Application of Statistical Approaches in Past Temperature Reconstruction

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PART I


Uncertainties
in Past Temperature Reconstruction
-Ensemble reconstruction
and
Bayesian Reconstrution
CLIMATE
CHANGE

Why care about the PAST temperature?

- Long time series of climate variables including temperature are required to understand the dynamics of climate change
- Direct observations of surface temperature is only available from 1850

How to get past temperatures?

Reconstruct the past temperature from indirect observations (proxies) such as Tree Ring, Pollen and Borehole and Radiative Forcings

A stats perspective

The problem:

Quantify the uncertainty in the temperature estimates from other kinds of observations (i.e proxies).

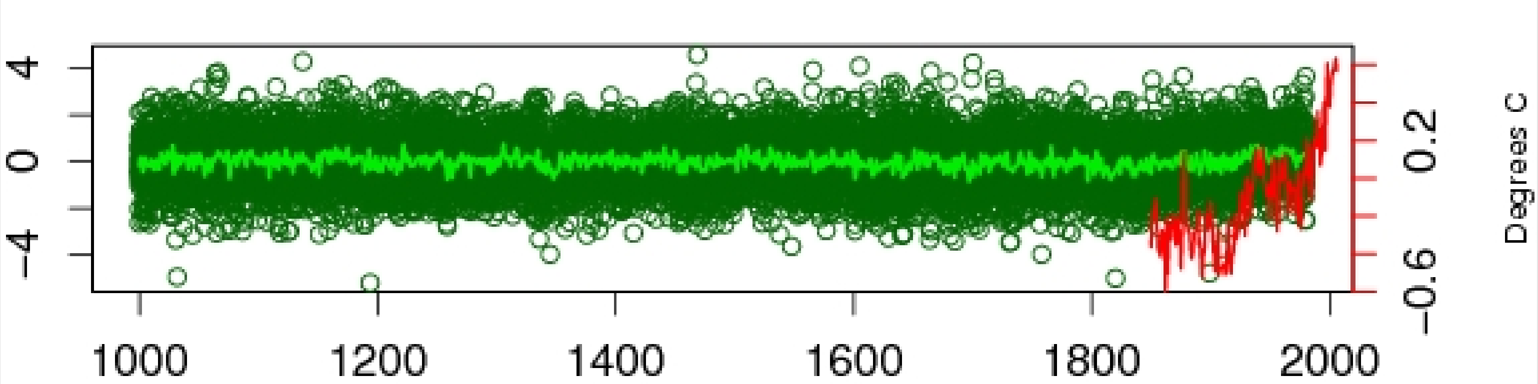
e.g. What is the uncertainty in the maximum decadal temperature estimates for the last 1000 years?

A statistical solution:

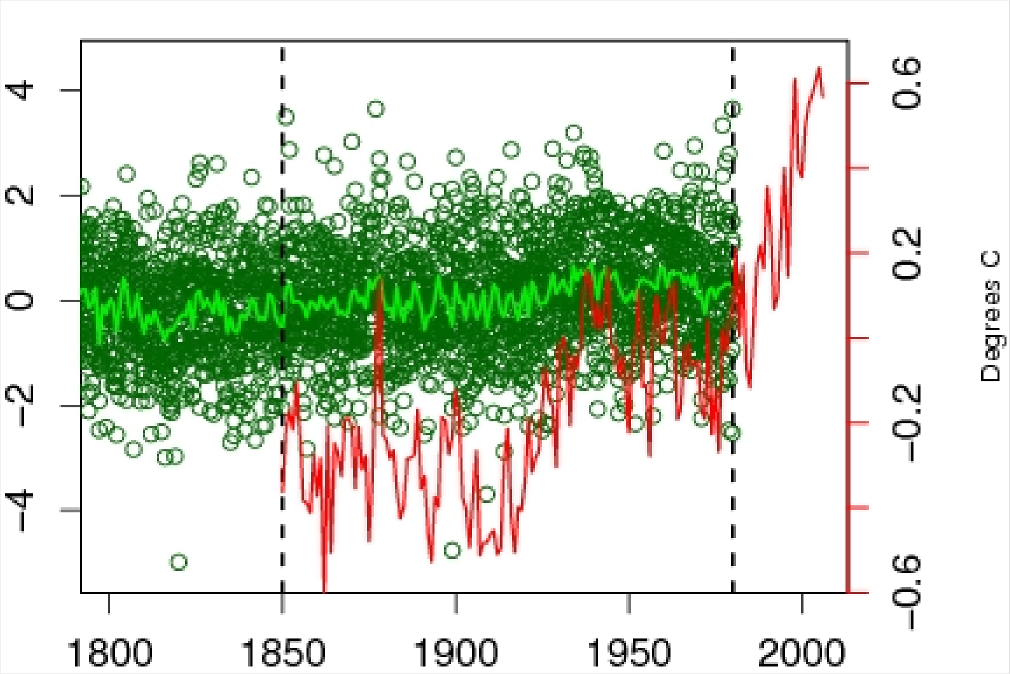
Find the distribution of the temperatures given the other observations.

Represent this distribution by an ensemble of possible reconstructions all statistically valid.

Long Climate Proxies and NH temperatures



Exploit where there is overlap in two sets of data.

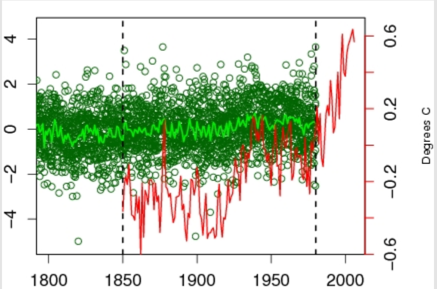


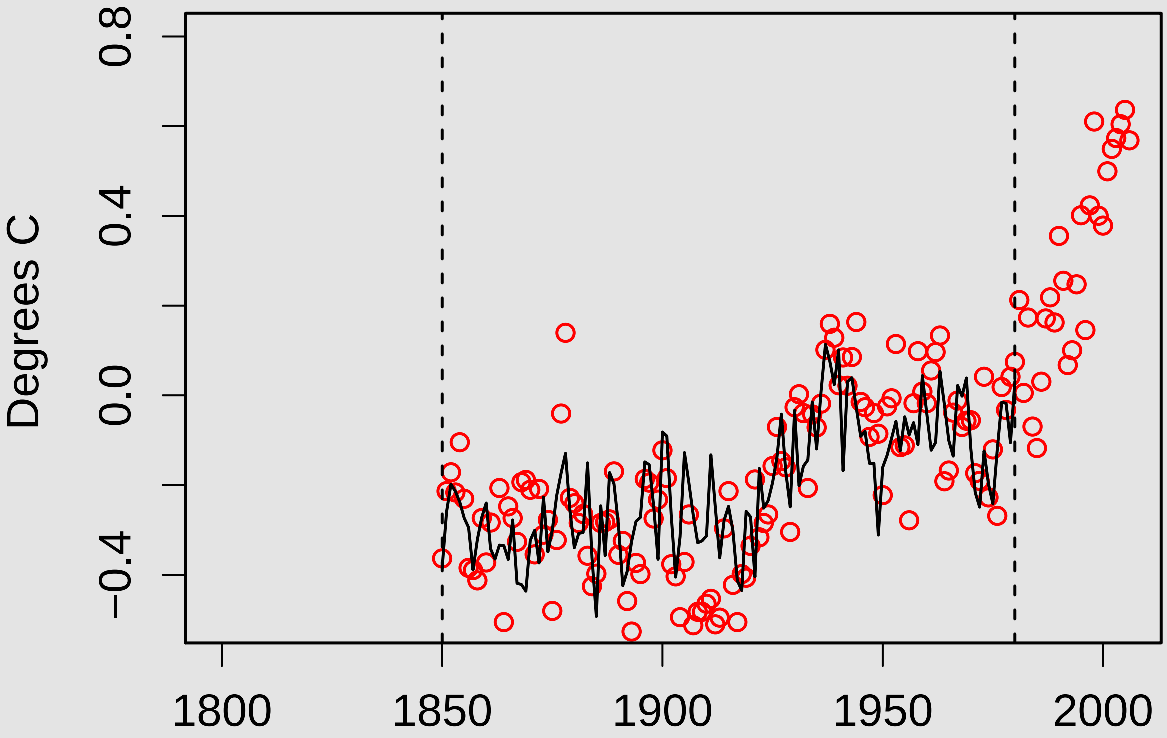
Statistical Relationship between NH temperature and the proxies.

A minimal recipe for generating ensembles:

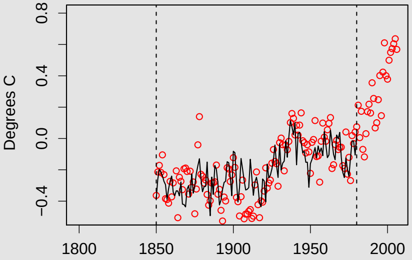
- A stationary linear relationship between the temperature and proxies
- the conditional distribution of temperature given proxies is normally distributed.
- Prediction errors are correlated in time
- Adjustment made for overfitting during calibration period using cross-validation
- Also include the uncertainty in the parameters.

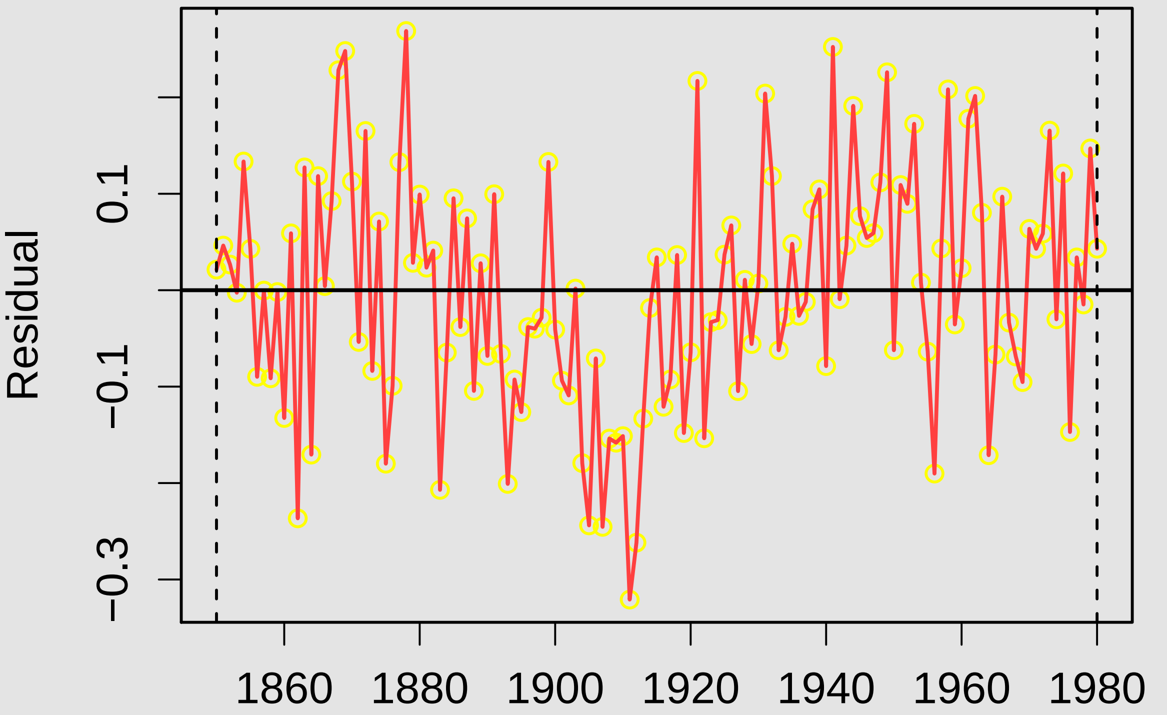
Linear prediction of the expected temperature based on proxies.





Linear prediction of the expected temperature based on proxies.





Clearly the errors are correlated.

Linear Model of NH temperature on proxies

The statistical model:

$$T_t = \mathrm{p}_t' \boldsymbol{\beta} + e_t$$

T_t, p_t: the temperature and proxies at time t.

β: a vector of regression coefficients

The vector of error e_t follows an AR(2) process:

$$e_t = \phi_1 e_{t-1} + \phi_2 e_{t-2} + \epsilon_t, \;\; \epsilon_t \sim \textbf{iid Normal}(0, \sigma^2)$$

Model fitting procedure: Generalized Least Squares

Overfitting and uncertainty of model parameter estimates

- Symptom of overfitting: The variance of observed prediction errors is greater than the prediction variability derived from the model.

