Geodynamo Simulation using Yin-Yang grid on Earth Simulator

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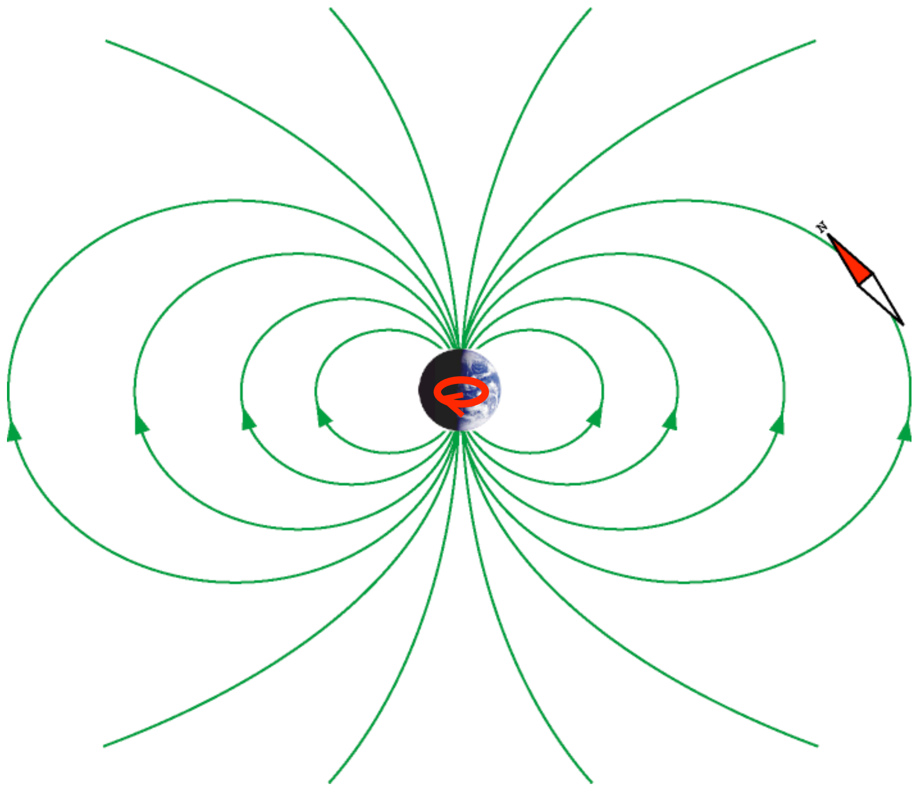
Visualization of Geomagnetic Field

Aurora taken from Space Shuttle, May 1991.

http://commons.wikimedia.org/wiki/Image:Aurora-SpaceShuttle-EO.jpg

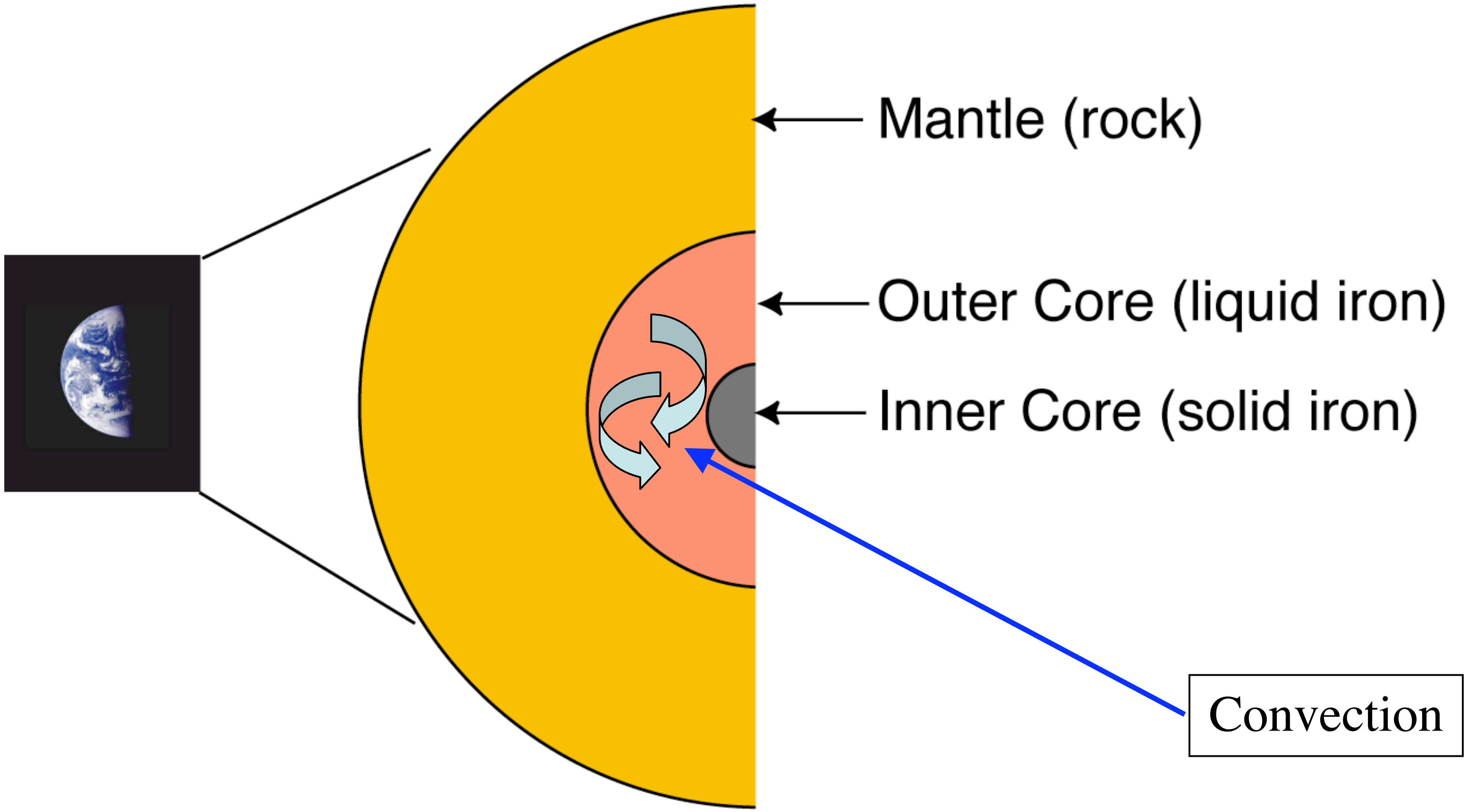
Geomagnetic Field

Ring current in the core



$I = 10^9$ A

Earth's structure



Outer Core as an MHD Fluid

	Non dimensional parameters
Outer core radius $R_o = 3.5 \times 10^6 m$	Rayleigh number $Ra \sim 10^{30}$
Thermal diffusivity $\kappa \sim 5 \times 10^{-6} m^2/sec$	Reynolds number $Re = V R_o/\nu \sim O(10^9)$
Viscosity $\nu \sim 1 \times 10^{-7} m^2/sec$	Magnetic Reynolds $Rm = V R_o/\eta \sim O(10^3)$
Electrical resistivity $\eta \sim 2\, m^2/sec$	Magnetic Prandtl $Pm = 5 \times 10^{-6}$
Mass density $\rho \sim 1 \times 10^4 kg/m^3$	Prandtl number $Pr = 0.2$
Poloidal magnetic field $B = O(10^{-4})\, T$	Ekman number $E = \nu/\Omega R_o^2 = O(10^{-15})$
Flow velocity $V \sim O(10^{-4})\, m/sec$	Elsasser number $\Lambda = B^2/2\Omega\mu_0\rho\eta \sim O(10)$
Sound velocity $V_s \sim O(10^4)\, m/sec$	Magnetic energy density / Flow energy density
Alfvén velocity $V_A \sim O(10^{-3})\, m/sec$	$= \left(\frac{B^2}{2\mu_0}\right) / \left(\frac{\rho V^2}{2}\right) = (V_A/V)^2 \sim O(10^2)$

Computer Simulation of Geodynamo

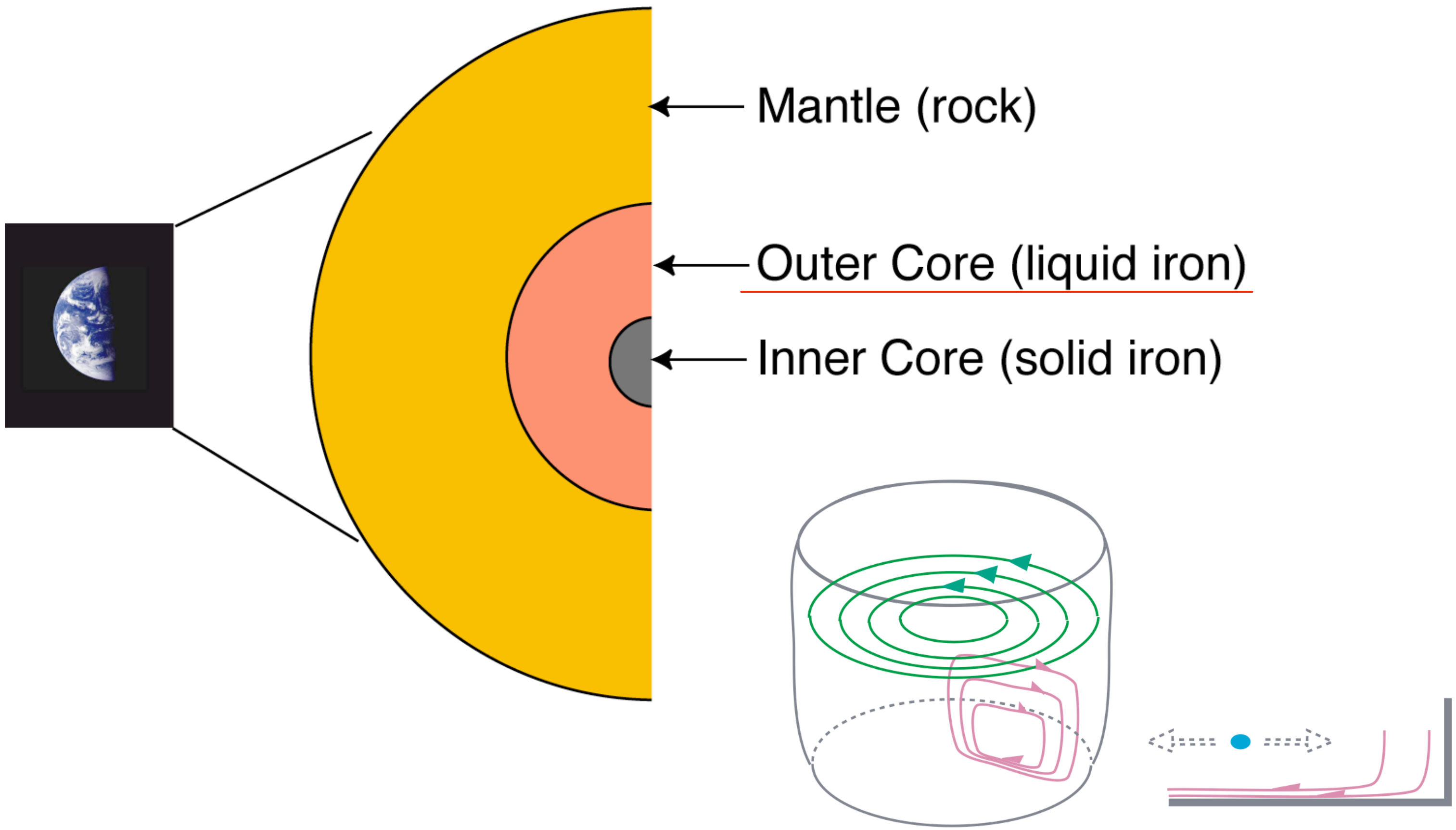
- MHD simulation of the outer core
- DNS with real parameters are impossible.
  - "Hyperdiffusivity" approach was popular, but not now.
  - DNS with *quite* higher viscosity.
- A barometer of the distance to the real core:
  - Ekman number *E*
    - 1995: E = O(10^-4)
    - 1998: E = O(10^-5)
    - 2005: E = O(10^-6)
    -
    -
    -
    - ????: E = O(10^-15) THE CORE

"Already" succeeded to reproduce:
- Dipole field generation
- Intermittent reversals

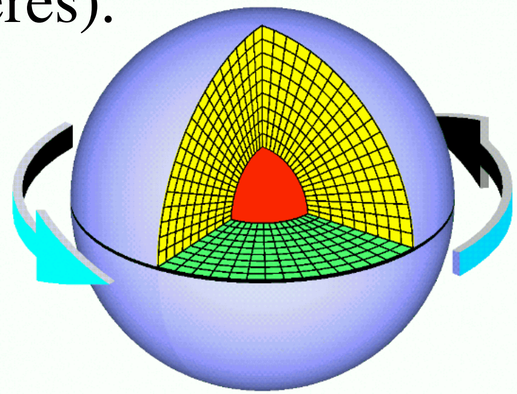
Marvel…

Ekman number

$$E = \nu/\Omega R_o^2 = O(10^{-15})$$



Computer Simulation of Geodynamo



- Outer core (a spherical shell between two spheres).
- Rotation.
- MHD fluid
- Temperature difference between the spheres.
- Gravity
- Initial condition: random & weak “seed” of magnetic field and temperature as a perturbation.
- MHD eqs.

⟹ Thermal convection of the MHD fluid

⟹ MHD dynamo

Equations

Compressible MHD equations

$$\frac{\partial \rho}{\partial t} = -\nabla \cdot \mathbf{f},$$

$$\frac{\partial \mathbf{f}}{\partial t} = -\nabla \cdot (\mathbf{v}\mathbf{f}) - \nabla p + \mathbf{j} \times \mathbf{B} + \rho \mathbf{g} + 2\rho \mathbf{v} \times \mathbf{\Omega} + \mu(\nabla^2 \mathbf{v} + \frac{1}{3}\nabla(\nabla \cdot \mathbf{v})),$$

$$\frac{\partial p}{\partial t} = -\mathbf{v} \cdot \nabla p - \gamma p \nabla \cdot \mathbf{v} + (\gamma - 1) K \nabla^2 T + (\gamma - 1)\eta \mathbf{j}^2 + (\gamma - 1)\Phi,$$

$$\frac{\partial \mathbf{A}}{\partial t} = -\mathbf{E},$$

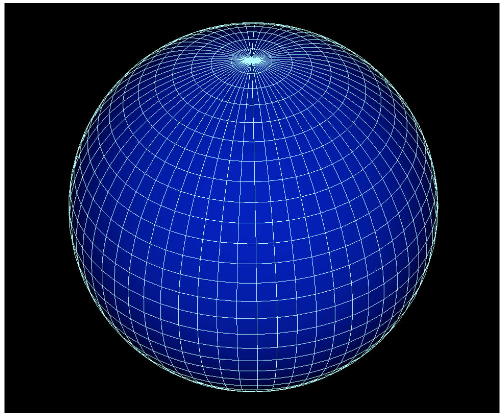
with

$$\mathbf{B} = \nabla \times \mathbf{A}, \mathbf{j} = \nabla \times \mathbf{B}, \mathbf{E} = -\mathbf{v} \times \mathbf{B} + \eta \mathbf{j},$$

$$p = \rho T, \mathbf{g} = -g_0/r^2 \hat{\mathbf{r}}, \Phi = 2\mu \left(\epsilon \cdot \epsilon - (\nabla \cdot \mathbf{v})^2 / 3\right).$$

Our Old Simulations in1995 -- 1997

- On the latitude-longitude (lat-lon) grid
- Finite Difference Method
- NEC SX-4 (vector processor)
- Typical grid size and params:

