# Extreme Values on Spatial Fields

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# Background

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- 2nd year with GSP
- Anticipated graduation date: Fall 2005

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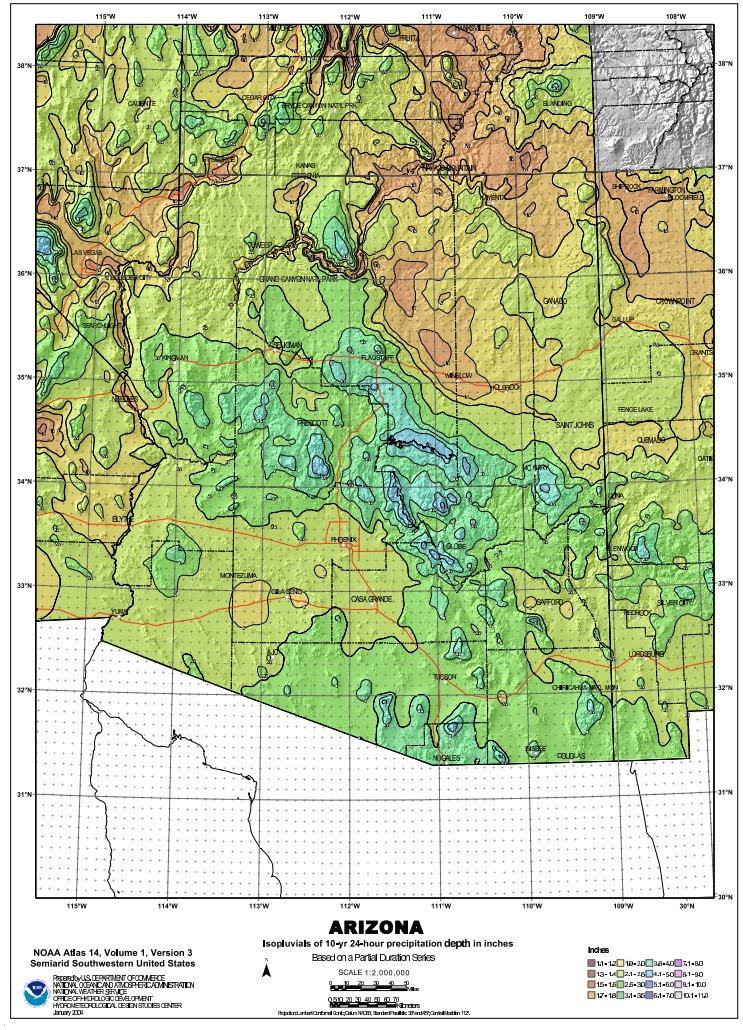
## Projects

- Extreme value model for Lichenometry
- Spatial dependence estimation and prediction for annual maxima
- Colorado Front Range Precipitation

# Colorado Precipitation Project

Goal: Create a map of precipitation return levels for Colorado's Front Range.



