Case Study III: Functional ANOVA

Markov Random Fields and Regional Climate Models

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Goals

- Describe the distribution of (regional) climate model output.

- Understanding sources of variation.
  - NARCCAP/PRUDENCE: GCM, RCM, GCM×RCM.
  - climateprediction.net: perturbed physics.
  - Others sources?

- Combining model output & weighting models.

- Recognizing model output represents spatial, temporal, or spatial-temporal fields ⇒ functional ANOVA.
NARCCAP

- North American Regional Climate Change Assessment Program (NARCCAP)
  - NCAR, ISU, CCCma, OURANOS, LLNL, GFDL, Hadley, Scripps, PNNL, USSC, UCDHSC, etc.
  - NSF, NOAA, DOE, etc.
  - www.narccap.ucar.edu

- Systematically investigate the uncertainties in regional scale projections of future climate.
## NARCCAP Design

- 4 GCMs provide boundary conditions for 6 RCMs

<table>
<thead>
<tr>
<th>RCM</th>
<th>GCM</th>
<th>GFDL</th>
<th>CGCM3</th>
<th>HADCM3</th>
<th>CCSM</th>
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<td>X</td>
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</tbody>
</table>
A Work in Progress

- Three regional models – ECPC, MRCC, and RCM3
- Boundary conditions supplied by reanalysis.
- 1980-1999 (20 years)
- Total seasonal precipitation – winter (DJF) and summer (JJA)
- Common grid: $123 \times 101 = 12,423$ grid boxes
A Statistical Model

- A hierarchical construction:
  - Data model: \( Y_{ij} \sim \mathcal{N}(\mu_i, \sigma_{1i}^2 V(\theta_1)) \), \( i = 1, 2, 3, \ldots, 20 \)
  - Process model: \( \mu_i \sim \mathcal{N}(\mu, \sigma_{2i}^2 V(\theta_2)) \)
  - Prior model: non-informative.

- An alternative formulation:

\[
Y_{ij} = \mu + \alpha_i + \epsilon_{ij} = \text{Common} + \text{RCM} + \text{Error}
\]
A Statistical Model

- Spatial correlation matrix $V(\theta) = R(\theta) \otimes C(\theta)$ where $R$ and $C$ are parameterized through 1-D “stationary” Markov random fields:

$$R(\theta) = \begin{bmatrix}
1 & -\theta \\
-\theta & 1 + \theta^2 & -\theta \\
-\theta & 1 + \theta^2 & -\theta \\
& \ddots & \ddots & \ddots \\
-\theta & 1 + \theta^2 & -\theta \\
-\theta & 1 + \theta^2 & -\theta \\
-\theta & 1 + \theta^2 & -\theta \\
-\theta & 1 + \theta^2 & -\theta \\
-\theta & 1 + \theta^2 & -\theta \\
-\theta & 1 \\
\end{bmatrix}^{-1}$$

- Computationally efficient: lower-dimensional + sparse precision matrices.

- Other choices: tapering, reduced-rank kriging, etc.
MRF Formulation

- The conditional weight structure for an interior point:

\[
\frac{1}{(1 + \theta^2)^2} \begin{pmatrix}
0 & 0 & \cdots & 0 \\
0 & \theta^2 & -\theta(1 + \theta^2) & 0 \\
0 & -\theta(1 + \theta^2) & -\theta(1 + \theta^2) & \theta^2 \\
0 & 0 & -\theta(1 + \theta^2) & 0 \\
\vdots & \vdots & \vdots & \ddots 
\end{pmatrix}
\]

- And the resulting correlation functions:

![Correlation Functions]
Another View

- Exponential-like behavior?
A Statistical Model

- A hierarchical construction:
  
  Data model: \( Y_{ij} \sim N(\mu_i, \sigma^2_1 V(\theta_1)) \), \( i = 1, 2, 3, j = 1, \ldots, 20 \)

  Process model: \( \mu_i \sim N(\mu, \sigma^2_2 V(\theta_2)) \)

  Prior model: non-informative.

- An alternative formulation:

  \[
  Y_{ij} = \mu + \alpha_i + \epsilon_{ij} = \text{Common} + \text{RCM} + \text{Error}
  \]
Parameter Estimation

- MCMC to estimate parameters, posterior inference, etc.
Posterior Means
Posterior Means
Posterior Means
Inference

- for i in 1 to “a big number” ...
  - sample \((\mu^*, \mu_1^*, \mu_2^*, \mu_3^*) \Rightarrow \alpha^* = (\alpha_1^*, \alpha_2^*, \alpha_3^*)\)
  - construct (for each grid box):
    * \(s_{\alpha}^2\) (model-to-model variation)
    * \(s^2\) (year-to-year variation)
  - identify and record grid boxes where \(s_{\alpha}^2\) is larger than \(s^2\).
- compute \(\hat{P}[s_{\alpha}^2 > s^2]\) for each grid box
Winter Precipitation
Summer Precipitation
A PRUDENCE Example

- 2x2 “experiment”
  - 2 GCMs, 2 RCMs
  - PRUDENCE
- 1961-1990
- JJA average temp
A Two-Factor Model

\[ Z_{ijt}(s) = \mu_{ijt}(s) + \epsilon_{ijt}(s) \]

Output of RCM i, GCM j, at time t and location s = “Climate” + Spatially correlated residual/“internal response variability”

\[ \mu_{ijt}(s) = \mu(s) + i\alpha(s) + j\beta(s) + ij(\alpha\beta)(s) + \gamma t, \]

= Common + RCM + GCM + Interaction + Time

- \( i, j = -1, 1 \) (contrast coding)
- Hierarchical model with Gaussian process priors used for each effect.
- MCMC used to estimate parameters, posterior inference, etc.
Posterior Means

- Estimates of spatial effects.
Functional ANOVA

- Ratios of variances.