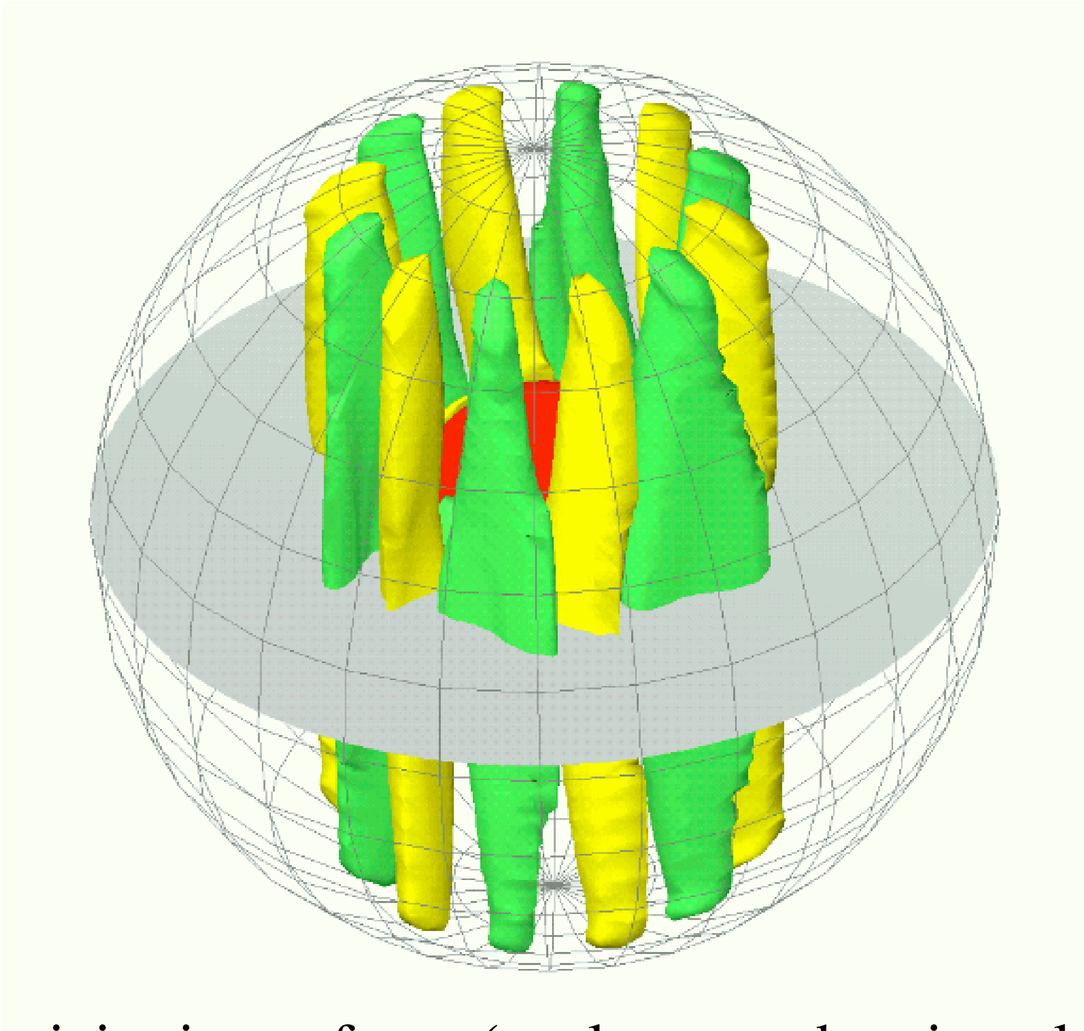


$$N_r \times N_\theta \times N_\phi = 50 \times 38 \times 128$$

$$Ek = 2 \times 10^{-4}$$

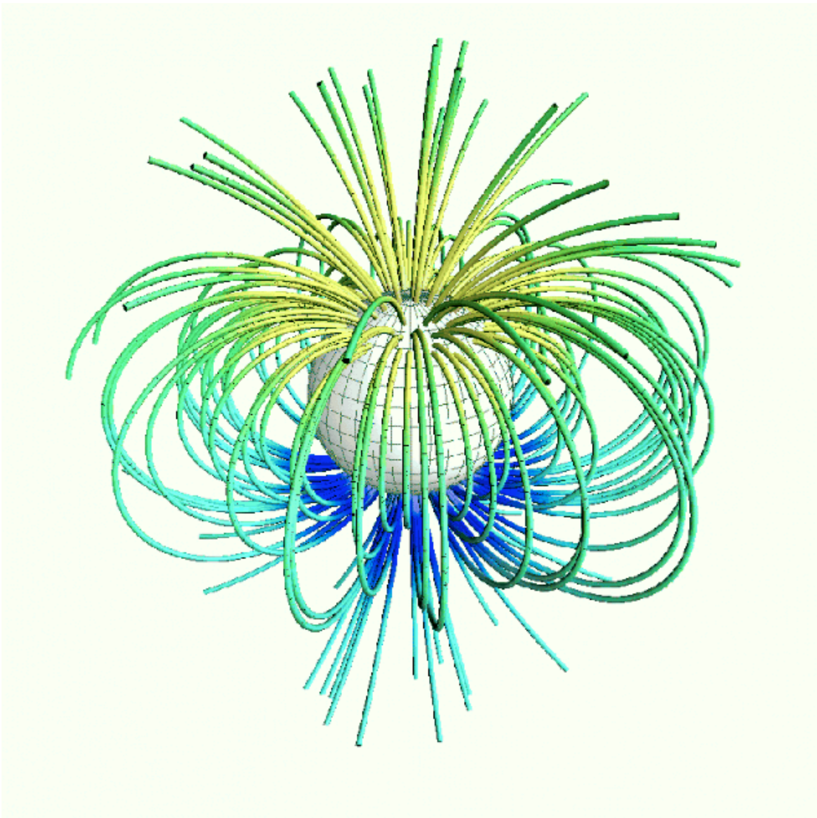
$$Re = O(10^2)$$

Simulation Results (moderate convection dynamo)

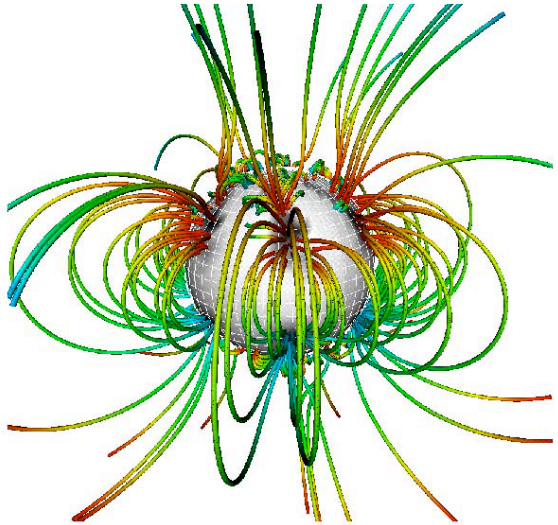


Vorticity isosurfaces (cyclones and anti-cyclones)

Generated magnetic field in our simulation

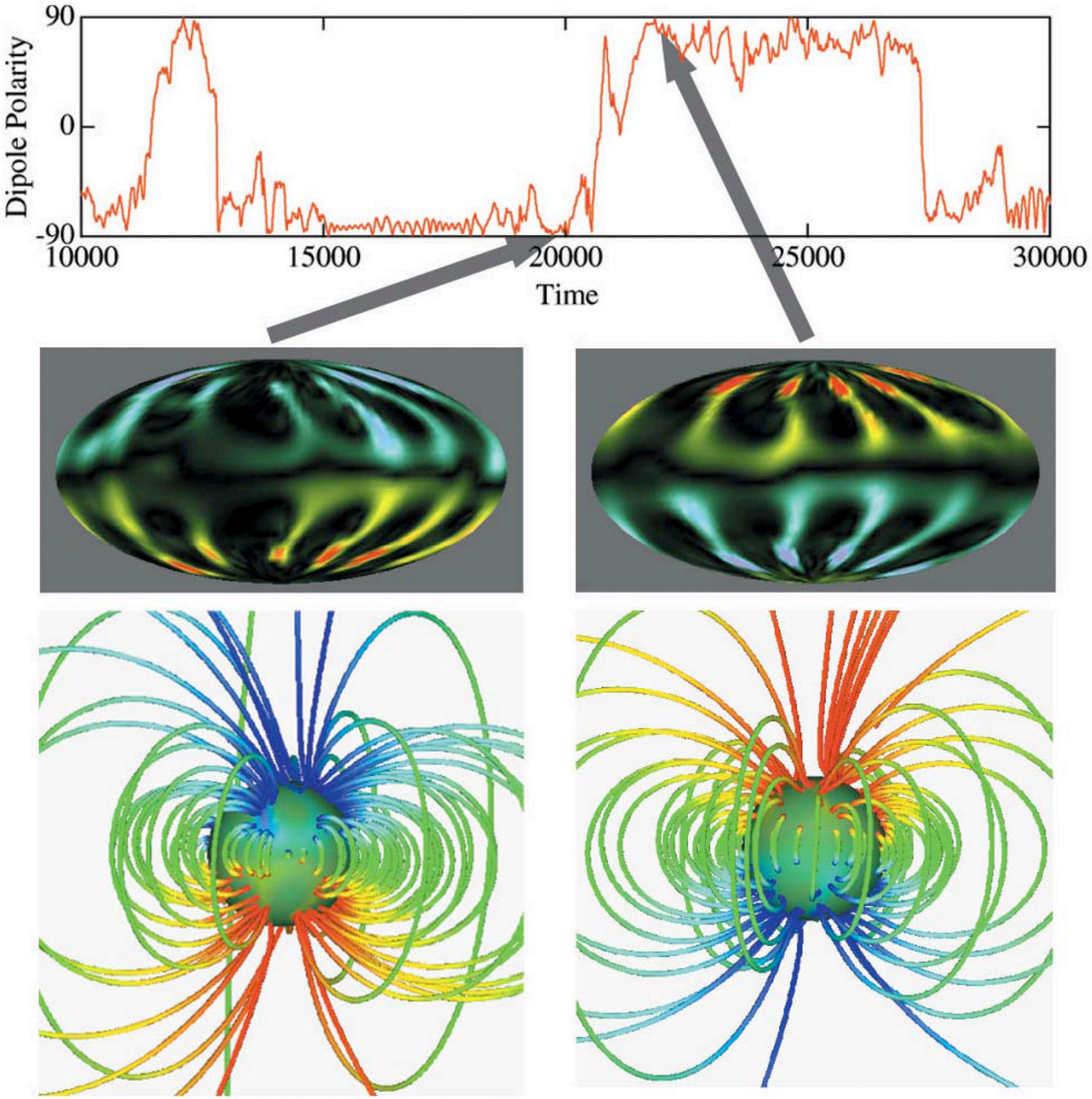


Magnetic field lines starting from the Earth's surface.

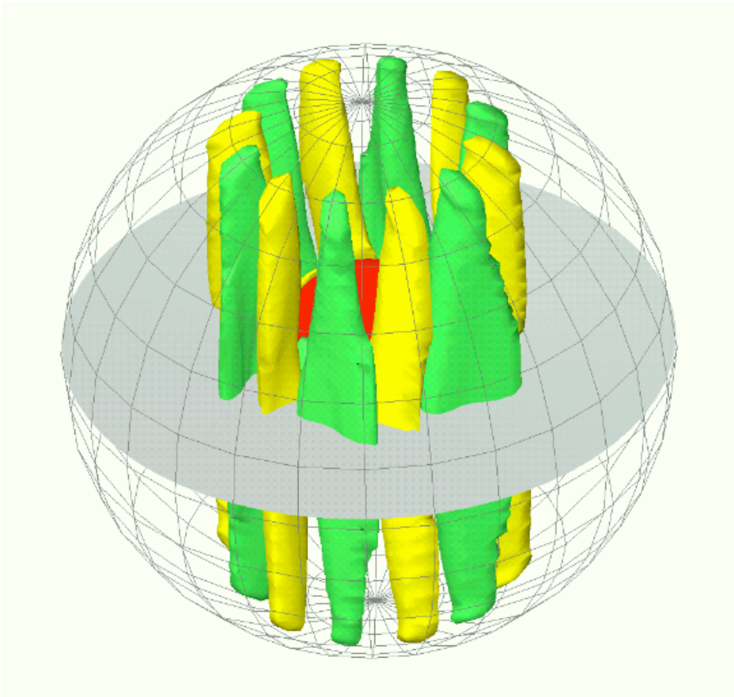


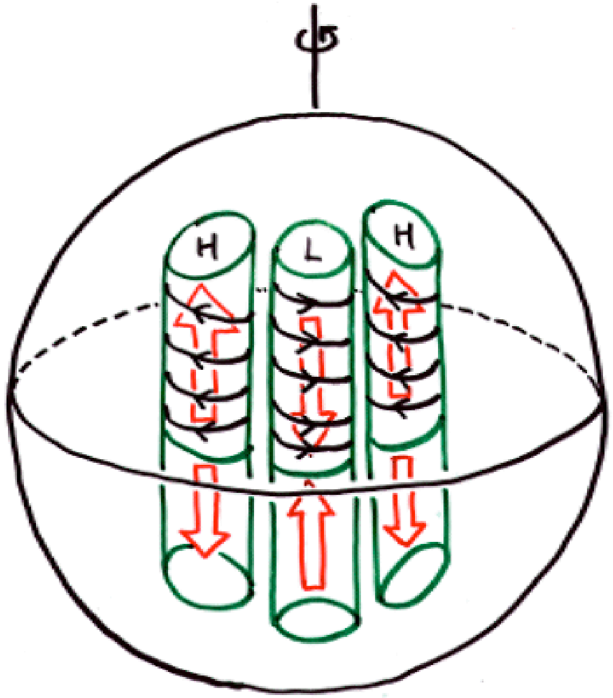
Magnetic field lines starting from the core surface.

