

Space-time modeling of stratospheric ozone



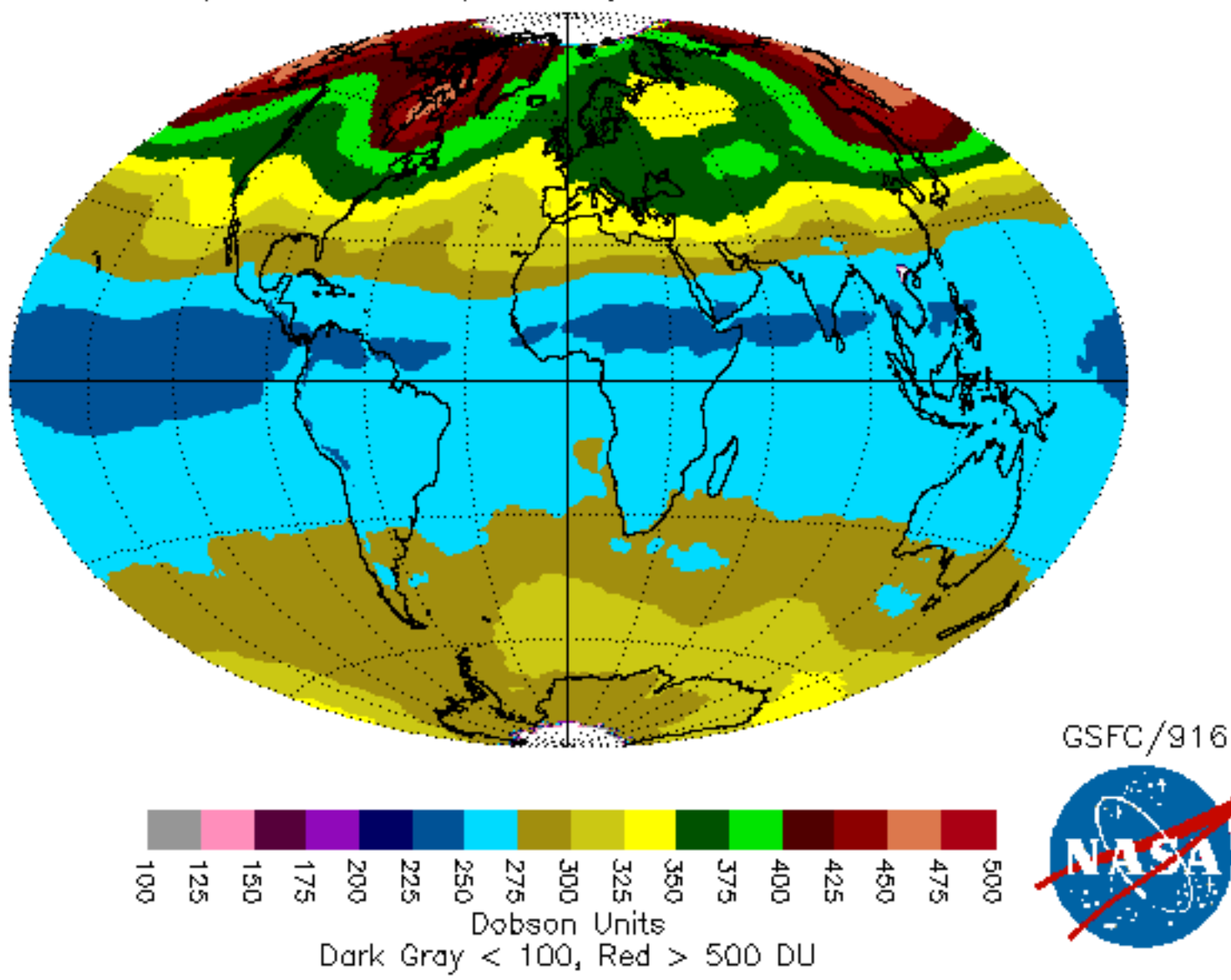
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Summary

Since the implementation of the international controls on ozone depleting chemicals, an important focus in studies of stratospheric ozone has been on the detection of a turnaround in the downward trend. This turning point in ozone is estimated to begin as soon as in the late 1990's according to eight models used in the latest international assessment. The statistical models which have been used to address this issue are regression models of ozone on a seasonal component, the solar flux, the QBO, and possibly optical thickness (a surrogate for stratospheric aerosols). Due to large-scale dynamical processes not explained by the former regression, the residuals are found to be autoregressive. In our study, we use the TOMS satellite data set from November 1978 to April 1993, averaged over longitude and month. We restrict our study to northern midlatitudes. In order to take advantage of the available information, we model jointly the variation in space and time. We find that a functional autoregressive time series model can explain some statistical properties of the residuals across latitudes.

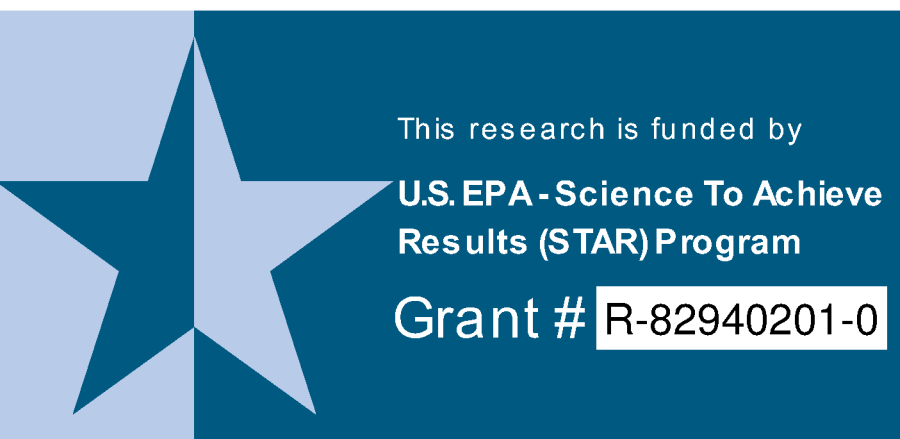
Data Set

Nimbus-7/TOMS Monthly Average Total Ozone March 1992



We use the Nimbus-7 (Total Ozone Mapping Spectrometer) monthly TOMS data set, averaged over longitude. Our study is restricted to 40.5-59.5°N for homogeneity purposes. We regress the measurements on monthly indicators (to remove seasonality), the solar flux, the QBO winds lagged 6 months, the optical thickness (a measure of aerosols, used to evaluate the volcanic activity [Reinsel 2002]), and four seasonal trends (Winter, Spring, Summer and Autumn). The time period is November 1978 to April 1993, i.e. 174 months.

The problem of ozone trend estimation was recently addressed by the World Meteorological Organization [WMO 2002]. An accurate modeling of the total column ozone evolution could lead to a small number of years of data records to detect a recovery.



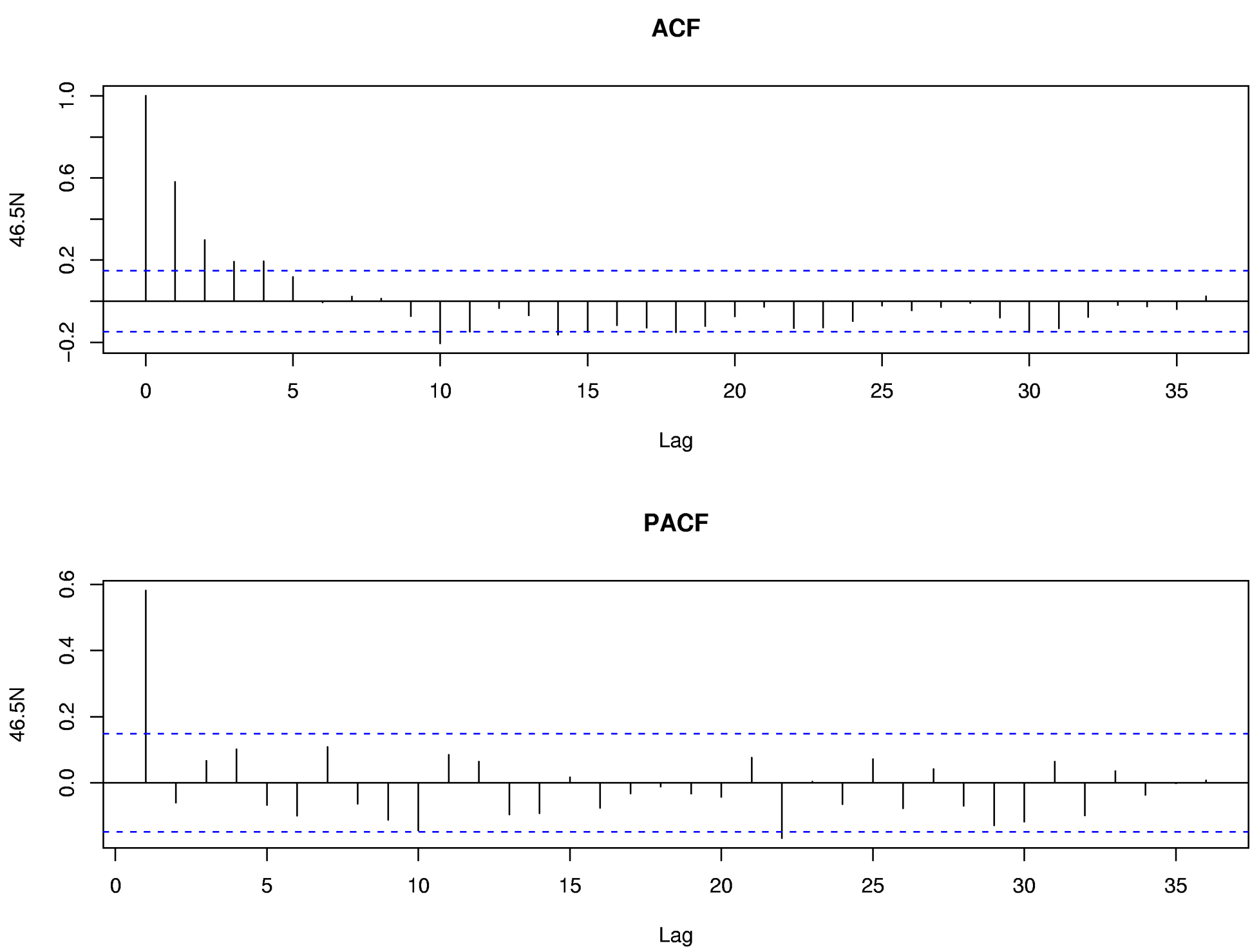
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This linear regression yields the following coefficient estimates.

