Case Study I: Combining Ensembles

Markov Random Fields and Regional Climate Models

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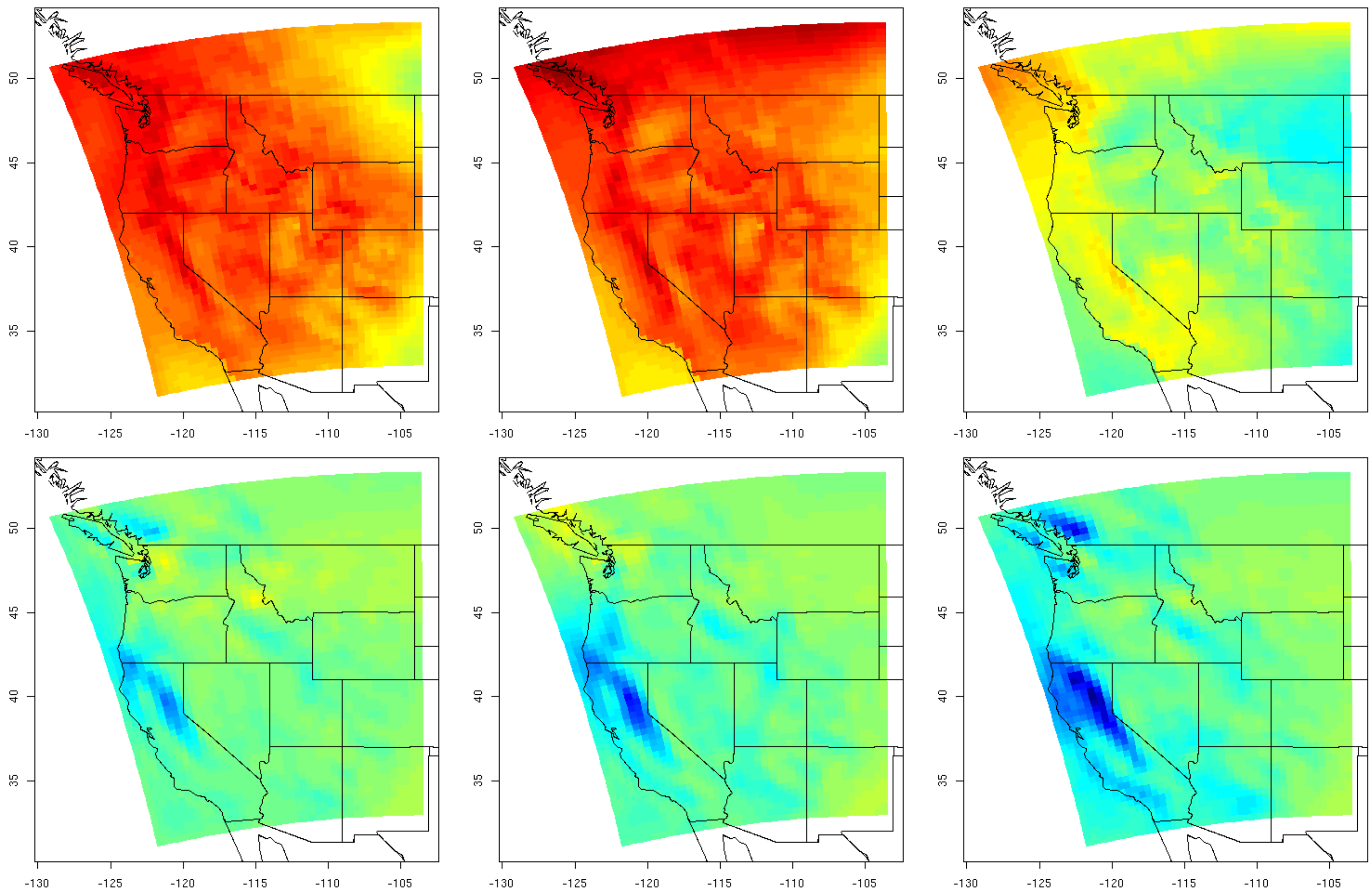
An RCM Example

- Driven by the NCAR/DOE Parallel Climate Model, the MM5 RCM was used to produce a control run and three future runs.
  - Control: 1995-2015
  - Future: 2040-2060
- Domain: western US, part of western Canada
- Climate scenario: "business as usual"
  - 1% yearly increase in greenhouse gasses
- Daily max/min temperature and precipitation

An RCM Example

- $n = 2464$ grid boxes on a regular lattice.
- Max/min temperature converted into midpoint (and range).
- 20-year winter (DJF) "averages" computed for each grid box.
- Two variables: Δ_{Tmid}, Δ_{Precip}
  - $\Delta = \text{Future}_i - \text{Control}, \; i = 1, 2, 3$

An RCM Example



A Hierarchical Model

- Data model:

$$\mathbf{y}_{rj} \sim \mathcal{N}\left(\mathbf{X}_1\boldsymbol{\alpha}_j + \mathbf{X}_2\boldsymbol{\beta}_{rj} + \mathbf{h}_{rj}, \boldsymbol{\Sigma}_j\right), \quad r = 1, \ldots, m, \quad j = 1, \ldots, p,$$

  - $\mathbf{X}$ includes intercept, longitude, latitude, and elevation.

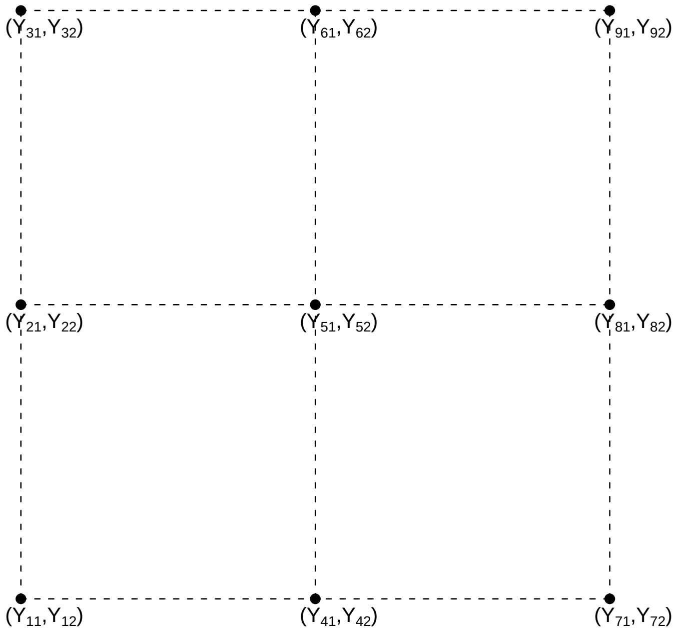
- Process model:

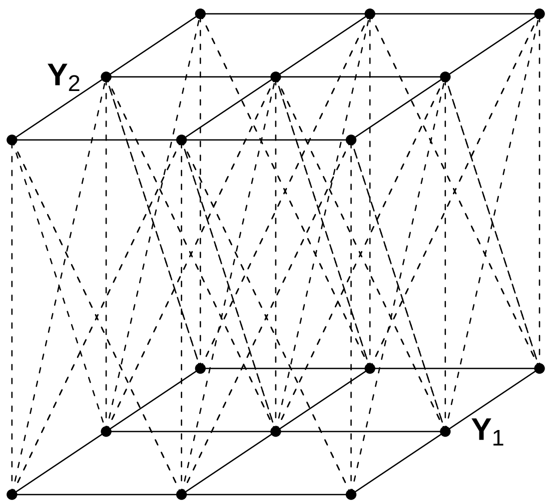
$$\begin{pmatrix} \boldsymbol{\beta}_{r1} \\ \vdots \\ \boldsymbol{\beta}_{rp} \end{pmatrix} \sim \mathcal{N}\left(\begin{pmatrix} \boldsymbol{\beta}_1 \\ \vdots \\ \boldsymbol{\beta}_p \end{pmatrix}, \boldsymbol{\Sigma}_b\right),$$

$$\begin{pmatrix} \mathbf{h}_{r1} \\ \vdots \\ \mathbf{h}_{rp} \end{pmatrix} \sim \mathcal{N}\left(\begin{pmatrix} \mathbf{h}_1 \\ \vdots \\ \mathbf{h}_p \end{pmatrix}, \mathbf{V}\left(\{\tau_j^2\}, \{\rho_{j\ell}\}, \{\phi_{j\ell}\}\right)\right).$$

- Priors: Non-informative.

A Different Approach





To improve interpretability and identifiability, rethink the lattice: rather than multivariate observations on a bivariate lattice, consider univariate observations on a stacked lattice.

A Different Approach

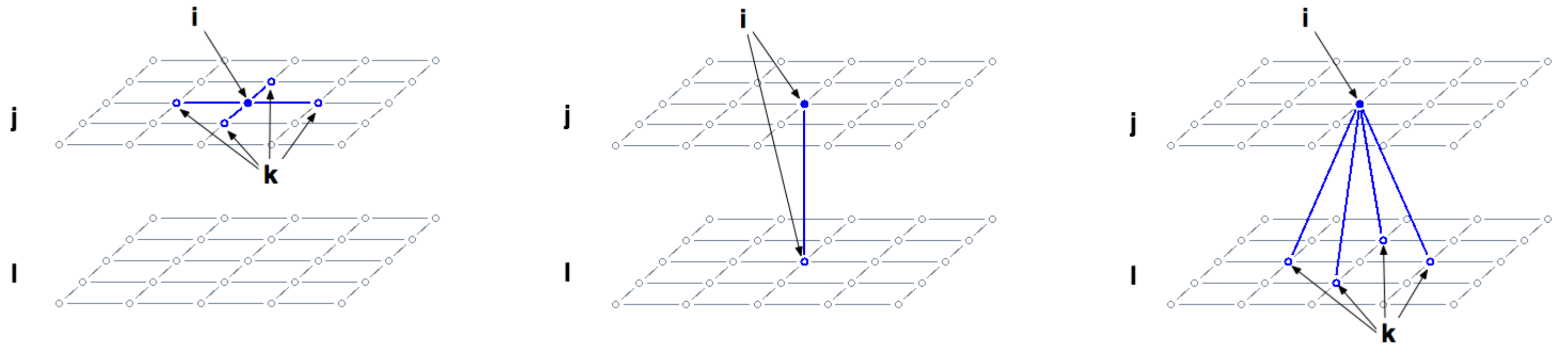
- Write the mean/variance of each conditional distribution as

$$
\begin{aligned}
\mathrm{E}[y_{ij}|y_{-\{ij\}}] = \mu_{ij} \;\; & + \;\; \sum_k b_{ijkj}(y_{kj} - \mu_{kj}) \qquad \text{(left)} \\
& + \;\; \sum_\ell b_{iji\ell}(y_{i\ell} - \mu_{i\ell}) \qquad \text{(middle)} \\
& + \;\; \sum_{k,\ell} b_{ijk\ell}(y_{k\ell} - \mu_{k\ell}) \qquad \text{(right)}
\end{aligned}
$$

and

$$
\mathrm{Var}[y_{ij}|y_{-\{ij\}}] = \tau_{ij}^2,
$$

A Different Approach



A Different Approach

- This formulation gives rise to a joint distribution of the form $\mathcal{N}(\boldsymbol{\mu}, \boldsymbol{\Sigma})$ where

$$\boldsymbol{\Sigma} = \left[\mathbf{I}_n \otimes \boldsymbol{\tau}^{1/2}\right] \left[\mathbf{I}_n \otimes \mathbf{A} - \begin{bmatrix} \mathbf{0} & & \mathbf{B} I_{ij} \\ & \ddots & \\ \mathbf{B}' I_{ij} & & \mathbf{0} \end{bmatrix}\right]^{-1} \left[\mathbf{I}_n \otimes \boldsymbol{\tau}^{1/2}\right],$$

where $\boldsymbol{\tau}^{1/2} = [\tau_1, \ldots, \tau_p]'$ and

$$\mathbf{A} = \begin{bmatrix} 1 & & -\rho_{j\ell} \\ & \ddots & \\ -\rho_{j\ell} & & 1 \end{bmatrix} \qquad \mathbf{B} = \begin{bmatrix} -\phi_{11} & & -\phi_{j\ell} \\ & \ddots & \\ -\phi_{\ell j} & & -\phi_{pp} \end{bmatrix}.$$

Bayesian Computation

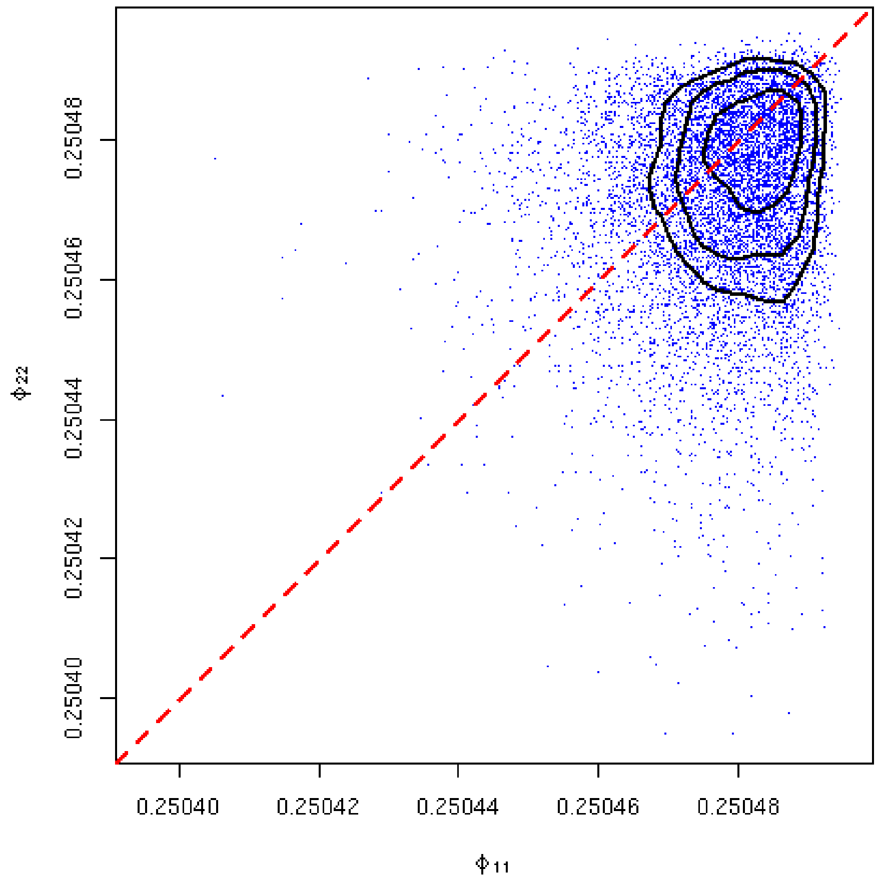
- Not much hope of computing posterior in closed form.

- Use computational methods (MCMC) to probe the posterior.
  - The Gibb's sampler.
  - Metropolis-Hastings – accept/reject method.

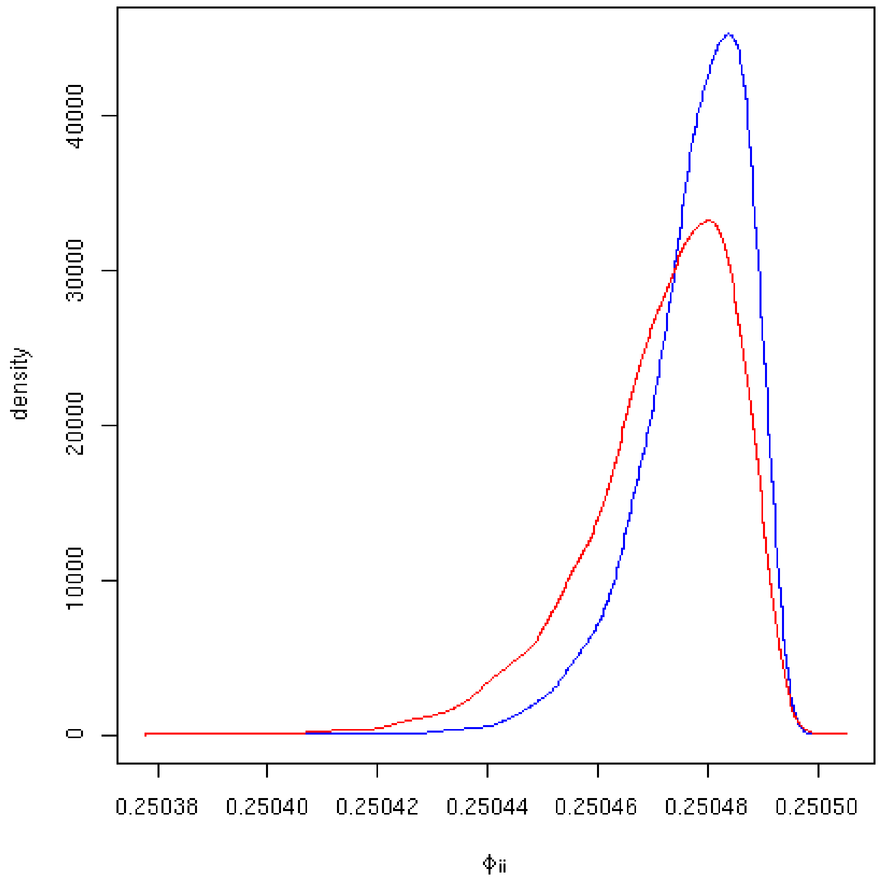
MCMC

- 3 regimes of 10K iterations each
  - Regime 1: all single variable M-H with periodic updates of the proposal distribution.
  - Regime 2: ρ, ϕ_{12}, and ϕ_{21} blocked M-H with periodic updates of the proposal distribution.
  - Regime 3: ρ, ϕ_{12}, and ϕ_{21} blocked M-H with no further updates.
- 10 chains run with random starts from the prior.
  - Choose ρ and $\boldsymbol{\phi}$ to yield positive-definite $\mathbf{H}$.
- Sparse matrix methods (`spam`) crucial to computation.

ϕ_{11}, ϕ_{22}

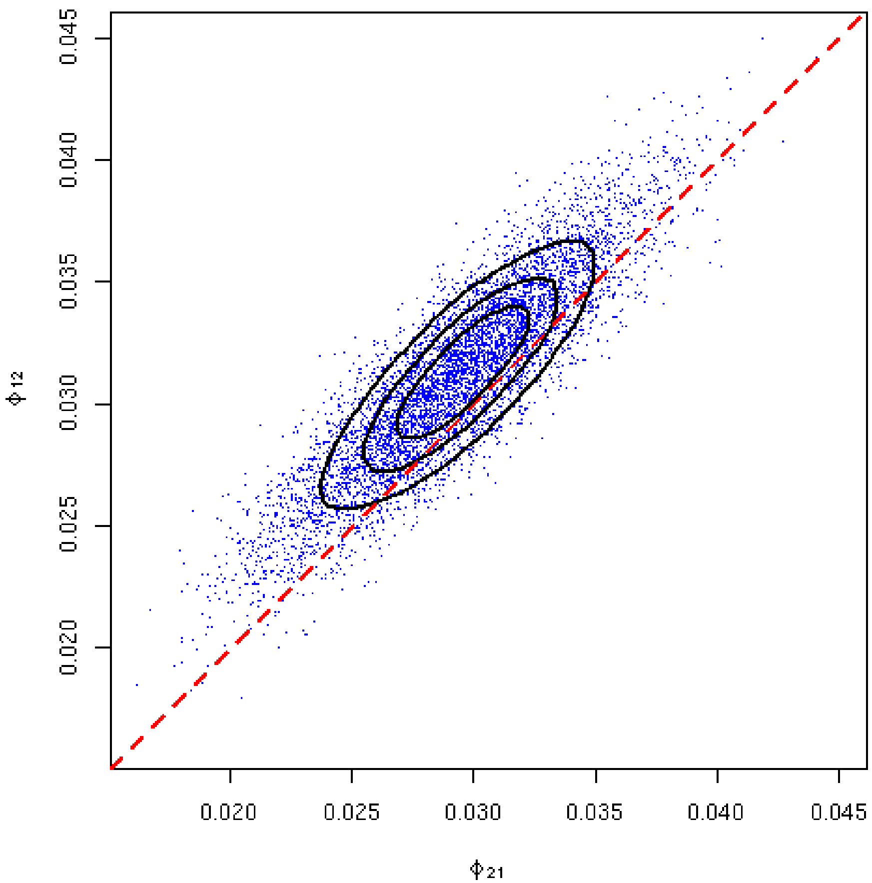


0.25040
0.25042
0.25044
0.25046
0.25048
ϕ11
0.25040
0.25042
0.25044
0.25046
0.25048
ϕ22