Dipole Field Reversals

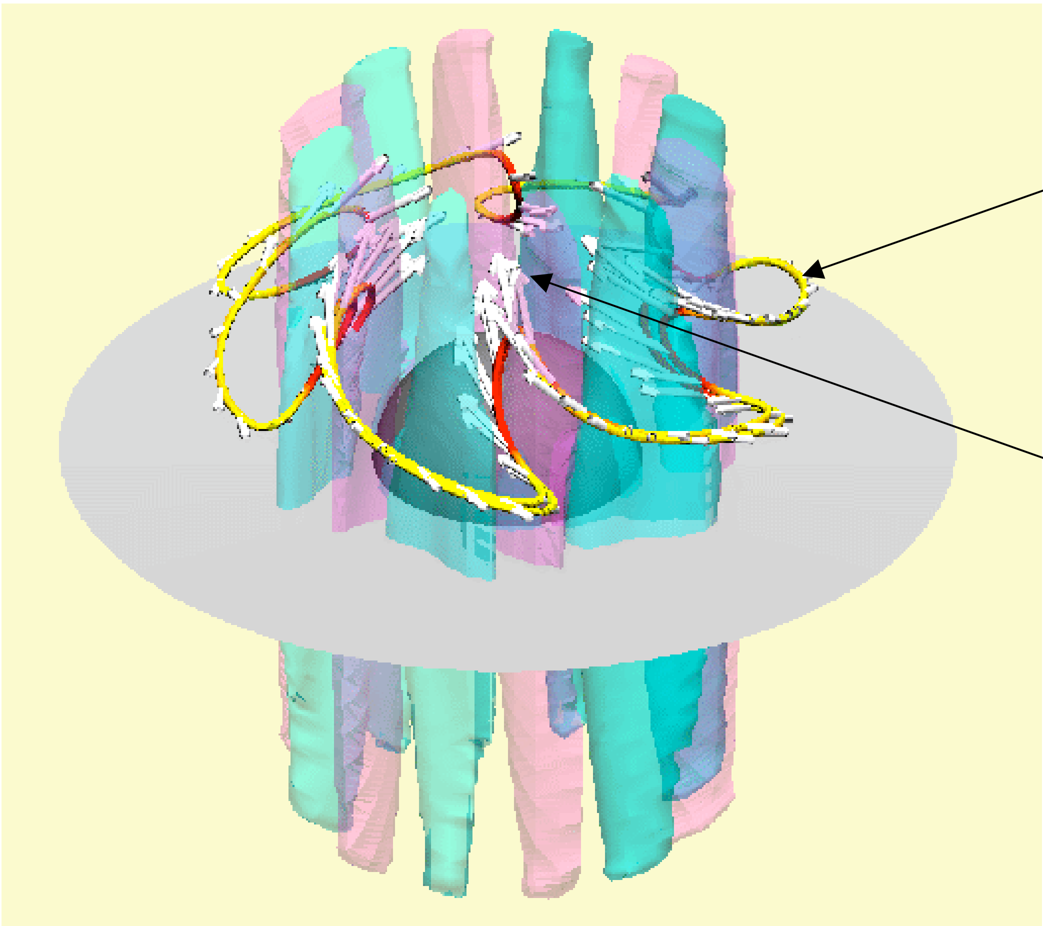


Generation Mechanism of Dipole Field





Generation Mechanism of Dipole Field

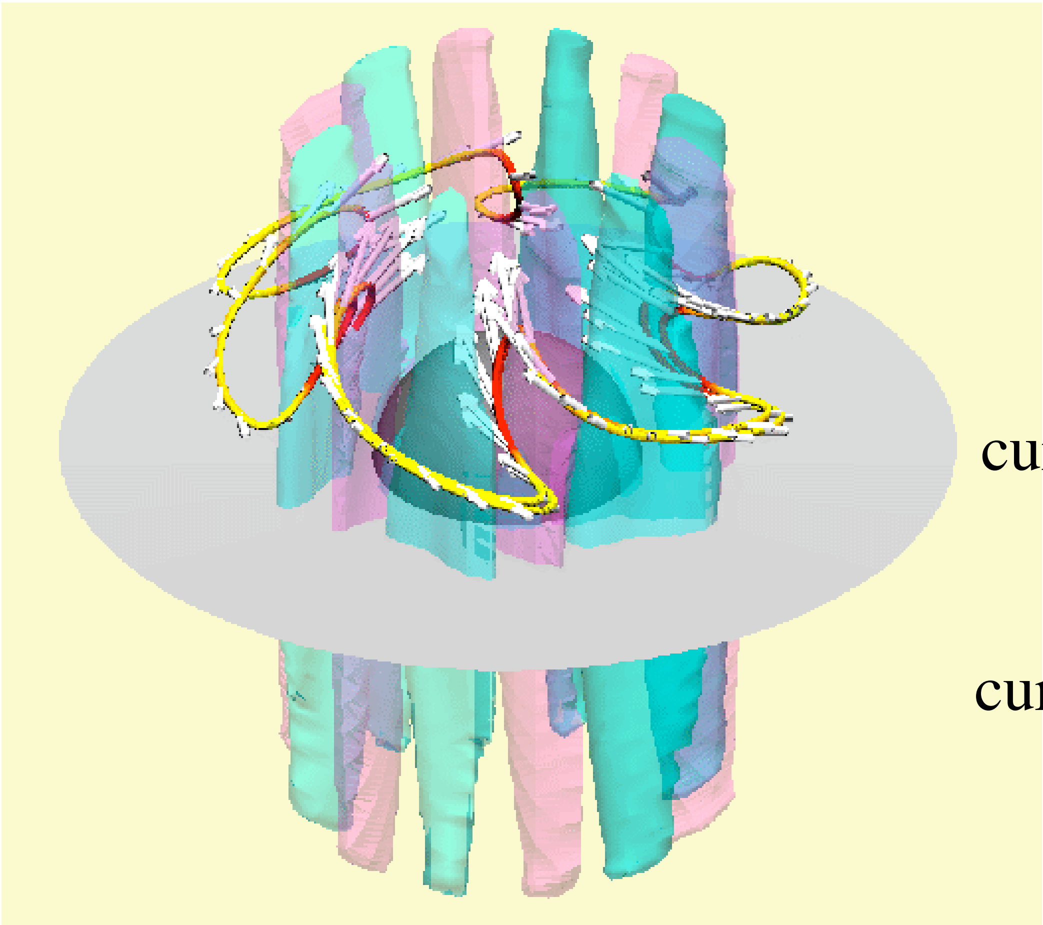


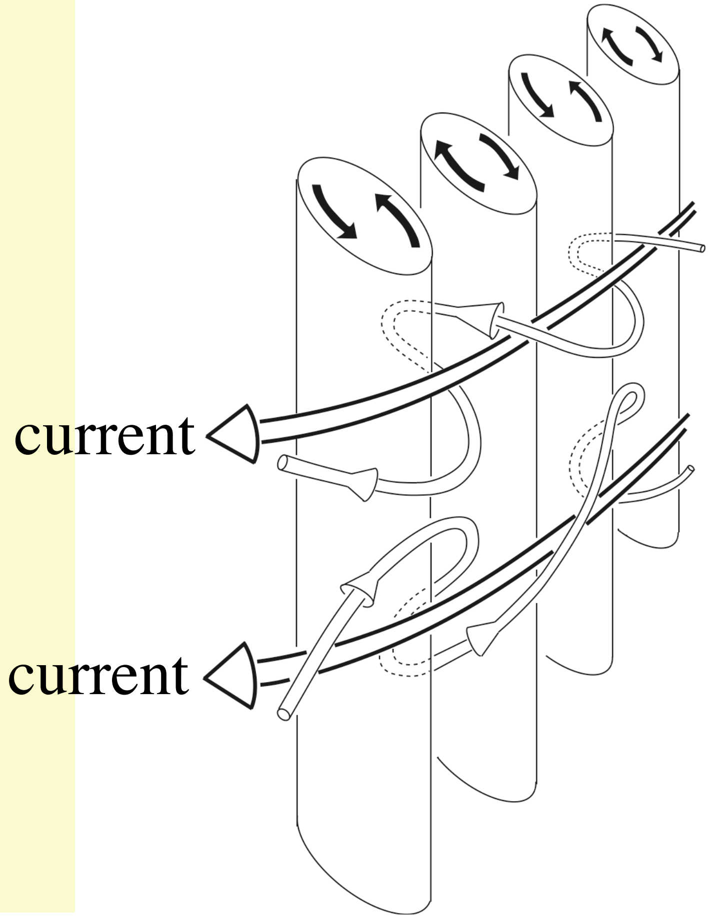
Colored line:
A magnetic field line

White arrows:
Convection flow vectors

Kageyama and Sato, Phys. Rev. E, 1997

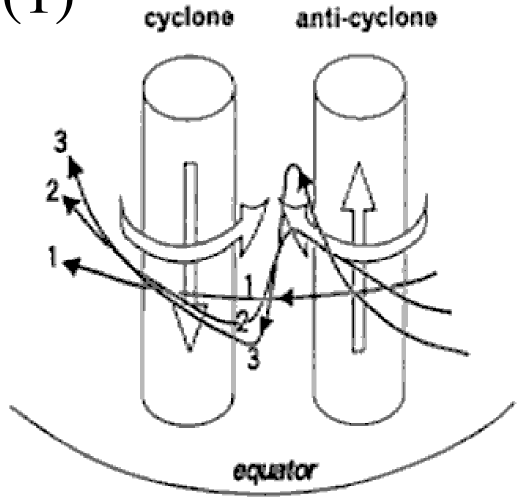
Generation Mechanism of Dipole Field



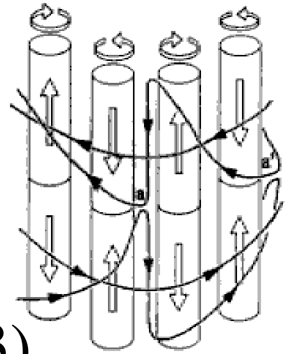


Generation Mechanism of Dipole Field

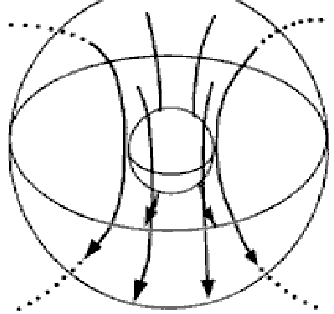
(1)



(2)



(3)



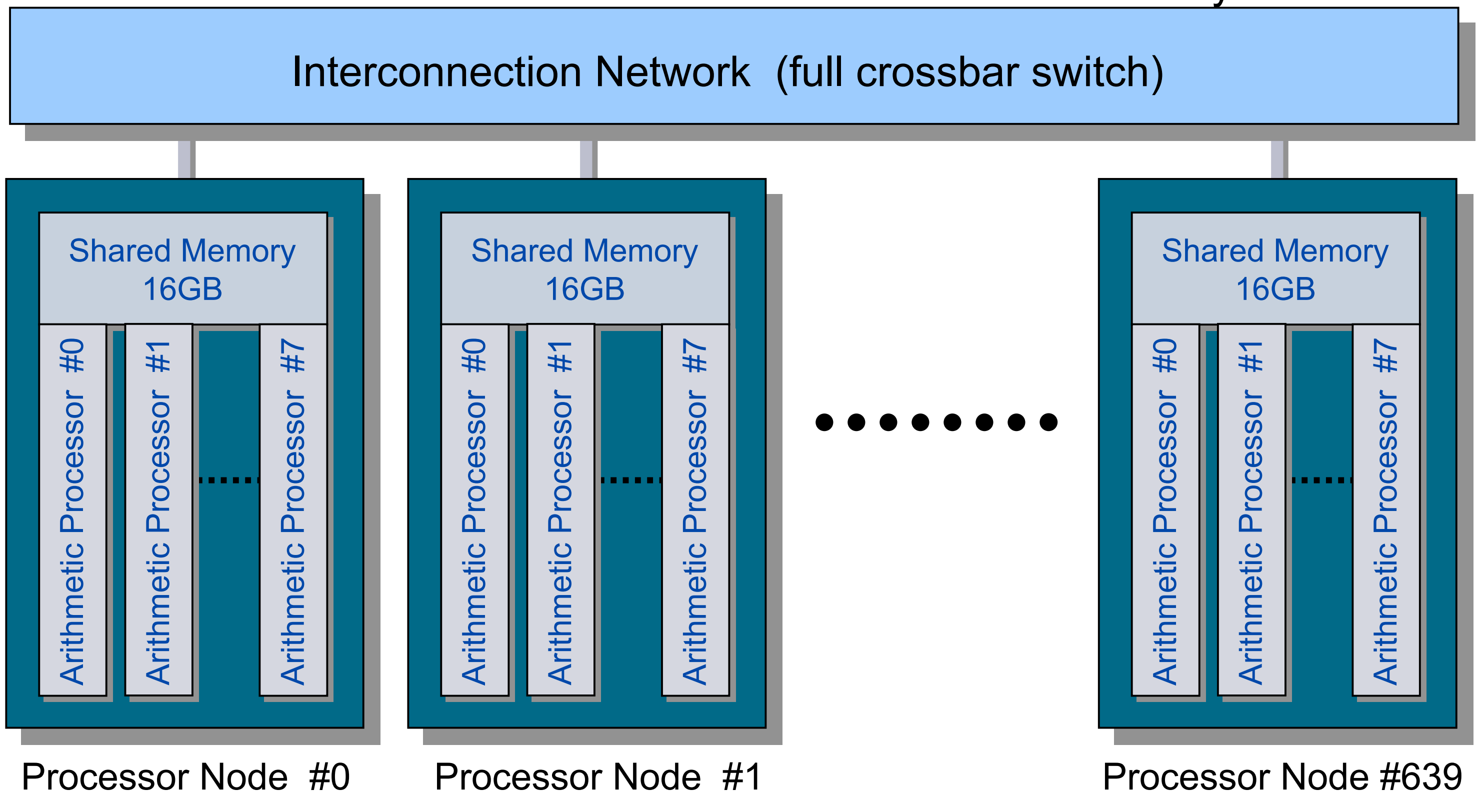
My plan was:
(1) Port this code to Earth Simulator
(2) Use maximum nodes of ES to
   - increase the resolution and
   - decrease Ekman number

Kageyama and Sato, Phys. Rev. E, 1997

Earth Simulator

- Peak performance/AP : 8Gflops
- Peak performance/PN : 64Gflops
- Shared memory/PN : 16GB

- Total number of APs : 5120
- Total number of PNs : 640
- Max usable PNs : 512
- Total peak performance : 40TFLOPS
- Total main memory : 10TB

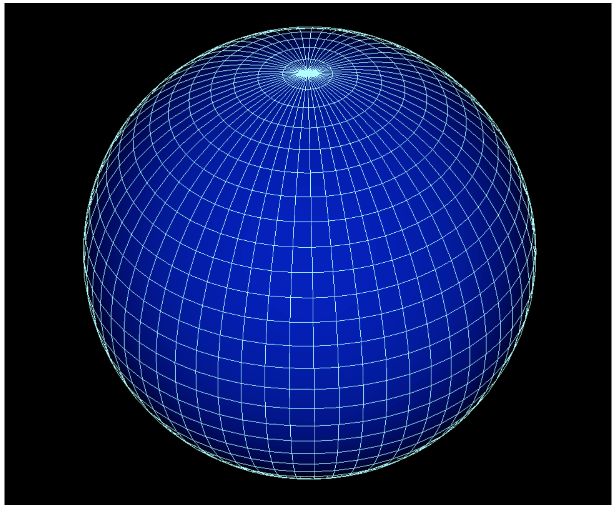


But…

- Encountered difficulty to achieve high performance on ES.
- The difficulty comes from the base grid system (lat-lon grid).

Numerical Problems of Lat-Lon Grid

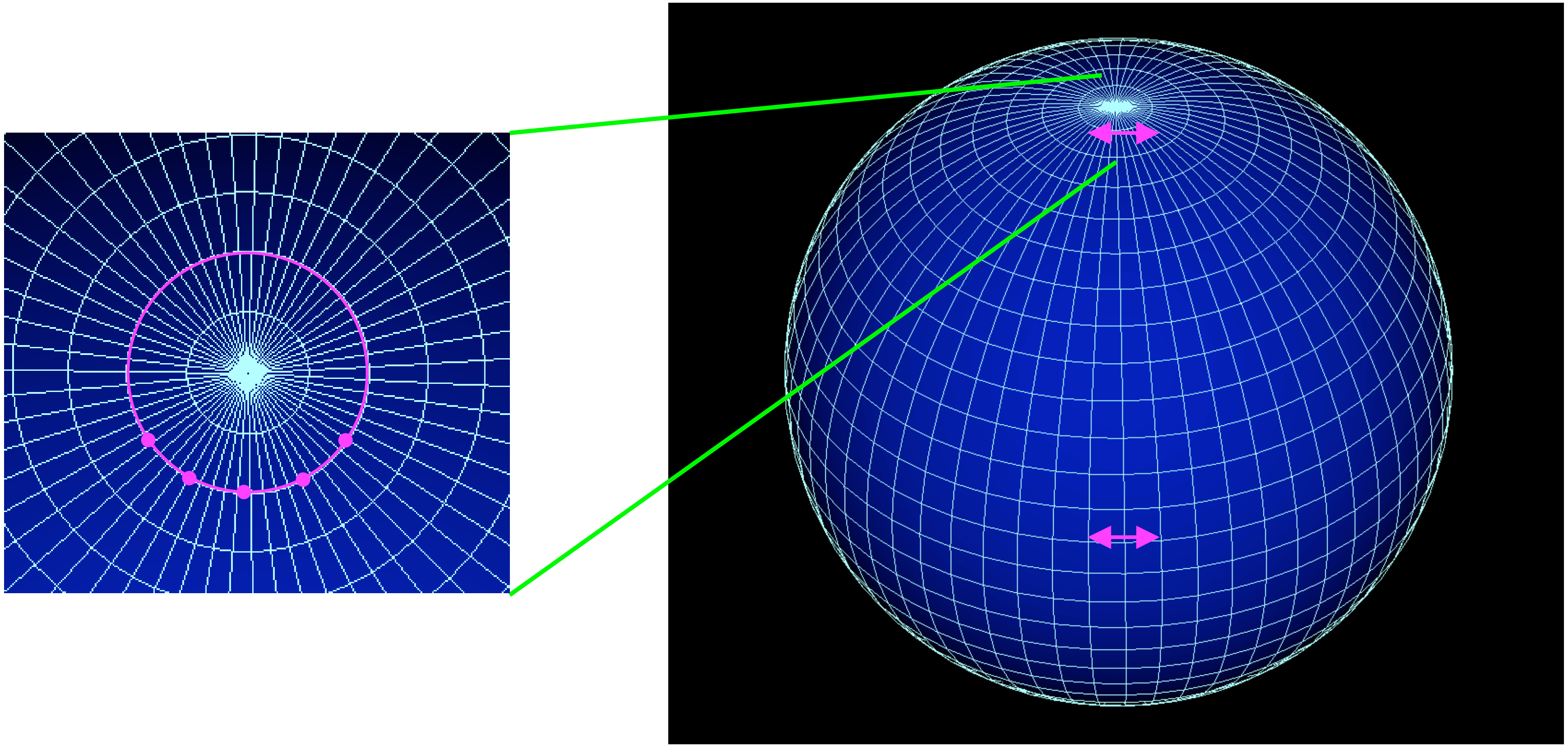
- On the poles
  - Coordinate singularity
  - No problem
  - L'Hospital's thereom
- Near the poles
  - Severe CFL condition
  - Serious problem
  - Needs spherical filter



Spherical Filter

Retain the grids, but drop useless information.

→ Filtering



Inefficiency of Lat-Lon Grid

- Needed optimized parallel spherical filter (with FFT).

  -- bottleneck

- Even if you could make highly optimized spherical filter, the lat-lon grid + spherical filter method is computationally inefficient.

  (1) Place many grid points near the poles, spoiling the low-latitude's resolution.

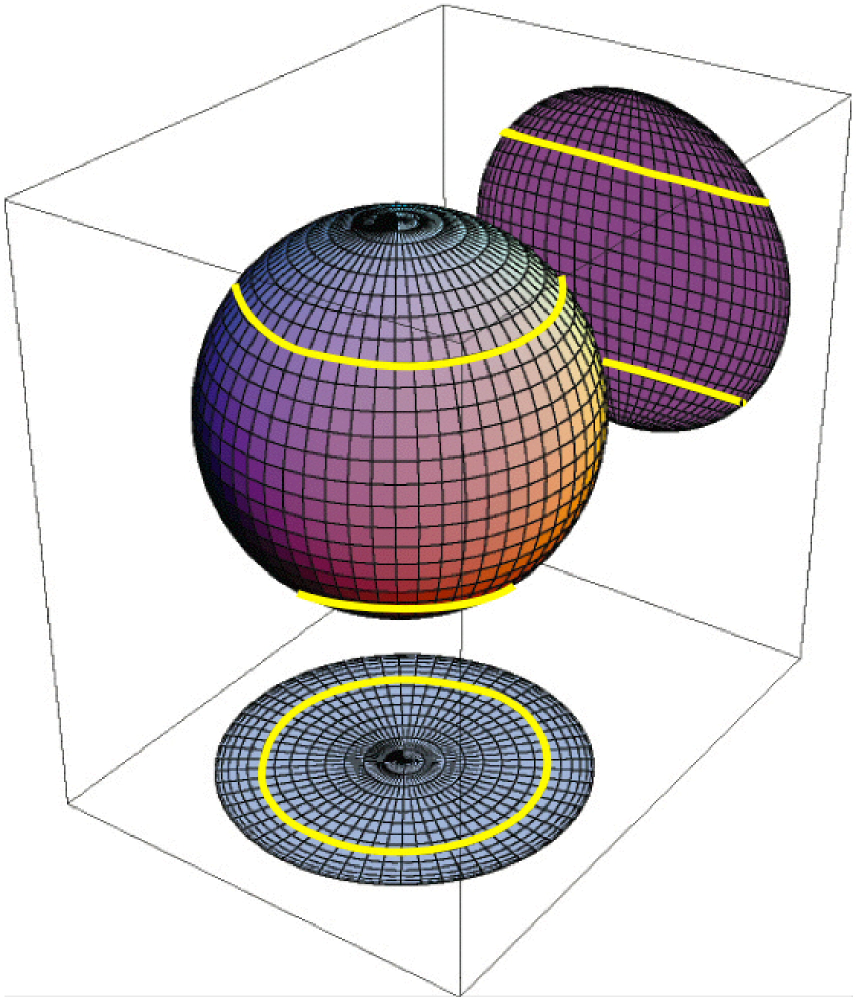
  (2) Work hard to calculate data on all the grids.

  (3) Throw away most of the data!

This is true for other spherical discretization methods:

- Double FFT spectral method (FFT both in latitude & longitude).
- Single FFT, hybrid method (FD in latitude & FFT in longitude).

Grid Convergence Near the Poles



84% of grid points are located in high-laritude part (>45^{o}N and S).

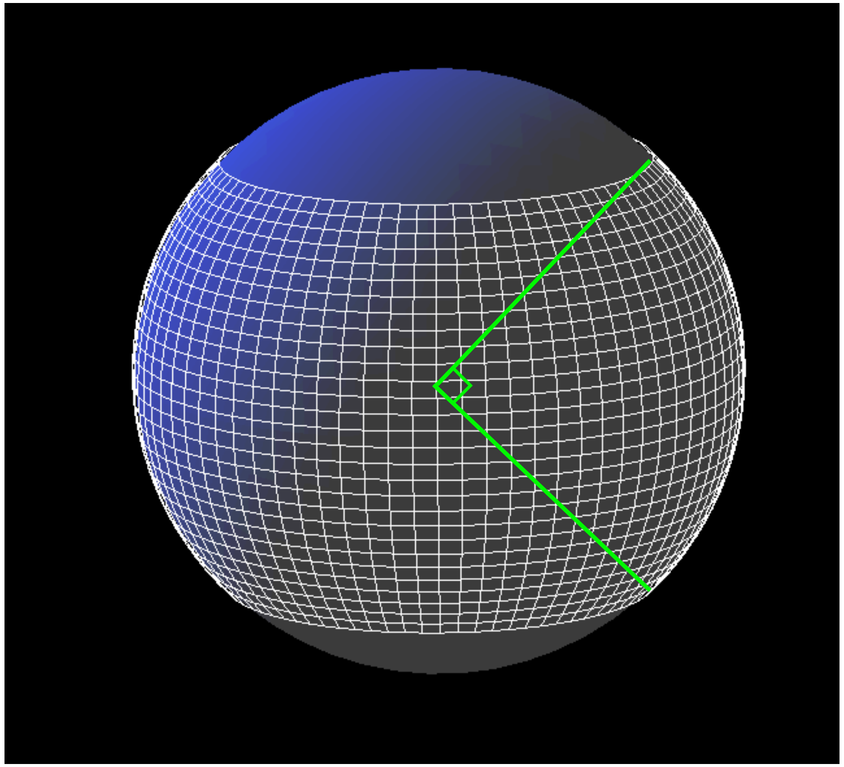
Only 16% grids cover the low-latitude part (between 45^{o}N and S)

Quest for new spherical grid

Let's re-view the lat-lon grid

It is almost ideal grid in the low latitude region.