  Solution: 10-fold cross-validation to estimate the inflation adjustment

- Uncertainty of model parameter estimates:

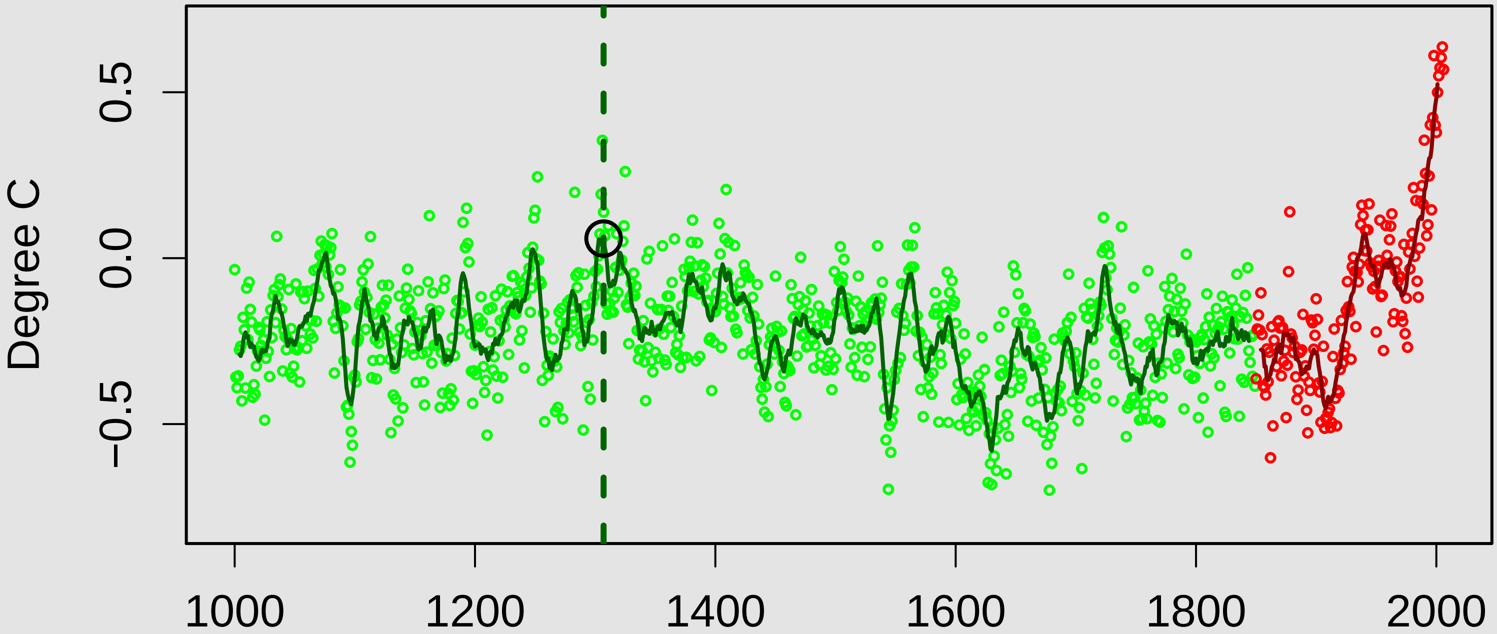
  Parametric bootstrap to estimate the sampling distribution of the parameter estimates.

- An ensemble:

$$\tilde{\mathrm{T}} = \mathrm{P}\tilde{\boldsymbol{\beta}} + (e_{1000}, ..., e_{1849})'|(e_{1850}, ..., e_{1980})'$$

Reconstructed temperatures

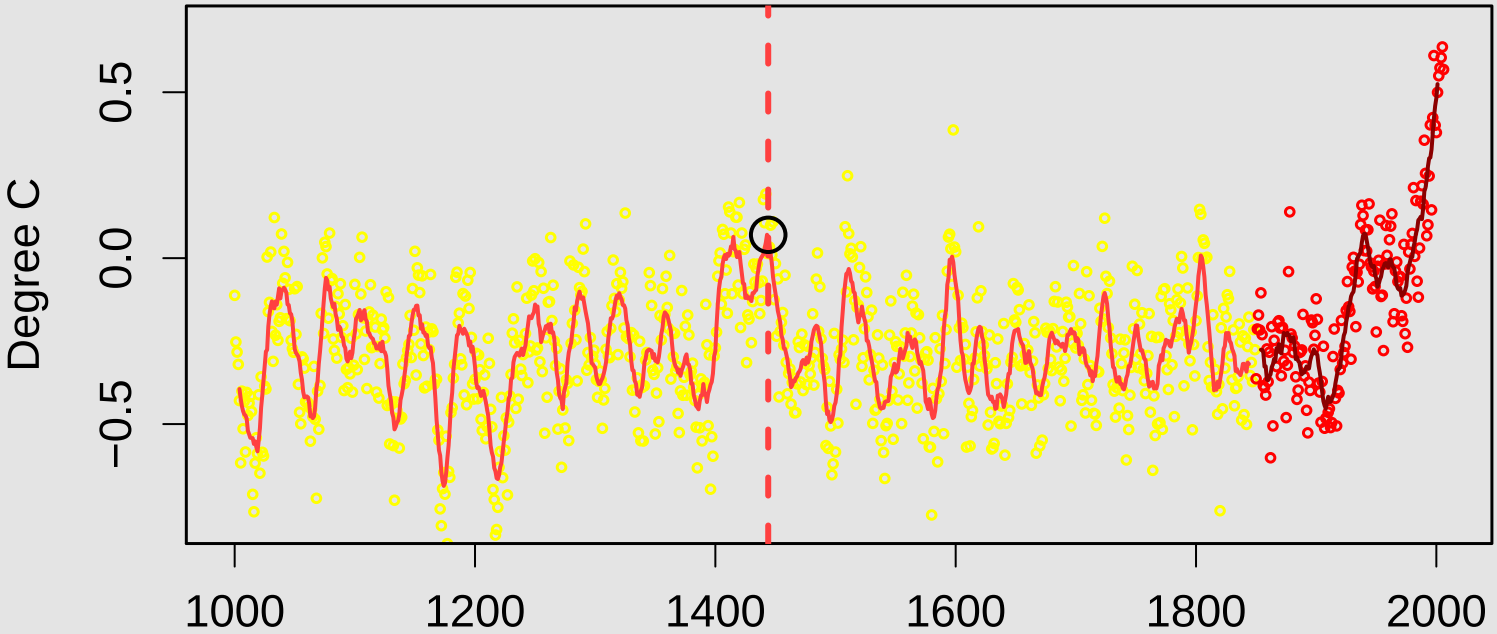
An ensemble member



A draw from the the conditional distribution of temperature given the values of the proxies.

Reconstructed temperatures

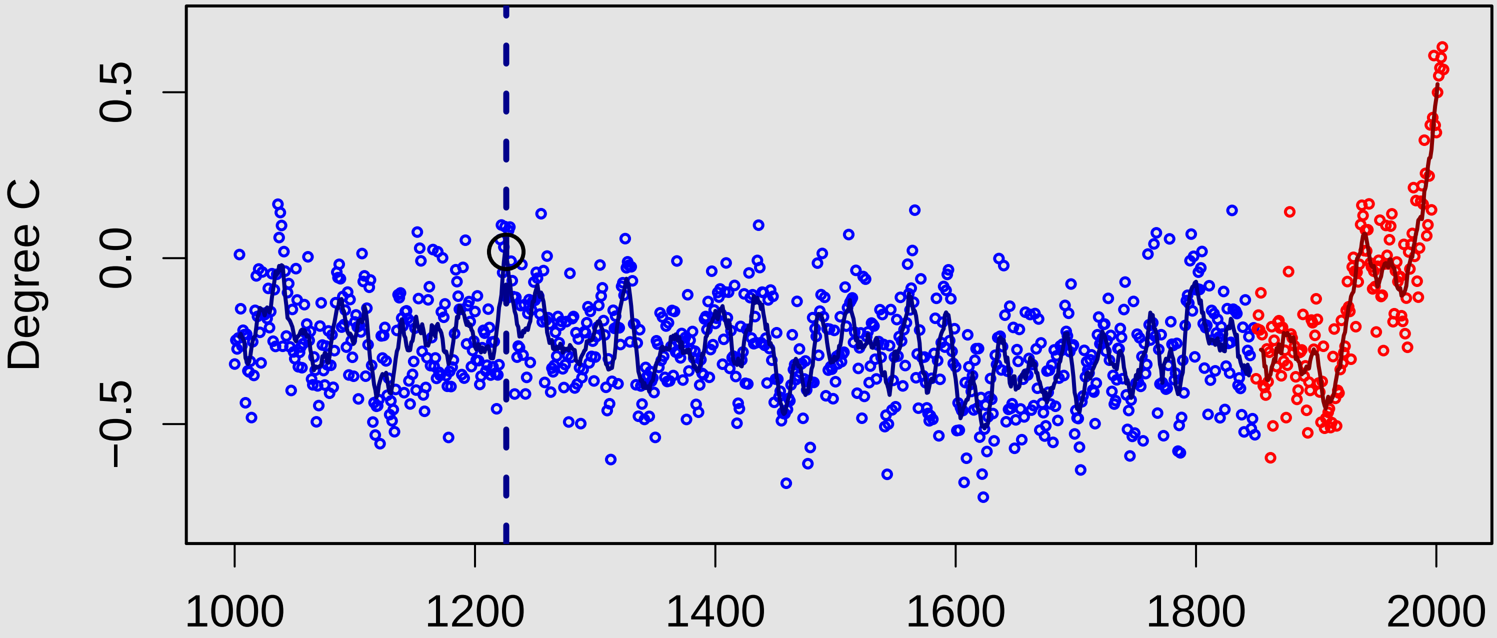
A second ensemble member



A draw from the the conditional distribution of temperature given the values of the proxies.

Reconstructed temperatures

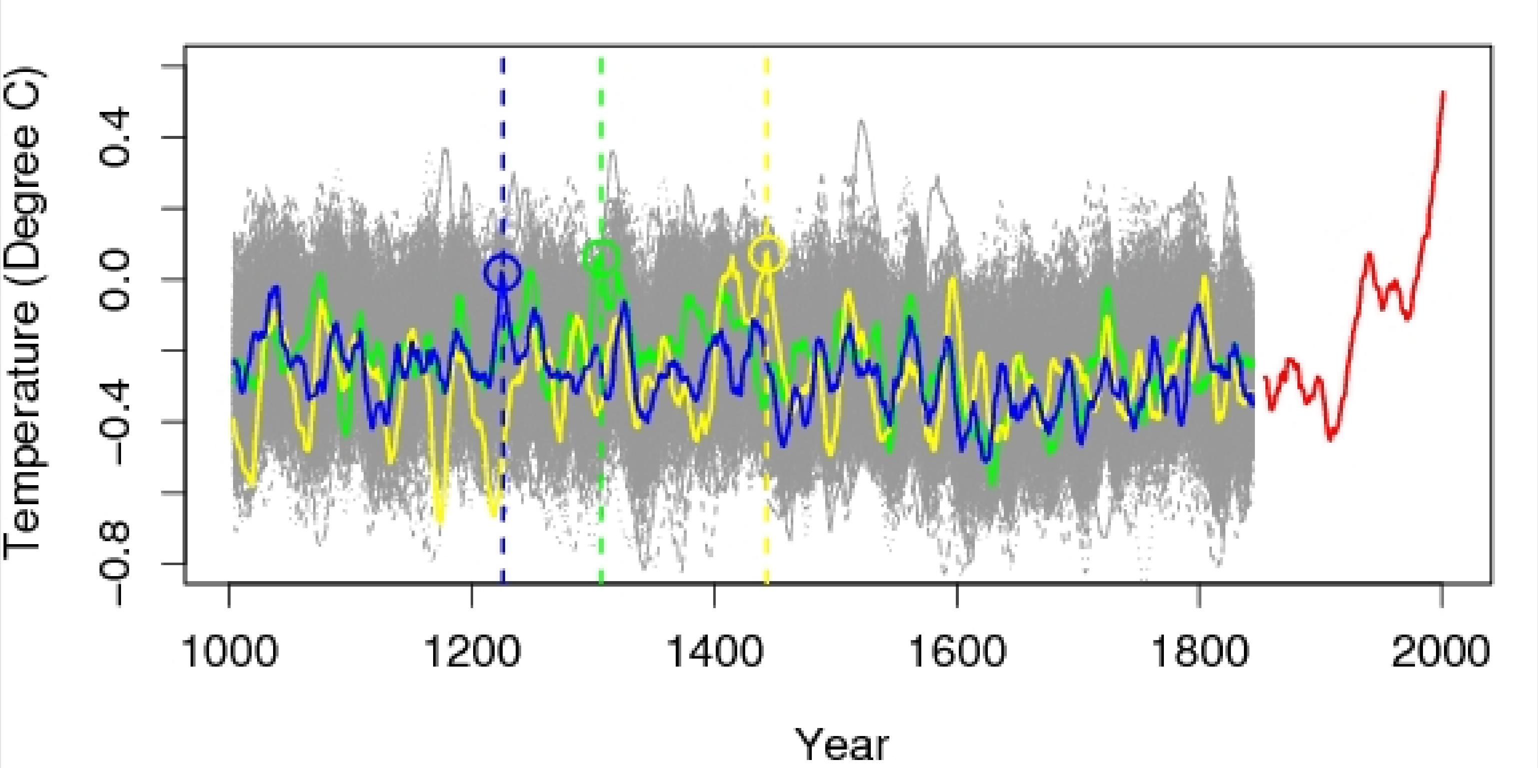
Still another ensemble member



A draw from the the conditional distribution of temperature given the values of the proxies.

