# Data

Data provided for the Fourth Assessment Report of IPCC:

- 21 models (CCSM, GFDL, HADCM, PCM, . . . )
- Around $2.8^\circ \times 2.8^\circ$ resolution (8192 data points, T42)
- Different scenarios (A2: "business as usual", A1B, B1)
- Temperature, precipitation, pressure, winds. . .

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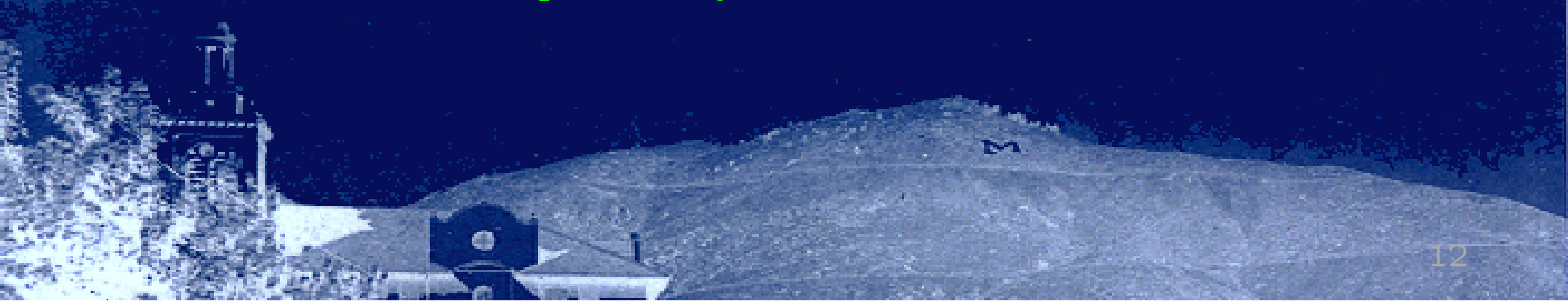
- 21 models (CCSM, GFDL, HADCM, PCM, . . . )
- Around $2.8^\circ \times 2.8^\circ$ resolution (8192 data points, T42)
  aggregate to $5^\circ \times 5^\circ$ and omit the "poles" (3264 points).
- Different scenarios (A2: "business as usual", A1B, B1)
- Temperature, precipitation, pressure, winds. . .
  seasonal averages over years 1980–1999 and 2080–2099



# Statistical Model

Given AOGCM output construct a statistical model to describe climate change probabilistically while accounting for all (most?) underlying uncertainties.

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For models $i = 1, \ldots, N$, stack the gridded seasonal temperature into vectors:

$$\mathbf{X}_i = \text{simulated present climate}_i$$
$$\mathbf{Y}_i = \text{simulated future climate}_i$$

PDF and probabilistic description of climate change

$$\mathbf{D}_i = \mathbf{Y}_i - \mathbf{X}_i$$

# Hierarchical Model

Separate the statistical modeling of a complex process into different levels consisting of:

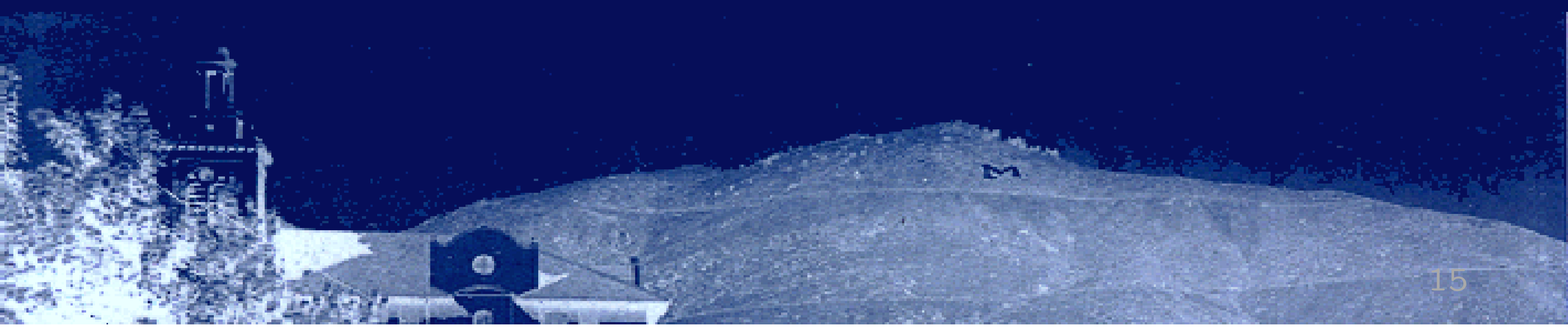
| | | |
|---|---|---|
| Data level: | Classical geostatistics | (variogram, kriging) |
| Process level: | Multivariate analysis | (EOF, PCA) |
| Prior level: | Bayesian statistics | (priors, MCMC) |