- It is orthogonal coordinates (simple metrics)
- Nearly uniform grid spacing



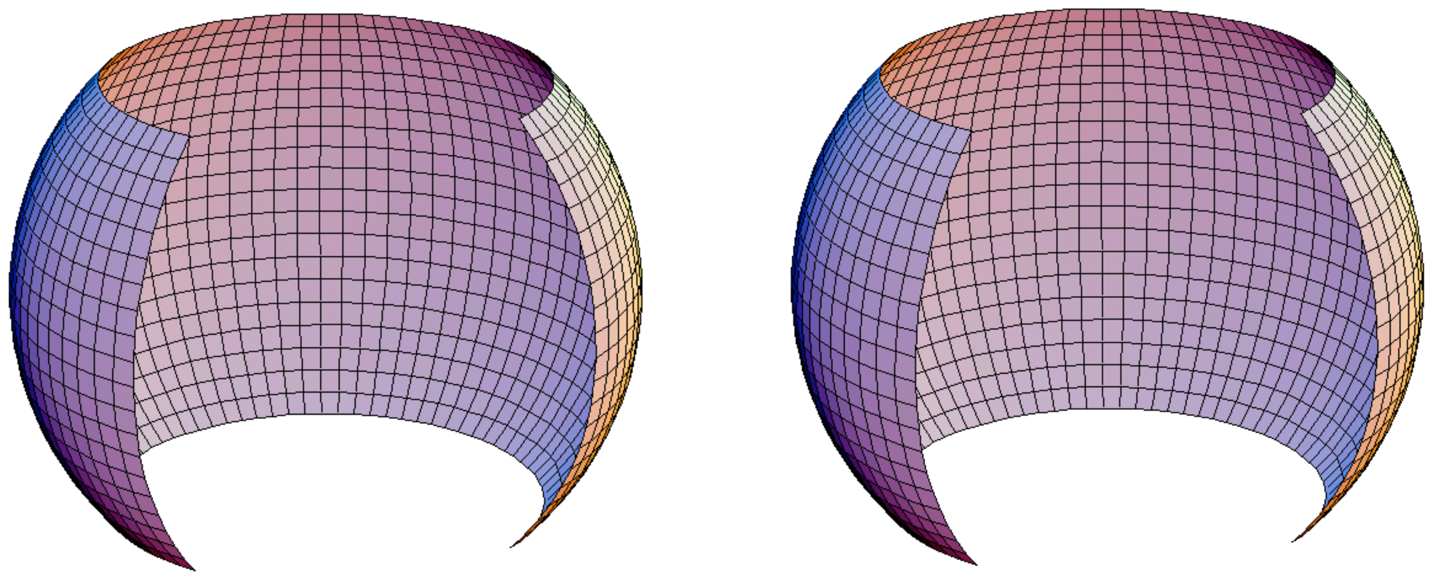
This picture reminds us a baseball!

Baseball

A spherical surface is covered by
a pair of two identical parts (patches).

It has only one seam.

Can we combine two identical component grids to cover a spherical surface, like the baseball?



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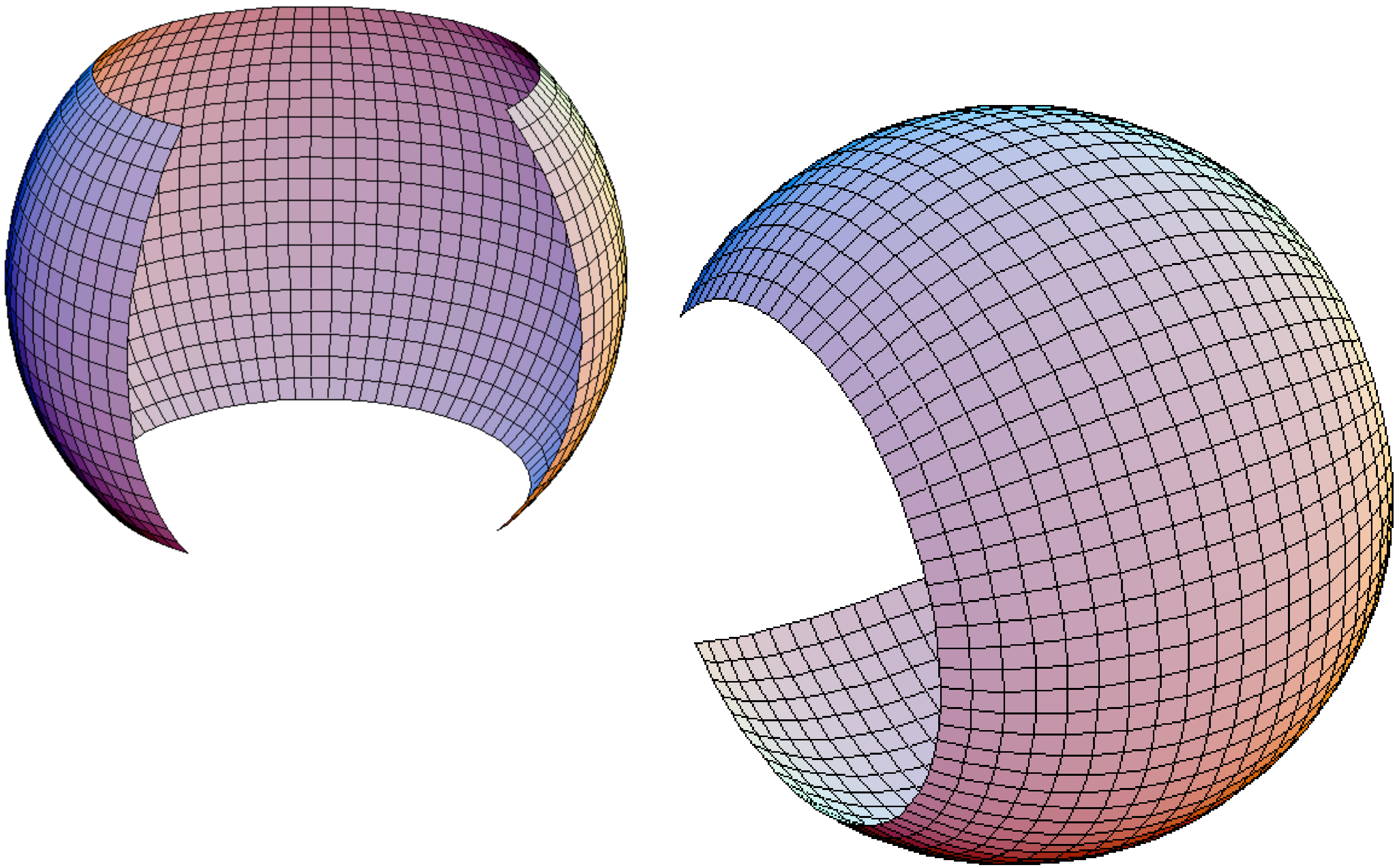
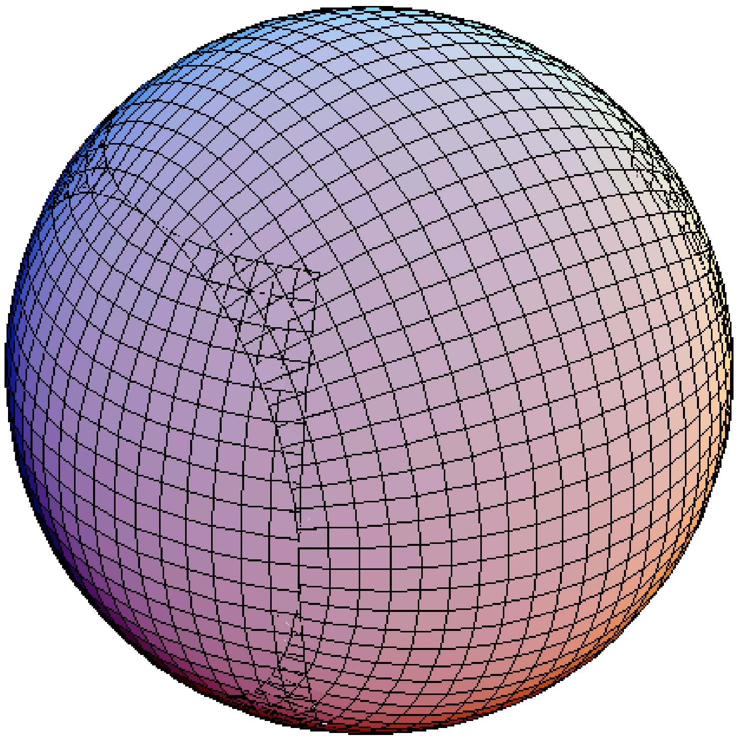
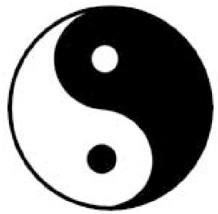


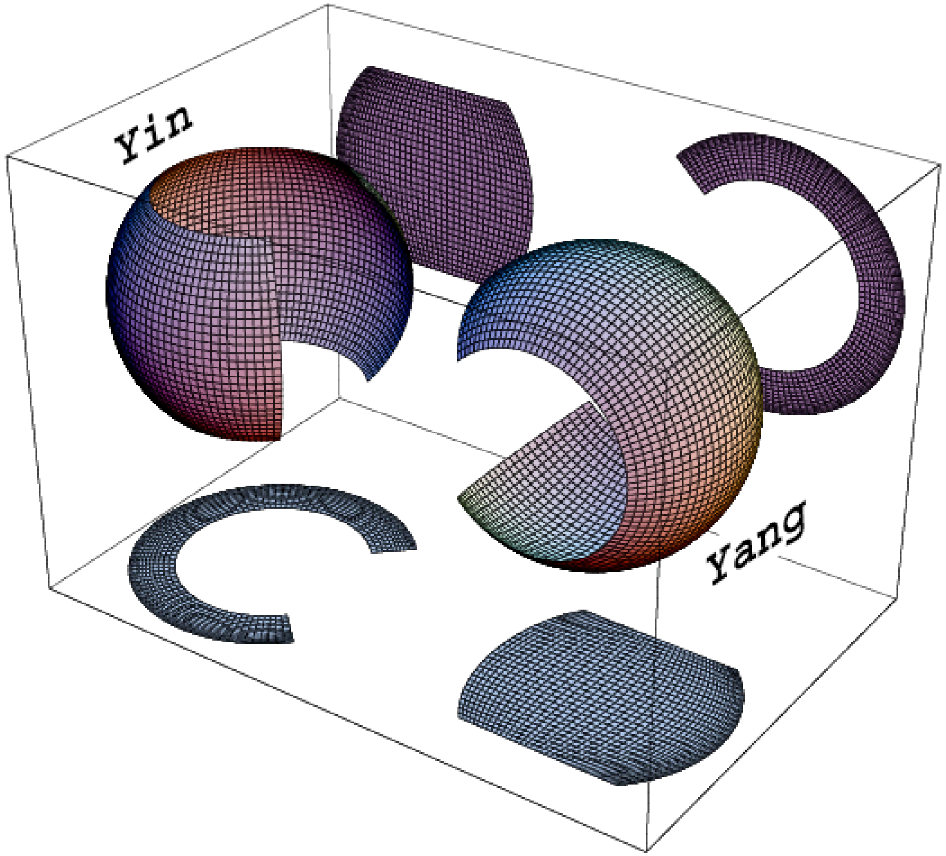
“Yin-Yang Grid”





The yin-yang symbol of complemental relation

Two Component Grids of Yin-Yang:
Yin grid & Yang grid



Suppose a point on the sphere with Yin (or n) coordinates given by

$$(x^n, y^n, z^n),$$

and with Yang (or e) coordinates given by

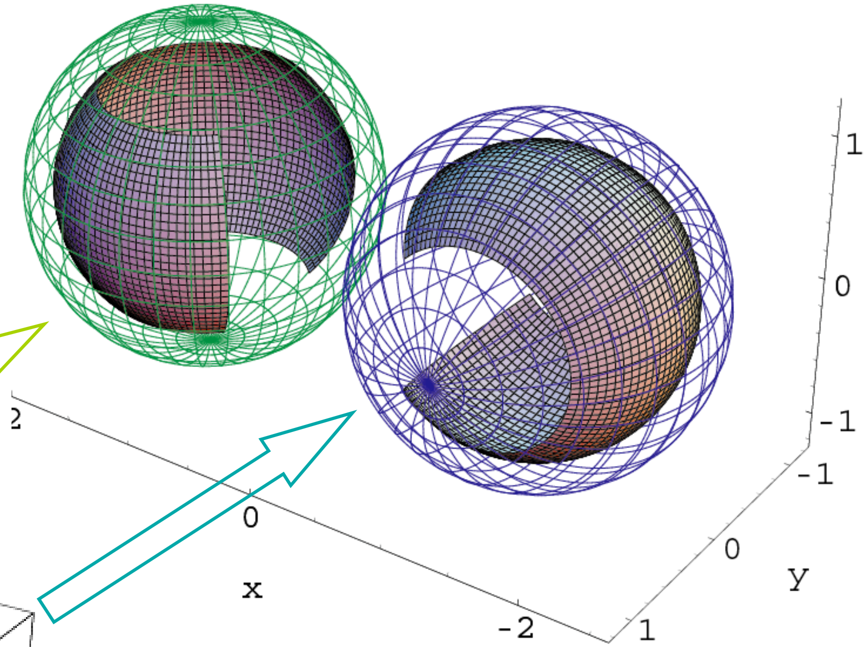
$$(x^e, y^e, z^e).$$

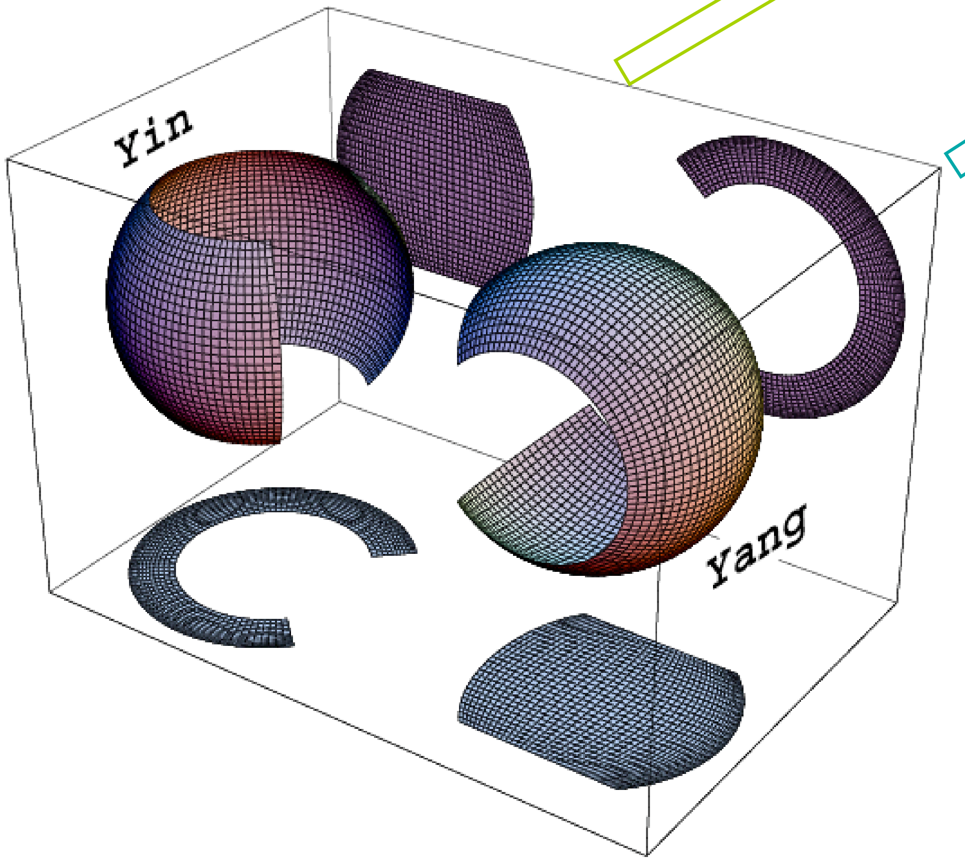
Their relation is given by

$$(x^n, y^n, z^n) = (-x^e, z^e, y^e),$$

or

$$(x^e, y^e, z^e) = (-x^n, z^n, y^n).$$

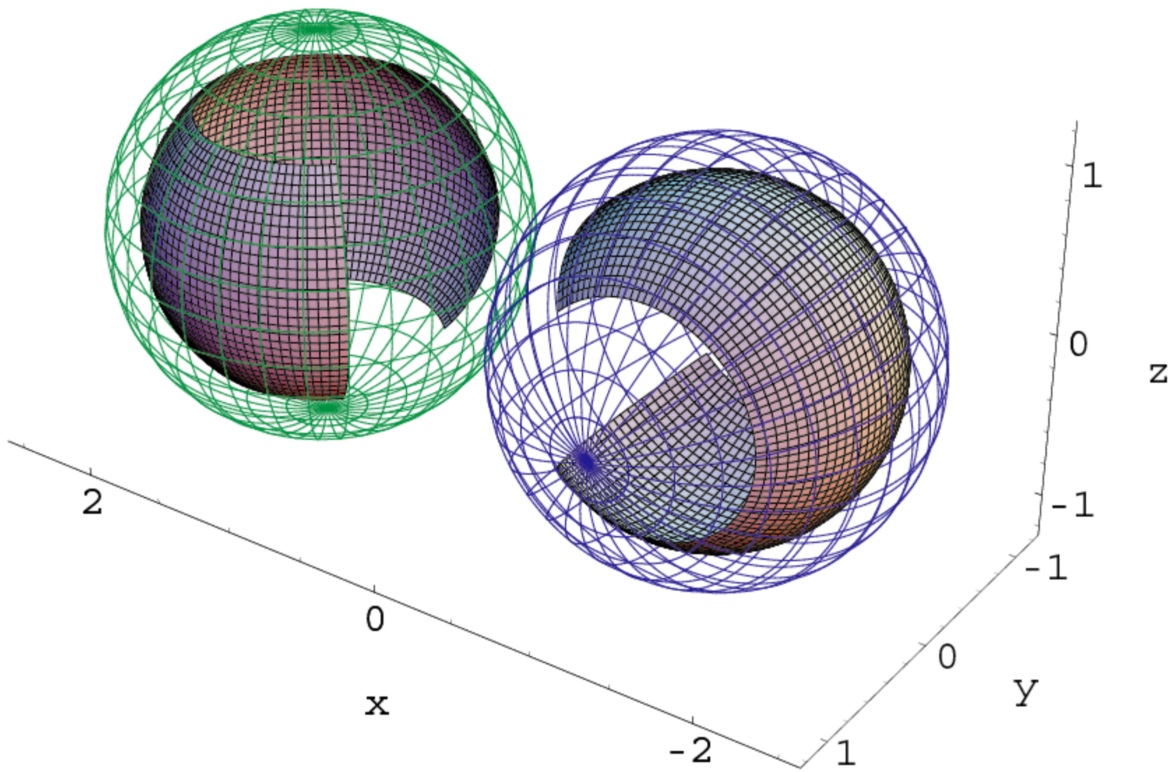




The coordinates relations are symmetric (complemental).

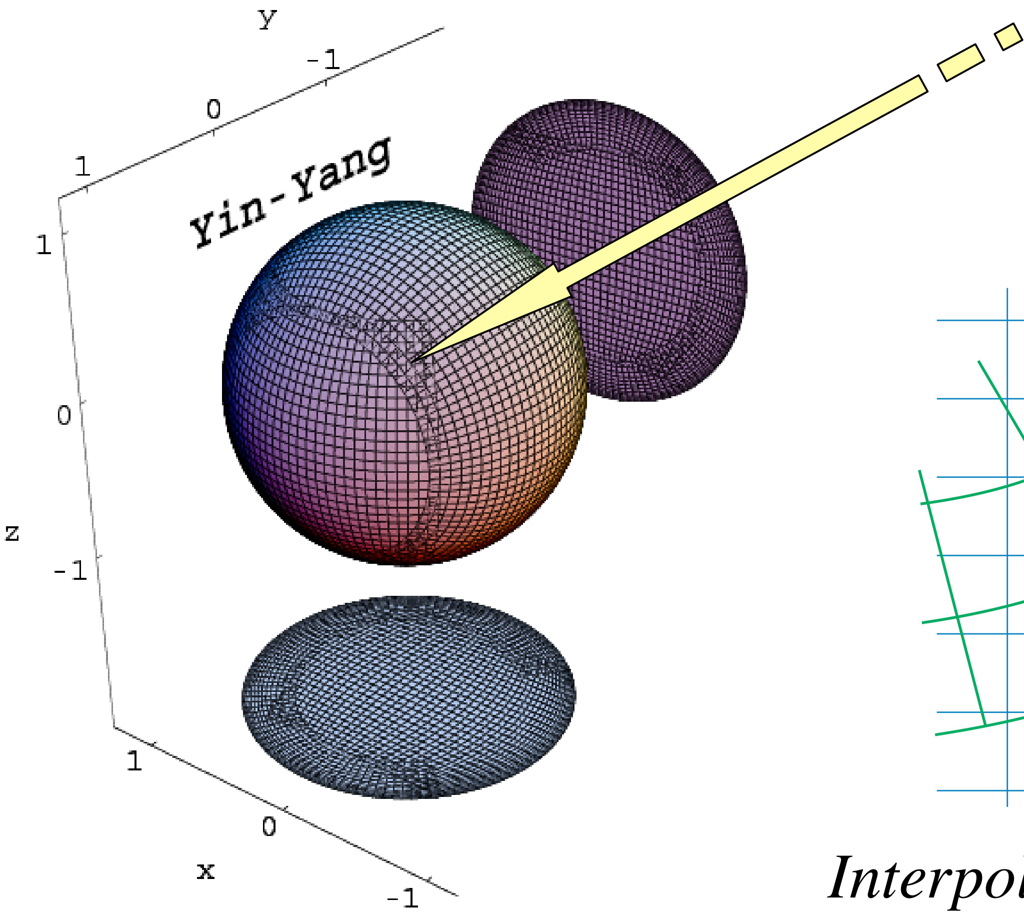
Concise Coding of Yin-Yang Grid

- Make one routine on the (partial) latitude-longitude grid.
- Recycle it for two times; one for Yin and one for Yang.

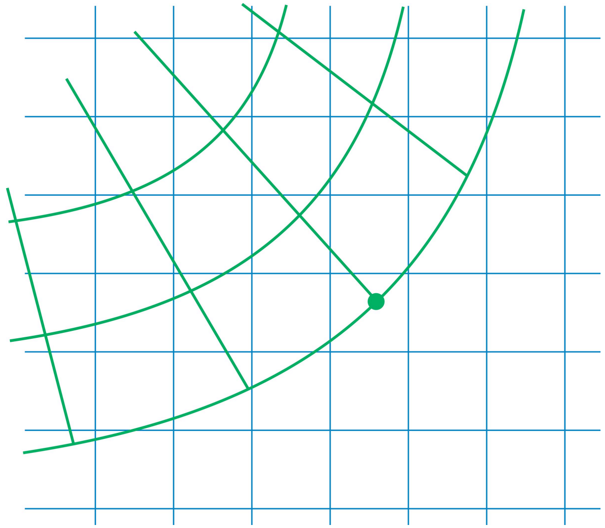


Routines for
- MHD solver
- boundary conditions
- interpolations

Yin-Yang grid is an Overset Grid in Spherical Geometry



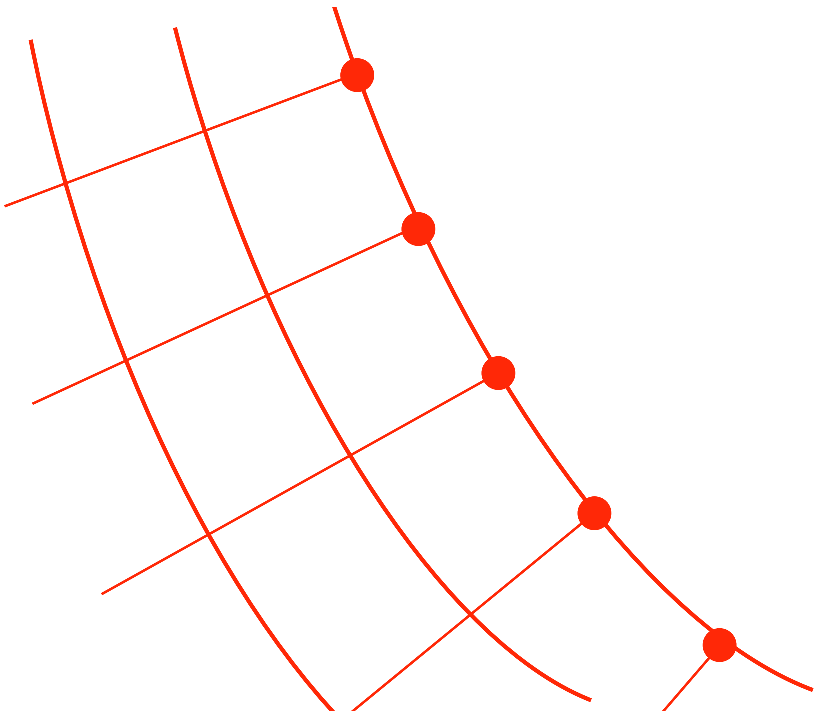
Partial overlap between Yin and Yang grids.



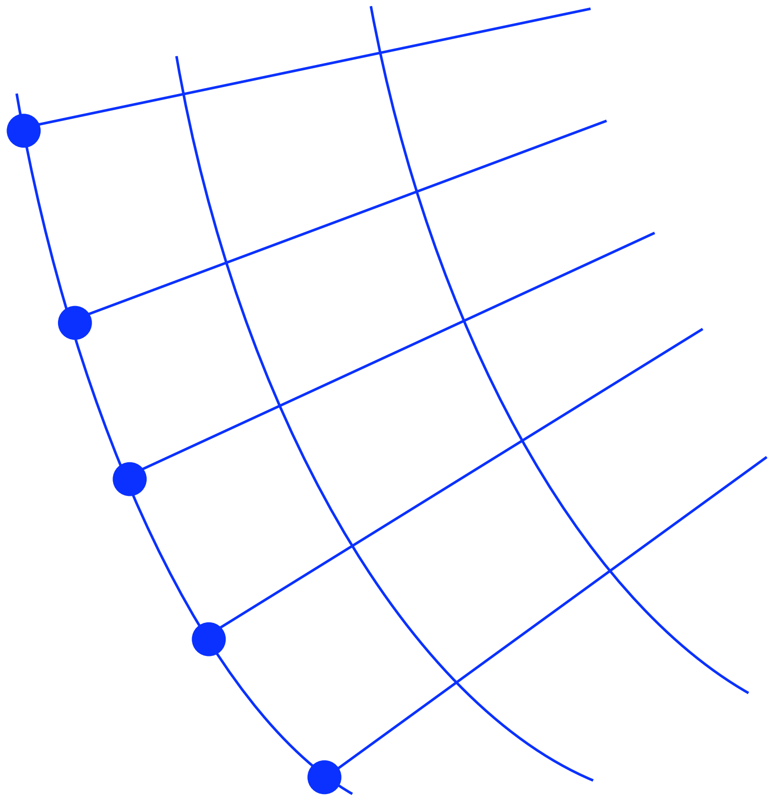
Interpolation

→ *boundary condition*

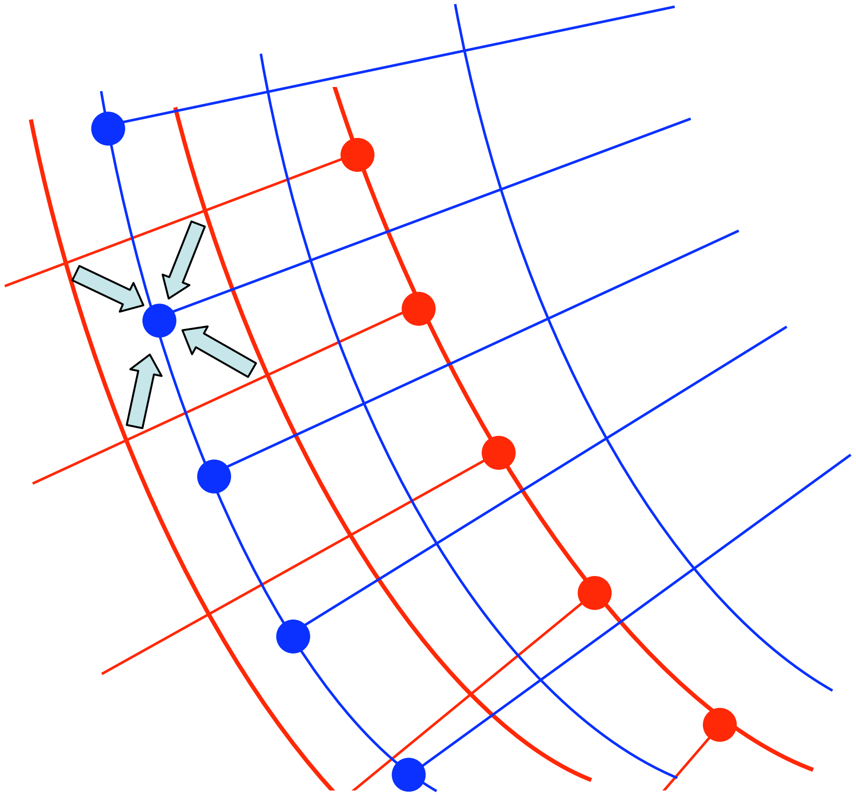
Two Independent Component Grids



Two independent grids



Overset grid (Chimera grid) method



- "Divide and conquer" approach

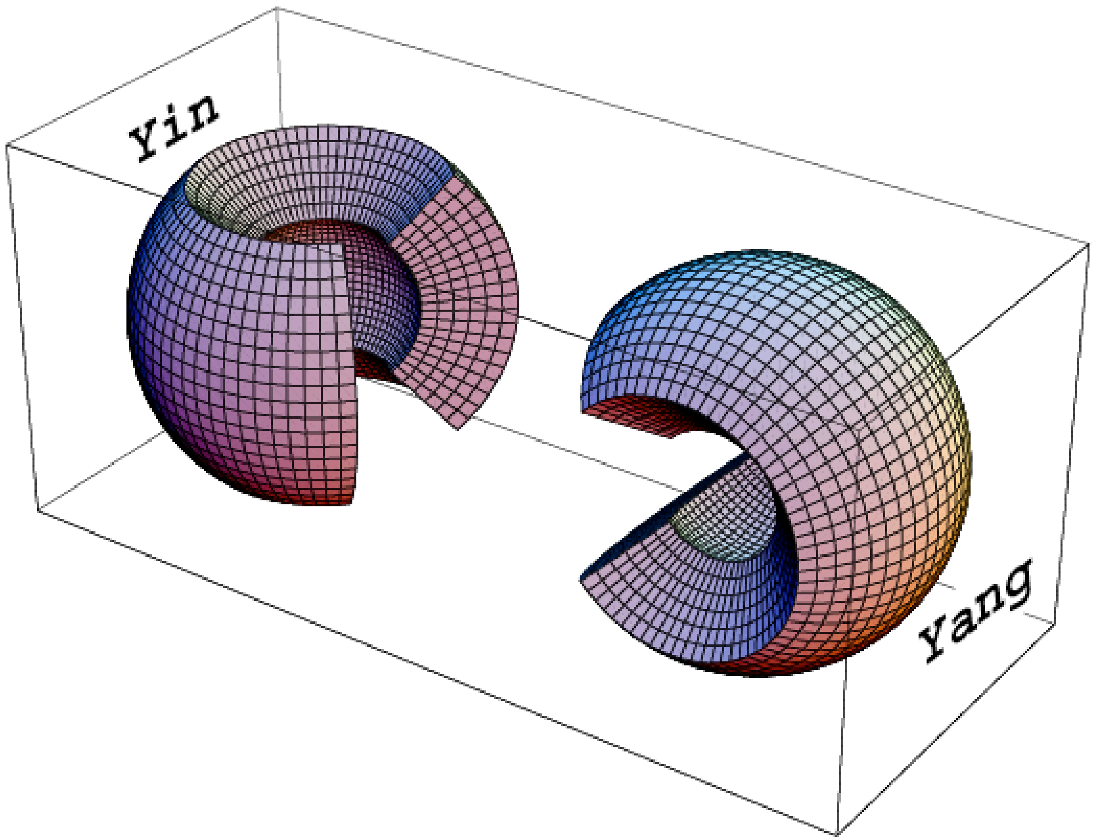
- Partially overlapped meshes.

- Setting boundary values by mutual interpolations.

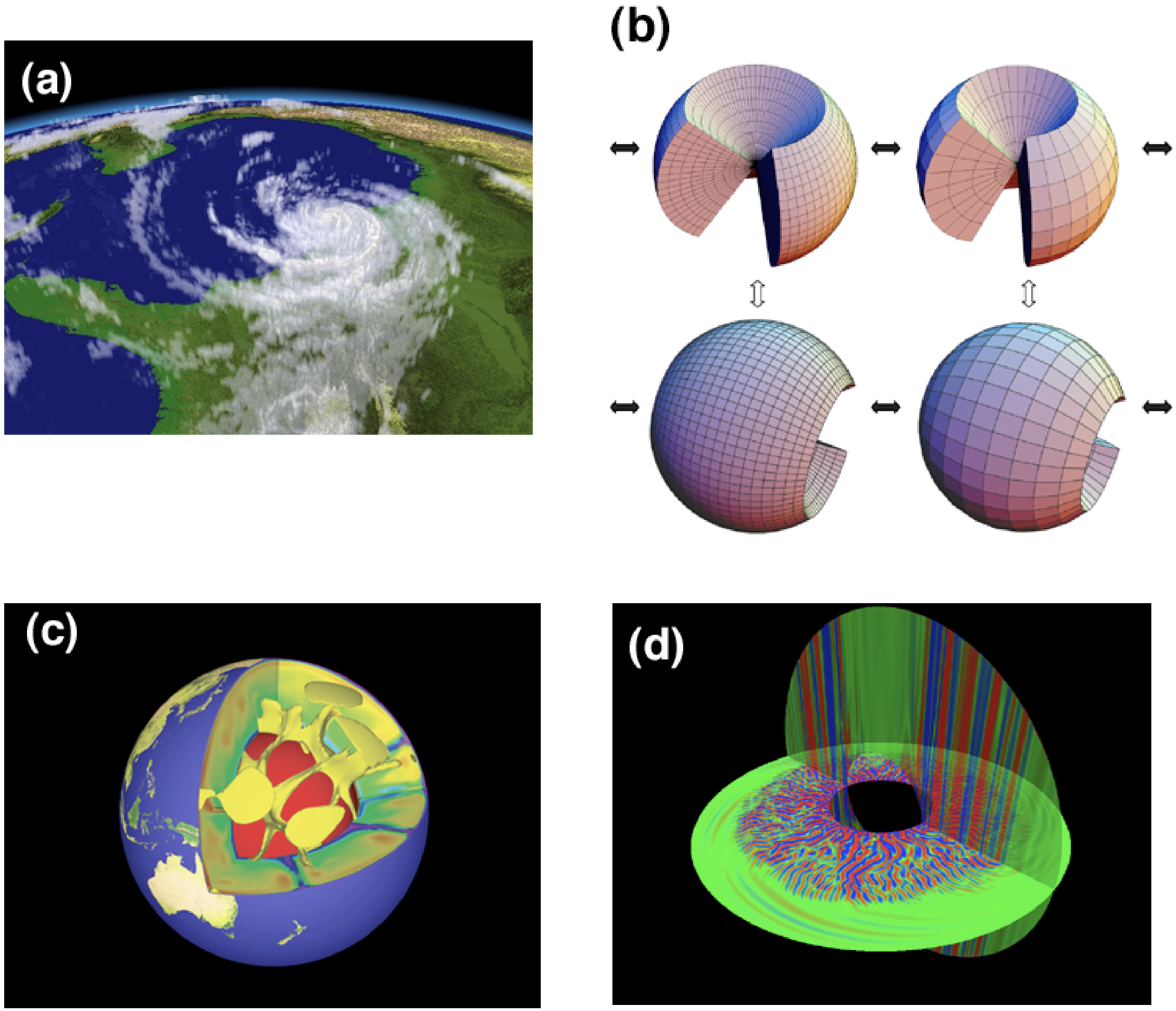
- Essentially parallel computation.

