Extreme Values on Spatial Fields

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Background

- 5th year graduate student
- 2nd year with GSP
- Anticipated graduation date: Fall 2005
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Projects

- Extreme value model for Lichenometry
- Spatial dependence estimation and prediction for annual maxima
- Colorado Front Range Precipitation
Colorado Precipitation Project

Goal: Create a map of precipitation return levels for Colorado’s Front Range.
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- project originated from ISSE, interested in flooding
- NWS has done maps for Southwest US (AZ, NM, UT, NV) and mid-Atlantic (OH, PA, MD, NJ, NC, WV)
- plans to do entire US (contingent on funding)
- handles spatial dependence and prediction differently
- present NWS with our method and results
Front Range Data

Data: hourly precipitation from 56 weather stations, 12-60 years of data, Apr1 - Oct 31.
Univariate Extreme Values

**GEV**: Used to model block (annual) maxima

\[
G(z; \mu, \sigma, \xi) = \exp \left[ - \left( 1 + \xi \left( \frac{z - \mu}{\sigma} \right) \right)^{-\frac{1}{\xi}} \right]
\]

- wasteful of data
- used by NWS to produce their maps
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**GPD:** Models exceedences above a threshold

\[ P\{X - u < y | X > u\} \approx 1 - \left(1 + \frac{\xi y}{\tilde{\sigma}}\right)^{-1/\xi} \]

where \( \tilde{\sigma} = \sigma + \xi(u - \mu) \)

- Less wasteful of data
- Must choose threshold \( u \)
- How to go spatial?
Bayesian Hierarchical Model

Let $X_{i,j}$ be observation $j$ from station $i$.

$$X_{i,j} \sim GPD(\tilde{\sigma}_i, \xi_i)$$
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$$\xi \sim MVN(c, D)$$

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- $B, D$ are functions of distance
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Plan: Use existing spatial techniques on the parameters, then convert to the desired return levels.
Covariates

To do spatial prediction, any covariate must be available for every location in the region.
First Model

\[ X_{i,j} \sim \text{GPD}(\tilde{\sigma}_i, \xi_i) \]

\[ \log \tilde{\sigma} \sim \text{MVN}(a, B) \]

\[ \xi \sim \text{MVN}(c, D) \]

\[ a = \alpha_{\sigma 1} + \alpha_{\sigma 2}(\text{elevation}) \quad B = \beta_{\sigma 1} \exp[-\beta_{\sigma 2}(\text{distance})] \]

\[ c = \alpha_{\xi 1} + \alpha_{\xi 2}(\text{elevation}) \quad D = \beta_{\xi 1} \exp[-\beta_{\xi 2}(\text{distance})] \]

\[ \alpha_{.} \sim \text{Unif}(-\infty, \infty) \quad \beta_{.} \sim \text{Unif}(-\infty, \infty) \]
Model Results

![Graphs showing the relationship between log(sigma) and elevation, and another graph showing xi against elevation.](image)
Model Results

\[ \log(\sigma) \]
Model Results

\[ \xi \]
Model Results

20-year Return Levels

Extreme Values on Spatial Fields – p. 13/1
Future Work

- Extend spatially
- Search for covariates
- Speed up MCMC method
- Examine covariance structure
- Test other models
- Model comparison (DIC)
- ???