$\leadsto$ hierarchical Bayesian modeling

# Data Level

{ Data level | Process level | Prior level }

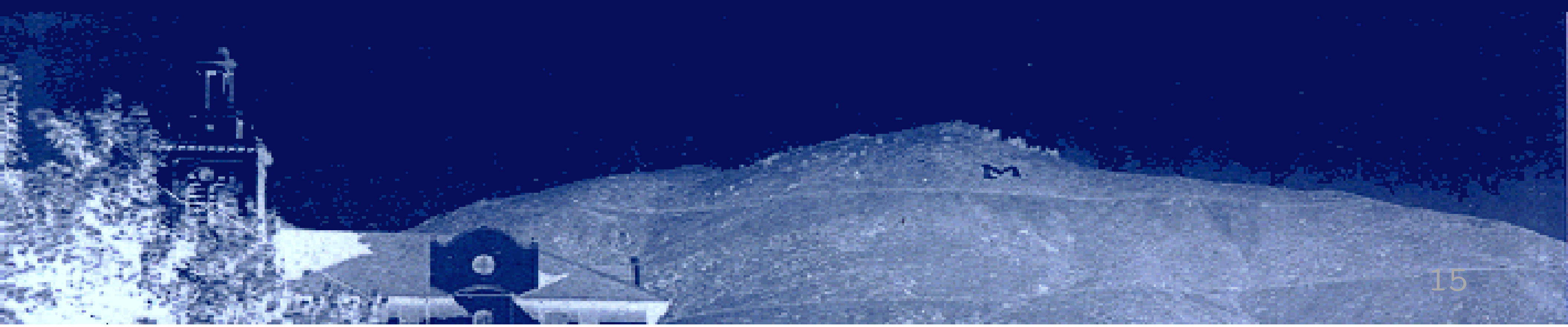
$$\mathbf{D}_i = \mathbf{Y}_i - \mathbf{X}_i = \text{simulated climate change}$$



# Data Level

{ Data level | Process level | Prior level }

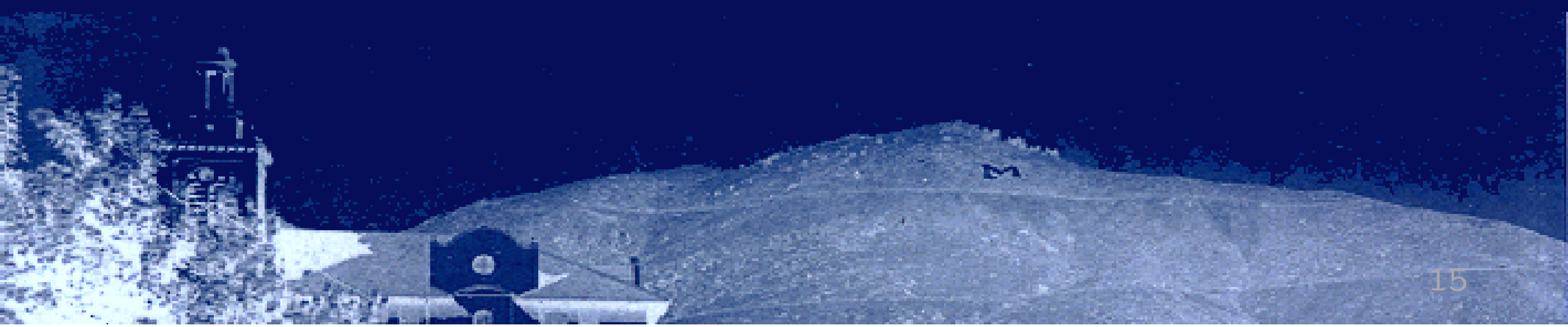
$$\begin{aligned} \mathbf{D}_i = \mathbf{Y}_i - \mathbf{X}_i &= \text{simulated climate change} \\ &= \text{large scale structure} + \text{small scale structure} \end{aligned}$$



# Data Level

{ Data level | Process level | Prior level }

$$
\begin{aligned}
\mathbf{D}_i = \mathbf{Y}_i - \mathbf{X}_i &= \text{simulated climate change} \\
&= \text{large scale structure} + \text{small scale structure} \\
&= \text{climate signal} + \text{model bias and internal variability}
\end{aligned}
$$



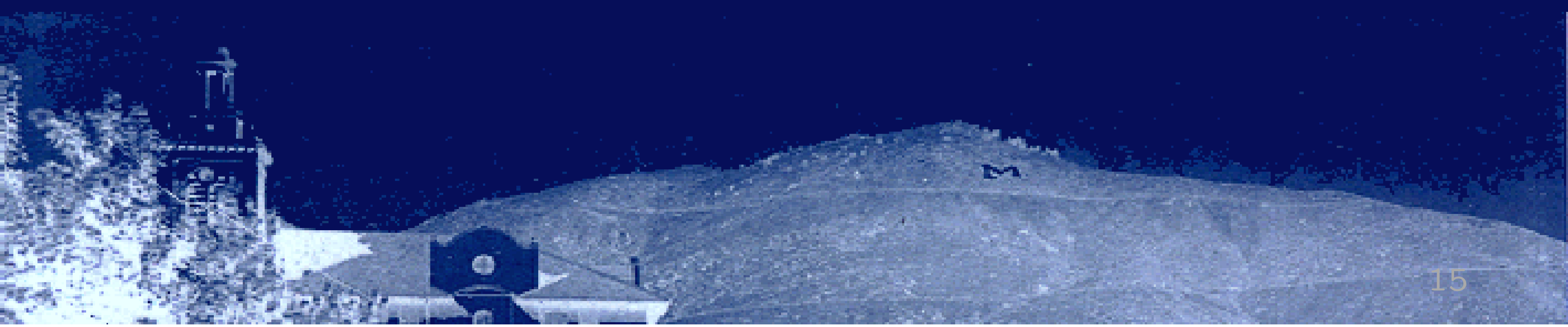
# Data Level

{ Data level | Process level | Prior level }

$$
\begin{aligned}
\mathbf{D}_i = \mathbf{Y}_i - \mathbf{X}_i &= \text{simulated climate change} \\
&= \text{large scale structure} + \text{small scale structure} \\
&= \text{climate signal} + \text{model bias and internal variability} \\
&= \qquad \boldsymbol{\mu}_i \qquad + \qquad \varepsilon_i
\end{aligned}
$$

$$
\mathbf{D}_i \mid \boldsymbol{\mu}_i, \phi_i \overset{\text{iid}}{\sim} \mathcal{N}_n(\boldsymbol{\mu}_i, \phi_i \boldsymbol{\Sigma}) \qquad \phi_i > 0 \qquad i = 1, \ldots, N
$$

for given $\boldsymbol{\Sigma}$

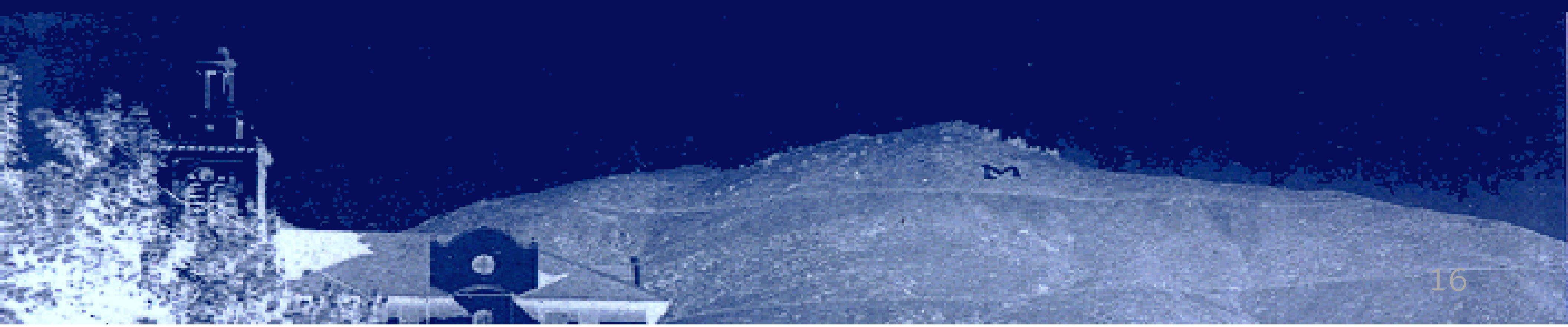


# Process Level

{ Data level | Process level | Prior level }

$$\boldsymbol{\mu}_i = \mathbf{M}\boldsymbol{\theta}_i$$

for given $\mathbf{M}$



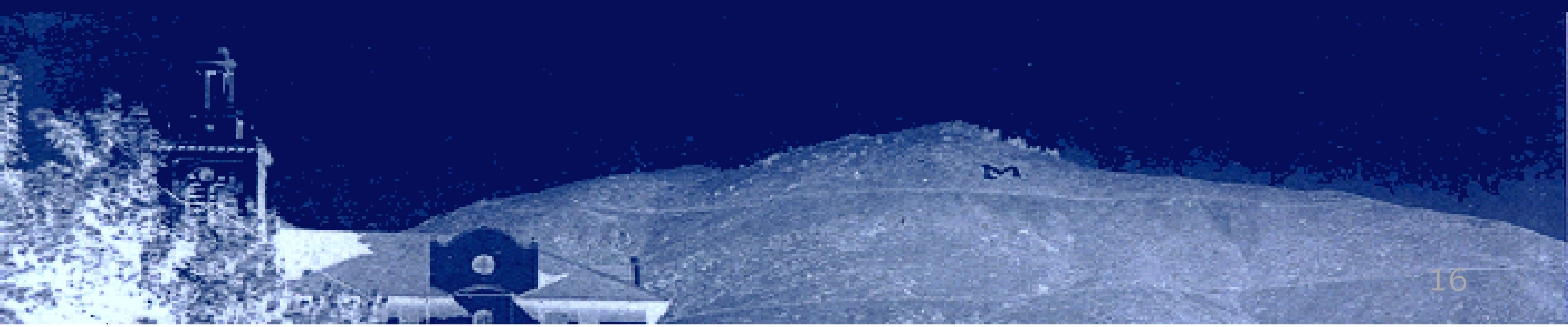
# Process Level

{ Data level | Process level | Prior level }

$$\boldsymbol{\mu}_i = \mathbf{M}\boldsymbol{\theta}_i$$

for given $\mathbf{M}$

$$\boldsymbol{\theta}_i \mid \boldsymbol{\nu},\, \psi_i \overset{\text{iid}}{\sim} \mathcal{N}_p(\, \boldsymbol{\nu},\, \psi_i \mathbf{I}\,) \qquad \psi_i > 0 \qquad i = 1, \ldots, N$$

