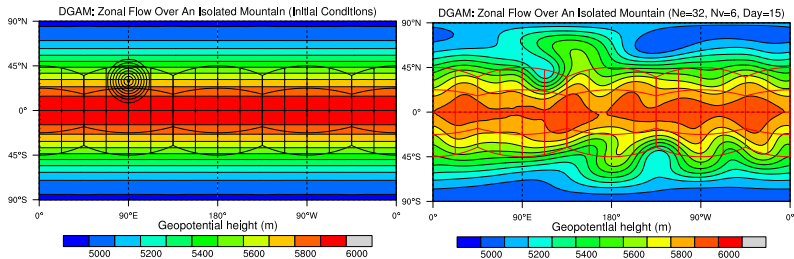
DGM: SW Test-2, Convergence



Steady state geostrophic flow ($\alpha = \pi/4$). Max height error is $\mathcal{O}(10^{-6})$ m.

SW Test-5: Flow over a Mountain *(Dennis et al. 2006)*

- No “spectral ringing” for the height fields

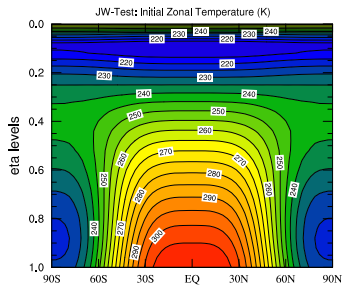
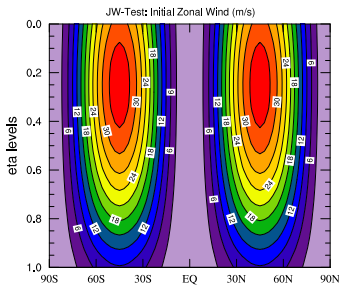


Flow over a mountain ($\approx 0.5^\circ$). Initial height field (left) initial and after 15 days of integration (right)

DG-3D: Baroclinic Instability Test

(JW-Test) Jablonowski & Williamson (*QJRM*, 2006)

- To assess the evolution of an idealized baroclinic wave in the Northern Hemisphere.
- The initial conditions are quasi-realistic and defined by analytic expressions. Analytic solutions do not exist.

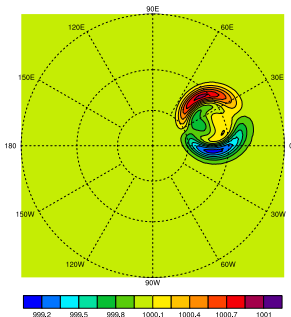


Initial Conditions

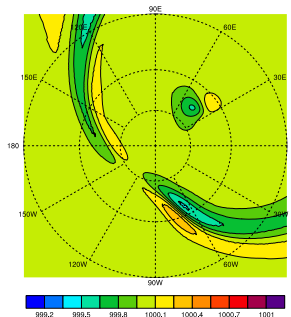
JW-Test: Evolution of Surface Pressure over the NH

- Baroclinic waves are triggered by perturbing the velocity field at (20°E, 40°N)
- This test case recommends up to 30 days of model simulation
- $N_e = N_v = 8$ (approx. 1.6°) with 26 vertical levels and $\Delta t = 30$ Sec.

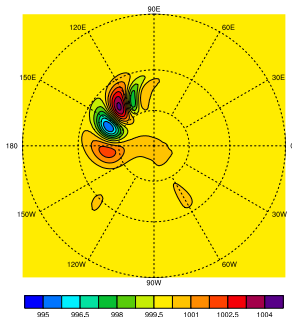
DG [Ne=8,Nv=8,NI=26], Ps(hPa) 1H



DG [Ne=8,Nv=8,NI=26], Ps(hPa) 24H

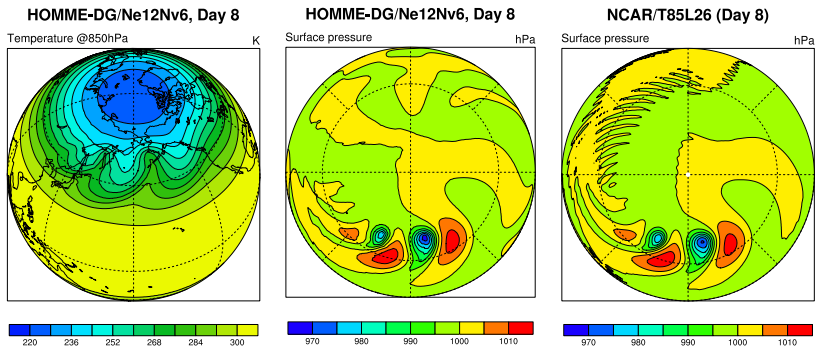


HOMME/DG: Ps(hPa) Day 6 (NH)



DG-3D Model Vs. NCAR Spectral Model

- The HOMME-DG dynamical core successfully simulates baroclinic instability.