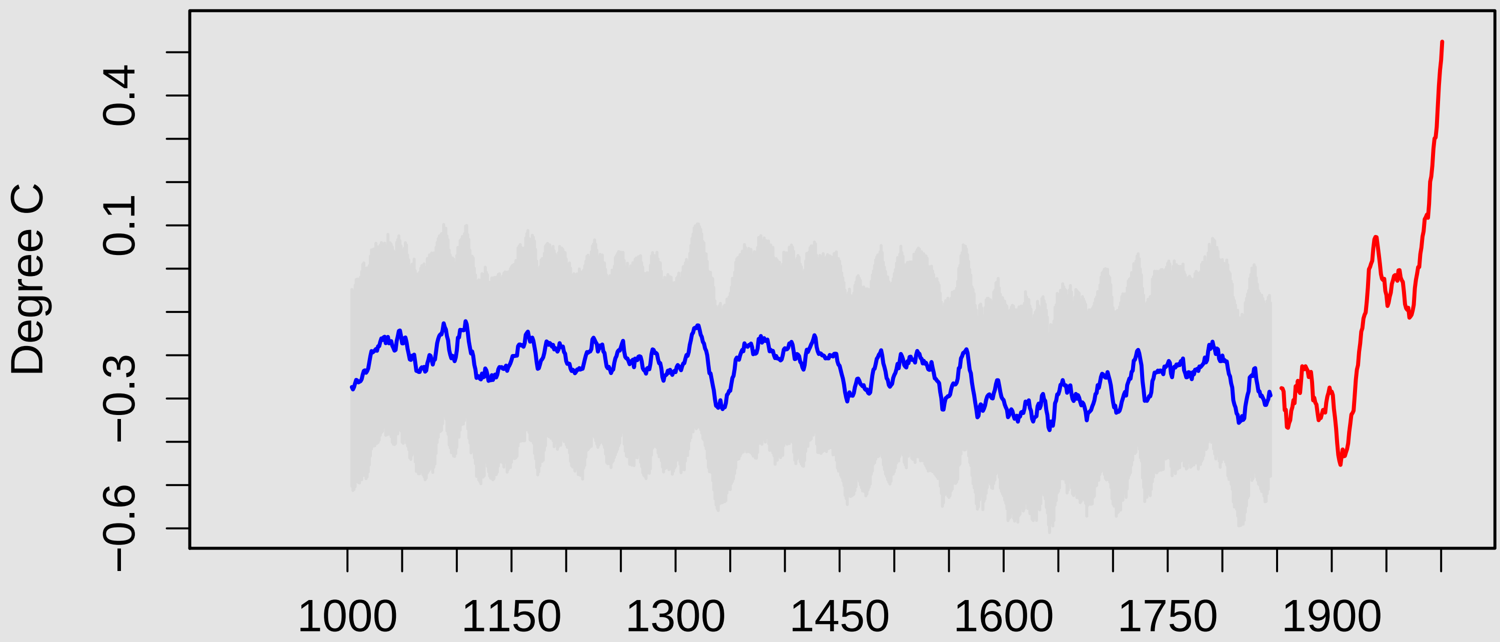
Reconstructed temperatures

Decadal means – three ensembles



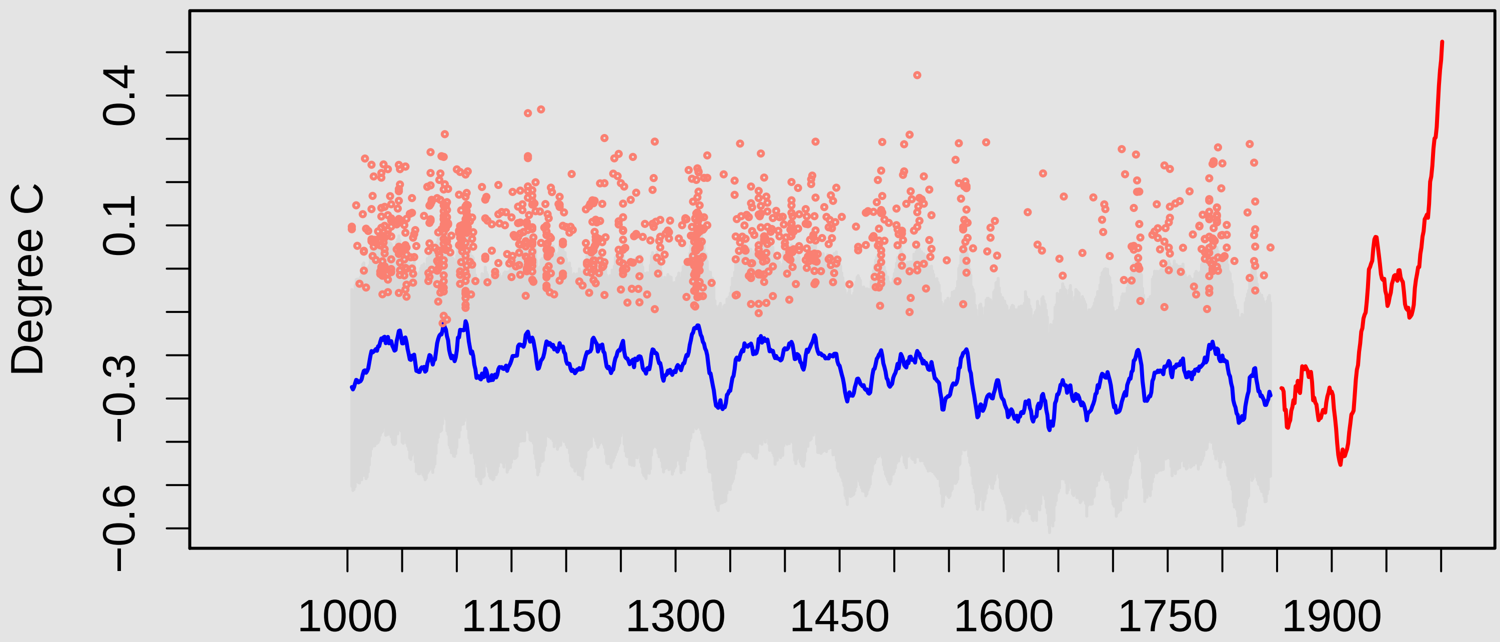
Reconstructed temperatures

95 % uncertainty bounds decadal means



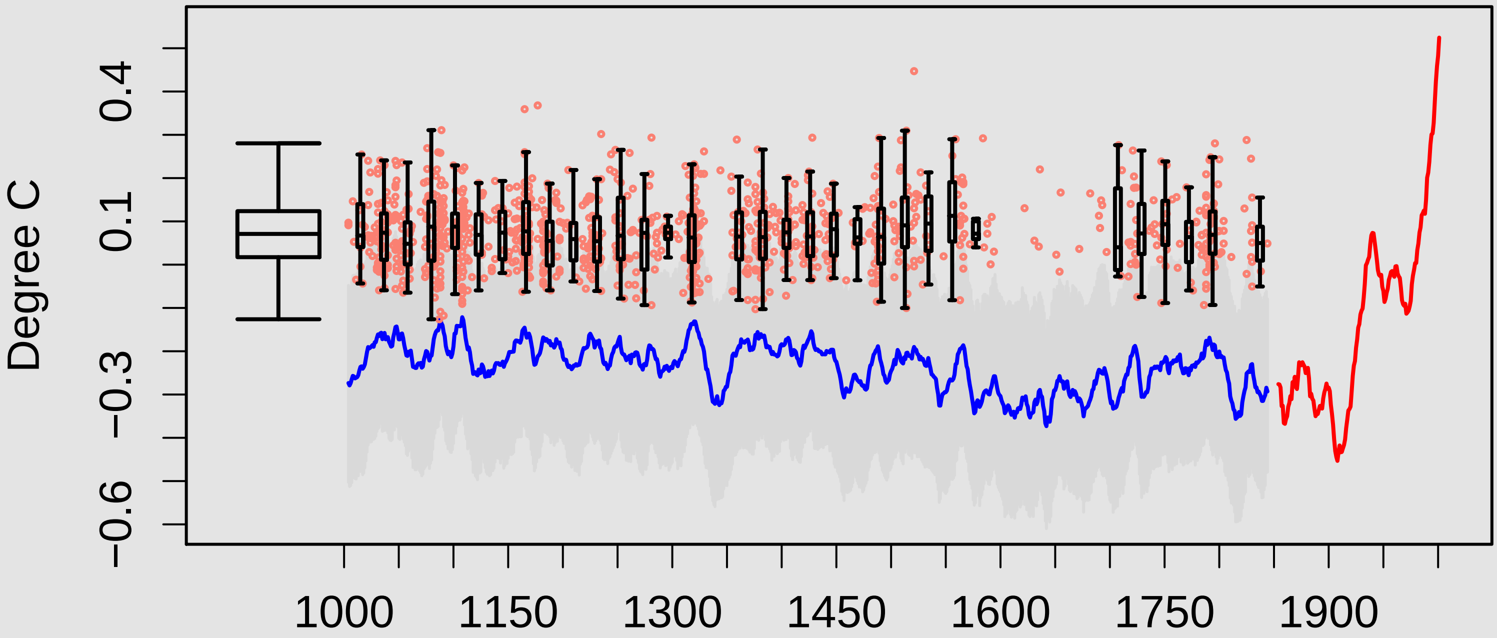
Reconstructed temperatures

Maxima uncertainty



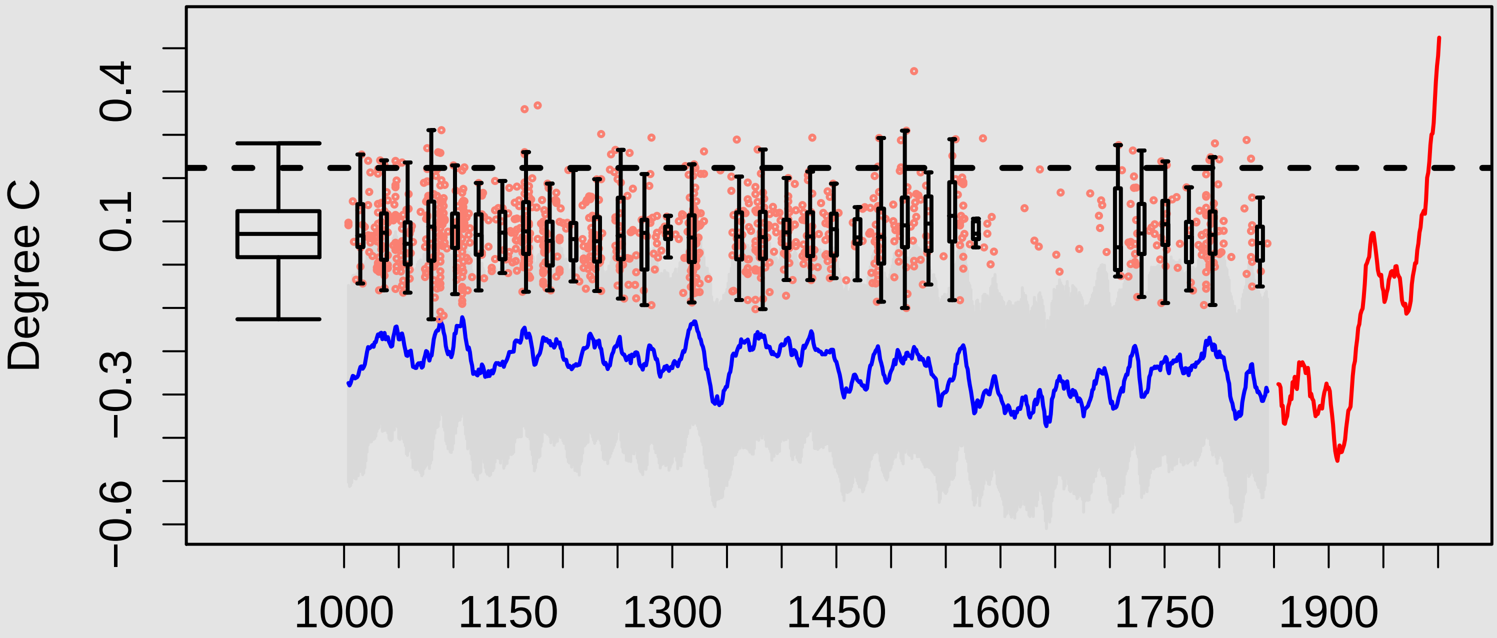
Reconstructed temperatures

Maxima uncertainty using box plots



Reconstructed temperatures

Maxima uncertainty 95% upper bound



Reconstructed temperatures

The Program:

- **Develop a statistical (perhaps complicated) relationship between temperatures and proxies.**
- **The prediction is the distribution of the temperatures given the observed values of the proxies.**

What can not be addressed:

Errors in the proxies and analysis outside the statistical model. (We don't know what we don't know.)