Chesshire, G., and W. D. Henshaw (1990),
Composite overlapping meshes for the solution of partial differential equations,
J. Comput. Phys., 90, 1－64,

3D Yin-Yang Grid for Spherical Shell Geometry

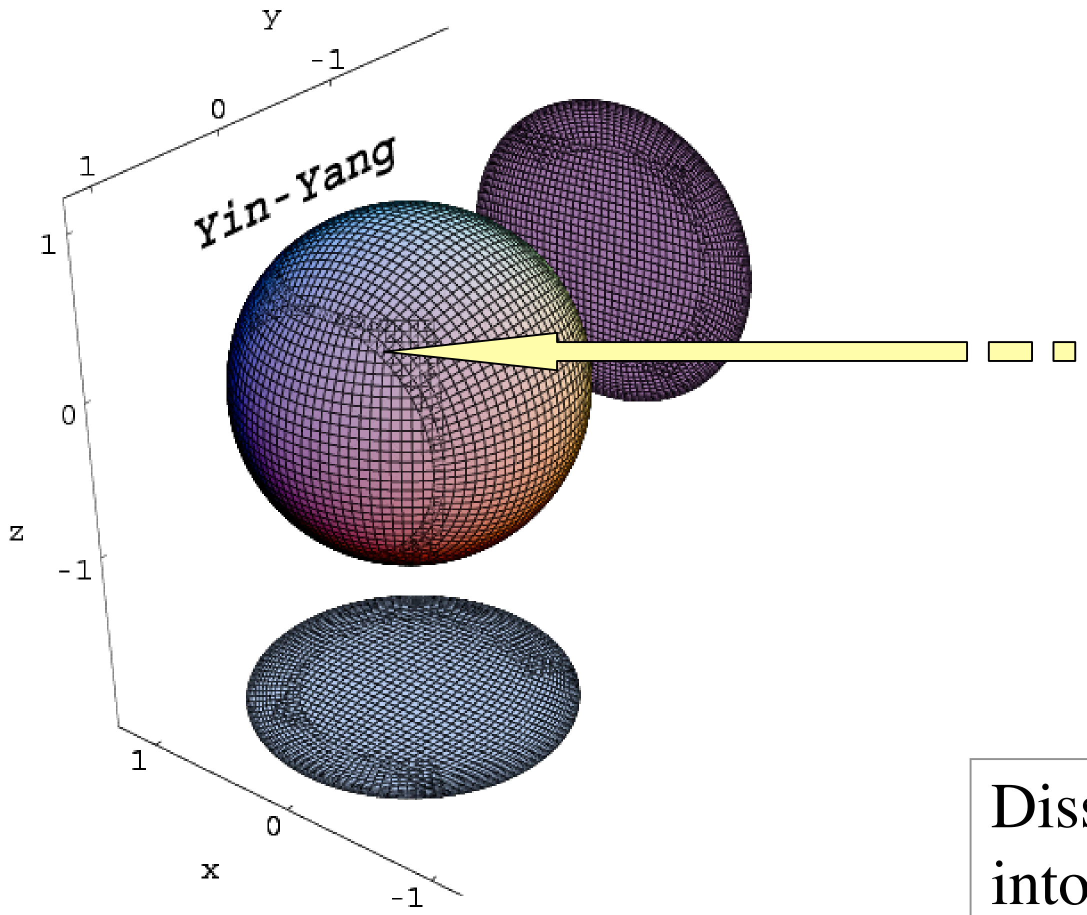


Applications of Yin-Yang Grid at Earth Simulator Center



(a) Coupled GCM of atmosphere & ocean
(b) Fast spherical Poisson eq. solver by multigrid on Yin-Yang grid
(c) Mantle convection simulation
(d) Geodynamo

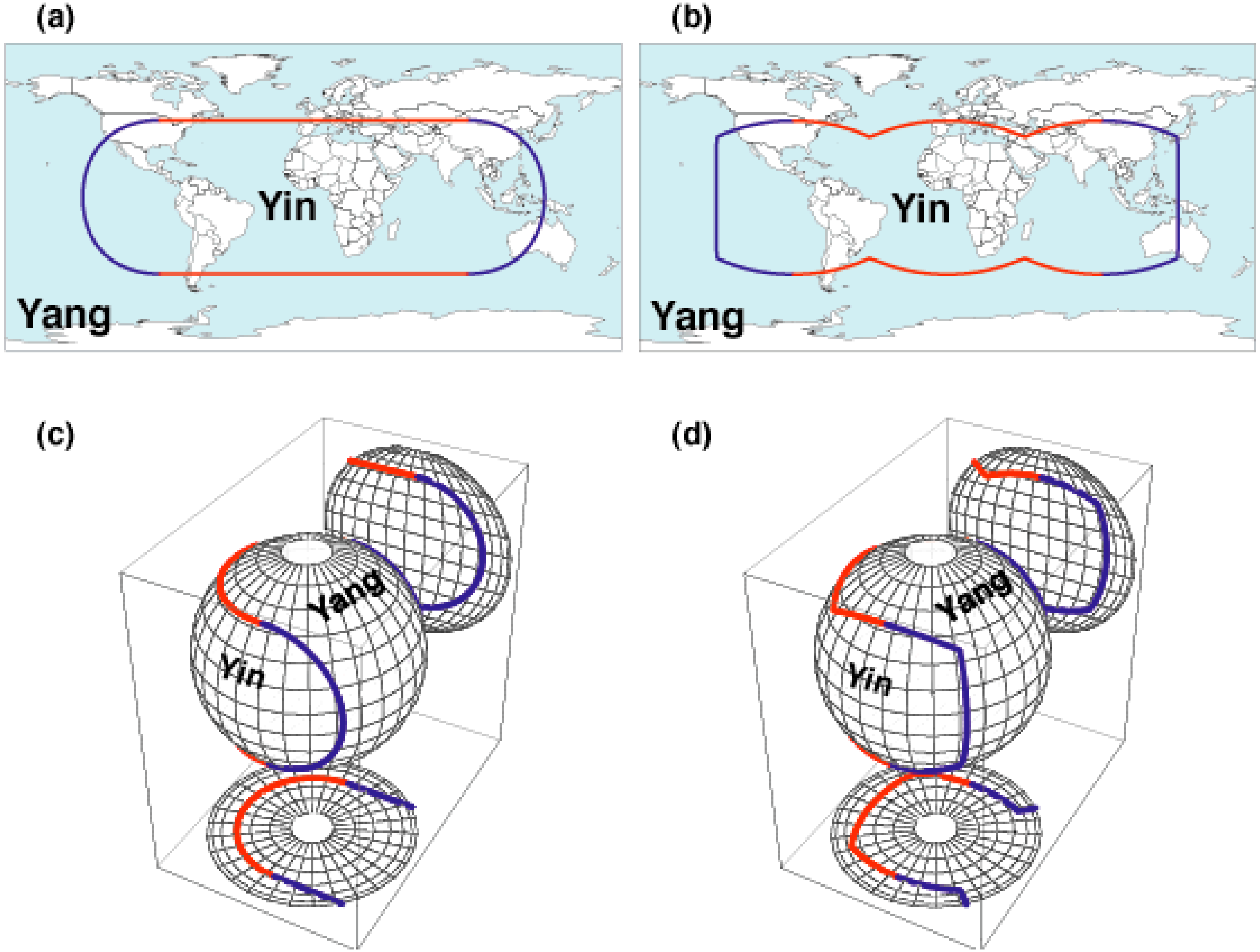
Yin-Yang Variations



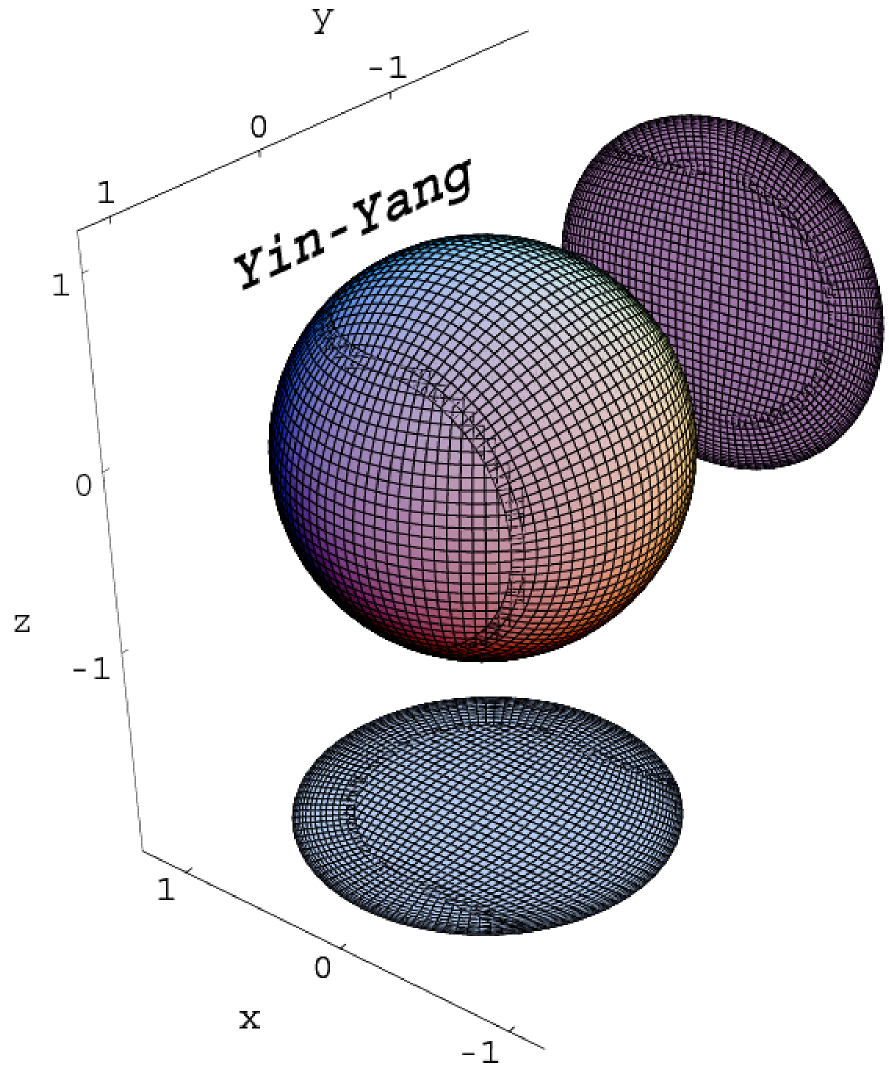
Minimize the partial overlap region.

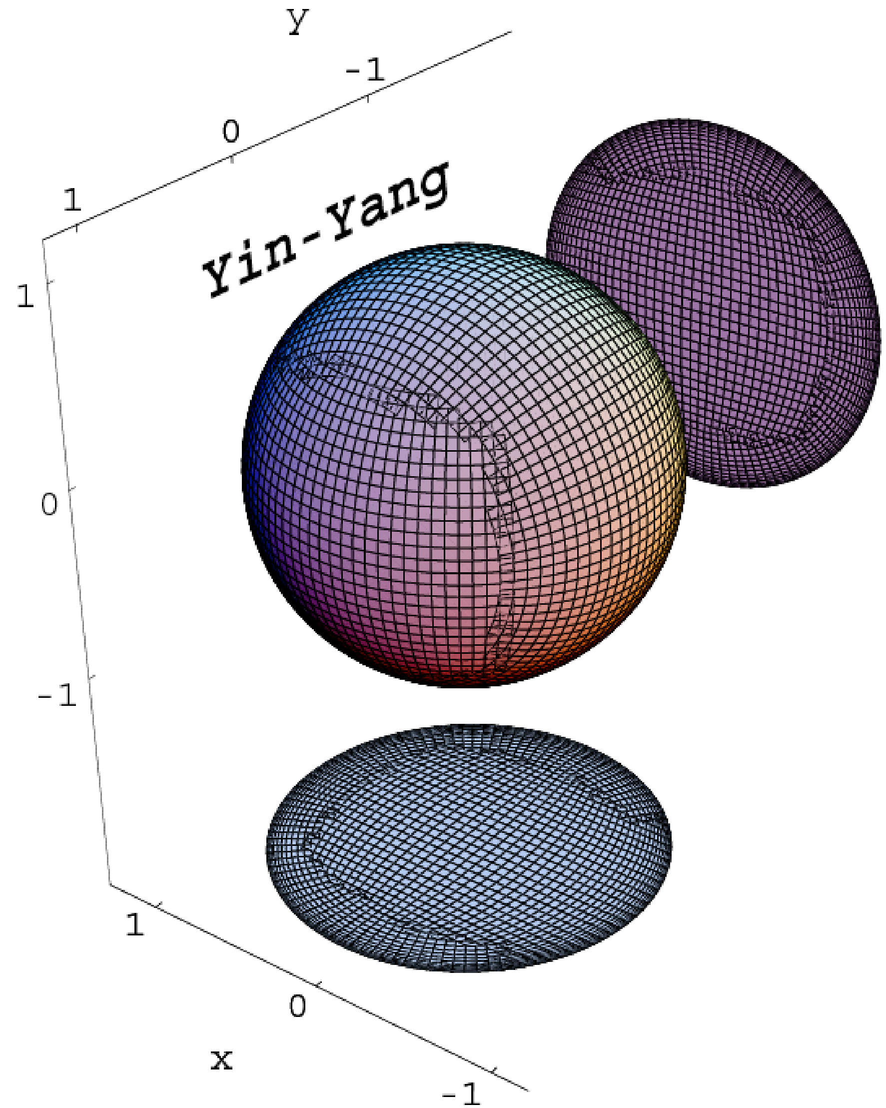
Dissection of a sphere into two identical parts.

Dissection of a Sphere into Two Identical Parts



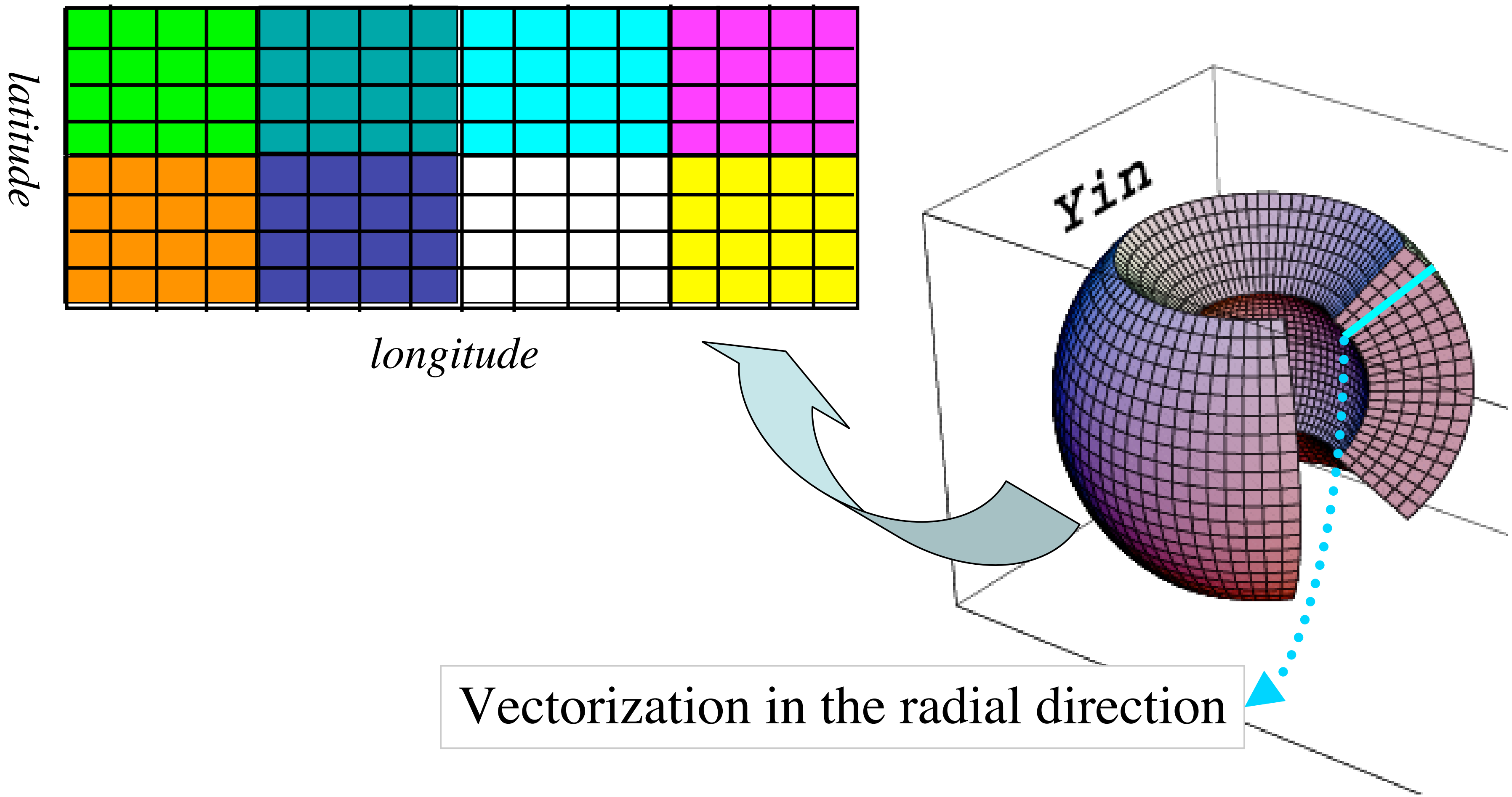
Yin-Yang grids with minimum overlap





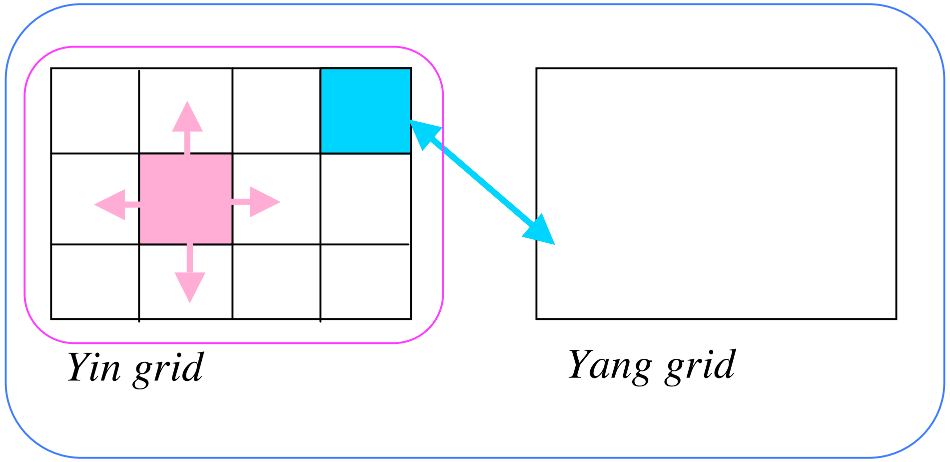
Vector-Parallel Processing on Yin-Yang Grid

2-dimensional domain decomposition in the horizontal directions.