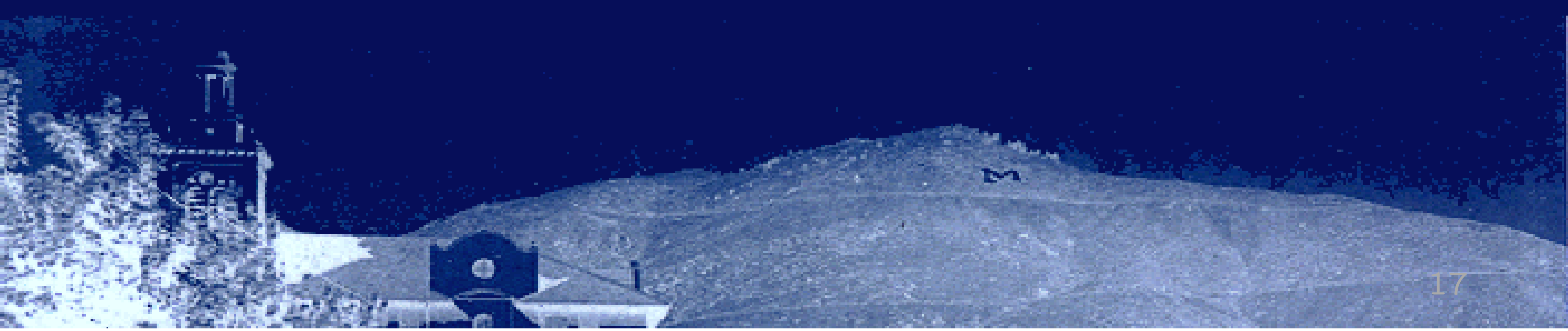


# Prior Level

{ Data level | Process level | Prior level }

$$
\begin{array}{lll}
\phi_i \overset{\text{iid}}{\sim} \mathrm{I\Gamma}(\, \xi_1,\, \xi_2 \,) & \xi_1,\, \xi_2 > 0 & i = 1, \ldots, N \\
\psi_i \overset{\text{iid}}{\sim} \mathrm{I\Gamma}(\, \xi_3,\, \xi_4 \,) & \xi_3,\, \xi_4 > 0 & i = 1, \ldots, N \\
\boldsymbol{\nu} \sim \mathcal{N}_p(\, \mathbf{0},\, \xi_5 \mathbf{I} \,) & \xi_5 > 0 &
\end{array}
$$

for given $\xi_1, \ldots, \xi_5$



# Initial Parameters

For the different levels we need to specify:

**Data level** Covariance model for $\phi_i \mathbf{\Sigma}$:
spatial coherence of internal variability and bias

**Process level** Basis functions used in **M**:
practical decompostion of possible signals,
dimension reduction

**Prior level** Hyperparameters $\xi_1, \xi_2, \quad \xi_3, \xi_4, \quad \xi_5$:
tuning parameters

# Covariance Model for $\phi_i\Sigma$

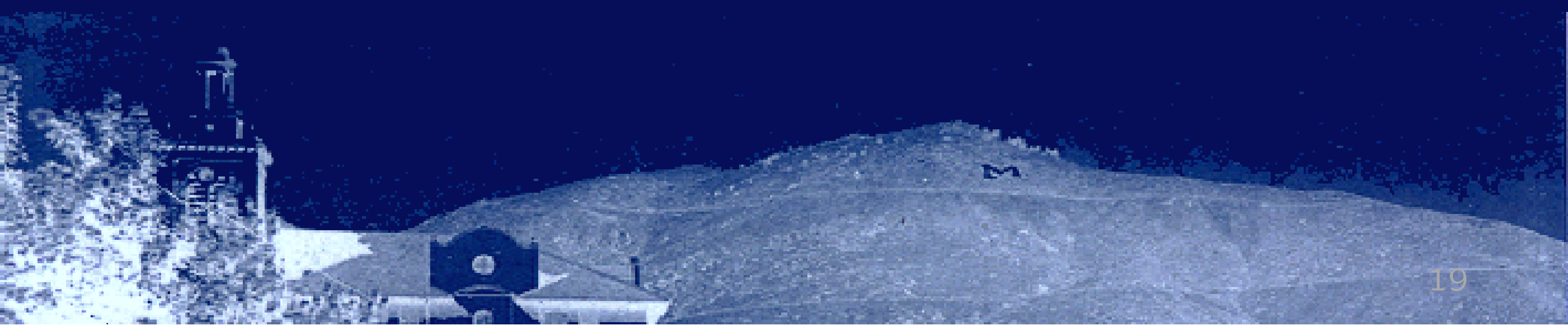
{ Data level | Process level | Prior level }

For the covariance matrices $\phi_i\mathbf{\Sigma}$, we need positive definite functions on the sphere (by restricting one on $\mathbb{R}^3$ to $\mathbb{S}^2$):

$$c(h;\phi_i,\tau) = \phi_i \exp\Big(-\tau \sin(h/2)\Big)$$

Individual variances $\phi_i$ are modelled.

Common range $\tau$ is choosen according to an "empirical Bayes" approach.



# Basis Functions Used in M

{ Data level | Process level | Prior level }

1. Spherical harmonics (here shown 4 out of 121)