Latitude	Winter	Spring	Summer	Autumn	Optical Thickness
40.5	-1.88	-1.78	-0.64	-0.39	-3.73
41.5	-1.93	-1.82	-0.65	-0.41	-7.16
42.5	-1.97	-1.90	-0.68	-0.41	-10.94
43.5	-1.98	-1.98	-0.67	-0.41	-16.46
44.5	-2.02	-2.02	-0.70	-0.46	-20.04
45.5	-2.01	-2.10	-0.71	-0.47	-27.13
46.5	-2.02	-2.14	-0.71	-0.52	-31.41
47.5	-2.01	-2.21	-0.71	-0.56	-36.06
48.5	-2.00	-2.25	-0.69	-0.55	-40.72
49.5	-1.94	-2.28	-0.71	-0.59	-45.27
50.5	-1.89	-2.29	-0.73	-0.58	-49.63
51.5	-1.84	-2.31	-0.73	-0.59	-53.90
52.5	-1.79	-2.34	-0.78	-0.60	-56.00
53.5	-1.71	-2.37	-0.78	-0.59	-60.14
54.5	-1.65	-2.39	-0.80	-0.64	-63.48
55.5	-1.57	-2.45	-0.82	-0.63	-65.90
56.5	-1.51	-2.48	-0.85	-0.65	-67.64
57.5	-1.38	-2.51	-0.87	-0.67	-71.14
58.5	-1.19	-2.56	-0.88	-0.68	-72.72
59.5	-1.09	-2.56	-0.88	-0.71	-74.6

The residuals from these regression are well estimated by a scalar AR(1) time series model [Tiao *et al.* 1990; Weatherhead *et al.* 1998]. This dependence structure is due to large scale dynamics.

For example, for 46.5°N, the ACF and PACF are:



Functional Model

Each curve representing the latitudinal left over profile is associated with a random variable taking its values in an Hilbert space H , e.g. $H = L^2[0, 20]$.

Let (ε_n) be a sequence of i.i.d. H -valued random variables. We will consider the following statistical model, AutoRegressive of order one ($ARH(1)$) [Bosq 2000]:

$$X_n = \rho(X_{n-1}) + \varepsilon_n. \quad (1)$$

where $\rho : H \rightarrow H$ is a linear operator to be estimated.

3 steps :

1. Compute, by a Principal Component Analysis, empirical estimators of the eigenelements of the covariance operator C associated with (X_n) .
2. Project the relation between the cross-covariance operator D and C

