Black diamond ski runs and the `fields` R package.

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Introduction

Estimating a surface and quantifying its uncertainty are basic problems in the analysis of geophysical data. Here we use spatial statistical models and the `fields` package to approach this problem. The application is to the digital elevations covering the Mary Jane/Winter Park ski resort in Colorado. A black diamond run typically has slopes in the range of 30 degrees – and is difficult to descend. A 40 degree slope is steep enough where you will still end up at the bottom of the pitch even if you fall!

The Problem(s)

Determine the slope at arbitrary locations and directions in a mountainous area based on a grid of elevations.

Quantify the uncertainty in the slope estimates.

A spatial statistical model

An additive observation model

Given scattered and possibly noisy observations of a function f

$$Y_k = f(\boldsymbol{x}_k) + e_k$$

find $\frac{\partial f(\boldsymbol{x})}{\partial \boldsymbol{x}}$ along a smooth curve.

A prior for f

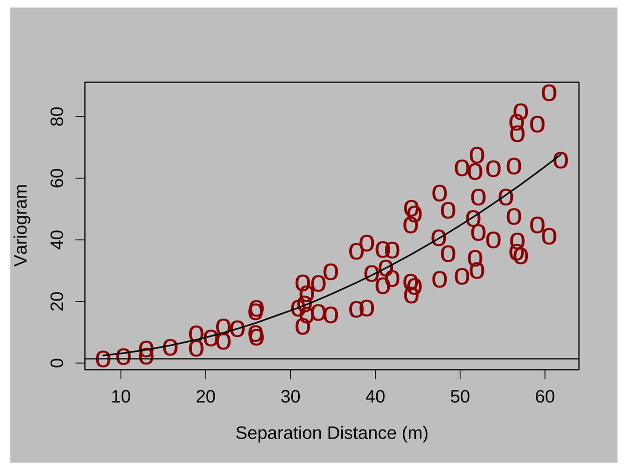
f = low order polynomial + Gaussian process.

The Data

- *Elevations:* Gridded elevations obtained by `ftp` from the USGS Seamless Data Distribution Service (SDDS).The region is approximately 1.5km X 2km with a grid spacing at 10m and comprising 146X196 = 28616 locations.
  This is a large data set for conventional spatial statistics.
- *Image:* 256 gray level digital photograph at 1m resolution also from SDDS.
- *Riflesight Notch ski trail:* Entered using the R `locator` function.

Spatial Analysis

Directional variogram suggests some modest nonstationarity but the shape is quadratic near zero distance with a very small nugget variance. The lack of nugget may suggest some post- processing of the elevations.



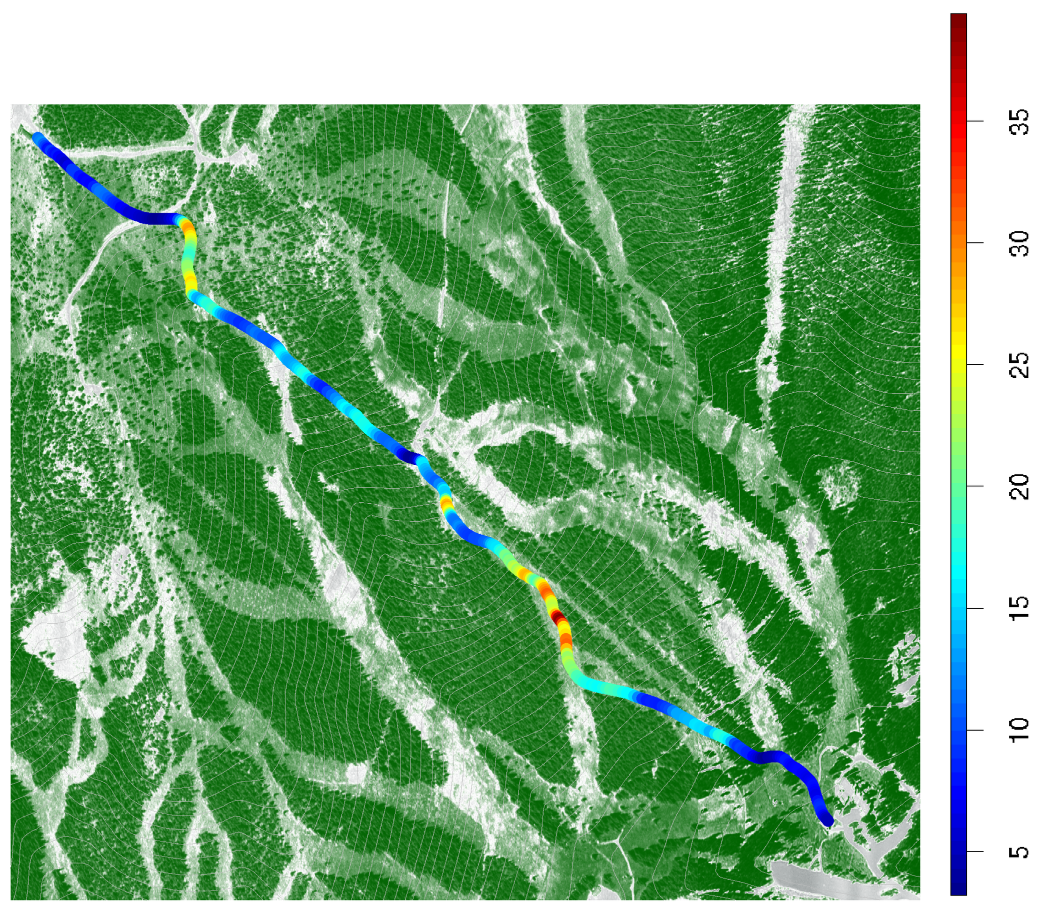
Variogram values at same distance represent different orientation of pairs of locations. The plotted line is a quadratic polynomial fit by least squares.