e.g. Proxies change in their relationship to temperature over time

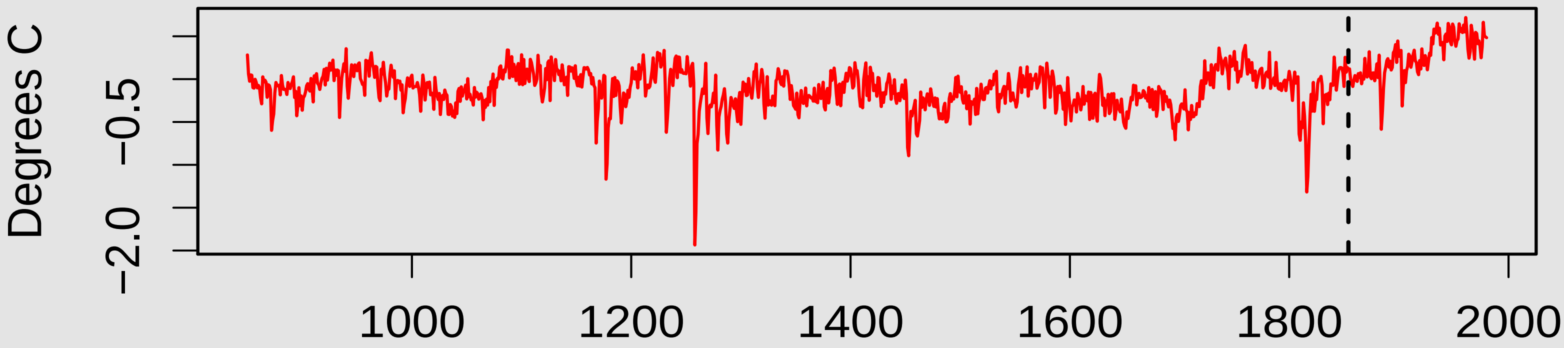
Hierarchical Bayesian Model (HBM)

Three hierarchies:

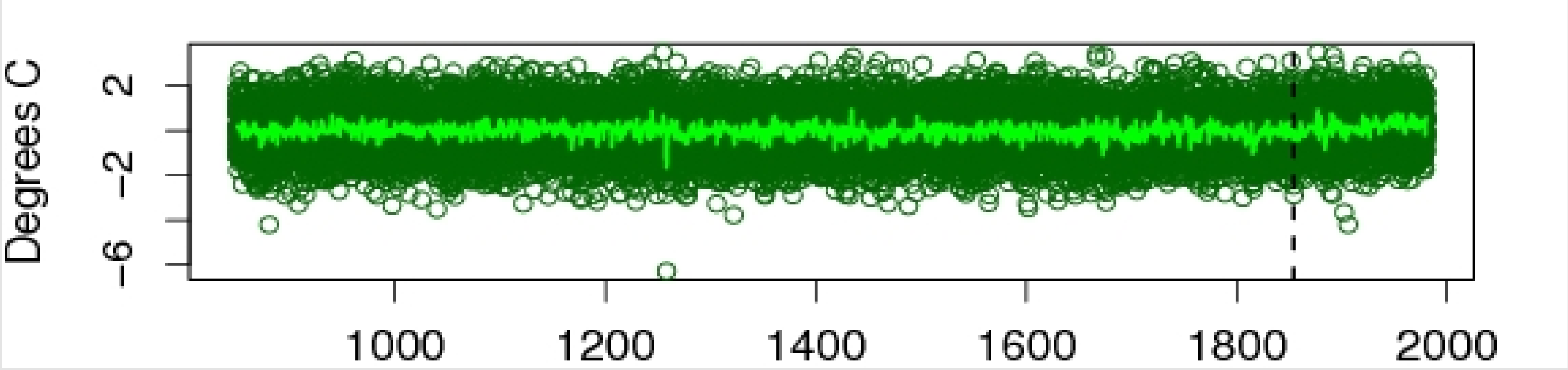
- **Data Stage:** [Proxies|Temperature, Parameters]
  Likelihood of Proxies given temperatures
- **Process Stage:** [Temperature|Parameters]
  Physical model of temperature process
- **Parameter Stage:** [Parameters]
  Specify the prior of parameters

An example of HBM: simulated numerical data

Output from global coupled climate model



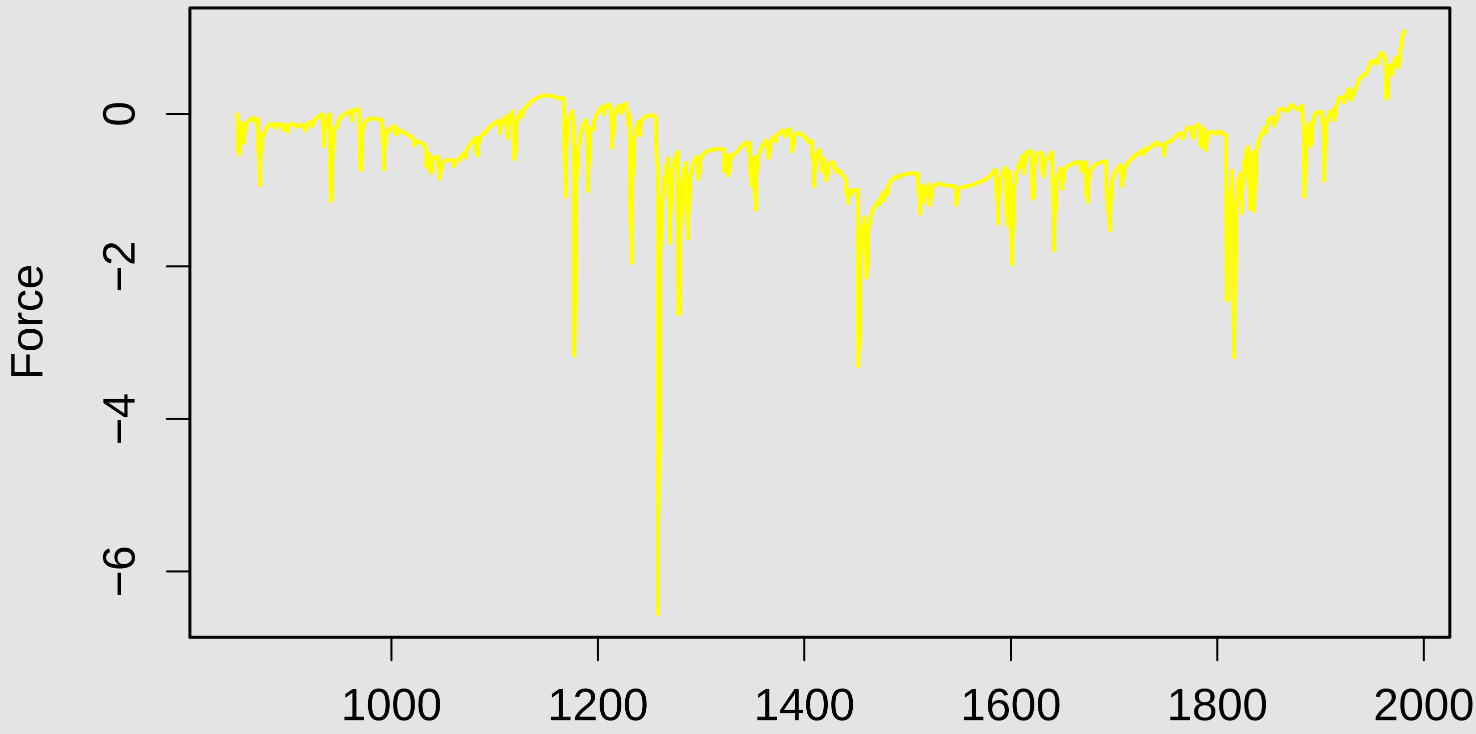
pseudo-proxy series sampled from 14 individual grid boxes in the climate model



Process Model

process model:

Radiative forcings: Explosive volcanism, Solar Activity Changes and Anthropogenic forcings



Process Model

$(\boldsymbol{\mu_{01}}1, \boldsymbol{\mu_{02}})^\top$: **Radiative forcings**

Let

$$\mu = \begin{pmatrix} \boldsymbol{\mu}_1 \\ \boldsymbol{\mu}_2 \end{pmatrix} = \begin{pmatrix} 1 & \boldsymbol{\mu_{01}} \\ 1 & \boldsymbol{\mu_{02}} \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix}$$

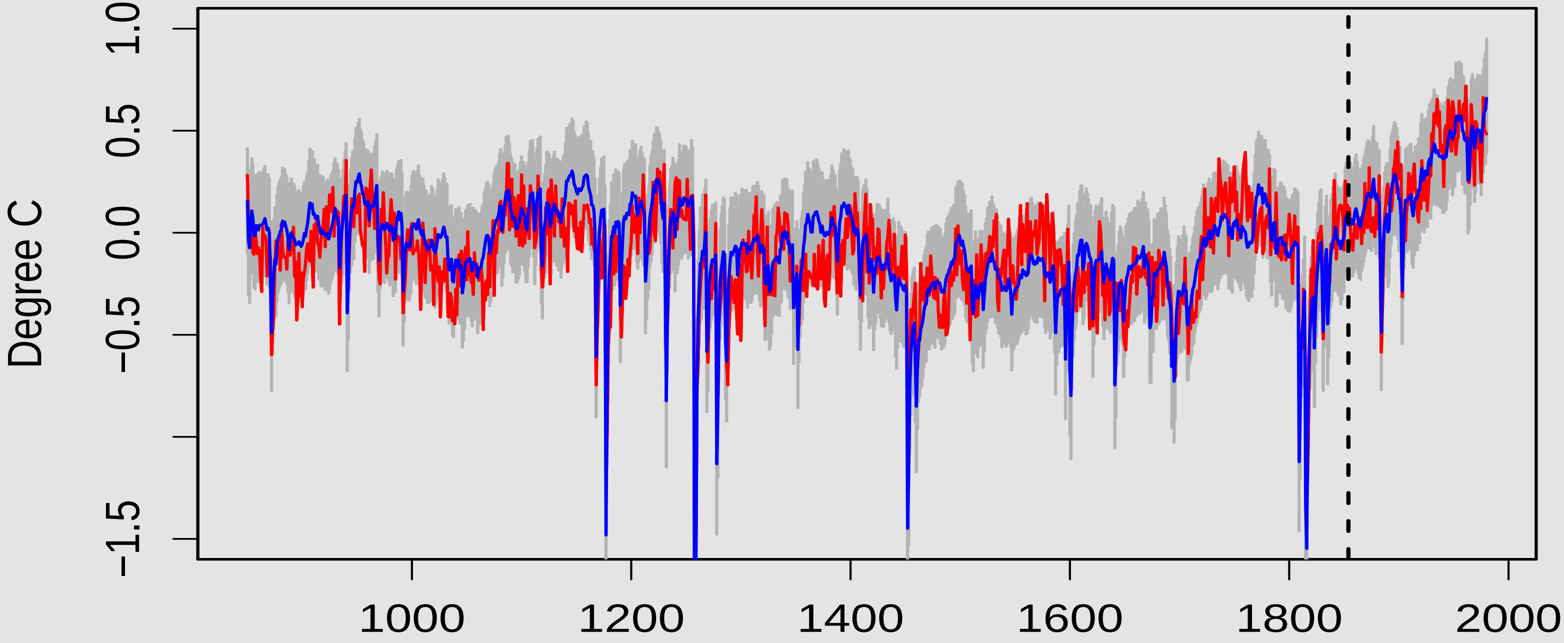
$$({T_1}^\top, {T_2}^\top)|\boldsymbol{\mu}^\top, \phi_1, \phi_2, \sigma_T^2 \sim MN(\boldsymbol{\mu}^\top, \boldsymbol{\Sigma}_{(\phi_1,\phi_2,\sigma_T^2)})$$

- μ_{01}: **External forces before 1850**
  μ_{02}: **External forces 1850-1980**
- $\phi_{1\epsilon}, \phi_{2\epsilon}, \sigma_P^2$: **first and second order time lag coefficient and variance parameter of the AR(2) model**

Simulated numerical data

Input: 14 pseudo-proxies and full model temperature 1854-1980

Reconstruct the past temperature (850-1853)