$$D = \rho C$$

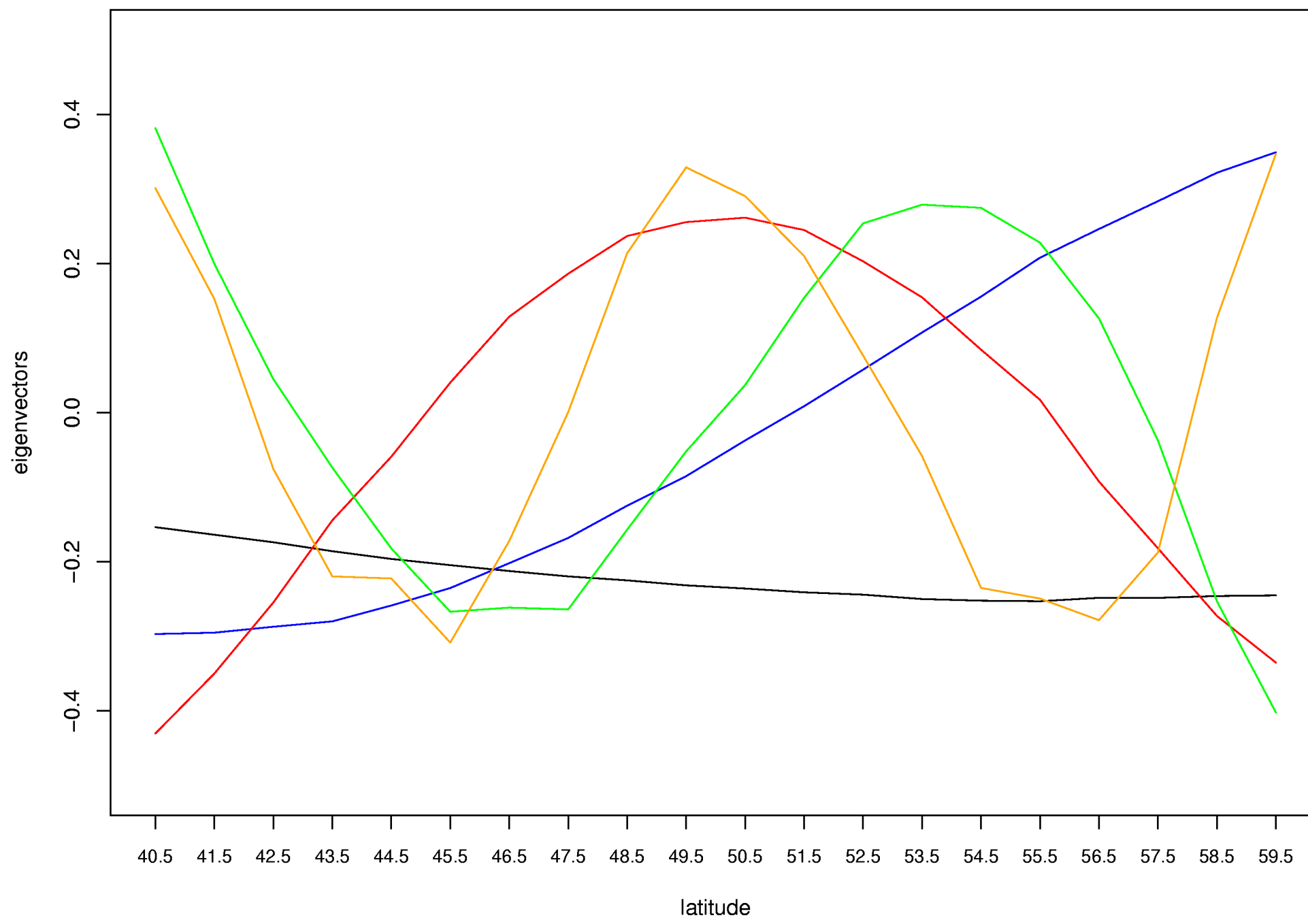
in the subspace spanned by the first k_n eigenvectors of C associated with the k_n greatest empirical eigenvalues. [Bosq 2000; Guillas 2001]

3. Find optimal k_n by a cross validation based on prediction of the last 20 curves.

4. select the optimal number of dimensions (here=5), and compute the autocorrelation operator:

$$\hat{\rho} = \begin{pmatrix} 0.689 & 0.168 & 0.672 & -0.364 & 2.590 \\ -0.056 & 0.307 & 0.585 & -0.848 & -1.400 \\ 0.023 & 0.021 & 0.132 & -0.271 & -1.040 \\ 0.002 & -0.015 & 0.014 & 0.233 & 0.196 \\ -0.001 & -0.002 & -0.011 & 0.073 & -0.013 \end{pmatrix}$$

The spectral decomposition of the covariance operator C gives the following first five eigenvalues: 49.168 7.620 0.943 0.203 0.039



The first five eigenvectors of the covariance operator are in black, blue, red, green, orange.

References

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k_n	L^2 Error
1	5.464791
2	5.215683
3	5.151730
4	5.087865
5	5.057268
6	5.066962
7	5.108127
8	5.236157
9	5.233813
10	5.260613
11	5.166464
12	5.199122
13	5.161209
14	5.116867
15	5.184319
16	5.227816
17	5.209815
18	5.387559
19	5.414432
20	5.417522