1 Nonstationary Structures

A Monte Carlo EM method of nonstationary covariance model and estimator proposed in the main text is illustrated with an application to surface ozone data from an irregularly distributed environmental monitoring network. It is useful to display the predicted covariances visually to convey the form of nonstationarity. Figure 1 shows nonstationary covariance structures of the surface ozone, obtained from 20 iterations of the EM analysis without smoothing of the fine-scale wavelet-coefficient variance (i.e., $\hat{H}_1$ described in the main text). The thresholding parameters for the wavelet-based covariance model are set to be $j_T = 2$ and $\tau = 98\%$. Each panel displays the covariance between the point indicated by red dot and the rest of grid points. Next, the method is applied to the same data but with smoothing of $\hat{H}_1$ and with different thresholding parameters ($j_T = 2$ and $\tau = 86\%$). This time the EM analysis converges after 1 iteration, and the result is shown in Figure 2. Both of the covariance surfaces are similar: an elongated correlation pattern along the urban coastal area and a rather isotropic pattern in the middle west. It is notable, from the wiggly contour of the covariance surfaces shown in Figure 1, that with the model with $j_T = 3$ there are small scale structures even in the area where there are no observations. This may be a result of Monte Carlo sampling error and/or over-fitting due to a larger degree of freedom in the model of $j_T = 3$. 
Figure 1:

Figure 2: