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Scale-similar models for large eddy simulations of a rotating convection-driven dynamo

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The scale-similar model of [1] *et al.* and a dynamic similarity model have been applied to a rotating convection-driven dynamo simulation. The results from the similarity model, using unit constant coefficients, are satisfactory: the large-scale magnetic/kinetic energies and $r.m.s$ magnetic/velocity field fluctuations are in much better agreement with the highly-resolved solution than with the low resolution simulation. The model is found to be much less sensitive to the filter scale than the *a priori* test. Implementation of a dynamic procedure to the similarity model gives better agreement provided that the two filtering scales are properly chosen. The model coefficients from the dynamic procedure are less than 1.0, in the range $[0.4, 0.8]$.

[A mathematical dynamo model]

Equations: The governing Boussinesq equations for a rotating convection-driven plane layer dynamo

$$\mathbf{e}_z \times \mathbf{v} = -\boldsymbol{\nabla} p + (\boldsymbol{\nabla} \times \mathbf{B}) \times \mathbf{B} + qRa\mathbf{e}_z T + Ek\nabla^2 \mathbf{v}, \tag{1}$$

$$\frac{\partial \mathbf{B}}{\partial t} = \boldsymbol{\nabla} \times (\mathbf{v} \times \mathbf{B}) + \nabla^2 \mathbf{B}, \tag{2}$$

$$\frac{\partial T}{\partial t} + \mathbf{v} \cdot \boldsymbol{\nabla} T = q\nabla^2 T + v_z, \tag{3}$$

$$\boldsymbol{\nabla} \cdot \mathbf{v} = 0\,, \qquad\qquad \boldsymbol{\nabla} \cdot \mathbf{B} = 0. \tag{4}$$

Here $\mathbf{v}$, $\mathbf{B}$ and T are the dimensionless velocity, magnetic field and temperature fluctuation, respectively. The dimensionless t is the magnetic diffusion time. The Roberts number $q = \kappa/\eta$, the modified Rayleigh number $Ra = g\bar{\alpha}\bar{\beta}d^2/2\Omega\kappa$, the Ekman number $Ek = \nu/2\Omega d^2$ and the magnetic Ekman number $Ek_\eta = \eta/2\Omega d^2$.

[Large-eddy simulation]

In the LES approach, the large-scale velocity $\overline{\mathbf{v}}$ is obtained by convolution, through a spatial filter function $G_\Delta(r, x)$ [2]:

$$\overline{\mathbf{v}}(\mathbf{x}, t) = \int G_\Delta(\mathbf{r}, \mathbf{x})\mathbf{v}(\mathbf{x} - \mathbf{r}, t)d\mathbf{r}\,. \tag{5}$$

where Δ is the filter width. The velocity $\mathbf{v}(\mathbf{x}, t)$ is decomposed into a large-scale (or resolved) part $\overline{\mathbf{v}}$ and a subgrid (or under-resolved) part $\mathbf{v}'$ as $\mathbf{v} = \overline{\mathbf{v}} + \mathbf{v}'$.

Equations: A LES representation of the dynamo model:

$$\mathbf{e}_z \times \overline{\mathbf{v}} = -\boldsymbol{\nabla}\overline{p} + \boldsymbol{\nabla} \times \overline{\mathbf{B}} \times \overline{\mathbf{B}} - \boldsymbol{\nabla} \cdot \boldsymbol{\tau} + qRa\overline{T}\mathbf{e}_z + Ek\nabla^2\overline{\mathbf{v}}, \tag{6}$$

$$\frac{\partial \overline{\mathbf{B}}}{\partial t} + \overline{\mathbf{v}} \cdot \boldsymbol{\nabla}\overline{\mathbf{B}} = \overline{\mathbf{B}} \cdot \boldsymbol{\nabla}\overline{\mathbf{v}} - \boldsymbol{\nabla} \cdot \boldsymbol{\tau}^B + \nabla^2\overline{\mathbf{B}}, \tag{7}$$

$$\frac{\partial \overline{T}}{\partial t} + \overline{\mathbf{v}} \cdot \boldsymbol{\nabla} \overline{T} = -\boldsymbol{\nabla} \cdot \mathbf{Q} + \overline{v}_z + q \nabla^2 \overline{T}, \tag{8}$$

$$\boldsymbol{\nabla} \cdot \overline{\mathbf{v}} = 0, \qquad\qquad \boldsymbol{\nabla} \cdot \overline{\mathbf{B}} = 0. \tag{9}$$

[The subgrid-scale(SGS) terms]

The influence of the subgrid-scales on the resolved scales is embedded in SGS terms:

1. *Reynolds stress:*

$$\boldsymbol{\tau} = Ek_\eta(\overline{\mathbf{v}\mathbf{v}} - \overline{\mathbf{v}}\,\overline{\mathbf{v}}) - (\overline{\mathbf{B}\mathbf{B}} - \overline{\mathbf{B}}\,\overline{\mathbf{B}}), \tag{10}$$

2. *Turbulent electromotive force (emf):*

$$\boldsymbol{\tau}^B = \overline{\mathbf{v}\mathbf{B}} - \overline{\mathbf{v}}\,\overline{\mathbf{B}} - (\overline{\mathbf{B}\mathbf{v}} - \overline{\mathbf{B}}\,\overline{\mathbf{v}}), \tag{11}$$

3. *Heat flux:*

$$\mathbf{Q} = \overline{\mathbf{v}T} - \overline{\mathbf{v}}\,\overline{T}. \tag{12}$$

In our investigation the inertial forces are negligible, the magnetic Ekman number $Ek_\eta = 2\Omega/\eta$ is zero, so the Reynolds stress $\boldsymbol{\tau}$ reduces to $\boldsymbol{\tau} = -\boldsymbol{\tau}^{Max}$ where $\boldsymbol{\tau}^{Max} = \overline{\mathbf{B}\mathbf{B}} - \overline{\mathbf{B}}\,\overline{\mathbf{B}}$ is generally called 'the Maxwell stress tensor'.

[The similarity model]

We model the SGS terms using the similarity model.

1. *Maxwell stress:*

$$\boldsymbol{\tau}^{Max}_{sim} = C_{mom}(\widetilde{\overline{\mathbf{B}}\,\overline{\mathbf{B}}} - \widetilde{\overline{\mathbf{B}}}\,\widetilde{\overline{\mathbf{B}}})\,, \tag{13}$$

2. *Turbulent electromotive force:*

$$\boldsymbol{\tau}^{B}_{sim} = C_{ind}(\widetilde{\overline{\mathbf{v}}\overline{\mathbf{B}}} - \widetilde{\overline{\mathbf{v}}}\,\widetilde{\overline{\mathbf{B}}} - (\widetilde{\overline{\mathbf{B}}\,\overline{\mathbf{v}}} - \widetilde{\overline{\mathbf{B}}}\,\widetilde{\overline{\mathbf{v}}}))\,, \tag{14}$$

3. *Heat flux:*

$$\mathbf{Q}_{sim} = C_T(\widetilde{\overline{\mathbf{v}}\,\overline{T}} - \widetilde{\overline{\mathbf{v}}}\,\widetilde{\overline{T}})\,. \tag{15}$$

where, $\tilde{}$ represents a second filtering operation at a scale λ with $\lambda/\Delta \geq 1$. The model coefficients $C_{mom} = C_{ind} = C_T = 1$ in the similarity model. In a dynamic similarity model , these models can be adjusted to vary in time and space, i.e. $C_{mom}(z,t), C_{ind}(z,t), C_T(z,t)$.

References

[1] J. Bardina, J.H. Ferziger and W.C. Reynolds 1980. *Improved subgrid scale models for large-eddy simulations*, Am. Inst. Aeronaut. Astronaut. J., **34**, 1111–1119.

[2] A. Leonard 1974. *Energy cascade in large-eddy simulations of turbulent flow*, Adv. Geophys., **18**, 237–248.