Vectorization in the radial direction

2 MPI Communicators



(1) Overall (world) communicator

(2) Yin/Yang communicator

-Yin's communicator

-Yang's communicator

Performance of the Yin-Yang geodynamo code on ES

processors	grid points	Tflops	efficiency
4096	$511 \times 514 \times 1538 \times 2$	15.2	46%
3888	$511 \times 514 \times 1538 \times 2$	13.8	44%
3888	$255 \times 514 \times 1538 \times 2$	12.1	39%
2560	$511 \times 514 \times 1538 \times 2$	10.3	50%
2560	$255 \times 514 \times 1538 \times 2$	9.17	45%
1200	$255 \times 514 \times 1538 \times 2$	5.40	56%

Flat MPI

Performance Comparison of Simulations on ES

Paper	Shingu[16]	Yokokawa[20]	Sakagami[15]	Komatitsch[8]	Kageyama et al.
Flops/PN	26.6T/640	16.4T/512	14.9T/512	5T/243	15.2T/512
efficiency	65%	50%	45%	32%	46%
grid points (g.p.)	7.1×10^8	8.6×10^9	1.7×10^{10}	5.5×10^9	8.1×10^8
g.p./AP	1.4×10^5	2.1×10^6	4.2×10^6	2.8×10^6	2.1×10^5
Flops/g.p.	38K	1.9K	0.87K	0.91K	19K
Simulation kind	fluid	fluid	fluid	wave propagation	fluid
Field	atmosphere	turbulence	inertial fusion	seismic wave	geodynamo
Method	spectral	spectral	finite volume	spectral element	finite difference
Parallelization	MPI-microtask	MPI-microtask	HPF (flat MPI)	flat MPI	flat MPI

TABLE III: Performances on the Earth Simulator reported at SC

↑ Yin-yang geodyamo code

Rules I took in the Code Development

- Avoid global communication
- Keep the model simple and symmetric
  - Grid system ==> Yin-Yang grid
  - Parallelization ==> Flat MPI
    - Use MPI for both intra- and inter- nodes communications
    - If inter-node network speed is high enough, this is the simplest programming model for the programmer.

No Global Communication

- Trade off
  - Un(geo)physical model
    - Boundary condition of the magnetic field
      - Magnetic field has only radial component
    - Compressibility
      - Most in the community use the Boussinesq model
- Effects of compressibility should be negligible when Mach number is small.
- We have compressible mode, but we effectively reduced the sound wave speed.

Simulation Parameters

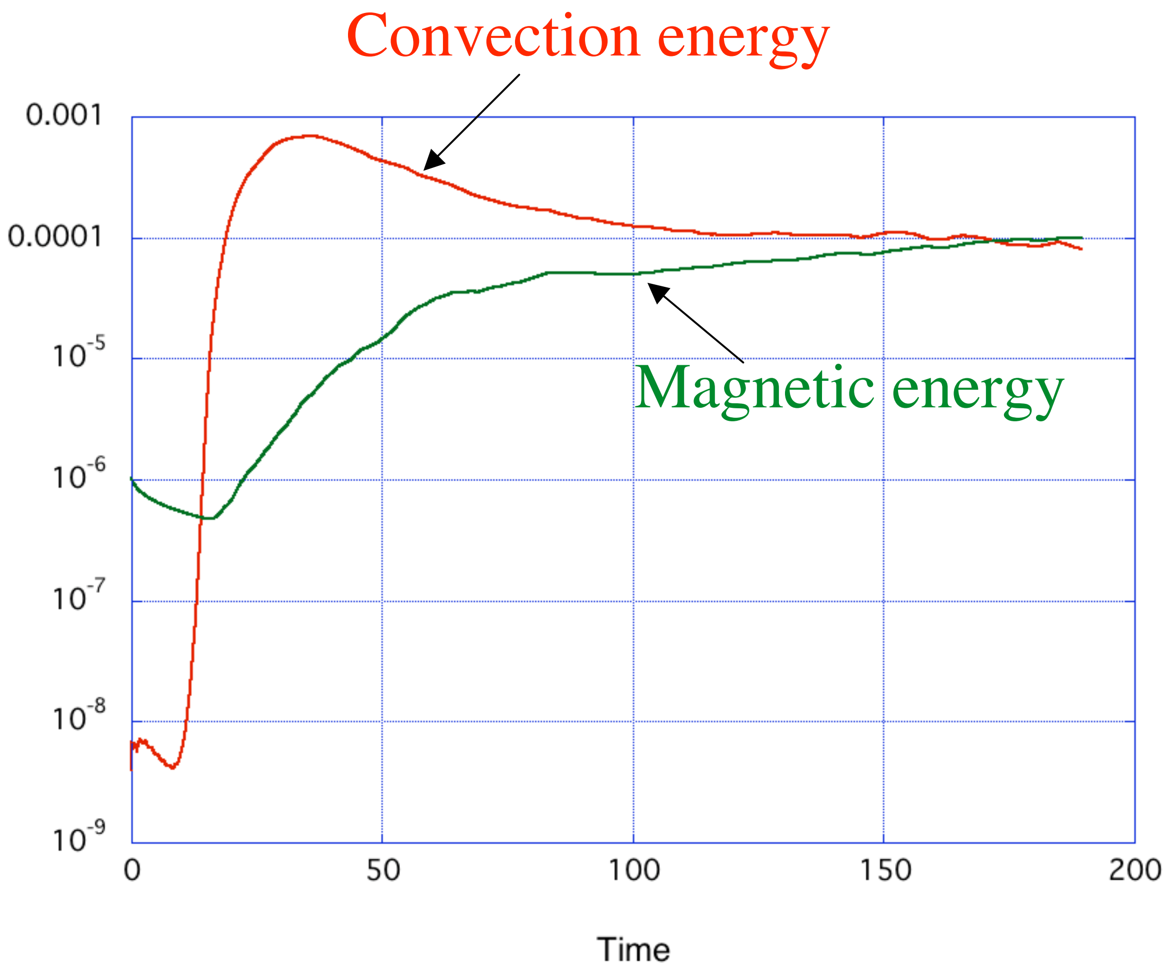
- Grid mesh (Yin-Yang grid)

$$N_r \times N_\theta \times N_\phi \times 2 = 511 \times 514 \times 1538 \times 2$$

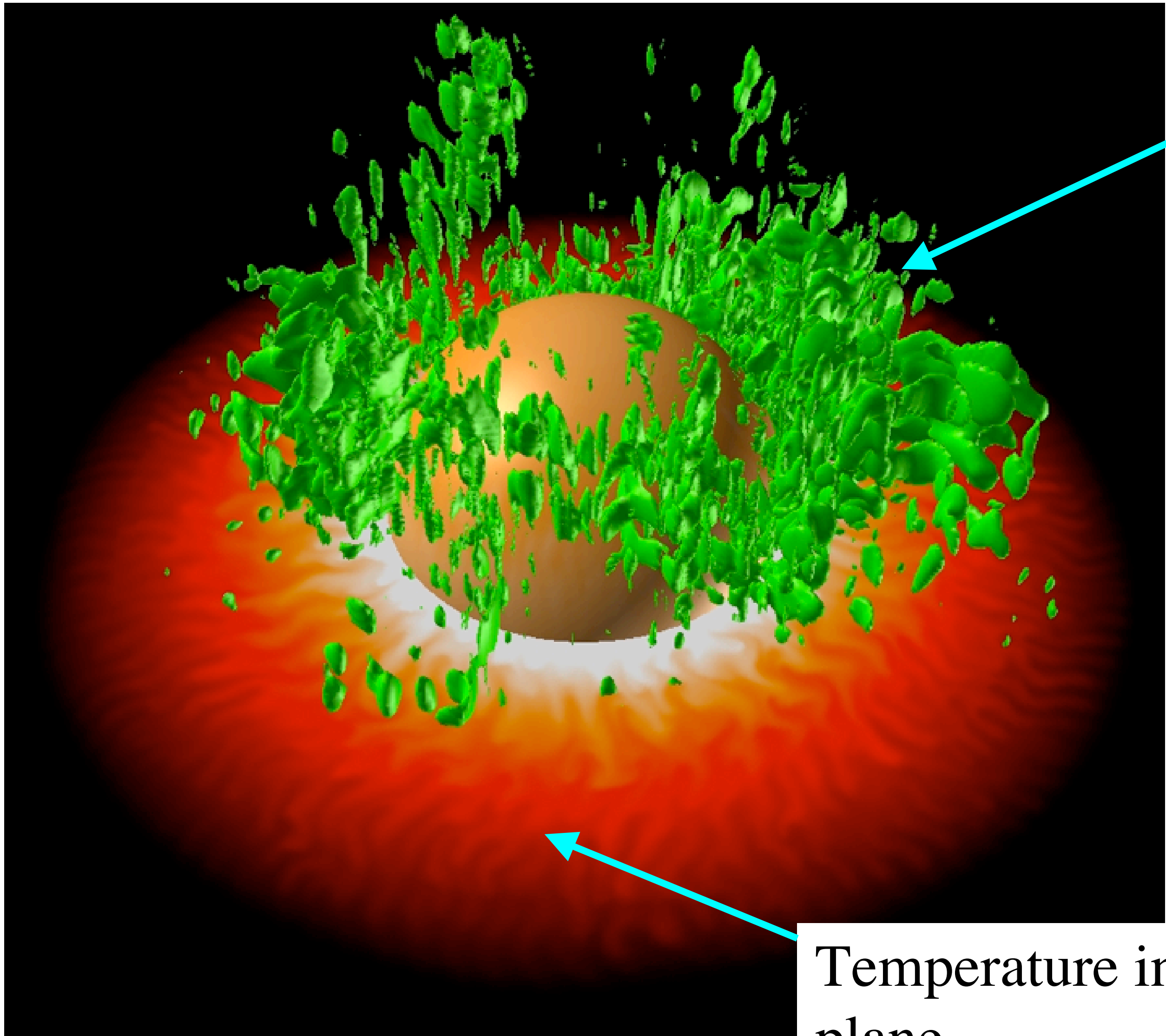
For 360 degree equator ==> 2045 grid points
==> on the core surcface: 1 grid = 10.75km

- Rayleigh number —— $Ra = 2 \times 10^8$
- Prandtl number —— $Pr = 1$
- Magnetic Prandtl number —— $Pm = 1$
- Ekman number —— $E = 4.6 \times 10^{-7}$