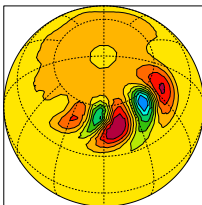


Simulated temperature (K) and surface pressure (hPa) at day 8 for a baroclinic instability test with the HOMME-DG model and the NCAR global spectral model (right). The horizontal resolution is approximately 1.4° .

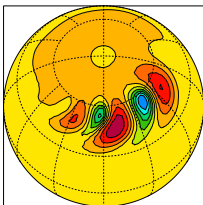
Note that the DG solution is free of "spectral ringing".

Jablonowski-Williamson Baroclinic Test (Convergence)

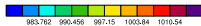
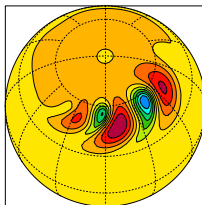
Ps: Ne=3,Nv=10,Nl=18 Day 9



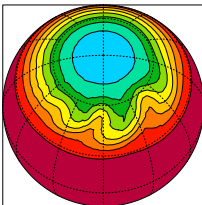
Ps: Ne=4,Nv=10,Nl=18 Day 9



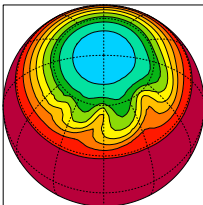
Ps: Ne=6,Nv=10,Nl=18 Day 9



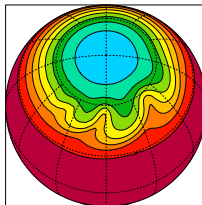
T@850mb: Ne=3, Nv=10, Day 9



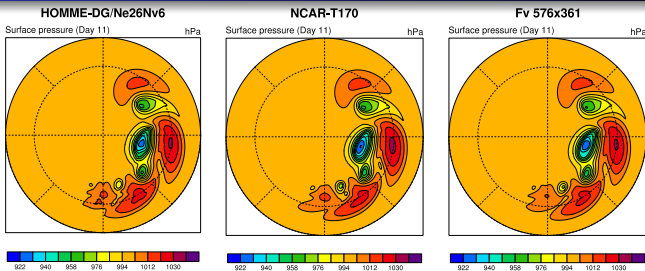
T@850mb: Ne=4, Nv=10, Day 9



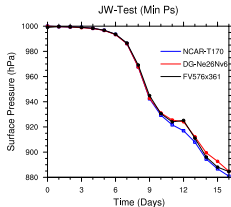
T@850mb: Ne=6, Nv=10, Day 9



DG Model Vs. NCAR Climate Models (*Nair & Tufo, 2007*)

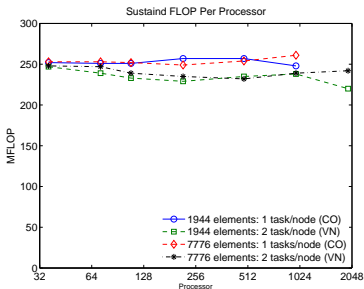


Simulated surface pressure at day 11 for a baroclinic instability test with DG model, and NCAR global spectral model and a FV model. The models use 26 vertical levels and with approximate horizontal resolution of 0.7° .

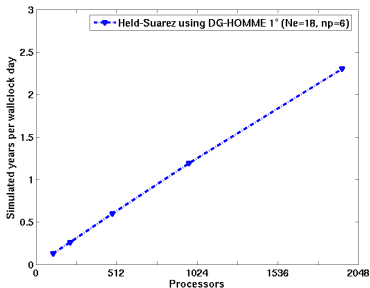


Parallel Performance (3D) - Frost [IBM BG/L]

- **DG-3D parallel performance:** Sustained Mflops on IBM BG/L (1024 DP nodes, 700 MHz PPC 440s): Approx. 9% peak
- **Held-Suarez (preliminary) test:** 800 days idealized climate simulation (1° resolution, 26 vertical levels, $\Delta t = 10$ Sec)



Parallel performance (strong scaling) results for JW-Test



Held-Suarez test (800 days)

Summary

- The DG-3D model successfully simulates the Baroclinic instability test and the results are comparable with that of the NCAR global spectral model.
- The preliminary scaling results are impressive and comparable to the SE version in HOMME.
- The explicit R-K time integration scheme is robust for the DG-3D model, but very time-step restrictive.
- **More efficient time integration schemes are required** for practical climate simulations. Possible approaches: Semi-implicit, IMEX-RK, Rosenbrock with optimized Schwarz, etc..
- **Future Work:** Coupling of the CAM/CCSM *physics* for the *real* climate simulations in HOMME. Targeting for large-scale parallelism with $\mathcal{O}(100K)$ processors.