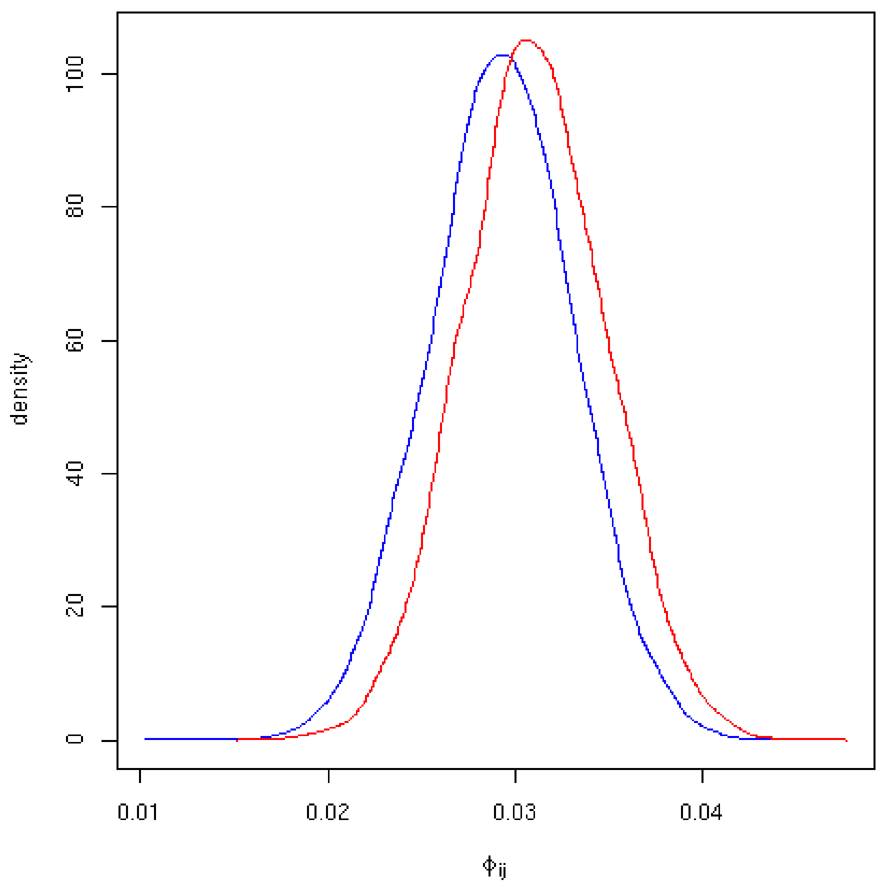


0.25038
0.25040
0.25042
0.25044
0.25046
0.25048
0.25050
ϕii
0
10000
20000
30000
40000
density