Conclusion from the example

- The posterior mean matches the trend of the numerical data

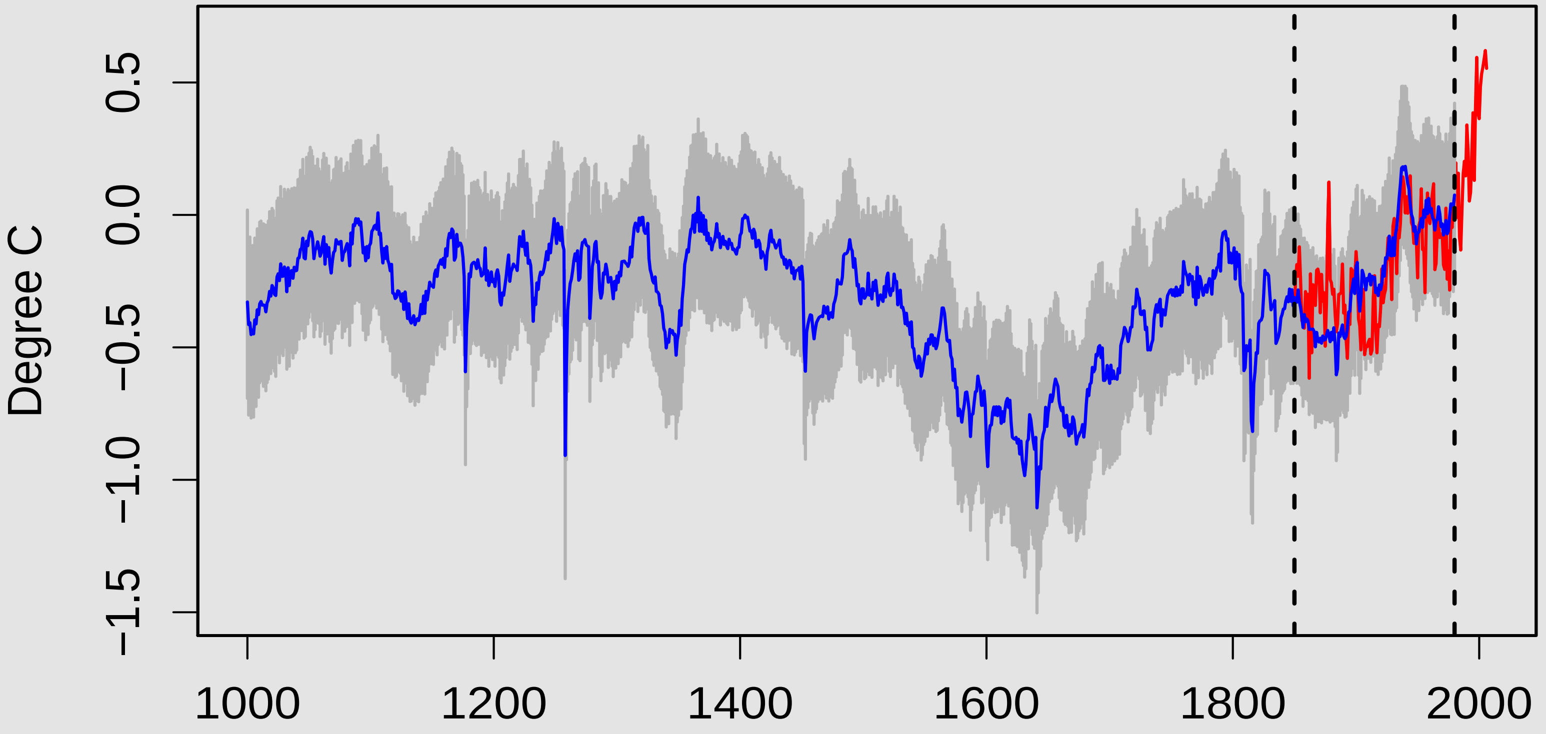
- The numerical data is within the 95% prediction band

- The posterior mean of parameters are close to those directly estimated from numerical data

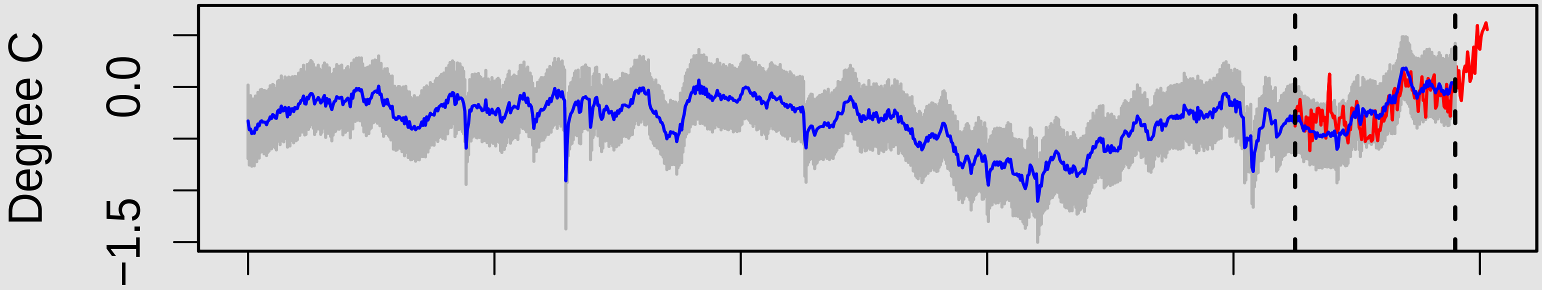
HBM works well in reconstructing the past temperature

Revisit the MBH 99 data using HBM

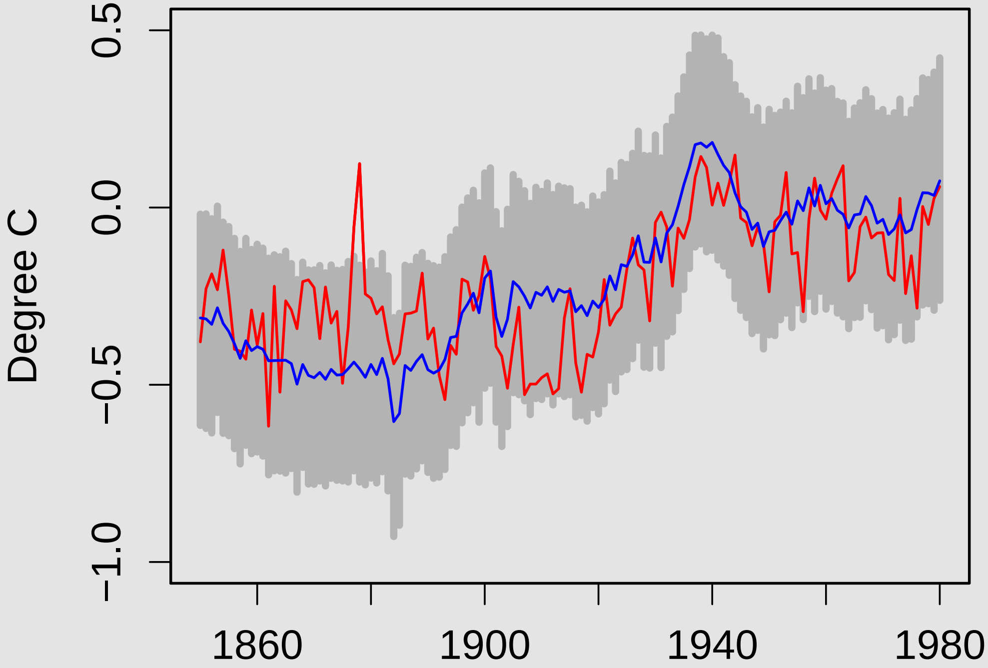
Input: 14 proxies and the instrumental temperature
Reconstruct the past temperature (1000-1849)



Reconstruct real world NH temperature

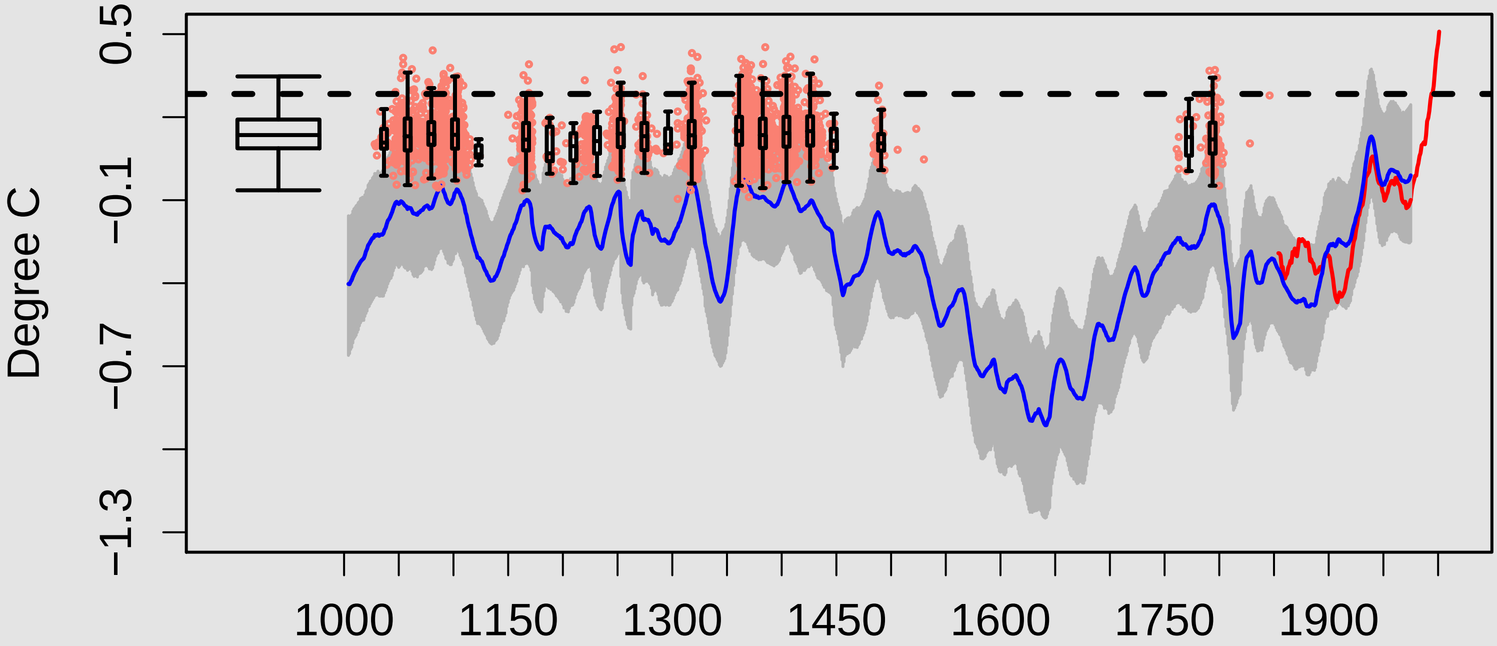


The period with instrumental records.



Reconstruct real world NH temperature

Maxima uncertainty 95% upper bound

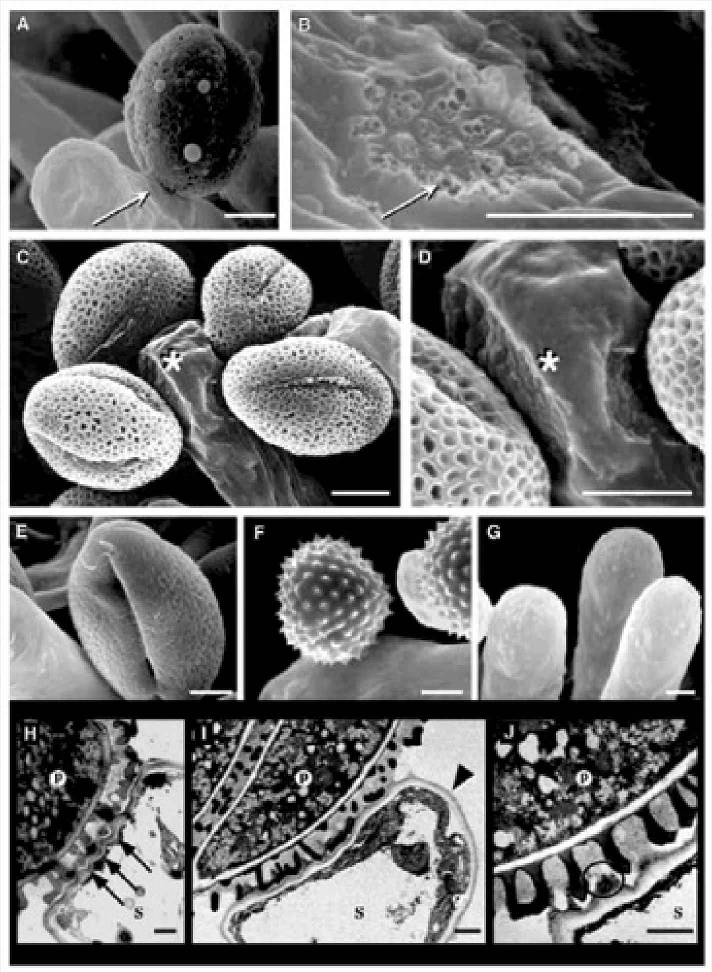


PART II

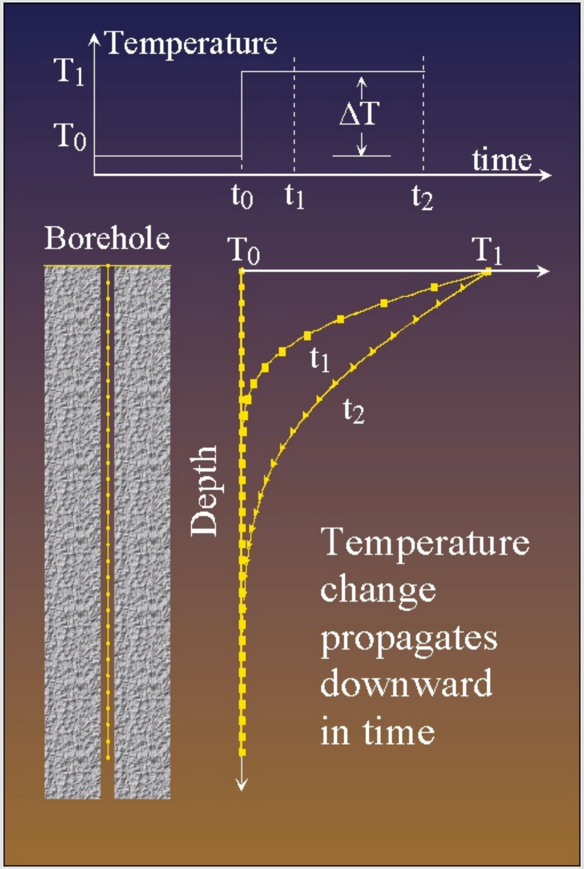
Combining Information from Different Sources