Spatial Inference

Estimate f using a Kriging estimate

- Wendland covariance function, compactly supported but quadratic at zero
- Partial derivatives found by differentiating the estimated surface.
- Uncertainty found by conditional simulation of a stationary Matern process but matched to the data variogram in the range [0,60] meters.

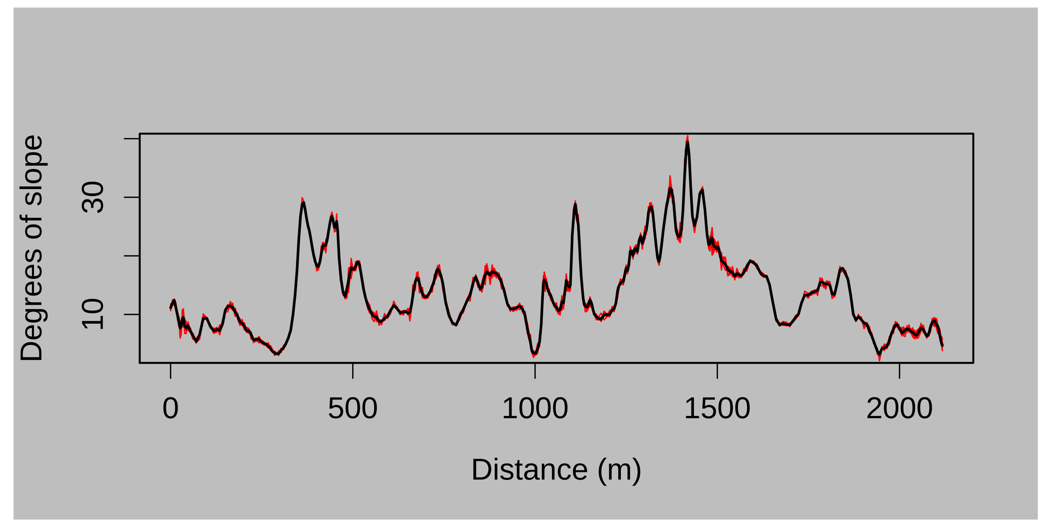
The slope estimate



The crux of Riflesight Notch trail is confirmed quantitatively by the high slopes (red) near the center of the image. (See also the next figure around 1400 m.)

Uncertainty in the estimated slope

10 realizations of the estimated slopes (red) with the posterior mode (black). Note that the transformation to degrees slope (atan) is a nonlinear transformation of the estimated elevation field. The uncertainty under the assumption that the elevations are measured without error is small — on the order of 1-2 degrees.



Computation

A key is to use compactly supported covariance functions and take advantage of sparse matrix linear algebra. The basic spatial analysis involves nearly 30,000 locations and is not possible with a direct, naive implementation of the Kriging estimator. But with sparse methods this analysis can be pursued in an interactive session in R using a laptop.

`fields:` Graphics and computation is implemented in the `fields` package for R. Some key functions are:
`drape.plot, ribbon.plot` for the perspective image and representing the slopes.
`vgram.matrix` for the directional variogram of observations from a regular grid.
`mKrig` a fast Kriging function that takes advantage of sparse covariance matrices
`sim.rf` simulation of stationary random fields on large grids using circulant embedding.

Web References: `www.image.ucar.edu/ nychka`

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