ϕ_{12}, ϕ_{21}

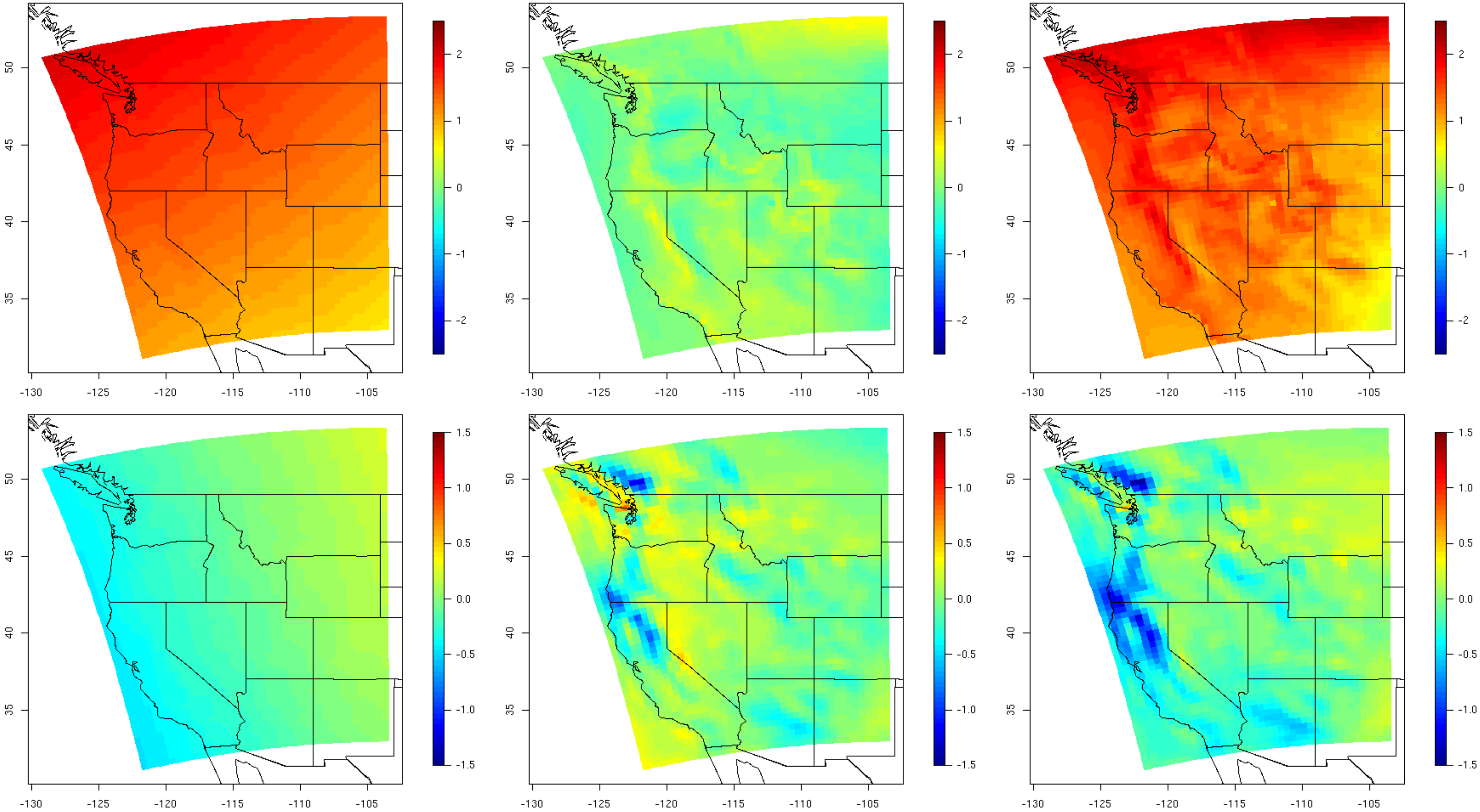


0.045
0.040
0.035
0.030
0.025
0.020
ϕ12
0.020
0.025
0.030
0.035
0.040
0.045
ϕ21

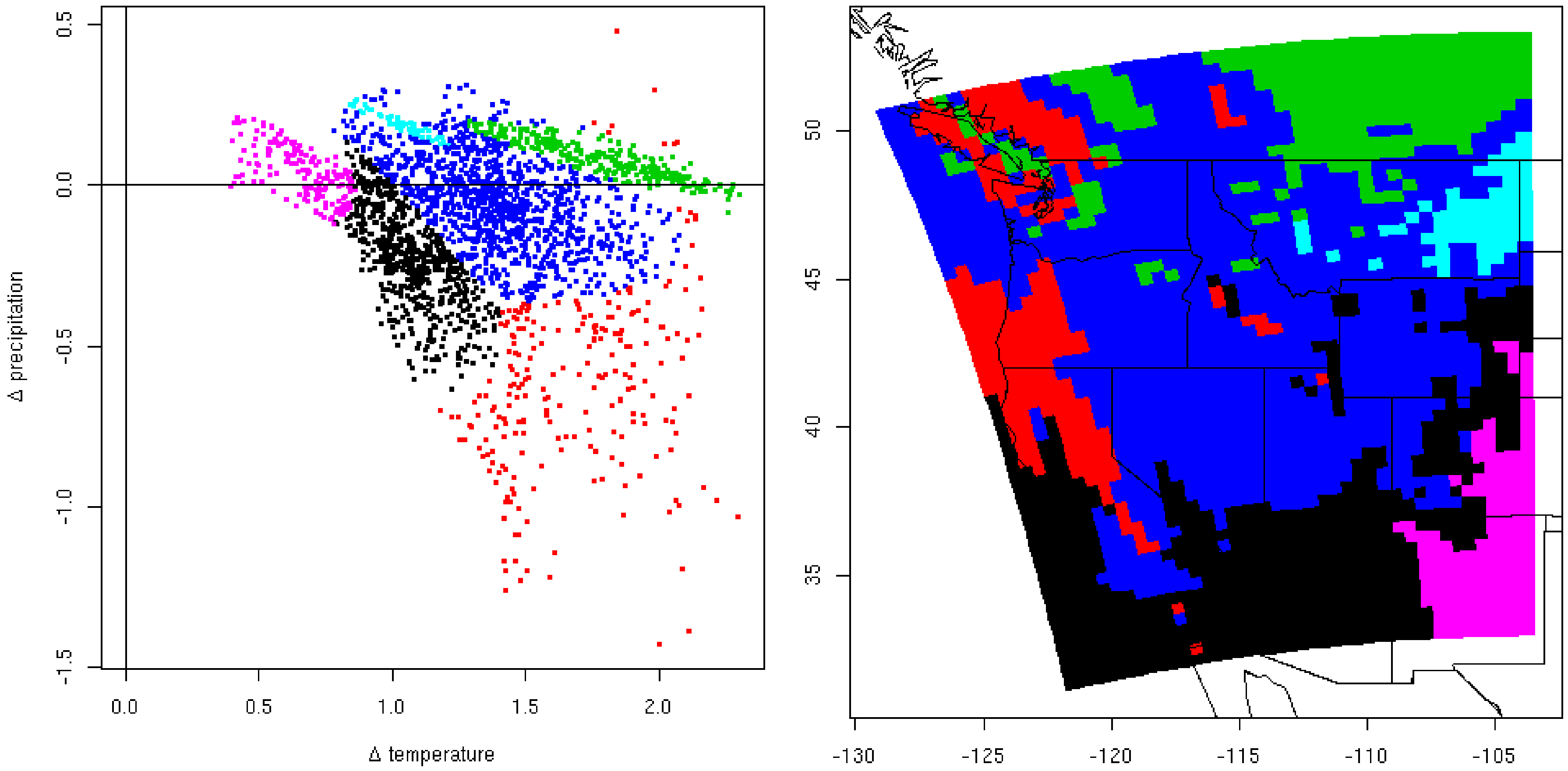


100
80
60
40
20
0
density
0.01
0.02
0.03
0.04
ϕij

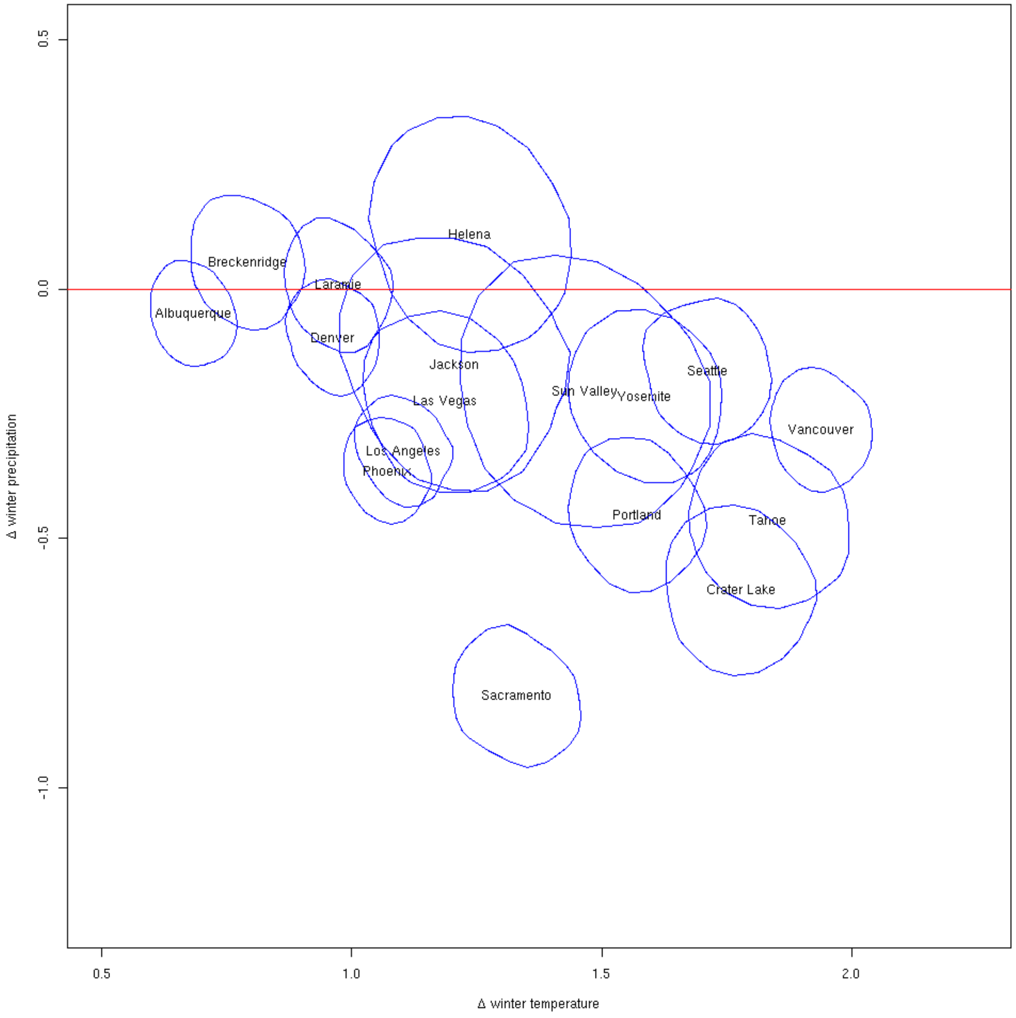
Posterior Means



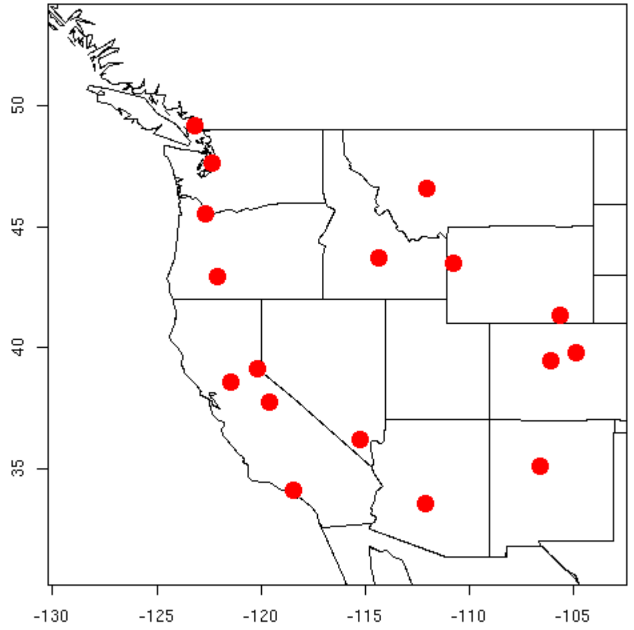
Clustering


0.5
0.0
-0.5
-1.0
-1.5
Δ precipitation
0.0
0.5
1.0
1.5
2.0
Δ temperature
50
45
40
35
-130
-125
-120
-115
-110
-105

Impacts

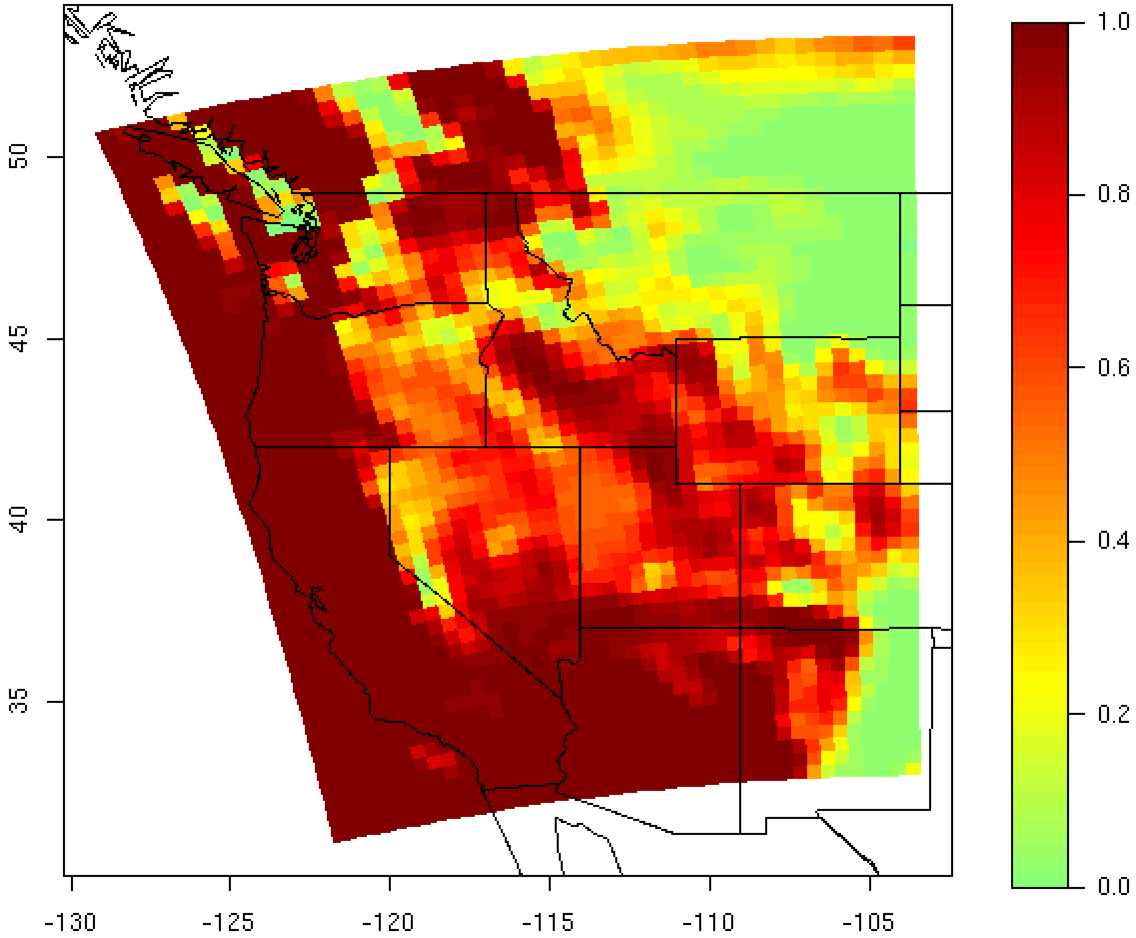


Helena
Breckenridge
Laramie
Albuquerque
Denver
Jackson
Las Vegas
Sun Valley
Yosemite
Seattle
Vancouver
Los Angeles
Phoenix
Portland
Tahoe
Crater Lake
Sacramento
0.5
0.0
-0.5
-1.0
0.5
1.0
1.5
2.0
Δ winter precipitation
Δ winter temperature

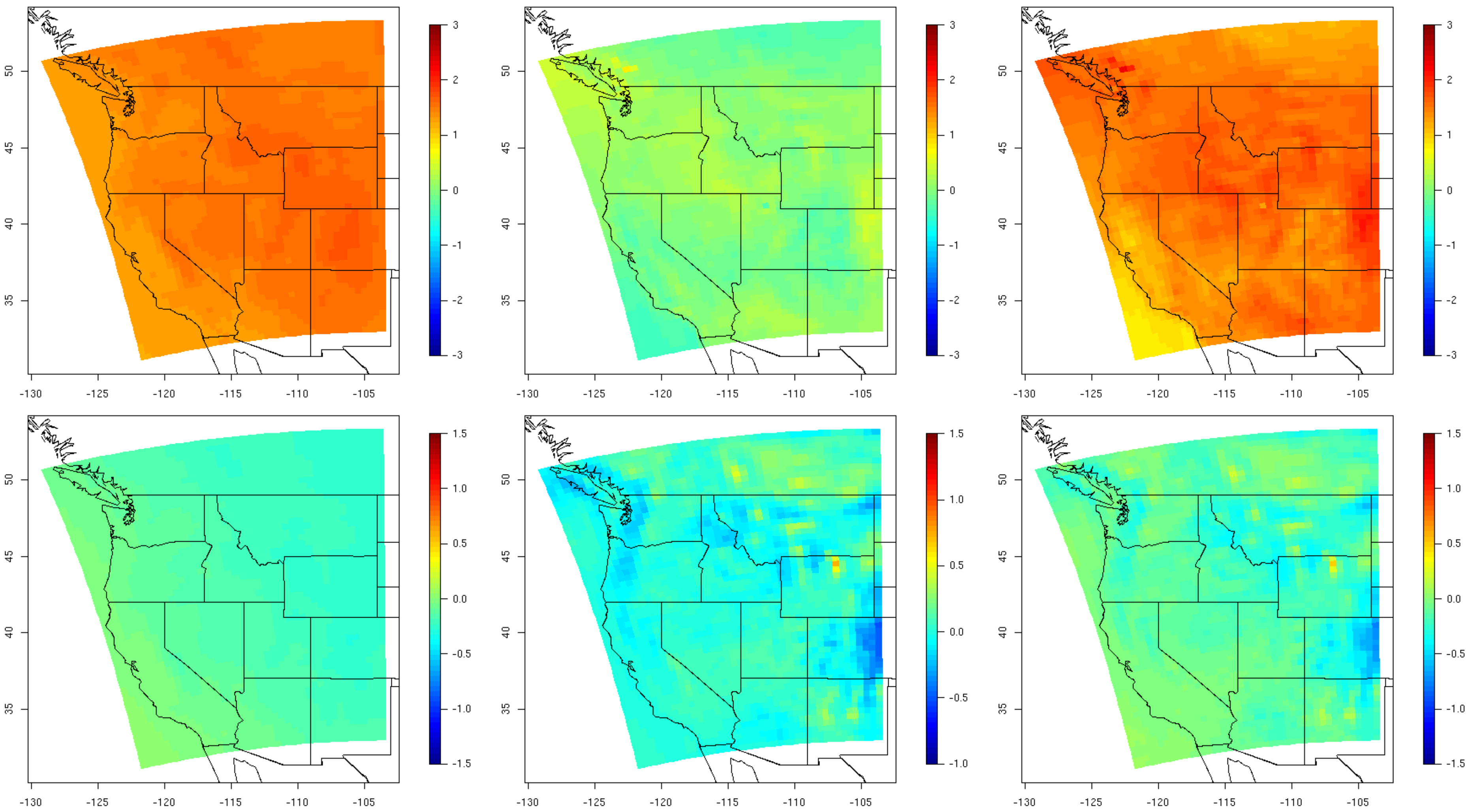


50
45
40
35
-130
-125
-120
-115
-110
-105

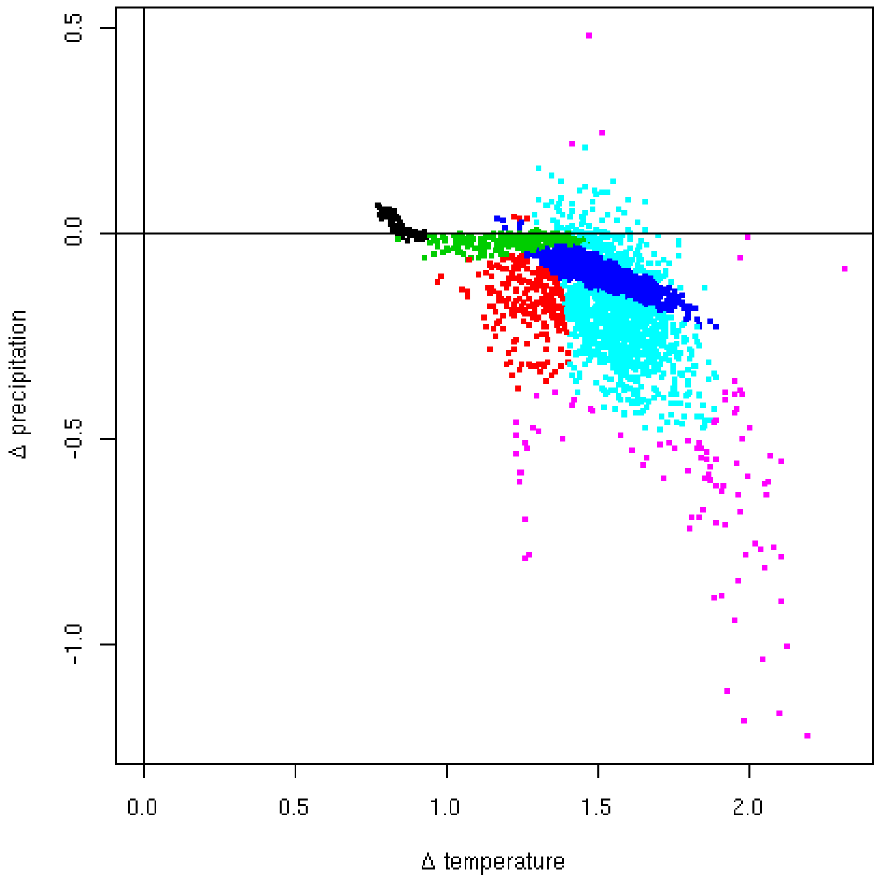
Impacts

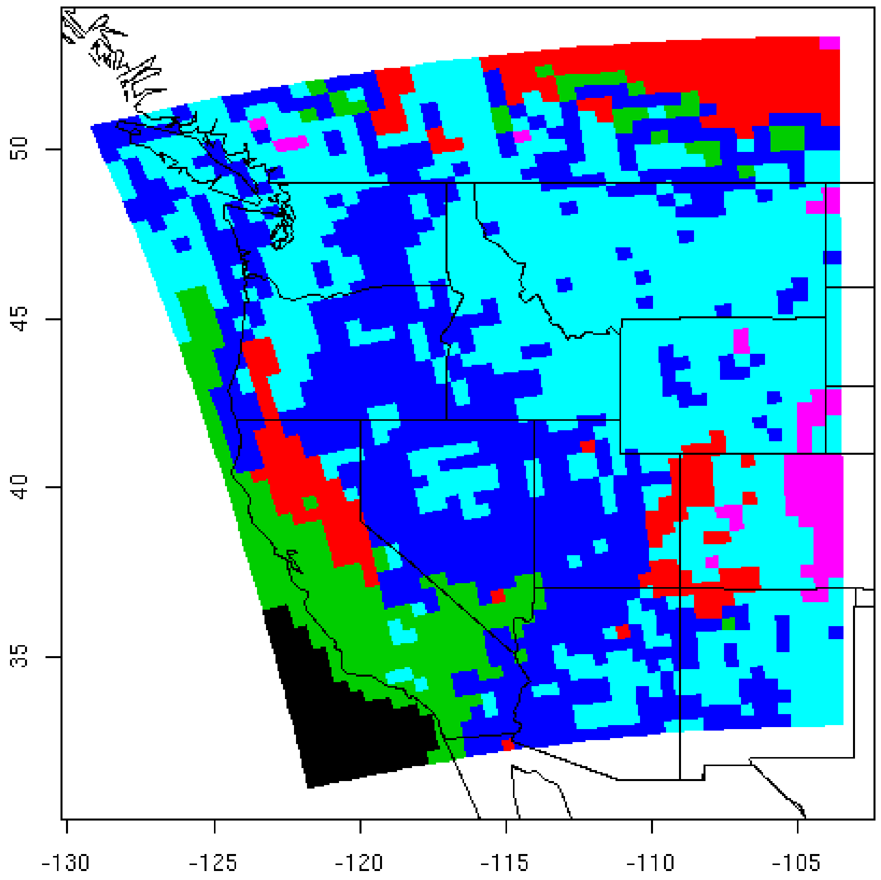


Posterior Means

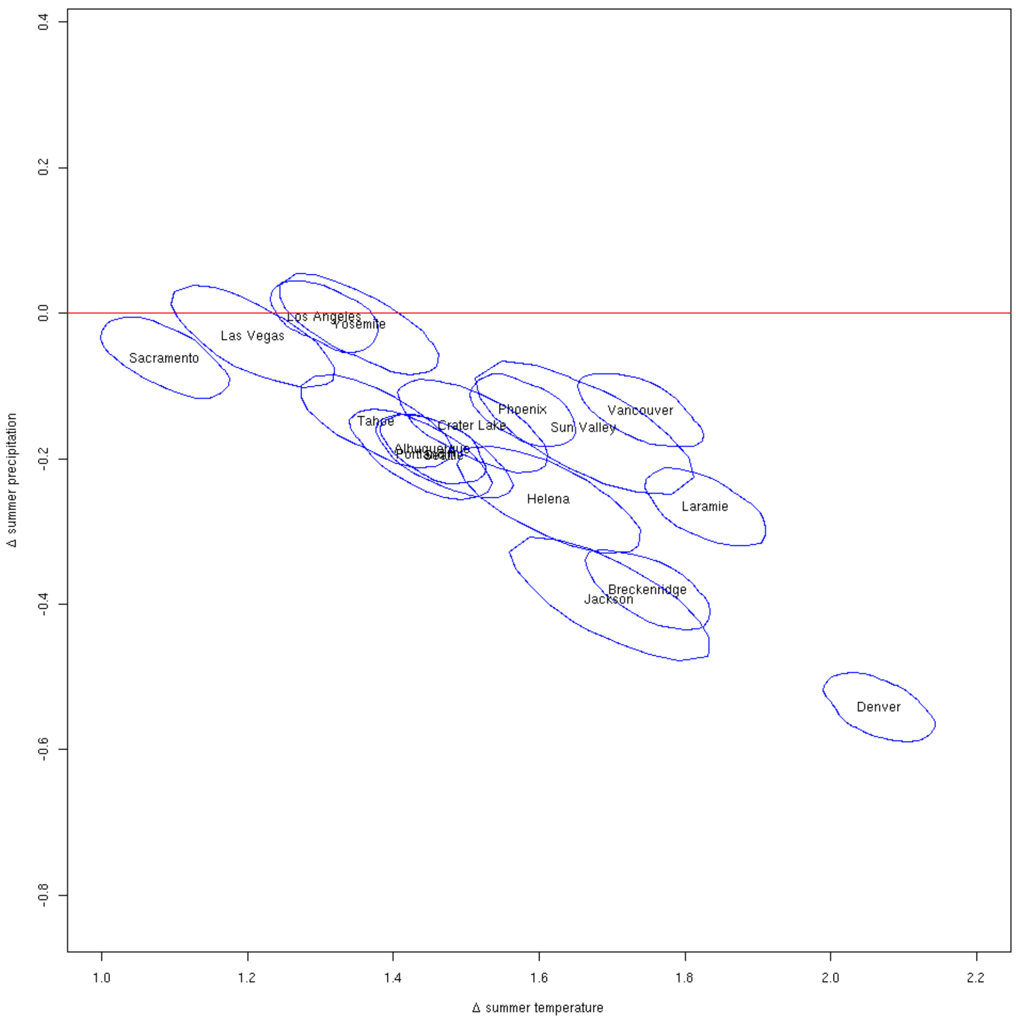


Clustering

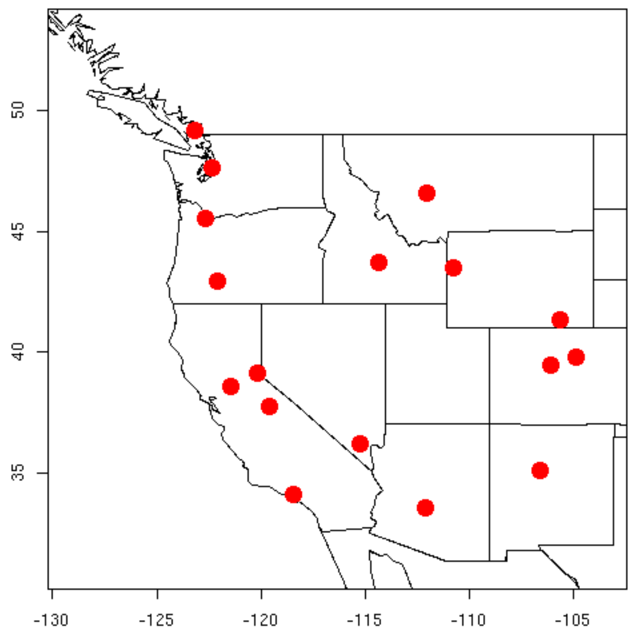

0.5
0.0
-0.5
-1.0
Δ precipitation
0.0
0.5
1.0
1.5
2.0
Δ temperature


50
45
40
35
-130
-125
-120
-115
-110
-105

Impacts



Δ summer precipitation
Δ summer temperature
0.4
0.2
0.0
-0.2
-0.4
-0.6
-0.8
1.0
1.2
1.4
1.6
1.8
2.0
2.2
Sacramento
Las Vegas
Los Angeles
Yosemite
Tahoe
Crater Lake
Albuquerque
Phoenix
Sun Valley
Vancouver
Helena
Laramie
Jackson
Breckenridge
Denver



50
45
40
35
-130
-125
-120
-115
-110
-105

Impacts