Time Development of Energies



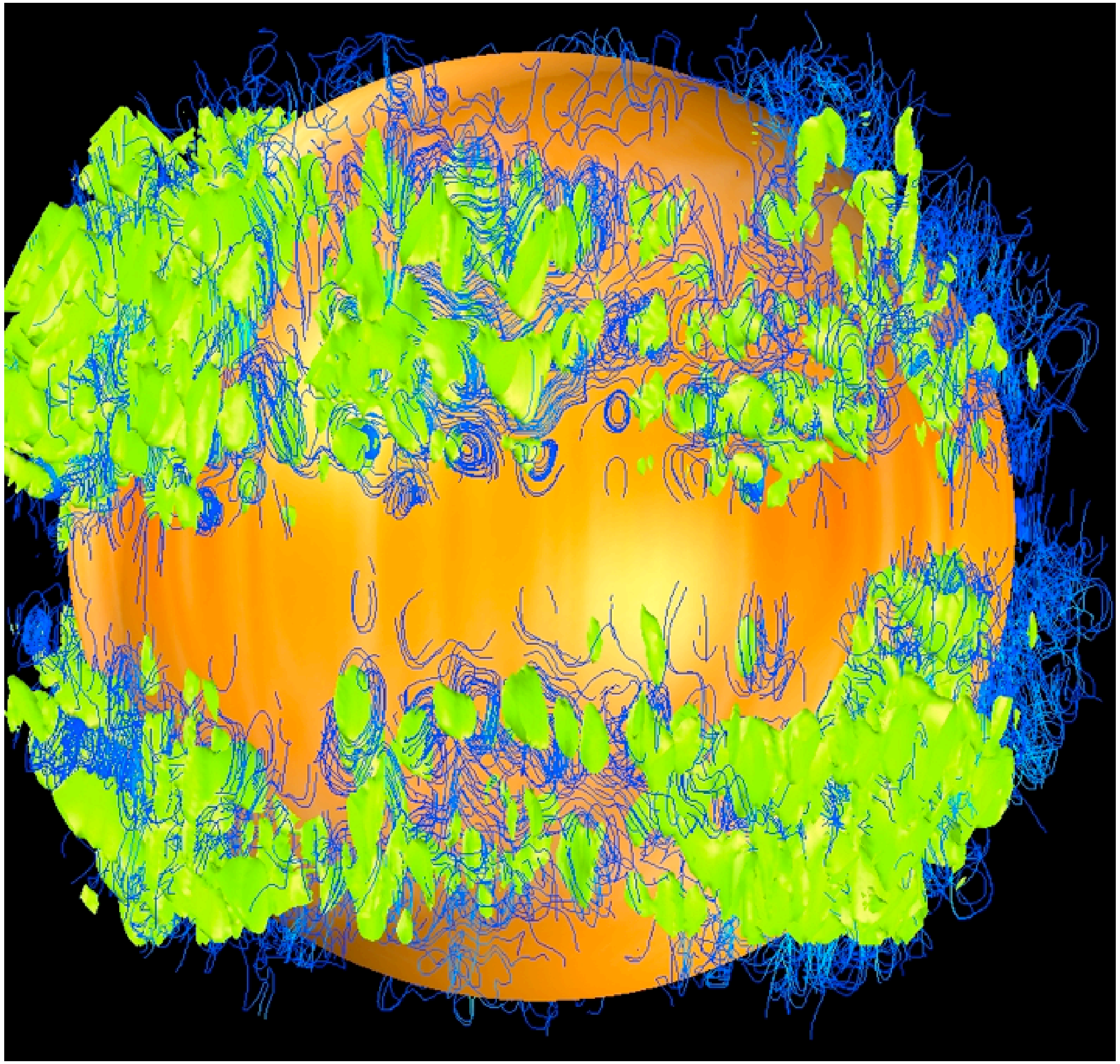
Distribution of Dynamo Source D



$$D = -\mathbf{v} \cdot (\mathbf{j} \times \mathbf{B})$$

Temperature in the equatorial plane

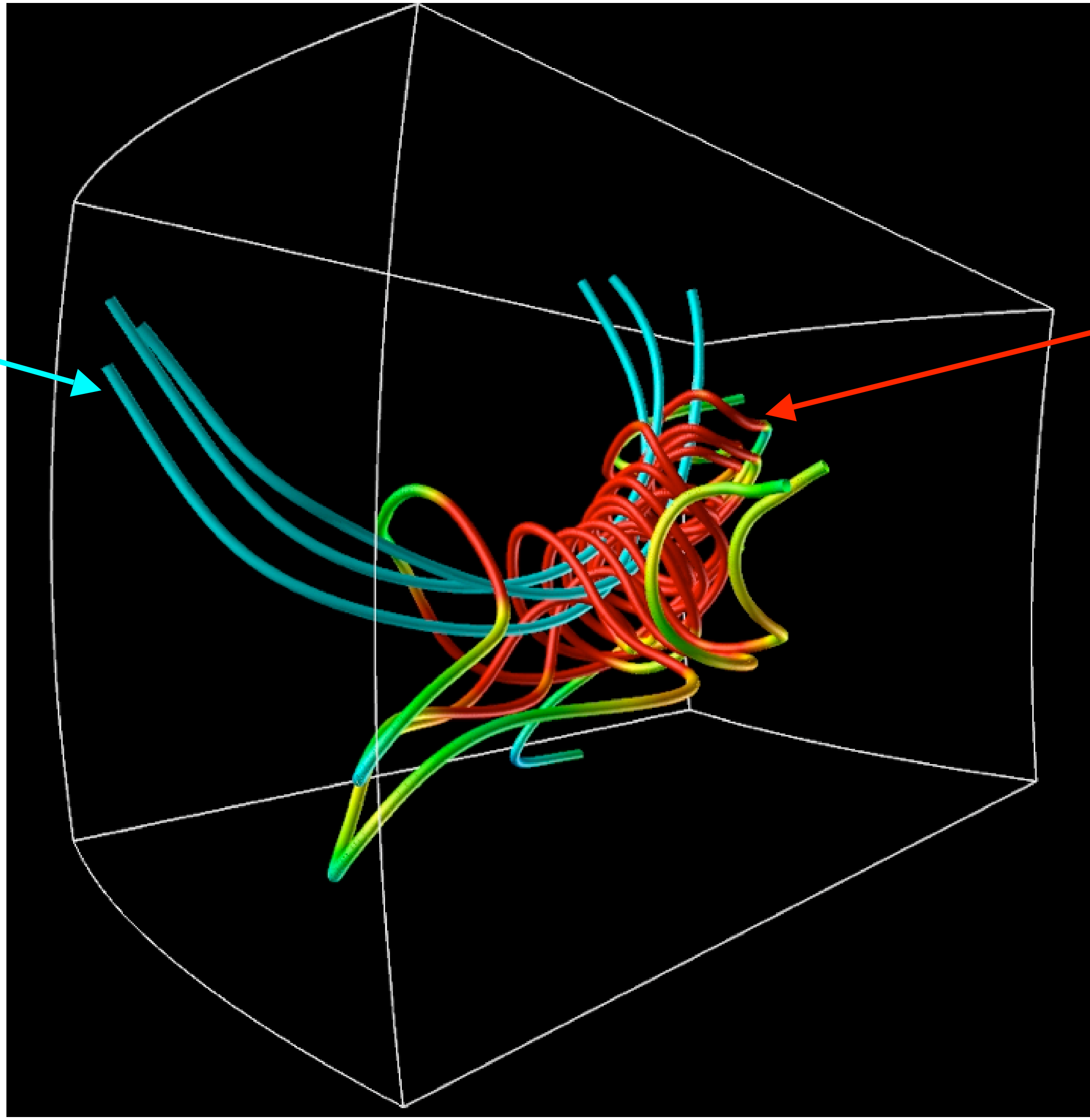
Distribution of *D* and ***j***



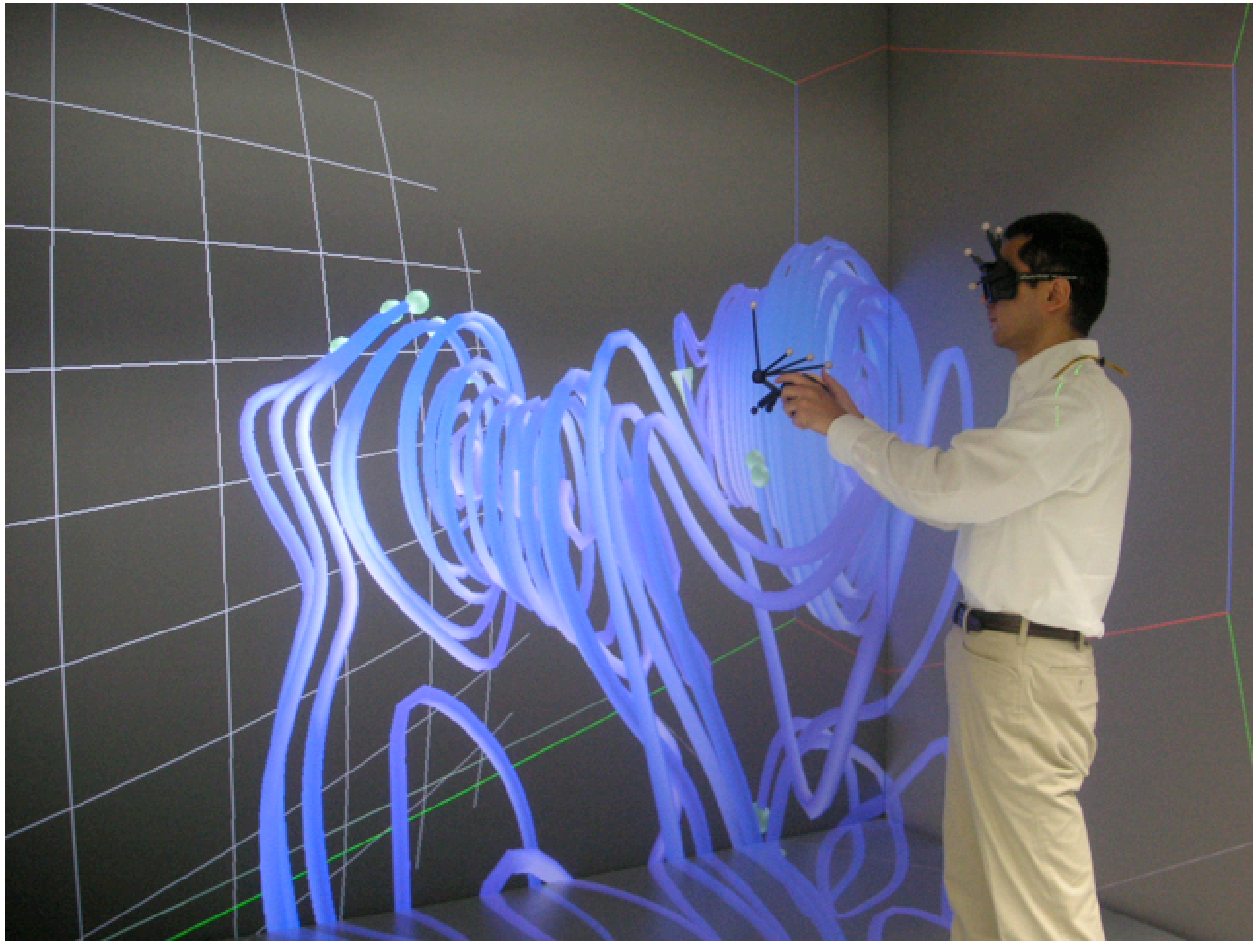
Dynamo source D (green) and electric current lines (blue).

Current coils

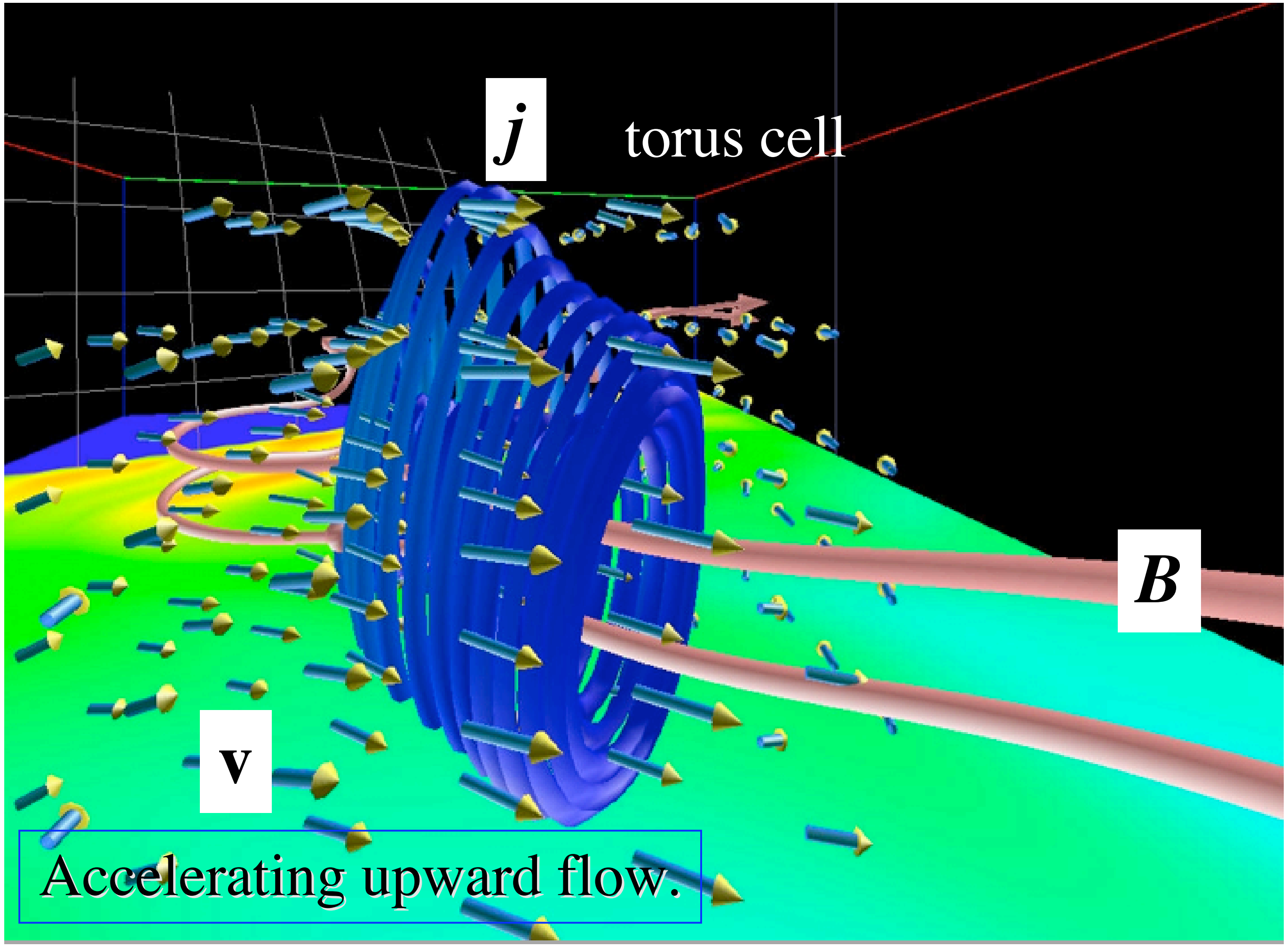
Current Coil and Magnetic Field Lines



Virtual Reality Visualization

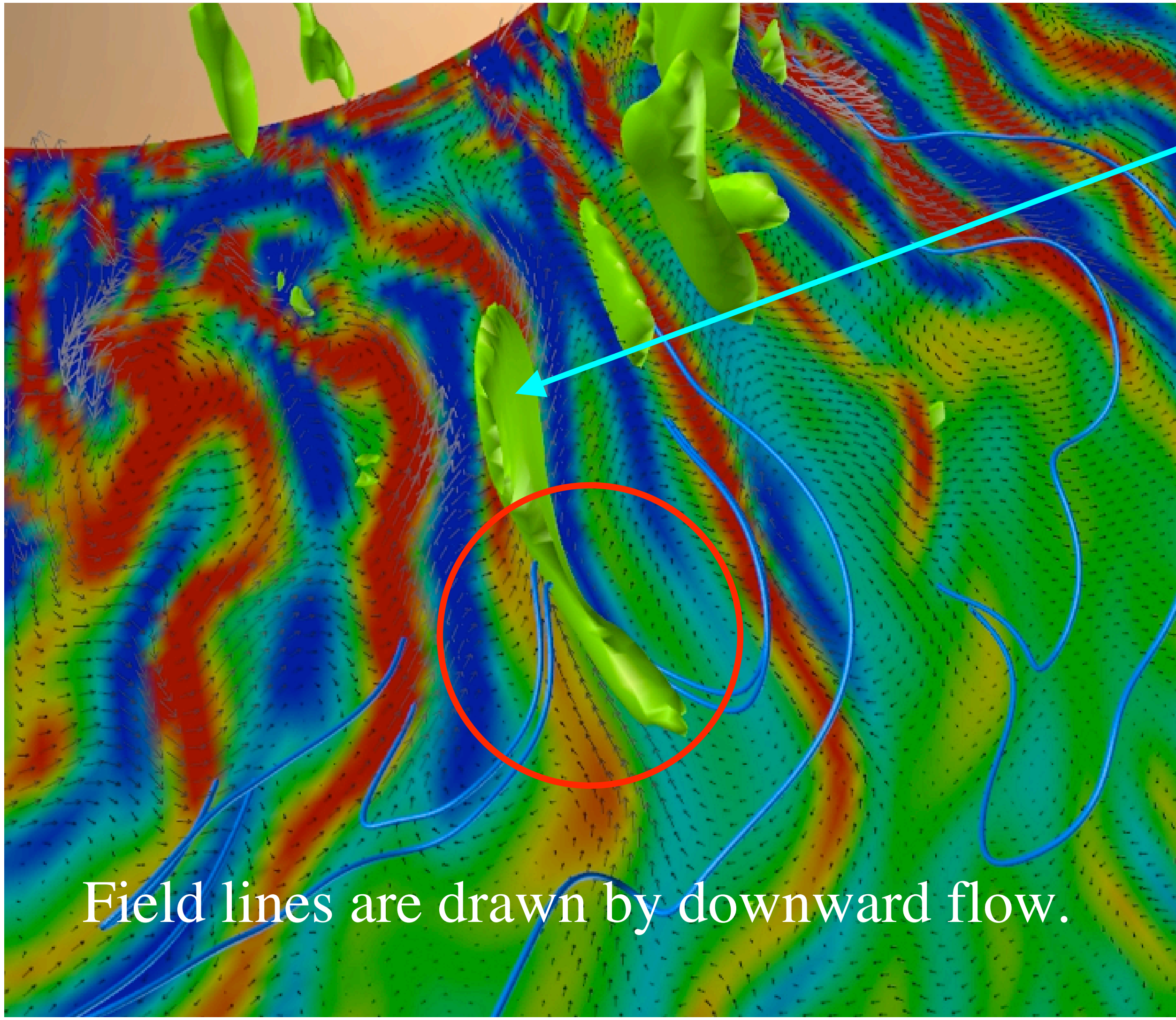


Dynamo Mechanism



Dynamo due to "field line stretching" by upward flow parallel to the magnetic field lines.

Another Dynamo by Downward Flow

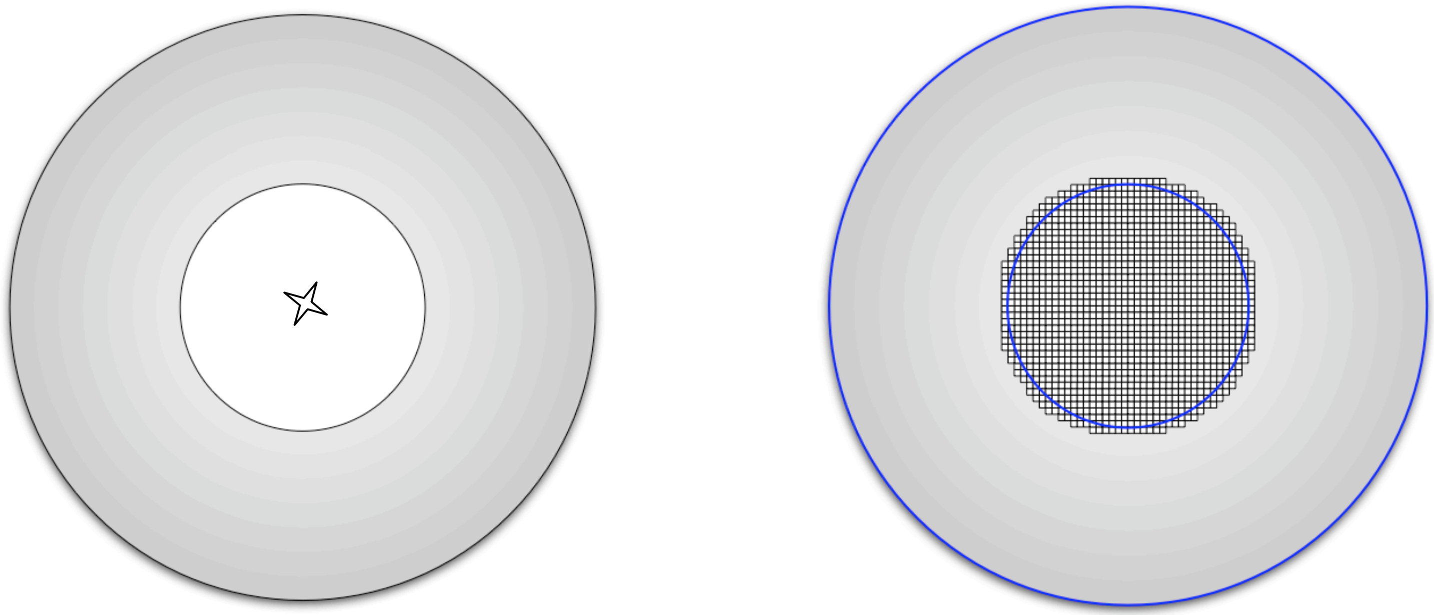


Green:
dynamo source *D*

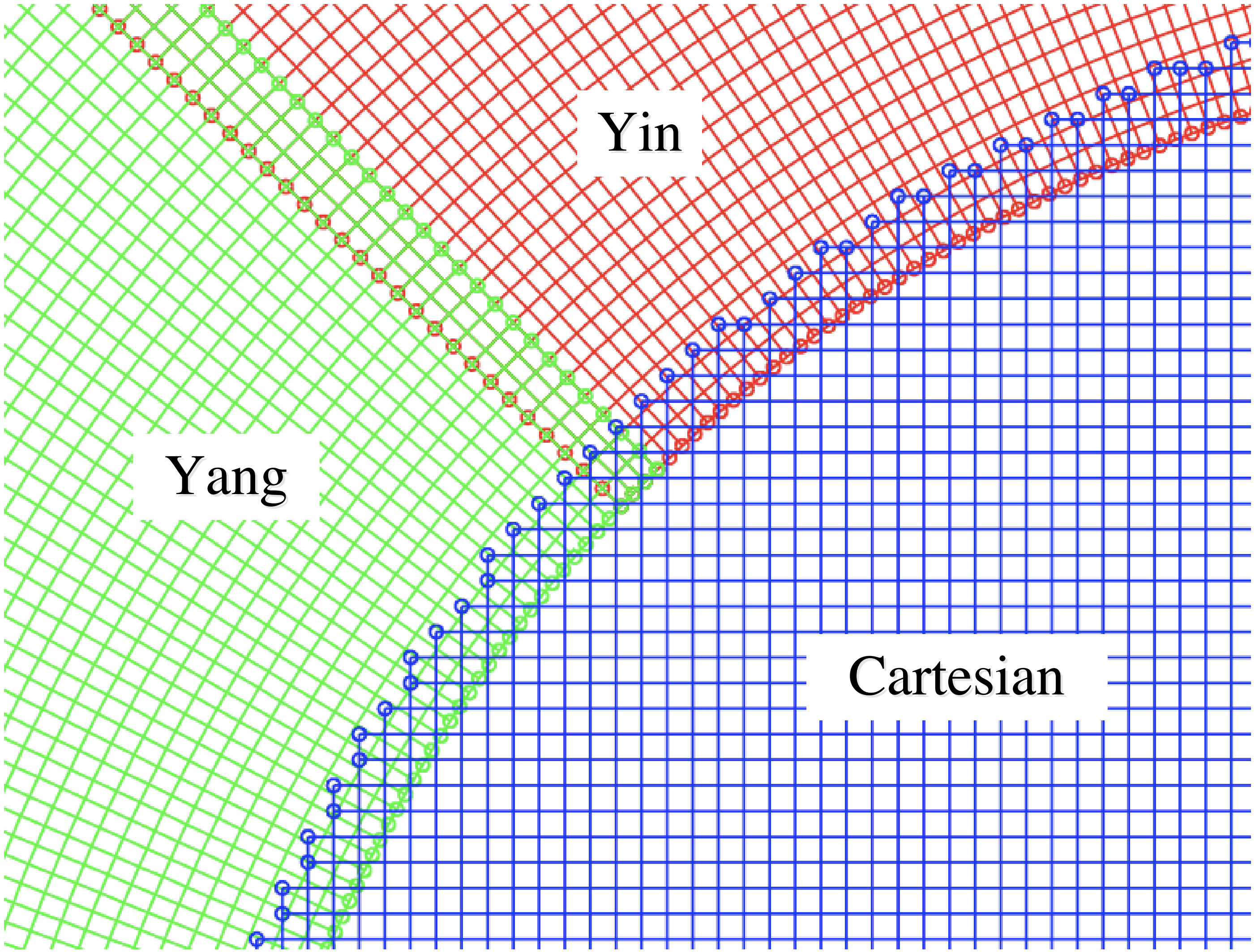
Recent Development of Yin-Yang Grid

- Inner core

  - Another coordinate singularity at the origin.
  - Chimera method for Yin-Yang and Cartesian.



Recent Development of Yin-Yang Grid



Summary and Lessons

- For a new computer of new architecture, re-design the simulation method from the beginning
  - Yin-Yang grid enables us to perform max node run on the Earth Simulator.
  - No other (spectral-based) geodynamo code cannot run on maximum nodes.
- Yin-Yang geodynamo code has led us to a new dynamo regime.
  - Lowest Ekman number ever achieved.
  - Current coils with straight flux tubes.
- For massively parallel computation… Avoid global communication.

To Avoid the Global Communication

- Re-formulate the model to reduce the maximum signal speed of the system.
- Nature has no global communication anyway. (Speed of light.)
- My idea: Go back or change the basic formulation to have no global communication, and reduce the “speed of the light” of the system.