Outline

- Kriging as a ridge regression
- ridge regression as Kriging
Kriging

We have a covariance function $\rho_k(x, x')$ and with $\lambda = \sigma^2 / \rho$ a basis $\{\phi_i\}$ for the fixed part
$g = \text{fixed part} + h$, $\hat{h}(x_0) = k_0(K + \lambda I)^{-1}(y - T\tilde{d})$

Kriging → ridge regression

STEP 1) Build basis functions from the covariance functions and observation locations:

$\phi_1(x) = \rho_k(x, x_1)$
$\phi_2(x) = \rho_k(x, x_2)$
...
$\phi_n(x) = \rho_k(x, x_n)$

STEP 2) Recognize that estimate is a sum of basis functions with coefficients found from the data!

$\hat{h}(x) = \sum_j \phi_j(x)c_j$, $\hat{c} = (K + \lambda I)^{-1}(y - T\tilde{d})$
STEP 3) Coefficients are really found by ridge regression!

\[ \hat{c} = (K + \lambda I)^{-1}(y - T\hat{d}) \]

\[ \hat{c} = (K^T K + \lambda K^T)^{-1} K^T (y - T\hat{d}) \]

\[ \hat{c} = (X^T X + \lambda H)^{-1} X^T (y - T\hat{d}) \]

**Conclusion:**

*This is a ridge regression estimator penalty matrix $H = K^T$ and $X = K$*

Note $K$ is symmetric so $H = K$. 
Ridge regression

Given the fixed basis \(\{\phi_i\}\), nonparametric basis \(\{\psi_j\}\) and penalty matrix \(H\).

\textit{Ridge regression} \rightarrow \textit{Kriging}

Create the covariance function:

\[
(1/\lambda)k(x, x') = \sum_{j,k} \psi_j(x) H_{jk}^{-1} \psi_k(x')
\]

\textbf{Conclusion:}

This is a spatial process estimate with covariance \((1/\lambda)k\) for \(h\).

At the observation points the covariance matrix is \(XH^{-1}X^T\)