Fluctuations of magnetic induction in von Kármán swirling flows, Application to dynamo

Philippe Odier, Romain Volk, Jean-François Pinton

Laboratoire de Physique
Ecole Normale Supérieure de Lyon
Dynamo capacity of VK flows

\[ \partial_t \vec{B} = \vec{\nabla} \times (\vec{u} \times \vec{B}) + \lambda \Delta \vec{B} \]

VKS2: a dynamo with the mean flow

\[ R_{m \text{ max}} = 55 > R_{m^c} = 43 \]
Several faces of VK flow

\[ \langle \vec{V}(t) \rangle_T = \frac{1}{T} \int_t^{t+T} \vec{V}(t') \, dt' \]
VKG Experiment

$1 < R_m < 5$, $Re \sim 10^6$, $N << 1$
A magnetic tool to probe the flow large scale fluctuations

Measurements in 8 points $\vec{B}(r_i, t)$

Hall probes, Sentron 1SA-1M

What does this probe see?

The induction equation ...

$$\partial_t \vec{B} = \left( (\vec{B} + \vec{B}_0) \cdot \vec{\nabla} \right) \vec{V} - (\vec{V} \cdot \vec{\nabla}) \vec{B} + \lambda \Delta \vec{B}$$

...using QS approximation and linear approximation ($Rm < 5$) ...

... gives: $\vec{B}(t) \sim -\frac{1}{\lambda} \Delta^{-1} \left[ B_0 \partial_z \vec{V}(t) \right]$ (suppose $B_0//Oz$)
VKG
Gallium Setup
Simulations:

\[ \partial_z V_\theta \quad \Omega - \text{effect} \]

differential rotation

\[ \langle B_\theta \rangle \]

\[ R_m = \frac{2 \pi R^2 \Omega}{\lambda} \]
Simulations:

Stretching effect
Fluctuations of the induction profiles

Ω-effect (toroidal)

\[ B_\theta(r_i, t) \]

\[ \bar{B}(t) \sim -\frac{1}{\lambda} \Delta^{-1} \left[ B_0 \partial_z \tilde{V}(t) \right] \]

Stretching (poloidal)

\[ B_z(r_i, t) \]
Strong spatial correlations

Space-time diagram shows global profile evolution

Correlation length comparable to experiment size
Distance to the mean profile

\[ E_k(t) = \sqrt{\frac{1}{N} \sum_{i=1}^{N} (B_k(r_i, t) - \langle B_k(r_i, t) \rangle_T)^2} \]

- \( <E> \) of the order of \( <B> \)
- \( \frac{E_{rms}}{\langle E \rangle} \sim 50\% \)
- stronger deviations for the field induced by the rotation flow
- more deviations for \( s_2t_2 \) than for \( s_1t_1 \)
Polynomial analysis of the profiles

\[ B(r, t) = a_0(t) + a_1(t)r + a_2(t)r^2 + a_3(t)r^3 \]

\[ a_1 \propto R_m \]

\[ a_0 \propto R_m \]

Ω-effect (toroidal)

stretching (poloidal)
Polynomial analysis of the profiles
Fluctuations of the coefficients

(a) Axial field: $\Omega$ effect

(b) Axial field: stretching effect

\[ \frac{a_{1,\text{rms}}}{\langle a_1 \rangle} \sim 114\% \]

$\Omega$-effect (toroidal)

\[ \frac{a_{0,\text{rms}}}{\langle a_0 \rangle} \sim 20\% \]
stretching (poloidal)
Loss of the $s_2t_2$ geometry

- **$\Omega$-effect (toroidal)**
  \[
  \langle B_\theta \rangle
  \]
  \[
  (a_1 \propto R_m)
  \]

- **By symmetry**: $a_0=0$ but at $\Omega=10$ Hz, $a_{0,rms}=1.25G \approx \langle B_{\text{induced}} \rangle$
- **$a_0$ and $a_1$ strongly correlated**
- **Combined probability**: $|a_0(t)|>0.5$ $a_{0,rms} = 50\%$
  & $|a_1(t)|>0.5$ $a_{1,rms}$
Conclusions

- \( B(r,t) \) is an image of velocity gradients (probably more complicated at higher \( Rm \) - non linear effects)

- In addition to the turbulent small scale structure, the VK flows \( (s_2t_2) \) have strong large scale fluctuations

- They spend about 50% of the time away from the mean structure, not only in terms of amplitude but also topology and symmetry

- Could that cause an increase of the dynamo threshold in the unconstrained experiments?

real \( Rmc \neq Rmc \) predicted with \( \langle V \rangle \) ?