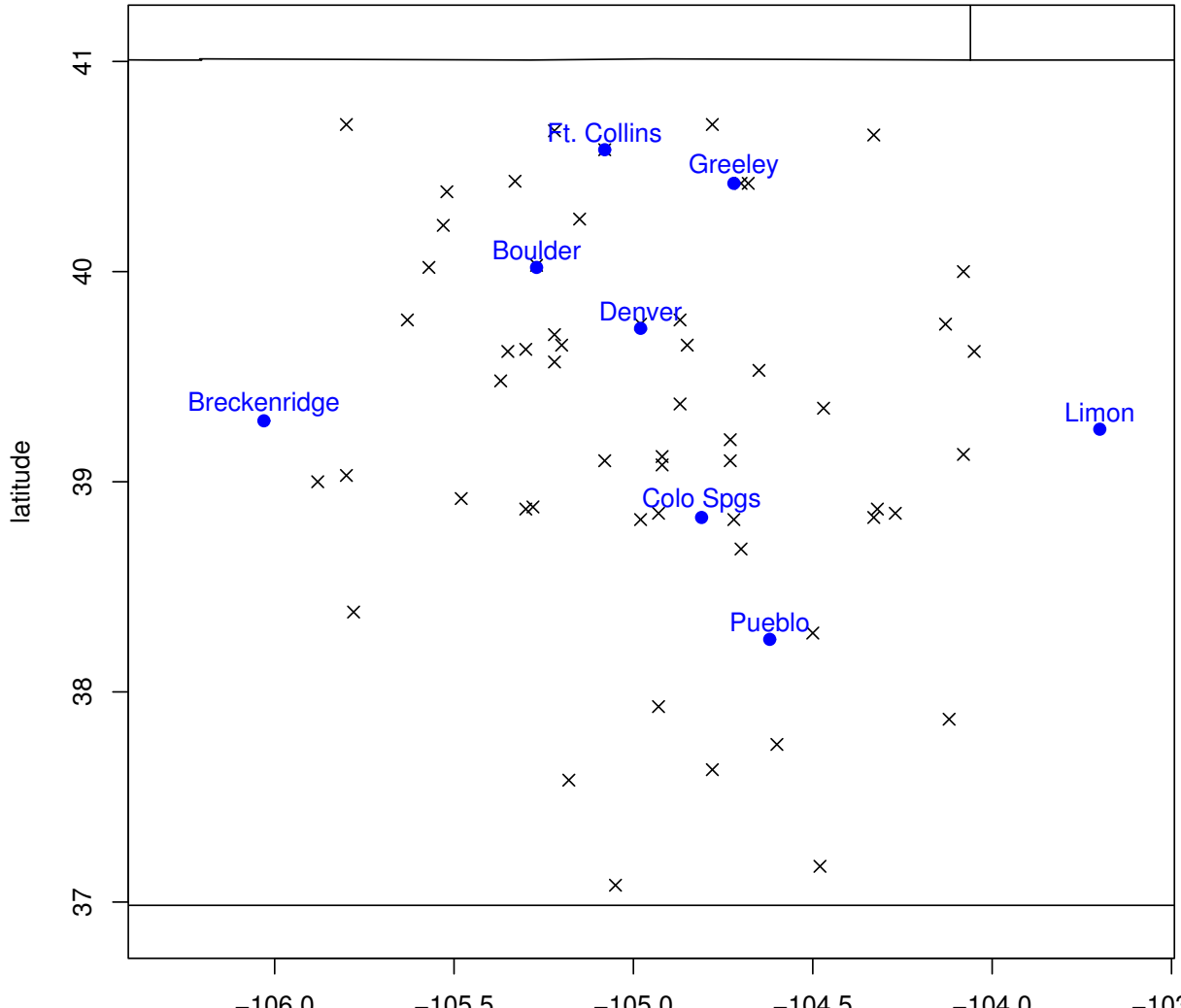
# Colorado Precipitation Project

Goal: Create a map of precipitation return levels for Colorado's Front Range.

- project originated from ISSE, interested in flooding
- NWS has done maps for Southwest US (AZ, NM, UT, NV) and mid-Atlantic (OH, PA, MD, NJ, NC, WV)
- plans to do entire US (contingent on funding)
- handles spatial dependence and prediction differently
- present NWS with our method and results

# Front Range Data

Data: hourly precipitation from 56 weather stations, 12-60 years of data, Apr1 - Oct 31.



# Univariate Extreme Values

GEV: Used to model block (annual) maxima

$$G(z;\mu,\sigma,\xi) = \exp\left[-\left[1+\xi\left(\frac{z-\mu}{\sigma}\right)\right]^{-\frac{1}{\xi}}\right]$$

- wasteful of data
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GPD: Models exceedences above a threshold

$$P\{X-u<y|X>u\} \approx 1-\left(1+\frac{\xi y}{\tilde{\sigma}}\right)^{-1/\xi}$$

where $\tilde{\sigma} = \sigma + \xi(u-\mu)$

- Less wasteful of data
- Must choose threshold $u$
- How to go spatial?

# Bayesian Hierarchical Model

Let $X_{i,j}$ be observation $j$ from station $i$.

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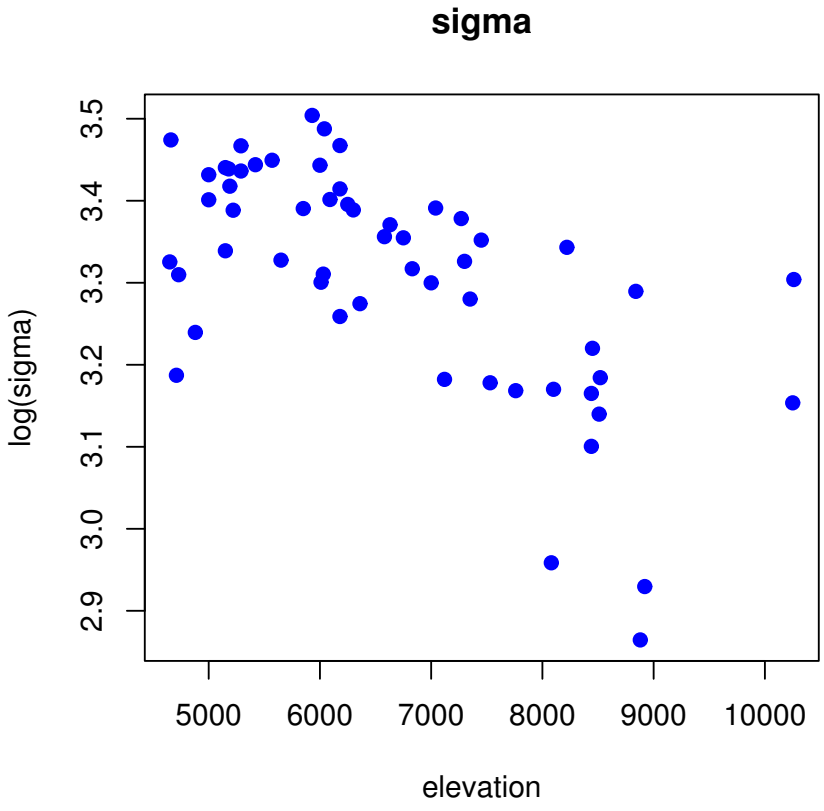
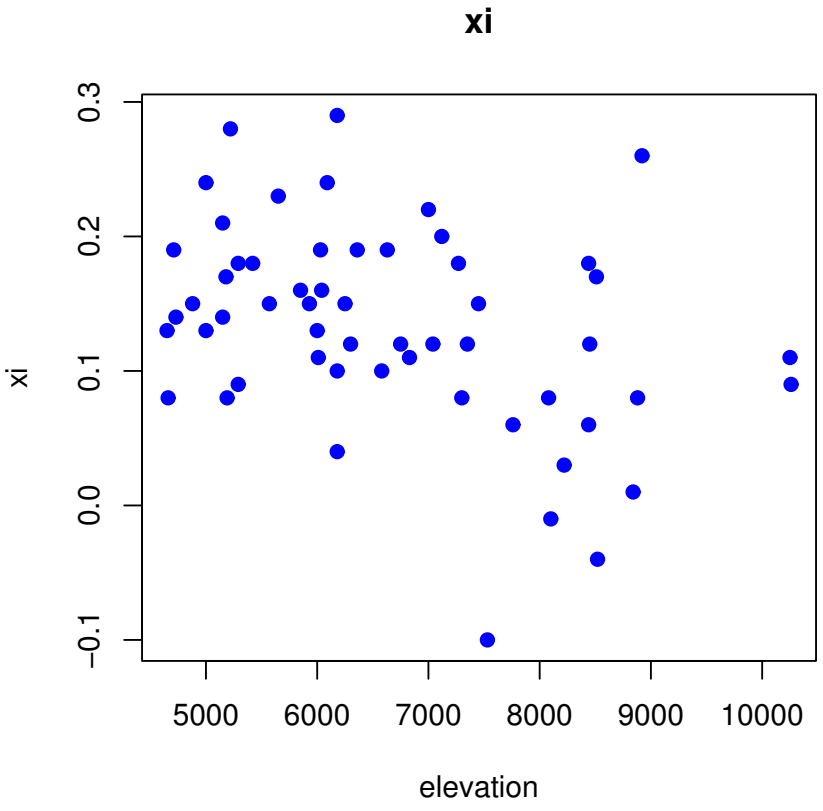
Plan: Use existing spatial techniques on the parameters, then convert to the desired return levels.

# Covariates

To do spatial prediction, any covariate must be available for every location in the region.



# First Model

$$X_{i,j} \sim GPD(\tilde{\sigma}_i, \xi_i)$$

$$\log \underline{\tilde{\sigma}} \sim MVN(\underline{a}, B)$$

$$\underline{\xi} \sim MVN(\underline{c}, D)$$

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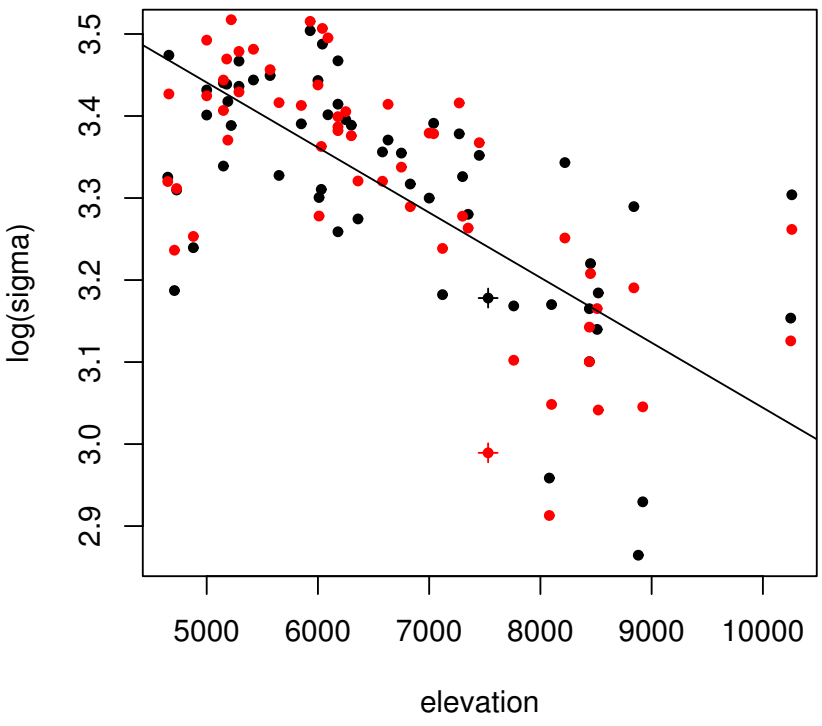
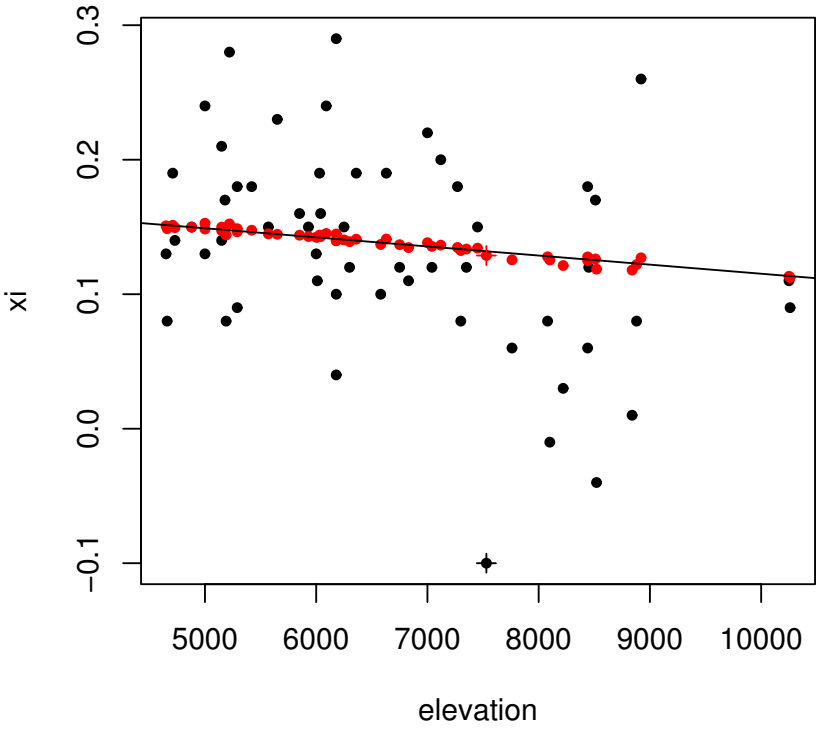
$$\underline{a} = \alpha_{\sigma 1} + \alpha_{\sigma 2}(\text{elevation}) \quad B = \beta_{\sigma 1} \exp[-\beta_{\sigma 2}(\text{distance})]$$

$$\underline{c} = \alpha_{\xi 1} + \alpha_{\xi 2}(\text{elevation}) \quad D = \beta_{\xi 1} \exp[-\beta_{\xi 2}(\text{distance})]$$

$$\alpha_{..} \sim Unif(-\infty, \infty) \qquad \beta_{..} \sim Unif(-\infty, \infty)$$

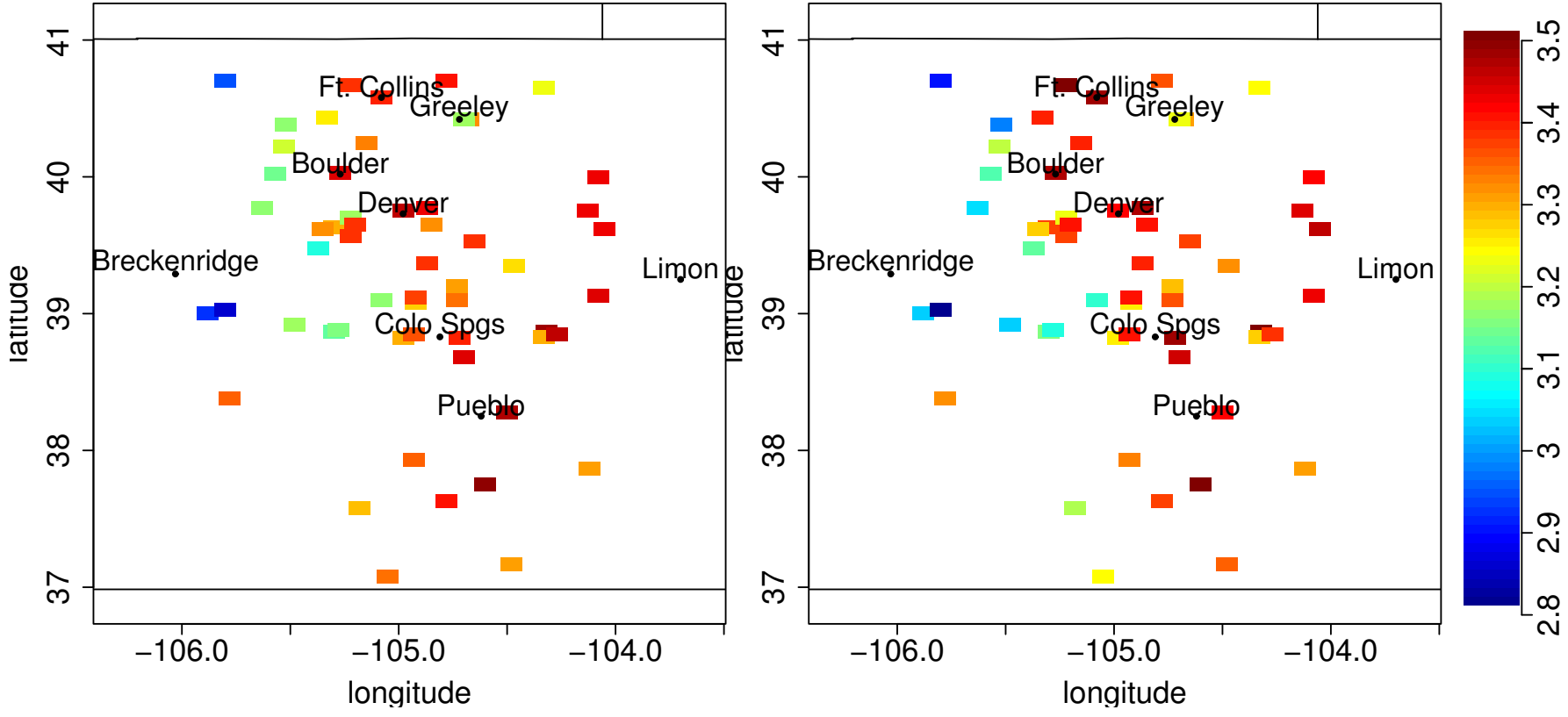
# Model Results



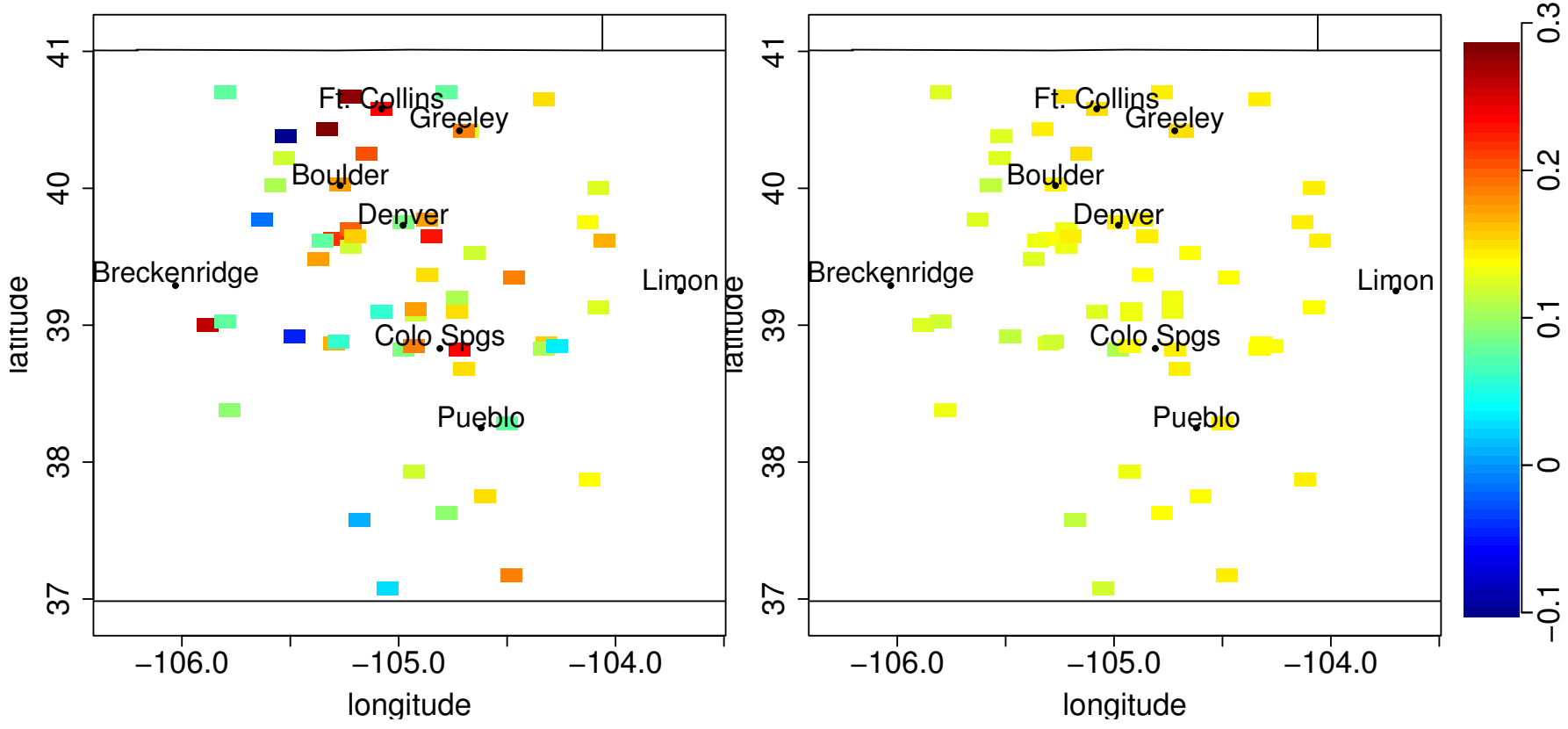
# Model Results

$$\log(\sigma)$$



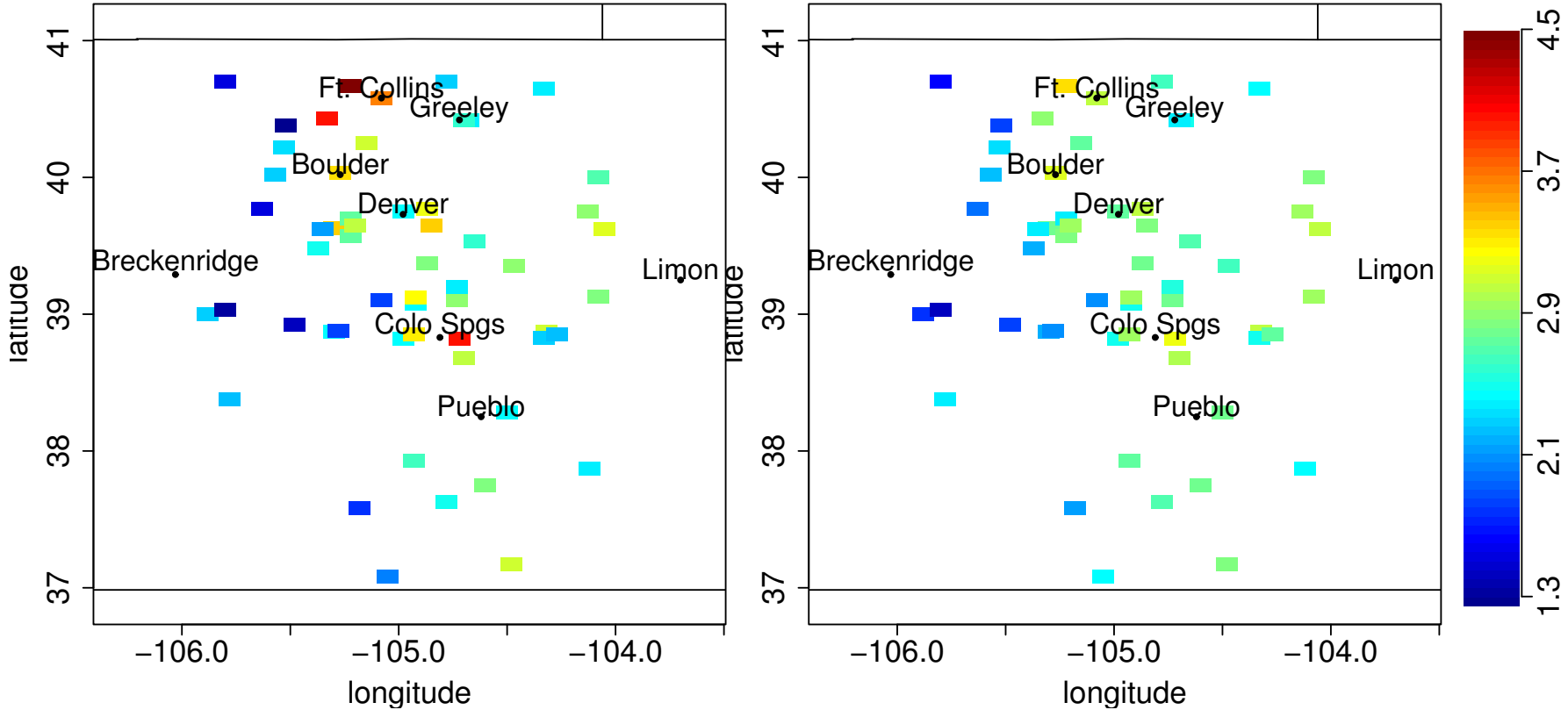
# Model Results

$\xi$



# Model Results

## 20-year Return Levels



# Future Work

- Extend spatially
- Search for covariates
- Speed up MCMC method
- Examine covariance structure
- Test other models
- Model comparison (DIC)
- ???