– New Application of Bayesian Hierarchical Models

Data - Tree Ring, Pollen and Borehole



Data - Tree Ring, Pollen and Borehole

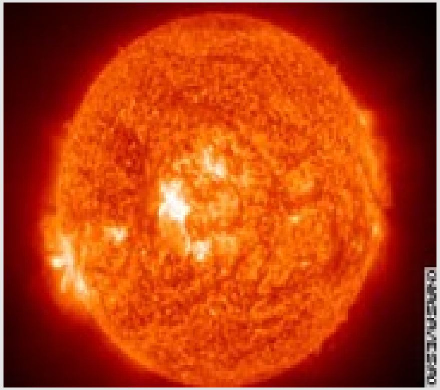


Energy Balance Model

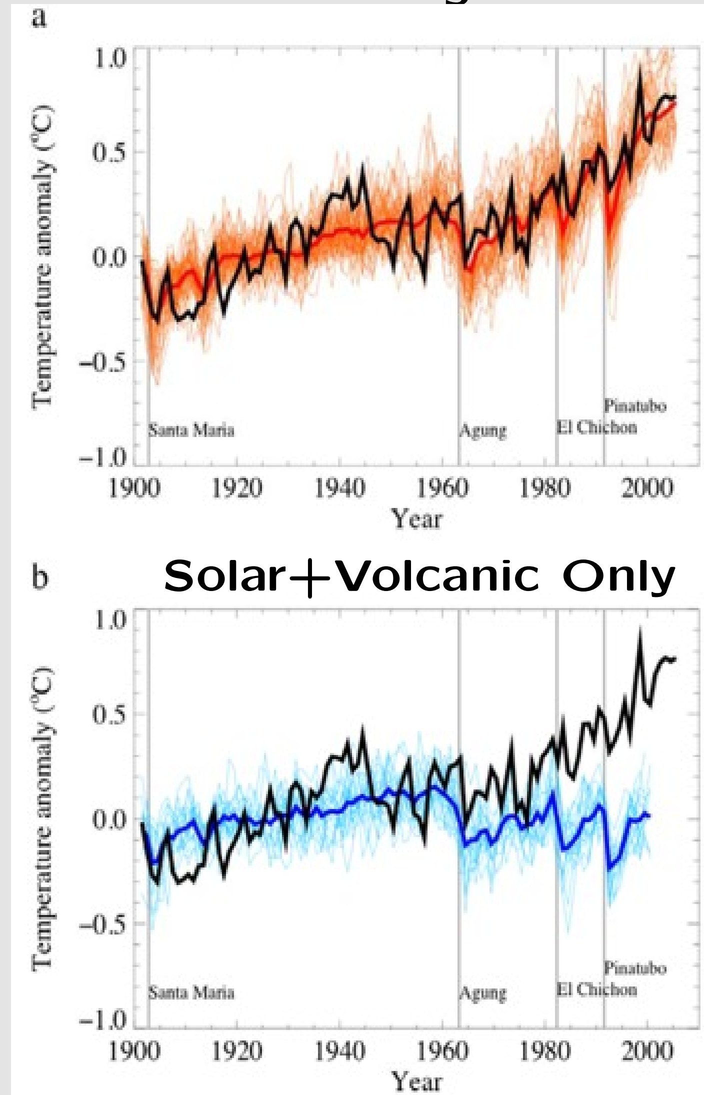
An object will warm or cool depending on its energy imbalances

Three forcings for global climate system:

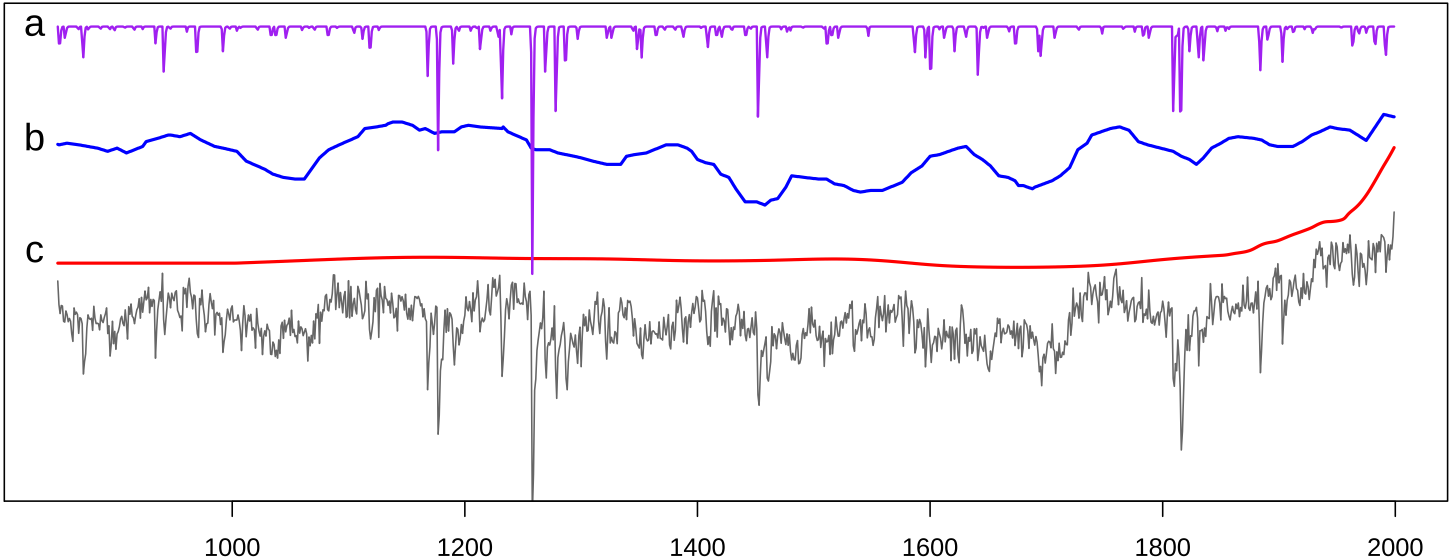
1. Solar radiation: A positive radiative forcing tends to warm the surface on average, whereas a negative one tends to cool it
2. Volcanism: block the solar radiation due to the large amounts of aerosols ejected by volcanic eruption into the atmosphere
3. Greenhouse gases: absorb infrared radiation, trap heat within the atmosphere.



All forcings



Forcings



a: Volcanism (contains substantial noise)

b: solar radiance

c: green house gases

Formulate the problem

Skill of each proxy and forcings

- **Tree ring: annual to decadal**
- **Pollen: bi-decadal to semi-centennial**
- **Borehole: centennial and onward**
- **Forcings: external drivers**

Goal: Reconstruct the 850-1849 temperature by all proxies, forcings and the 1850-1999 temperature

Bayesian Hierarchical Model (BHM) to thread all proxies, forcings and temperatures

Bayesian Hierarchical Model (BHM)

Bayes' theorem

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)}$$

$P(A|B)$: the posterior probability because it is derived from or depends upon the specified value of B.

$P(B)$: the prior or marginal probability of B, and acts as a normalizing constant.

Intuitively, Bayes' theorem describes the way in which one's beliefs about observing 'A' are updated by having observed 'B'.

BHM

A natural framework: Three hierarchies:

- **Data Stage:**

  [Proxies|Geophysical Process, Parameters]

  Likelihood of Proxies given underlying process

- **Process Stage: [Geophysical Process|Parameters]**

  Physical model of Geophysical Process

- **Parameter Stage: [Parameters]**

  Specify the prior of parameters

BHM

Data Stage: **[Data|Geophysical Process, Parameters]**

Forward models for tree ring (R) and pollen (P):

$$R_i|(T_1,T_2) \; = \beta_{01_i} + \beta_{11_i}\mathrm{M}_1(T_1,T_2) + \epsilon_{1i}, \quad \epsilon_{1i} \sim \textbf{AR}(2)$$

$$P_i|(T_1,T_2) \; = \beta_{02_i} + \beta_{12_i}\mathrm{M}_2(T_1,T_2) + \epsilon_{2i}, \quad \epsilon_{2i} \sim \textbf{AR}(2)$$

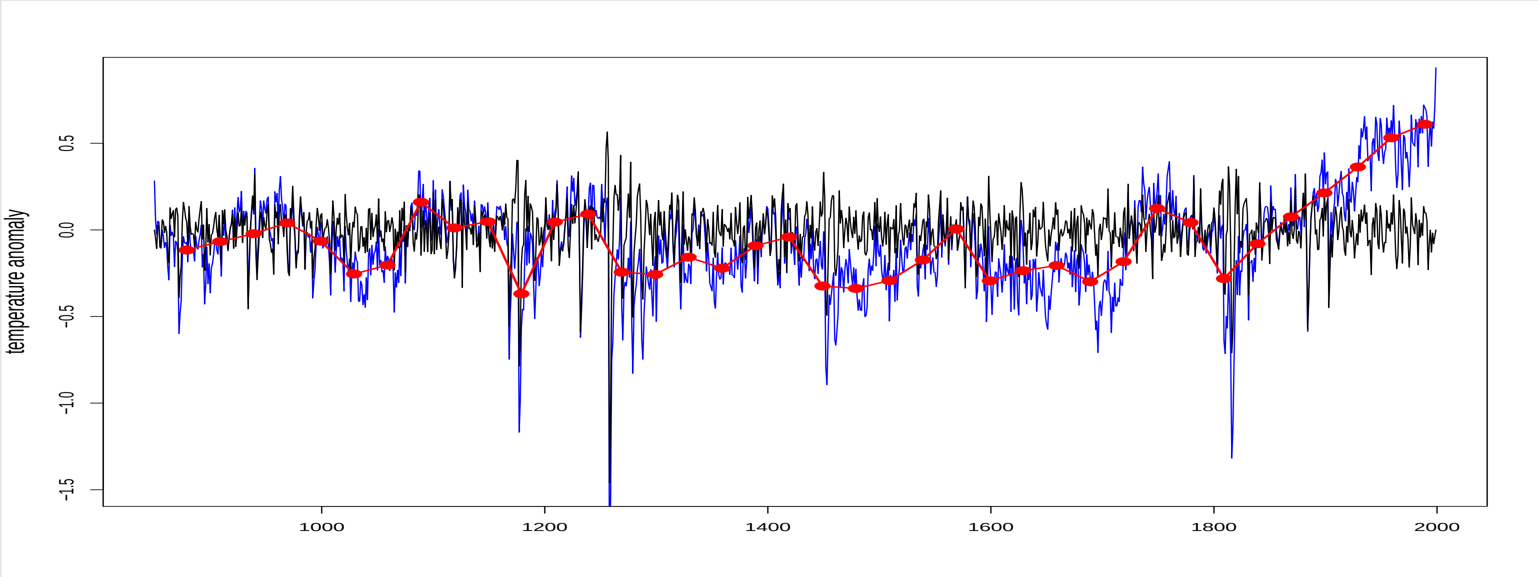
$$\epsilon_{1i} \sim \textbf{AR}(2)(\sigma_1^2, \phi_{11}, \phi_{21}); \quad \epsilon_{2i} \sim \textbf{AR}(2)(\sigma_2^2, \phi_{12}, \phi_{22})$$

T_1: **unknown temperature for the past millenium**

T_2: **observed temperature**

M_1 **and** M_2: **transformation matrices fore tree ring and pollen, respectively**

Tree Ring and Pollen:



Blue: temperature; Black: tree ring;

Red: Pollen, observed at every 30 years

BHM

Data Stage: **Forward models for Borehole (B):**

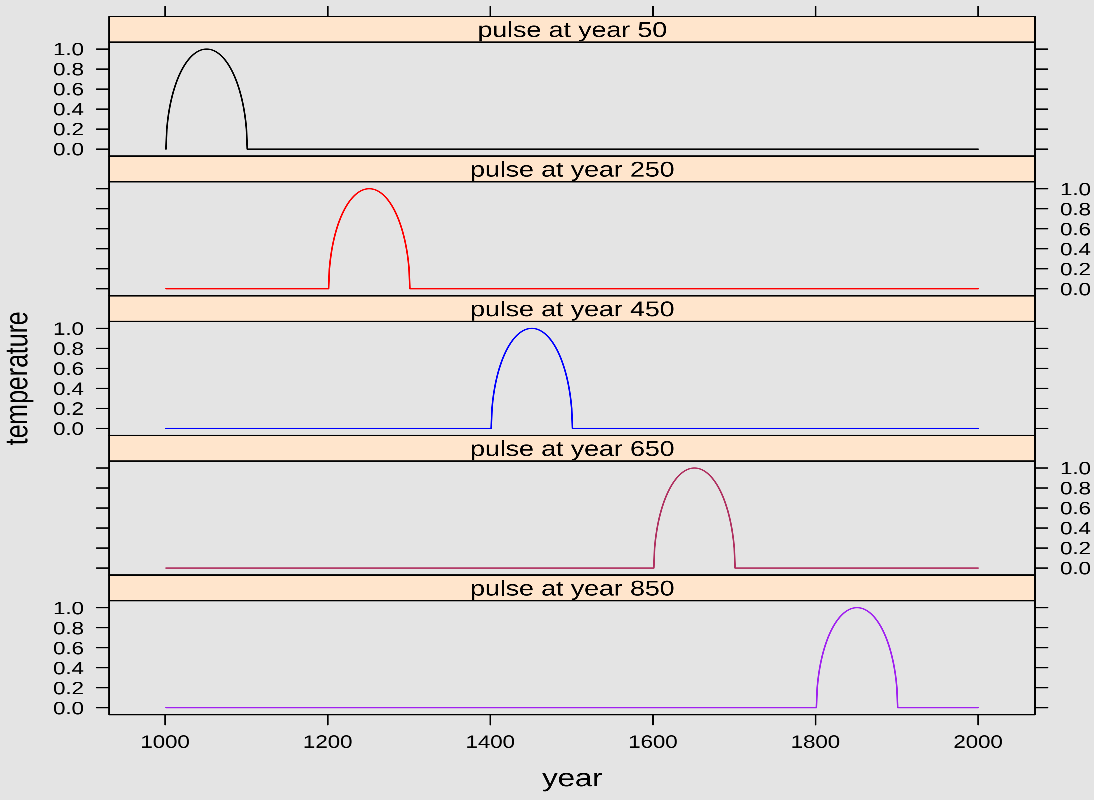
$$B_i|(T_1, T_2) = \mathrm{M}_3\{\beta_{03_i} + \beta_{13_i}(T_1, T_2) + \epsilon_{3i}\}, \quad \epsilon_{3i} \sim \textbf{iid Normal}$$

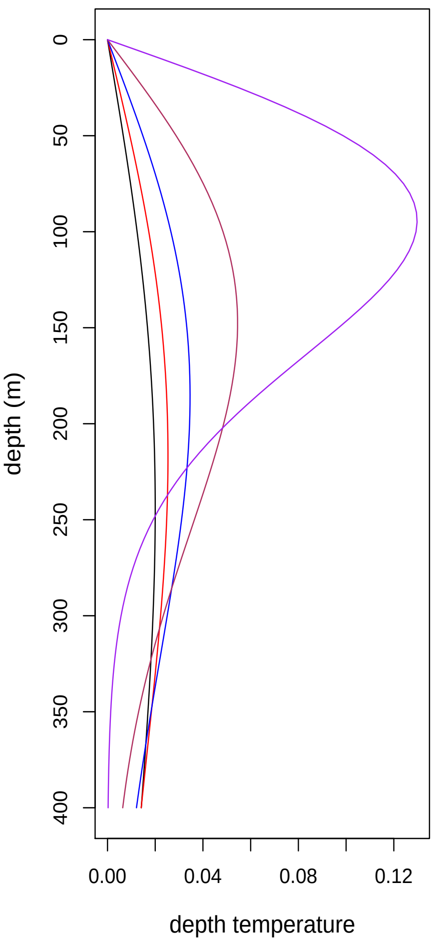
$$\epsilon_{3i} \sim \textbf{iid } N(0, \sigma_3^2)$$

M_3: **transformation matrix for borehole**

- **Developed from pre-observation mean–surface air temperature (POM–SAT) model based on heating equation**
- **Respect the smoothness of depth profile**

Borehole


pulse at year 50
pulse at year 250
pulse at year 450
pulse at year 650
pulse at year 850
temperature
0.0
0.2
0.4
0.6
0.8
1.0
1000
1200
1400
1600
1800
2000
year


depth (m)
0
50
100
150
200
250
300
350
400
0.00
0.04
0.08
0.12
depth temperature

BHM

Data Stage:

Volcanism contain considerable noise among three forcings

$$V|V' = (1 + \epsilon_4)V', \quad \epsilon_4 \sim \textbf{iid } N(0, 1/64)$$

V: **observed volcanism**

V': **ideal volcanism without noise**

The magnitude of noise in V depends on mean V'

BHM

Process Stage: [Geophysical Process|Parameters]

$$(T_1, T_2)|(S, V', C) = \beta_0 + \beta_1 S + \beta_2 V' + \beta_3 C + \epsilon_5, \quad \epsilon_5 \sim \textbf{AR}(2)$$

$$\epsilon_5 \sim \textbf{AR}(2)(\sigma_5^2, \phi_{13}, \phi_{23})$$

S: solar radiance

V': Volcanism

C: green house gases, mainly include CO_2, methane

Can be replace by more complicated dynamic model, e.g., General Circulate Model (GCM)

BHM

Priors:

Conjugate priors allow for an explicit full conditional posterior distribution

$\beta_{jk_i} \sim N(\tilde{\mu}_{jk_i}, \tilde{\sigma}^2_{jk_i})$ $j = 0, 1$; $k = 1, 2, 3$,

$\beta_i \sim N(\tilde{\mu}_i, \tilde{\sigma}_i^2)$, $i = 0, 1, 2, 3$,

$\sigma_i^2 \sim IG(\tilde{q}_i, \tilde{r}_i)$, $i = 1, ..., 5$

Guarantees their corresponding AR process to be stationary and causal (Shumway and Stoffer, 1999, ch. 3)

$\phi_{2i} \sim \textbf{unif}(-1, 1)$, $\phi_{1i}|\phi_{2i} \sim (1 - \phi_{2i}) \times \textbf{unif}(-1, 1)$, $i = 1, 2, 3$,

BHM

Posterior distribution:

$$[T_1, \beta_{01_i}, \beta_{11_i}, \beta_{02_i}, \beta_{12_i}, \beta_{03_i}, \beta_{13_i}, \beta_i, \phi_{1i}, \phi_{2i}, \sigma_i^2, V' | R_i, P_i, B_i, S, V, C, T_2]$$

$$\propto [R_i|(T_1,T_2)|\beta_{01i}, \beta_{11i}, \phi_{11}, \phi_{21}, \sigma_1][P_i|(T_1,T_2)|\beta_{02i}, \beta_{12i}, \phi_{12}, \phi_{22}, \sigma_2]$$
$$[B_i|(T_1,T_2)|\beta_{03i}, \beta_{13i}, \sigma_3][(T_1,T_2)|(S,V',C)][V|V']$$

One example:

$$[T_1|\bullet] = N(\mathbf{Ab}, \mathbf{A})$$

By completing square:

$$\mathbf{A} = (\textstyle\sum_{k=1}^3 \beta_{1ki}^2 M_k^\top V_k^{-1} M_k + V_T^{-1})^{-1}$$

$$\mathbf{b} = \textstyle\sum_{k=1}^3 \beta_{1ki} M_k^\top V_k^{-1} (\textbf{Proxy}_k - \beta_{0ki}) + V_T^{-1}(\beta_0 + \beta_1 S + \beta_2 V' + \beta_3 C)$$

BHM

Normal and Inverse Gamma:

Inverse Gamma:

$$f(x;\alpha,\beta) = \frac{1}{\beta^{\alpha}\Gamma(\alpha)} x^{-(\alpha+1)} e^{-\frac{1}{x\beta}}$$

$$E(x) = \frac{1}{(\alpha-1)\beta}$$

$$\mathbf{var}(x) = \frac{1}{(\alpha-1)^2(\alpha-2)\beta^2}$$

Multivariate Normal:

$$f(\mathrm{y};\boldsymbol{\mu},\Sigma) = \frac{1}{\sqrt{(2\pi)^n|\Sigma|}} e^{-\frac{1}{2}(\mathrm{y}-\boldsymbol{\mu})^{\mathsf{T}}\Sigma^{-1}(\mathrm{y}-\boldsymbol{\mu})} \tag{1}$$

BHM

If Σ can be written as $\sigma^2\widetilde{\Sigma}$, i.e., a product of constant variance σ^2 and correlation matrix $\widetilde{\Sigma}$, (1) becomes into:

$$f(\mathbf{y};\boldsymbol{\mu},\sigma^2,\widetilde{\Sigma}) = \frac{1}{\sqrt{(2\pi)^n\sigma^{2n}|\widetilde{\Sigma}|}}e^{-\frac{1}{2\sigma^2}(\mathbf{y}-\boldsymbol{\mu})^\top\widetilde{\Sigma}^{-1}(\mathbf{y}-\boldsymbol{\mu})}.$$

Suppose $\sigma^2 \sim IG(q,r)$,

$$[\mathbf{y}|\boldsymbol{\mu},\sigma^2,\widetilde{\Sigma}][\sigma^2] \propto (\sigma^2)^{-n/2}e^{-\frac{1}{2\sigma^2}(\mathbf{y}-\boldsymbol{\mu})^\top\widetilde{\Sigma}^{-1}(\mathbf{y}-\boldsymbol{\mu})}(\sigma^2)^{-(q+1)}e^{-\frac{1}{r\sigma^2}}$$

$$= (\sigma^2)^{-(q+n/2+1)}e^{-\frac{1}{\sigma^2}\{\frac{1}{r}+\frac{(\mathbf{y}-\boldsymbol{\mu})^\top\widetilde{\Sigma}^{-1}(\mathbf{y}-\boldsymbol{\mu})}{2}\}}$$

So the posterior $[\sigma^2|\cdot]$ is

$IG(q+n/2,\{1/r+0.5(\mathbf{y}-\boldsymbol{\mu})^\top\widetilde{\Sigma}^{-1}(\mathbf{y}-\boldsymbol{\mu})\}^{-1}$

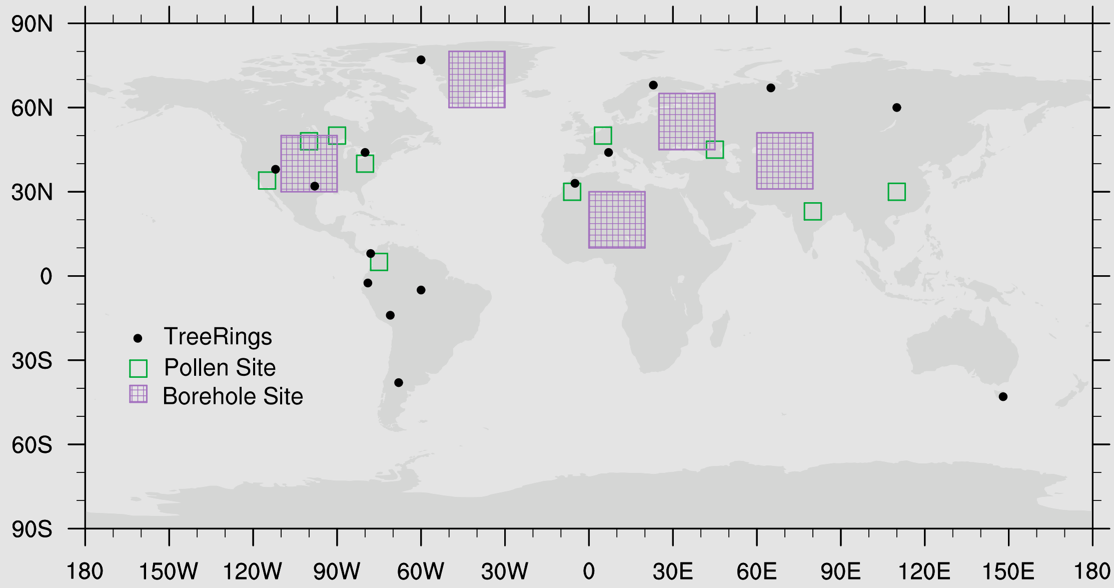
MCMC

generate posteriors by alternating Gibbs sampler and Metropolis-Hasting algorithm

1. $T_1|\cdot$
2. $P|\cdot$, $B|\cdot$, $S|\cdot$
3. $(\beta_{01}, \beta_{11})|\cdot$, $(\beta_{02}, \beta_{12})|\cdot$, $(\beta_{03}, \beta_{13})|\cdot$, $(\beta_0, \beta_1, \beta_2, \beta_3)|\cdot$
4. $\sigma_1^2|\cdot$, $\sigma_2^2|\cdot$, $\sigma_3^2|\cdot$, $\sigma_4^2|\cdot$, $\sigma_5^2|\cdot$, $\sigma_6^2|\cdot$, $\sigma_t^2|\cdot$
5. $(\phi_{1r}, \phi_{2r})|\cdot$, $\phi_p|\cdot$, $\phi_b|\cdot$, $(\phi_{1t}, \phi_{2t})|\cdot$ **(M-H Algorithm because the conditional distribution has no closed form)**

Scrambling the order does NOT affect the result

Implement BHM using Climate from Models



Generate pseudo proxies:

15 tree rings: remove 10 year smoothing average

10 pollen: 10 year smoothing average at every 30 years

5 borehole: POM-SAT forward model

Interesting questions

- What is the skill of forcings?
- What is the skill of each proxy?
- How does the noise in proxies affect the reconstruction?
  - Perfect proxies: proxies directly generated from the local/regional temperatures
  - Contaminated proxies: The signal to noise ratio is chosen 1:4 in terms of their variance

Variations of the full model

- **Compare reconstructions with forcings and without forcings:**

$$(T_1, T_2) = \beta_0 + \epsilon_5, \quad \epsilon_5 \sim \mathbf{AR}(2)(\sigma_5^2, \phi_{13}, \phi_{23})$$

- **Oracle experiment:**
  Reconstruction using all the original local temperatures rather than their transformed proxies as a baseline to evaluate reconstruction performance:

$$Tl_i|(T_1, T_2) = \beta_{0_i} + \beta_{1_i}(T_1, T_2) + \epsilon_i, \quad \epsilon_i \sim \mathbf{AR}(2)(\sigma^2, \phi_1, \phi_2),$$

Experiments

(a) Perfect proxies With forcings
(b) Perfect proxies Without forcings
(c) Contaminated proxies With forcings
(d) Contaminated proxies Without forcings

For each (a), (b), (c) and (d):

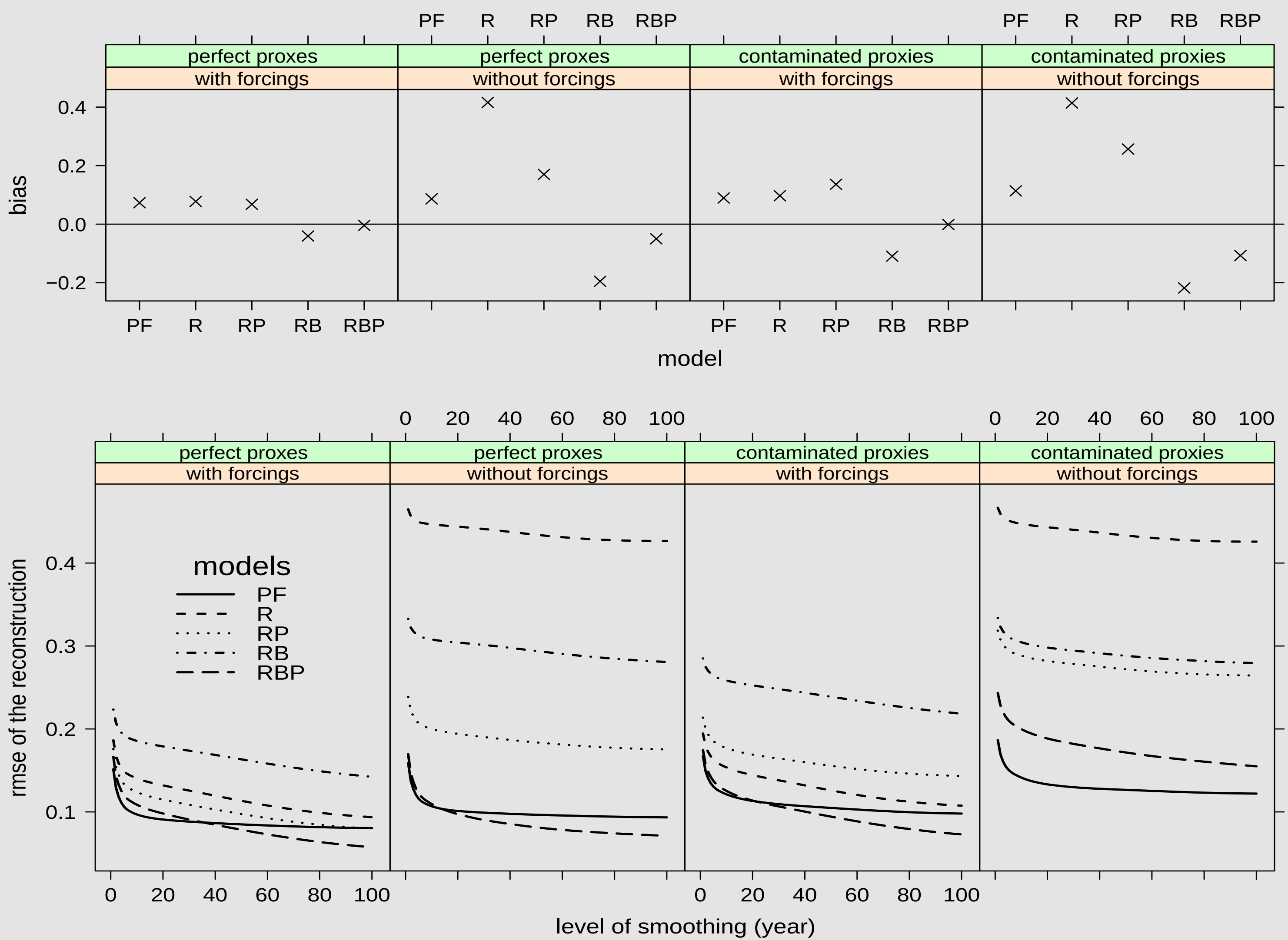
(i) local/regional temperature series (PF)

(ii) Tree ring only (R)

(iii) Tree ring + Pollen (RP)

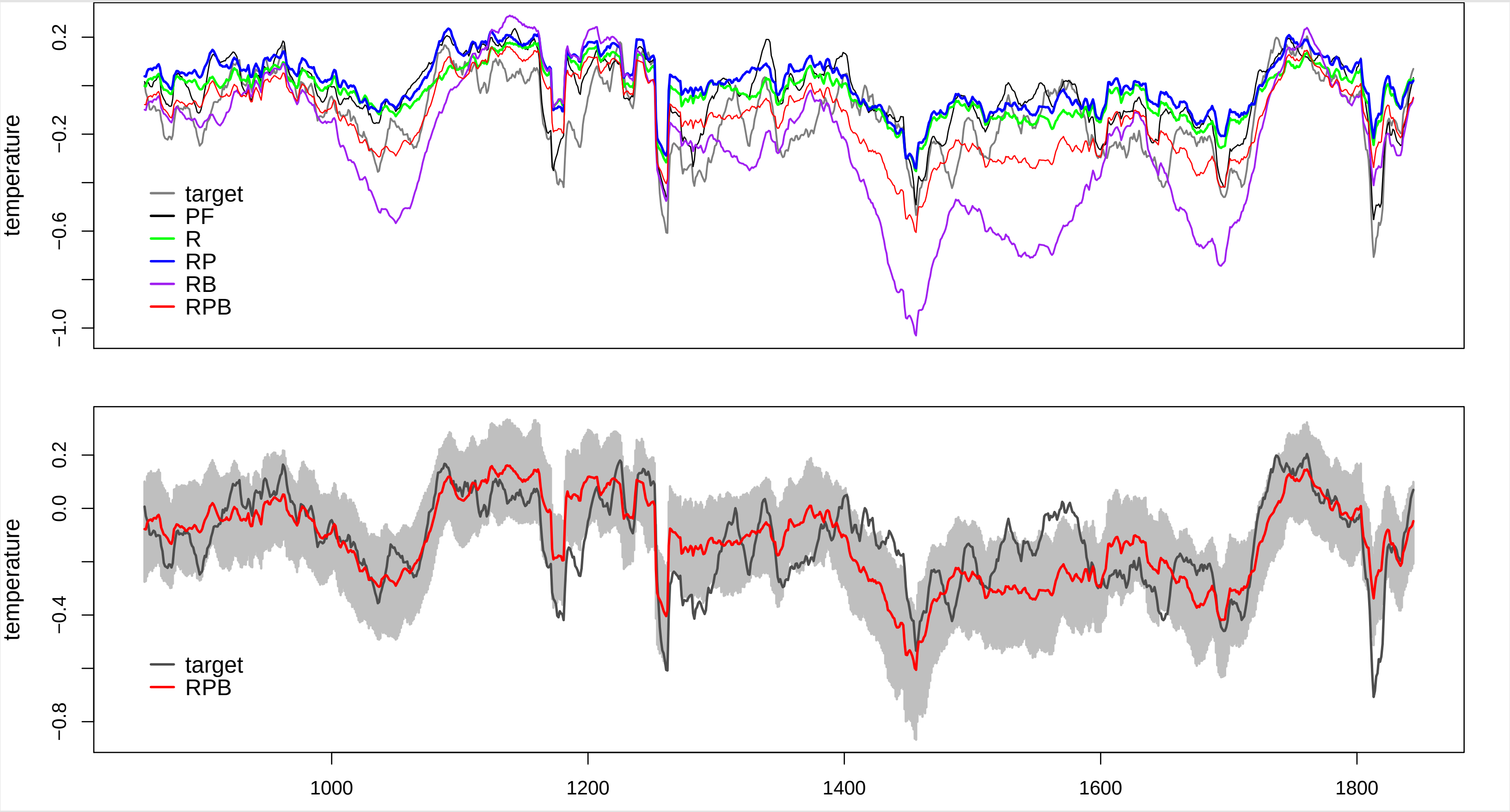
(iv) Tree ring + Borehole (RB)

(v) Tree ring + Borehole + Pollen (RBP)

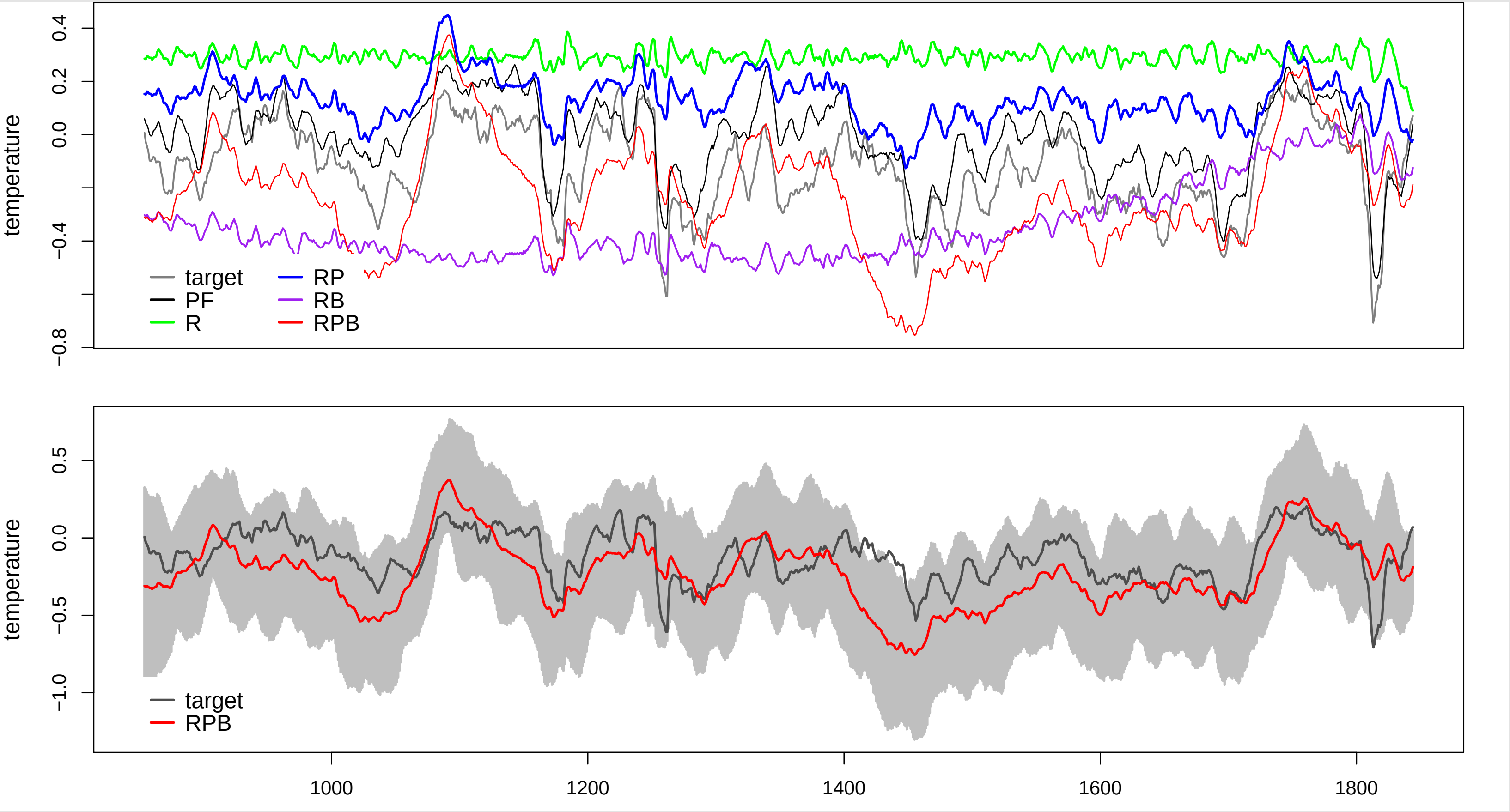


perfect proxes
with forcings
perfect proxes
without forcings
contaminated proxies
with forcings
contaminated proxies
without forcings
bias
0.4
0.2
0.0
−0.2
PF
R
RP
RB
RBP
model
rmse of the reconstruction
0.1
0.2
0.3
0.4
models
PF
R
RP
RB
RBP
0
20
40
60
80
100
level of smoothing (year)

Contaminated proxies With Forcings



Contaminated proxies Without Forcings



Discussion

- Forcings play an important role in the temperature reconstruction
- Pollen and borehole together can greatly improve the calibration of the reconstruction especially when forcings are not available
- BHM provides a flexible framework to integrate information from different sources
- The results are based on the climate from one model run, so they may not replicate very well given a different model run
- Borehole model needs further investigation